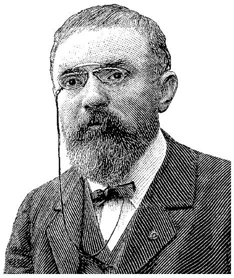

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Asymptotic sampling formulae for Λ-coalescents

Julien Berestycki[a,1], Nathanaël Berestycki[b,2] and Vlada Limic[c,3]

[a] *LPMA/UMR 7599, Université Pierre et Marie Curie (P6), 75252 Paris Cedex 05, France*
[b] *Statistical Laboratory, Centre for Mathematical Sciences, Wilberforce Road, Cambridge CB3 0WB, England*
[c] *Laboratoire de Mathématiques, UMR 8628 – Bâtiment 425, Université Paris-Sud, 91405 Orsay Cedex, France*

Abstract. We present a robust method which translates information on the speed of coming down from infinity of a genealogical tree into sampling formulae for the underlying population. We apply these results to population dynamics where the genealogy is given by a Λ-coalescent. This allows us to derive an exact formula for the asymptotic behavior of the site and allele frequency spectrum and the number of segregating sites, as the sample size tends to ∞. Some of our results hold in the case of a general Λ-coalescent that comes down from infinity, but we obtain more precise information under a regular variation assumption. In this case, we obtain results of independent interest for the time at which a mutation uniformly chosen at random was generated. This exhibits a phase transition at $\alpha = 3/2$, where $\alpha \in (1, 2)$ is the exponent of regular variation.

Résumé. Nous présentons une méthode robuste qui permet de traduire des informations sur la vitesse de descente de l'infini d'un arbre généalogique en formules d'échantillonnages pour la population sous-jacente. Nous appliquons cette méthode au cas où la génélaogie est donnée par un Λ-coalescent. Nous en déduisons une formule exacte pour le comportement asymptotique du spectre des fréquences alléliques et du nombre de sites de ségrégation, lorsque la taille de l'échantillon tend vers l'infini. Certains de ces résultats sont valides dans le cas général où le coalescent descend de l'infini, tandis que d'autres plus précis sont obtenus sous une hypothèse de variation régulière. Dans ce cas nous obtenons également des résultats, dont l'intérêt dépasse ce contexte, sur le temps auquel une mutation choisie uniformément au hasard est apparue. Il apparaît que cette quantité connaît une transition de phase autour de la valeur $\alpha = 3/2$, où α est l'exposant de variation régulière.

MSC: 60J25; 60F99; 92D25

Keywords: Λ-coalescents; Speed of coming down from infinity; Exchangeable coalescents; Sampling formulae; Infinite allele model; Genetic variation

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Genealogy of flows of continuous-state branching processes via flows of partitions and the Eve property

Cyril Labbé

Laboratoire de Probabilités et Modèles Aléatoires, Université Pierre et Marie Curie, Paris, France. E-mail: cyril.labbe@upmc.fr

Abstract. We encode the genealogy of a continuous-state branching process associated with a branching mechanism Ψ – or Ψ-CSBP in short – using a stochastic flow of partitions. This encoding holds for all branching mechanisms and appears as a very tractable object to deal with asymptotic behaviours and convergences. In particular we study the so-called Eve property – the existence of an ancestor from which the entire population descends asymptotically – and give a necessary and sufficient condition on the Ψ-CSBP for this property to hold. Finally, we show that the flow of partitions unifies the lookdown representation and the flow of subordinators when the Eve property holds.

Résumé. Nous construisons la généalogie d'un processus de branchement à espace d'états et temps continus associé à un mécanisme de branchement Ψ – ou Ψ-CSBP – à l'aide d'un flot stochastique de partitions. Cette construction est valable quel que soit le mécanisme de branchement et permet de définir un objet remarquablement efficace pour étudier les comportements asymptotiques et les convergences. En particulier, nous étudions la propriété d'Eve – l'existence d'un ancêtre dont descend asymptotiquement toute la population – et donnons une condition nécessaire et suffisante sur le Ψ-CSBP pour que cette propriété soit vérifiée. Finalement, nous montrons que le flot de partitions unifie la représentation lookdown et le flot de subordinateurs lorsque la propriété d'Eve est vérifiée.

