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GLASSY PHASE AND FREEZING OF LOG-CORRELATED GAUSSIAN POTENTIALS

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In this paper, we consider the Gibbs measure associated to a logarithmically correlated random potential (including two-dimensional free fields) at low temperature. We prove that the energy landscape freezes and enters in the so-called glassy phase. The limiting Gibbs weights are integrated atomic random measures with random intensity expressed in terms of the critical Gaussian multiplicative chaos constructed in [Ann. Probab. **42** (2014) 1769–1808 and Comm. Math. Phys. (2013) To appear]. This could be seen as a first rigorous step in the renormalization theory of super-critical Gaussian multiplicative chaos.

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DYNAMIC RANDOM NETWORKS AND THEIR GRAPH LIMITS

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We study a broad class of stochastic process models for dynamic networks that satisfy the minimal regularity conditions of (i) exchangeability and (ii) càdlàg sample paths. Our main theorems characterize these processes through their induced behavior in the space of graph limits. Under the assumption of time-homogeneous Markovian dependence, we classify the discontinuities of these processes into three types, prove bounded variation of the sample paths in graph limit space and express the process as a mixture of time-inhomogeneous, exchangeable Markov processes with càdlàg sample paths.

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THE MEAN EULER CHARACTERISTIC AND EXCURSION PROBABILITY OF GAUSSIAN RANDOM FIELDS WITH STATIONARY INCREMENTS¹

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Let $X = \{X(t), t \in \mathbb{R}^N\}$ be a centered Gaussian random field with stationary increments and $X(0) = 0$. For any compact rectangle $T \subset \mathbb{R}^N$ and $u \in \mathbb{R}$, denote by $A_u = \{t \in T : X(t) \geq u\}$ the excursion set. Under $X(\cdot) \in C^2(\mathbb{R}^N)$ and certain regularity conditions, the mean Euler characteristic of A_u , denoted by $\mathbb{E}\{\varphi(A_u)\}$, is derived. By applying the Rice method, it is shown that, as $u \rightarrow \infty$, the excursion probability $\mathbb{P}\{\sup_{t \in T} X(t) \geq u\}$ can be approximated by $\mathbb{E}\{\varphi(A_u)\}$ such that the error is exponentially smaller than $\mathbb{E}\{\varphi(A_u)\}$. This verifies the expected Euler characteristic heuristic for a large class of Gaussian random fields with stationary increments.

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CONDITIONED, QUASI-STATIONARY, RESTRICTED MEASURES AND ESCAPE FROM METASTABLE STATES

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We study the asymptotic hitting time $\tau^{(n)}$ of a family of Markov processes $X^{(n)}$ to a target set $G^{(n)}$ when the process starts from a “trap” defined by very general properties. We give an explicit description of the law of $X^{(n)}$ conditioned to stay within the trap, and from this we deduce the exponential distribution of $\tau^{(n)}$. Our approach is very broad—it does not require reversibility, the target G does not need to be a rare event and the traps and the limit on n can be of very general nature—and leads to explicit bounds on the deviations of $\tau^{(n)}$ from exponentially. We provide two nontrivial examples to which our techniques directly apply.

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A SYSTEM OF QUADRATIC BSDES ARISING IN A PRICE IMPACT MODEL

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We consider a financial model where the prices of risky assets are quoted by a representative market maker who takes into account an exogenous demand. We characterize these prices in terms of a system of BSDEs with quadratic growth. We show that this system admits a unique solution for every bounded demand if and only if the market maker's risk-aversion is sufficiently small. The uniqueness is established in the natural class of solutions, without any additional norm restrictions. To the best of our knowledge, this is the first study that proves such (global) uniqueness result for a system of fully coupled quadratic BSDEs.

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MSC2010 subject classifications. 60H10, 91B24, 91G80.

Key words and phrases. Liquidity, price impact, multi-dimensional quadratic BSDE.

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A WEAK APPROXIMATION WITH ASYMPTOTIC EXPANSION AND MULTIDIMENSIONAL MALLIAVIN WEIGHTS¹

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This paper develops a new efficient scheme for approximations of expectations of the solutions to stochastic differential equations (SDEs). In particular, we present a method for connecting approximate operators based on an asymptotic expansion with multidimensional Malliavin weights to compute a target expectation value precisely. The mathematical validity is given based on Watanabe and Kusuoka theories in Malliavin calculus. Moreover, numerical experiments for option pricing under local and stochastic volatility models confirm the effectiveness of our scheme. Especially, our weak approximation substantially improves the accuracy at deep Out-of-The-Moneys (OTMs).