MSC: Primary 60J80; secondary 60G09; 60J25

Keywords: Continuous-state branching process; Measure-valued process; Genealogy; Partition; Stochastic flow; Lookdown process; Subordinator; Eve

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Small positive values for supercritical branching processes in random environment

Vincent Bansaye[a] and Christian Böinghoff[b]

[a]CMAP, Ecole Polytechnique, Route de Saclay, 91128 Palaiseau Cedex, France. E-mail: vincent.bansaye@polytechnique.edu
[b]Department of Mathematics, Goethe-University Frankfurt/Main, Deutschland. E-mail: boeinghoff@math.uni-frankfurt.de

Abstract. Branching Processes in Random Environment (BPREs) $(Z_n\colon n \geq 0)$ are the generalization of Galton–Watson processes where in each generation the reproduction law is picked randomly in an i.i.d. manner. In the supercritical case, the process survives with positive probability and then almost surely grows geometrically. This paper focuses on rare events when the process takes positive but small values for large times.

We describe the asymptotic behavior of $\mathbb{P}(1 \leq Z_n \leq k | Z_0 = i)$, $k, i \in \mathbb{N}$ as $n \to \infty$. More precisely, we characterize the exponential decrease of $\mathbb{P}(Z_n = k | Z_0 = i)$ using a spine representation due to Geiger. We then provide some bounds for this rate of decrease.

If the reproduction laws are linear fractional, this rate becomes more explicit and two regimes appear. Moreover, we show that these regimes affect the asymptotic behavior of the most recent common ancestor, when the population is conditioned to be small but positive for large times.

Résumé. Les processus de branchement en environnement aléatoire $(Z_n\colon n \geq 0)$ sont une généralisation des processus de Galton Watson où à chaque génération, la reproduction est choisie de manière i.i.d. Dans le régime surcritique, ces processus survivent avec probabilité positive et croissent alors géométriquement. Ce papier considère l'événement rare où le processus prend des valeurs non nulles mais bornées en temps long.

Nous décrivons ainsi le comportement asymptotique de $P(1 \leq Z_n \leq k | Z_0 = i)$ quand $n \to \infty$. Plus précisément, nous caractérisons la vitesse exponentielle àlaquelle $\mathbb{P}(Z_n = k | Z_0 = i)$ tend vers zéro en utilisant une représentation en épine due à Geiger. Nous donnons alors des bornes pour cette vitesse.

Si la loi de reproduction est linéaire fractionnaire, la vitesse devient plus explicite et deux régimes apparaissent. Nous montrons par ailleurs que ces régimes affectent le comportement asymptotique de l'ancêtre commun le plus récent de la population en vie à l'instant n quand cette dernière est conditionnée à prendre de petites valeurs en temps long.

MSC: 60J80; 60K37; 60J05; 60F17; 92D25

Keywords: Supercritical branching processes; Random environment; Large deviations; Phase transitions

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Deviation inequalities and moderate deviations for estimators of parameters in bifurcating autoregressive models

S. Valère Bitseki Penda and Hacène Djellout

Laboratoire de Mathématiques, CNRS UMR 6620, Université Blaise Pascal, 24 Avenue des Landais, BP80026, 63177 Aubière, France.
E-mail: Valere.Bitsekipenda@math.univ-bpclermont.fr; Hacene.Djellout@math.univ-bpclermont.fr

Abstract. The purpose of this paper is to investigate the deviation inequalities and the moderate deviation principle of the least squares estimators of the unknown parameters of general pth-order asymmetric bifurcating autoregressive processes, under suitable assumptions on the driven noise of the process. Our investigation relies on the moderate deviation principle for martingales.

Résumé. L'objetcif de ce papier est d'établir des inégalités de déviations et les principes de déviations modérées pour les estimateurs des moindres carrés des paramètres inconnus d'un processus bifurcant autorégressif asymétrique d'ordre p, sous certaines conditions sur la suite des bruits. Les preuves reposent sur les principes de déviations modérées des martingales.

MSC: 60F10; 62F12; 60G42; 62M10; 62G05

Keywords: Deviation inequalities; Moderate deviation principle; Bifurcating autoregressive process; Martingale; Limit theorems; Least squares estimation

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Process-level large deviations for nonlinear Hawkes point processes[1]

Lingjiong Zhu

Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, NY 10012, USA. E-mail: ling@cims.nyu.edu

Abstract. In this paper, we prove a process-level, also known as level-3 large deviation principle for a very general class of simple point processes, i.e. nonlinear Hawkes process, with a rate function given by the process-level entropy, which has an explicit formula.

Résumé. Dans cet article nous prouvons un principe de grandes déviations de niveau trois pour une classe très générale de processus ponctuels, c'est à dire les processus de Hawkes non-linéaires ; nous obtenons une formule explicite pour la fonctionnelle de taux, donnée par l'entropie au niveau du processus.

MSC: 60G55; 60F10

Keywords: Large deviations; Rare events; Point processes; Hawkes processes; Self-exciting processes

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A quenched weak invariance principle

Jérôme Dedecker[a], Florence Merlevède[b] and Magda Peligrad[c,1]

[a] *Université Paris Descartes, Sorbonne Paris Cité, Laboratoire MAP5 and CNRS UMR 8145, 45 rue des saints pères, 75006 Paris, France.*
E-mail: jerome.dedecker@parisdescartes.fr
[b] *Université Paris Est, LAMA (UMR 8050), UPEMLV, CNRS, UPEC, 5 boulevard Descartes, 77454 Champs-sur-Marne, France.*
E-mail: florence.merlevede@univ-mlv.fr
[c] *University of Cincinnati, Department of Mathematical Sciences, PO Box 210025, Cincinnati, OH 45221-0025, USA.*
E-mail: peligrm@ucmail.uc.edu

Abstract. In this paper we study the almost sure conditional central limit theorem in its functional form for a class of random variables satisfying a projective criterion. Applications to strongly mixing processes and nonirreducible Markov chains are given. The proofs are based on the normal approximation of double indexed martingale-like sequences, an approach which has interest in itself.

Résumé. Dans cet article, nous étudions le théorème central limite conditionnel presque sûr, ainsi que sa forme fonctionnelle, pour des suites stationnaires de variables aléatoires réelles satisfaisant une condition de type projectif. Nous donnons des applications de ces résultats aux processus fortement mélangeants ainsi qu'à des chaînes de Markov nonirréductibles. Les preuves sont essentiellement basées sur une approximation normale de suites doublement indexées de variables aléatoires de type martingale.

MSC: 60F05; 60F17; 60J05

Keywords: Quenched central limit theorem; Weak invariance principle; Strong mixing; Markov chains

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Convergence to the Brownian Web for a generalization of the drainage network model

Cristian Coletti[a,1] and Glauco Valle[b,2]

[a] *UFABC - Centro de Matemática, Computação e Cognição, Avenida dos Estados, 5001, Santo André - São Paulo, Brasil.*
E-mail: cristian.coletti@ufabc.edu.br
[b] *UFRJ - Departamento de métodos estatísticos do Instituto de Matemática, Caixa Postal 68530, 21945-970, Rio de Janeiro, Brasil.*
E-mail: glauco.valle@im.ufrj.br

Abstract. We introduce a system of one-dimensional coalescing nonsimple random walks with long range jumps allowing paths that can cross each other and are dependent even before coalescence. We show that under diffusive scaling this system converges in distribution to the Brownian Web.

Résumé. Nous introduisons un système de marches aléatoires coalescentes unidimensionnelles, avec des sauts à longue portée. Ce système autorise des chemins qui se croisent, et qui sont dépendants, même avant leur coalescence. Après une renormalisation diffusive, nous montrons que ce système converge en loi vers le réseau brownien.

MSC: 60K35

Keywords: Brownian Web; Coalescing Brownian motions; Coalescing random walks; Drainage network; Scaling limit; Invariance principle; Interacting particle systems

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Gradient flows of the entropy for jump processes

Matthias Erbar[1]

Institute for Applied Mathematics, University of Bonn, Endenicher Allee 60, 53115 Bonn, Germany. E-mail: erbar@iam.uni-bonn.de

Abstract. We introduce a new transport distance between probability measures on $\mathbb{R}^d$ that is built from a Lévy jump kernel. It is defined via a non-local variant of the Benamou–Brenier formula. We study geometric and topological properties of this distance, in particular we prove existence of geodesics. For translation invariant jump kernels we identify the semigroup generated by the associated non-local operator as the gradient flow of the relative entropy w.r.t. the new distance and show that the entropy is convex along geodesics.

Résumé. On considère une nouvelle distance entre les mesures de probabilité sur $\mathbb{R}^n$. Elle est construite à partir d'un processus de saut par une variante non-locale de la formule de Benamou–Brenier. Pour les processus de Lévy on démontre que le semigroupe engendré par l'opérateur non-local associé est le flot de gradient de l'entropie par rapport à la nouvelle distance. On démontre aussi que l'entropie est convexe le long des géodésiques dans ce cas.