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A NEW COALESCENT FOR SEED-BANK MODELS

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We identify a new natural coalescent structure, which we call the *seed-bank coalescent*, that describes the gene genealogy of populations under the influence of a strong seed-bank effect, where “dormant forms” of individuals (such as seeds or spores) may jump a significant number of generations before joining the “active” population. Mathematically, our seed-bank coalescent appears as scaling limit in a Wright–Fisher model with geometric seed-bank age structure if the average time of seed dormancy scales with the order of the total population size N . This extends earlier results of Kaj, Krone and Lascoux [*J. Appl. Probab.* **38** (2011) 285–300] who show that the genealogy of a Wright–Fisher model in the presence of a “weak” seed-bank effect is given by a suitably time-changed Kingman coalescent. The qualitatively new feature of the seed-bank coalescent is that ancestral lineages are independently blocked at a certain rate from taking part in coalescence events, thus strongly altering the predictions of classical coalescent models. In particular, the seed-bank coalescent “does not come down from infinity,” and the time to the most recent common ancestor of a sample of size n grows like $\log \log n$. This is in line with the empirical observation that seed-banks drastically increase genetic variability in a population and indicates how they may serve as a buffer against other evolutionary forces such as genetic drift and selection.

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Key words and phrases. Wright–Fisher model, seed-bank, coalescent, coming down from infinity, age structure.

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QUANTITATIVE PROPAGATION OF CHAOS FOR GENERALIZED KAC PARTICLE SYSTEMS

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We study a class of one-dimensional particle systems with true (Bird type) binary interactions, which includes Kac’s model of the Boltzmann equation and nonlinear equations for the evolution of wealth distribution arising in kinetic economic models. We obtain explicit rates of convergence for the Wasserstein distance between the law of the particles and their limiting law, which are linear in time and depend in a mild polynomial manner on the number of particles. The proof is based on a novel coupling between the particle system and a suitable system of nonindependent nonlinear processes, as well as on recent sharp estimates for empirical measures.

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Key words and phrases. Propagation of chaos, Kac equation, wealth distribution equations, stochastic particle systems, Wasserstein distance, optimal coupling.

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CUTOFF FOR THE NOISY VOTER MODEL

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Given a continuous time Markov Chain $\{q(x, y)\}$ on a finite set S , the associated noisy voter model is the continuous time Markov chain on $\{0, 1\}^S$, which evolves in the following way: (1) for each two sites x and y in S , the state at site x changes to the value of the state at site y at rate $q(x, y)$; (2) each site rerandomizes its state at rate 1. We show that if there is a uniform bound on the rates $\{q(x, y)\}$ and the corresponding stationary distributions are *almost* uniform, then the mixing time has a sharp cutoff at time $\log |S|/2$ with a window of order 1. Lubetzky and Sly proved cutoff with a window of order 1 for the stochastic Ising model on toroids; we obtain the special case of their result for the cycle as a consequence of our result. Finally, we consider the model on a star and demonstrate the surprising phenomenon that the time it takes for the chain started at all ones to become close in total variation to the chain started at all zeros is of smaller order than the mixing time.

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MULTI-LEVEL STOCHASTIC APPROXIMATION ALGORITHMS

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This paper studies multi-level stochastic approximation algorithms. Our aim is to extend the scope of the multi-level Monte Carlo method recently introduced by Giles [*Oper. Res.* **56** (2008) 607–617] to the framework of stochastic optimization by means of stochastic approximation algorithm. We first introduce and study a two-level method, also referred as statistical Romberg stochastic approximation algorithm. Then its extension to a multi-level method is proposed. We prove a central limit theorem for both methods and give optimal parameters. Numerical results confirm the theoretical analysis and show a significant reduction in the initial computational cost.

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MSC2010 subject classifications. 60F05, 62K12, 65C05, 60H35.

Key words and phrases. Multi-level Monte Carlo methods, stochastic approximation, Ruppert–Polyak averaging principle, Euler scheme.

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CONNECTIVITY OF SOFT RANDOM GEOMETRIC GRAPHS

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Consider a graph on n uniform random points in the unit square, each pair being connected by an edge with probability p if the inter-point distance is at most r . We show that as $n \rightarrow \infty$ the probability of full connectivity is governed by that of having no isolated vertices, itself governed by a Poisson approximation for the number of isolated vertices, uniformly over all choices of p, r . We determine the asymptotic probability of connectivity for all (p_n, r_n) subject to $r_n = O(n^{-\varepsilon})$, some $\varepsilon > 0$. We generalize the first result to higher dimensions and to a larger class of connection probability functions.

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A CONSISTENCY ESTIMATE FOR KAC'S MODEL OF ELASTIC COLLISIONS IN A DILUTE GAS

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An explicit estimate is derived for Kac's mean-field model of colliding hard spheres, which compares, in a Wasserstein distance, the empirical velocity distributions for two versions of the model based on different numbers of particles. For suitable initial data, with high probability, the two processes agree to within a tolerance of order $N^{-1/d}$, where N is the smaller particle number and d is the dimension, provided that $d \geq 3$. From this estimate we can deduce that the spatially homogeneous Boltzmann equation is well posed in a class of measure-valued processes and provides a good approximation to the Kac process when the number of particles is large. We also prove in an appendix a basic lemma on the total variation of time-integrals of time-dependent signed measures.

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Key words and phrases. Kac process, law of large numbers, Wasserstein distance, Boltzmann equation.

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STOCHASTIC PERRON FOR STOCHASTIC TARGET GAMES¹

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We extend the stochastic Perron method to analyze the framework of stochastic target games, in which one player tries to find a strategy such that the state process almost surely reaches a given target no matter which action is chosen by the other player. Within this framework, our method produces a viscosity sub-solution (super-solution) of a Hamilton–Jacobi–Bellman (HJB) equation. We then characterize the value function as a viscosity solution to the HJB equation using a comparison result and a byproduct to obtain the dynamic programming principle.

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Key words and phrases. The stochastic target problem, stochastic Perron method, viscosity solutions, geometric dynamic programming principle.

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ON THE CONVERGENCE OF ADAPTIVE SEQUENTIAL MONTE CARLO METHODS

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In several implementations of Sequential Monte Carlo (SMC) methods it is natural and important, in terms of algorithmic efficiency, to exploit the information of the history of the samples to optimally tune their subsequent propagations. In this article we provide a carefully formulated asymptotic theory for a class of such *adaptive* SMC methods. The theoretical framework developed here will cover, under assumptions, several commonly used SMC algorithms [Chopin, *Biometrika* **89** (2002) 539–551; Jasra et al., *Scand. J. Stat.* **38** (2011) 1–22; Schäfer and Chopin, *Stat. Comput.* **23** (2013) 163–184]. There are only limited results about the theoretical underpinning of such adaptive methods: we will bridge this gap by providing a weak law of large numbers (WLLN) and a central limit theorem (CLT) for some of these algorithms. The latter seems to be the first result of its kind in the literature and provides a formal justification of algorithms used in many real data contexts [Jasra et al. (2011); Schäfer and Chopin (2013)]. We establish that for a general class of adaptive SMC algorithms [Chopin (2002)], the asymptotic variance of the estimators from the adaptive SMC method is *identical* to a “limiting” SMC algorithm which uses ideal proposal kernels. Our results are supported by application on a complex high-dimensional posterior distribution associated with the Navier–Stokes model, where adapting high-dimensional parameters of the proposal kernels is critical for the efficiency of the algorithm.

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Key words and phrases. Adaptive sequential Monte Carlo, CLT, MCMC.

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RATE OF CONVERGENCE AND ASYMPTOTIC ERROR DISTRIBUTION OF EULER APPROXIMATION SCHEMES FOR FRACTIONAL DIFFUSIONS