MSC: Primary 60J75; secondary 35S10; 45K05; 49J45; 60G51

Keywords: Jump process; Lévy process; Gradient flow; Entropy; Optimal transport

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Conditional distributions, exchangeable particle systems, and stochastic partial differential equations

Dan Crisan[a], Thomas G. Kurtz[b] and Yoonjung Lee[c]

[a]*Department of Mathematics, Imperial College London, 180 Queen's Gate, London SW7 2AZ, UK. E-mail: d.crisan@imperial.ac.uk*
[b]*Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Drive, Madison, WI 53706-1388, USA. E-mail: kurtz@math.wisc.edu*
[c]*Department of Statistics, Harvard University, 605 Science Center, One Oxford Street, Cambridge, MA 02138-2901, USA. E-mail: ylee@stat.harvard.edu*

Abstract. Stochastic partial differential equations (SPDEs) whose solutions are probability-measure-valued processes are considered. Measure-valued processes of this type arise naturally as de Finetti measures of infinite exchangeable systems of particles and as the solutions for filtering problems. In particular, we consider a model of asset price determination by an infinite collection of competing traders. Each trader's valuations of the assets are given by the solution of a stochastic differential equation, and the infinite system of SDEs, assumed to be exchangeable, is coupled through a common noise process and through the asset prices. In the simplest, single asset setting, the market clearing price at any time t is given by a quantile of the de Finetti measure determined by the individual trader valuations. In the multi-asset setting, the prices are essentially given by the solution of an assignment game introduced by Shapley and Shubik. Existence of solutions for the infinite exchangeable system is obtained by an approximation argument that requires the continuous dependence of the prices on the determining de Finetti measures which is ensured if the de Finetti measures charge every open set. The solution of the SPDE satisfied by the de Finetti measures can be interpreted as the conditional distribution of the solution of a single stochastic differential equation given the common noise and the price process. Under mild nondegeneracy conditions on the coefficients of the stochastic differential equation, the conditional distribution is shown to charge every open set, and under slightly stronger conditions, it is shown to be absolutely continuous with respect to Lebesgue measure with strictly positive density. The conditional distribution results are the main technical contribution and can also be used to study the properties of the solution of the nonlinear filtering equation within a framework that allows for the signal noise and the observation noise to be correlated.

Résumé. On considère des équations aux dérivées partielles stochastiques (EDPS) dont les solutions sont des processus à valeurs dans les mesures de probabilité. Des processus à valeurs mesures de ce type apparaissent naturellement comme des mesures de De Finetti de systèmes infinis de particules échangeables et comme solutions de problèmes de filtrage. En particulier nous considérons un modèle de détermination du prix d'un actif par une famille de traders en compétition. L'évaluation de chaque trader sur l'actif est donnée par la solution d'une équation différentielle stochastique et ce système infini d'EDSs, supposé échangeable, est couplé par un bruit commun et par les prix des actifs. Dans le cadre le plus simple à un seul actif, le prix d'équilibre du marché à tout temps t est donné par un quantile de la mesure de De Finetti déterminé par les évaluations du trader individuel. Dans le cadre à plusieurs actifs, les prix sont donnés essentiellement par la solution d'un problème d'attribution introduit par Shapley et Shubik. L'existence de solutions pour le système échangeable infini est obtenue par un argument d'approximation qui nécessite la dépendance continue des distributions des prix par rapport à la mesure de De Finetti associée. Ceci est vrai si la mesure de De Finetti donne une masse positive à tout ouvert non-vide. La solution de l'EDPS satisfaite par la mesure de De Finetti peut être interprétée comme la distribution conditionnelle de la solution d'une seule EDS donnée par le bruit commun et par le processus du prix. Sous des conditions faibles de non-dégénérescence des coefficients de l'EDS, on montre que la distribution conditionnelle donne une masse positive à tout ouvert non-vide, et sous des conditions légèrement plus fortes, on prouve qu'elle est absolument continue par rapport à la mesure de Lebesgue avec une densité strictement positive. Les résultats sur la distribution conditionnelle constituent la contribution technique principale et ils peuvent être aussi utilisés pour étudier les propriétés de la solution de l'équation de filtrage non-linéaire dans un cadre où le bruit du signal et celui de l'observation sont corrélés.