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For a stochastic differential equation(SDE) driven by a fractional Brownian motion(fBm) with Hurst parameter $H > \frac{1}{2}$, it is known that the existing (naive) Euler scheme has the rate of convergence n^{1-2H} . Since the limit $H \rightarrow \frac{1}{2}$ of the SDE corresponds to a Stratonovich SDE driven by standard Brownian motion, and the naive Euler scheme is the extension of the classical Euler scheme for Itô SDEs for $H = \frac{1}{2}$, the convergence rate of the naive Euler scheme deteriorates for $H \rightarrow \frac{1}{2}$. In this paper we introduce a new (modified Euler) approximation scheme which is closer to the classical Euler scheme for Stratonovich SDEs for $H = \frac{1}{2}$, and it has the rate of convergence γ_n^{-1} , where $\gamma_n = n^{2H-1/2}$ when $H < \frac{3}{4}$, $\gamma_n = n/\sqrt{\log n}$ when $H = \frac{3}{4}$ and $\gamma_n = n$ if $H > \frac{3}{4}$. Furthermore, we study the asymptotic behavior of the fluctuations of the error. More precisely, if $\{X_t, 0 \leq t \leq T\}$ is the solution of a SDE driven by a fBm and if $\{X_t^n, 0 \leq t \leq T\}$ is its approximation obtained by the new modified Euler scheme, then we prove that $\gamma_n(X^n - X)$ converges stably to the solution of a linear SDE driven by a matrix-valued Brownian motion, when $H \in (\frac{1}{2}, \frac{3}{4}]$. In the case $H > \frac{3}{4}$, we show the L^p convergence of $n(X_t^n - X_t)$, and the limiting process is identified as the solution of a linear SDE driven by a matrix-valued Rosenblatt process. The rate of weak convergence is also deduced for this scheme. We also apply our approach to the naive Euler scheme.

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Key words and phrases. Fractional Brownian motion, stochastic differential equations, Euler scheme, fractional calculus, Malliavin calculus, fourth moment theorem.

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BACKWARD SDE REPRESENTATION FOR STOCHASTIC CONTROL PROBLEMS WITH NONDOMINATED CONTROLLED INTENSITY

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We are interested in stochastic control problems coming from mathematical finance and, in particular, related to model uncertainty, where the uncertainty affects both volatility and intensity. This kind of stochastic control problem is associated to a fully nonlinear integro-partial differential equation, which has the peculiarity that the measure $(\lambda(a, \cdot))_a$ characterizing the jump part is not fixed but depends on a parameter a which lives in a compact set A of some Euclidean space $\mathbb{R}^q$. We do not assume that the family $(\lambda(a, \cdot))_a$ is dominated. Moreover, the diffusive part can be degenerate. Our aim is to give a BSDE representation, known as a nonlinear Feynman–Kac formula, for the value function associated with these control problems. For this reason, we introduce a class of backward stochastic differential equations with jumps and a partially constrained diffusive part. We look for the minimal solution to this family of BSDEs, for which we prove uniqueness and existence by means of a penalization argument. We then show that the minimal solution to our BSDE provides the unique viscosity solution to our fully nonlinear integro-partial differential equation.

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OPTIMAL STOPPING UNDER MODEL UNCERTAINTY: RANDOMIZED STOPPING TIMES APPROACH

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In this work, we consider optimal stopping problems with conditional convex risk measures of the form

$$\rho_t^\Phi(X) = \sup_{Q \in \mathcal{Q}_t} \left(\mathbb{E}_Q[-X | \mathcal{F}_t] - \mathbb{E} \left[\Phi \left(\frac{dQ}{dP} \right) | \mathcal{F}_t \right] \right),$$

where $\Phi : [0, \infty] \rightarrow [0, \infty]$ is a lower semicontinuous convex mapping and $\mathcal{Q}_t$ stands for the set of all probability measures Q which are absolutely continuous w.r.t. a given measure P and $Q = P$ on $\mathcal{F}_t$. Here, the *model uncertainty risk* depends on a (random) divergence $\mathbb{E}[\Phi(\frac{dQ}{dP}) | \mathcal{F}_t]$ measuring the distance between a hypothetical probability measure we are uncertain about and a reference one at time t . Let $(Y_t)_{t \in [0, T]}$ be an adapted nonnegative, right-continuous stochastic process fulfilling some proper integrability condition and let $\mathcal{T}$ be the set of stopping times on $[0, T]$; then without assuming any kind of time-consistency for the family (ρ_t^Φ) , we derive a novel representation

$$\sup_{\tau \in \mathcal{T}} \rho_0^\Phi(-Y_\tau) = \inf_{x \in \mathbb{R}} \left\{ \sup_{\tau \in \mathcal{T}} \mathbb{E}[\Phi^*(x + Y_\tau) - x] \right\},$$

which makes the application of the standard dynamic programming based approaches possible. In particular, we generalize the additive dual representation of Rogers [*Math. Finance* **12** (2002) 271–286] to the case of optimal stopping under uncertainty. Finally, we develop several Monte Carlo algorithms and illustrate their power for optimal stopping under Average Value at Risk.

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