MSC: 60H15; 60G09; 60G35; 60J25

Keywords: Exchangeable systems; Conditional distributions; Stochastic partial differential equations; Quantile processes; Filtering equations; Measure-valued processes; Auction based pricing; Assignment games

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Odd cutsets and the hard-core model on $\mathbb{Z}^d$

Ron Peled[a,1] and Wojciech Samotij[b,2]

[a]*School of Mathematical Sciences, Tel Aviv University, Tel Aviv, 69978, Israel. E-mail: peledron@post.tau.ac.il; url: http://www.math.tau.ac.il/~peledron*

[b]*School of Mathematical Sciences, Tel Aviv University, Tel Aviv, 69978, Israel; and Trinity College, Cambridge CB2 1TQ, UK. E-mail: samotij@post.tau.ac.il; url: http://www.math.tau.ac.il/~samotij*

Abstract. We consider the hard-core lattice gas model on $\mathbb{Z}^d$ and investigate its phase structure in high dimensions. We prove that when the intensity parameter exceeds $Cd^{-1/3}(\log d)^2$, the model exhibits multiple hard-core measures, thus improving the previous bound of $Cd^{-1/4}(\log d)^{3/4}$ given by Galvin and Kahn. At the heart of our approach lies the study of a certain class of edge cutsets in $\mathbb{Z}^d$, the so-called odd cutsets, that appear naturally as the boundary between different phases in the hard-core model. We provide a refined combinatorial analysis of the structure of these cutsets yielding a quantitative form of concentration for their possible shapes as the dimension d tends to infinity. This analysis relies upon and improves previous results obtained by the first author.

Résumé. Nous étudions la structure de phase, en grande dimension, d'un modèle de sphères dures sur le réseau $\mathbb{Z}^d$. Nous prouvons que le modèle présente plusieurs mesures lorsque le paramètre de densité dépasse $Cd^{-1/3}(\log d)^2$, améliorant ainsi la borne de $Cd^{-1/4}(\log d)^{3/4}$ obtenue par Galvin et Kahn. Notre approche repose sur l'étude de certaines classes d'ensembles séparateurs dans $\mathbb{Z}^d$, constituées d'ensembles impaires, qui délimitent la frontière entre différentes phases du modèle de sphères dures. Nous faisons une analyse combinatoire précise de la structure de ces ensembles séparateurs et obtenons une forme quantitative de la concentration des différentes formes possibles prises par ces ensembles lorsque la dimension d tend vers l'infini. Cette analyse repose sur des méthodes obtenues auparavant par le premier auteur, tout en les améliorant.

MSC: 82B20; 60C05; 05C30; 05A16

Keywords: Edge cutsets; Gibbs measures; Hard-core model; Integer lattice

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Cycle structure of percolation on high-dimensional tori

Remco van der Hofstad[a] and Artëm Sapozhnikov[b]

[a] *Department of Mathematics and Computer Science, Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands. E-mail: rhofstad@win.tue.nl*

[b] *Max-Planck Institute for Mathematics in the Sciences, Inselstrasse 22, 04103 Leipzig, Germany. E-mail: artem.sapozhnikov@mis.mpg.de*

Abstract. In the past years, many properties of the largest connected components of critical percolation on the high-dimensional torus, such as their sizes and diameter, have been established. The order of magnitude of these quantities equals the one for percolation on the complete graph or Erdős–Rényi random graph, raising the question whether the scaling limits of the largest connected components, as identified by Aldous (1997), are also equal.

In this paper, we investigate the *cycle structure* of the largest critical components for high-dimensional percolation on the torus $\{-\lfloor r/2\rfloor, \ldots, \lceil r/2\rceil - 1\}^d$. While percolation clusters naturally have many *short* cycles, we show that the *long* cycles, i.e., cycles that pass through the boundary of the cube of width $r/4$ centered around each of their vertices, have length of order $r^{d/3}$, as on the critical Erdős–Rényi random graph. On the Erdős–Rényi random graph, cycles play an essential role in the scaling limit of the large critical clusters, as identified by Addario-Berry, Broutin and Goldschmidt (2010).

Our proofs crucially rely on various new estimates of probabilities of the existence of open paths in critical Bernoulli percolation on $\mathbb{Z}^d$ with constraints on their lengths. We believe these estimates are interesting in their own right.

Résumé. Plusieurs propriétés du comportement des grandes composantes connexes de la percolation critique sur le tore en dimensions grandes ont été récemment établies, telles la taille et le diamétre. L'ordre de grandeur de ces quantités est égal à celle de la percolation sur le graphe complet ou sur le graphe aléatoire de Erdős–Rényi. Ce résultat suggère la question de savoir si les limites d'échelles des plus grandes composantes connexes, telles qu'identifiées par Aldous (1997), sont aussi égales.

Dans ce travail, nous étudions la structure des cycles des plus grandes composantes connexes pour la percolation critique en grande dimension sur le tore $\{-\lfloor r/2\rfloor, \ldots, \lceil r/2\rceil - 1\}^d$. Alors que les amas de percolation ont plusieurs cycles courts, nous montrons que les cycles longs, c'est-à-dire ceux qui passent à travers la frontière de chacun des cubes de largeur $r/4$ centrés aux sommets du cycle, ont une longueur de l'ordre $r^{d/3}$, comme dans le cas du graphe aléatoire critique d'Erdős–Rényi. Sur ce dernier, les cycles jouent un rôle essentiel dans la limite d'échelle des grands amas critiques tels qu'identifiés par Addario-Berry, Broutin and Goldschmidt (2010).

Les preuves sont basées de manière cruciale sur de nouvelles estimations de la probabilités d'existence de chemins ouverts dans la percolation critique de type Bernouilli sur $\mathbb{Z}^d$ avec des contraintes sur leurs longueurs. Ces estimations sont potentiellement intéressantes en soi.

MSC: 05C80; 60K35; 82B43

Keywords: Random graph; Phase transition; Critical behavior; Percolation; Torus; Cycle structure

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Estimation of the transition density of a Markov chain

Mathieu Sart

[a]Université de Nice Sophia-Antipolis, Laboratoire J-A Dieudonné, Parc Valrose, 06108 Nice cedex 02, France. E-mail: msart@unice.fr

Abstract. We present two data-driven procedures to estimate the transition density of an homogeneous Markov chain. The first yields a piecewise constant estimator on a suitable random partition. By using an Hellinger-type loss, we establish non-asymptotic risk bounds for our estimator when the square root of the transition density belongs to possibly inhomogeneous Besov spaces with possibly small regularity index. Some simulations are also provided. The second procedure is of theoretical interest and leads to a general model selection theorem from which we derive rates of convergence over a very wide range of possibly inhomogeneous and anisotropic Besov spaces. We also investigate the rates that can be achieved under structural assumptions on the transition density.

Résumé. Nous présentons deux procédures pour estimer la densité de transition d'une chaîne de Markov homogène. Dans la première procédure, nous construisons un estimateur constant par morceaux sur une partition aléatoire bien choisie. Nous établissons des bornes de risque non-asymptotiques pour une perte de type Hellinger lorsque la racine carrée de la densité de transition appartient à un espace de Besov inhomogène dont l'indice de régularité peut être petit. Nous illustrons ces résultats par des simulations numériques. La deuxième procédure est d'intérêt théorique. Elle permet d'obtenir un théorème de sélection de modèle à partir duquel nous déduisons des vitesses de convergence sur des espaces de Besov inhomogènes anisotropes. Nous étudions finalement les vitesses qui peuvent être atteintes sous des hypothèses structurelles sur la densité de transition.

MSC: 62M05; 62G05

Keywords: Adaptive estimation; Markov chain; Model selection; Robust tests; Transition density

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Evaluating default priors with a generalization of Eaton's Markov chain

Brian P. Shea and Galin L. Jones

School of Statistics, University of Minnesota, Minneapolis, MN, 55455, USA. E-mail: bshea@stat.umn.edu; galin@umn.edu

Abstract. We consider evaluating improper priors in a formal Bayes setting according to the consequences of their use. Let Φ be a class of functions on the parameter space and consider estimating elements of Φ under quadratic loss. If the formal Bayes estimator of every function in Φ is admissible, then the prior is strongly admissible with respect to Φ. Eaton's method for establishing strong admissibility is based on studying the stability properties of a particular Markov chain associated with the inferential setting. In previous work, this was handled differently depending upon whether $\varphi \in \Phi$ was bounded or unbounded. We consider a new Markov chain which allows us to unify and generalize existing approaches while simultaneously broadening the scope of their potential applicability. We use our general theory to investigate strong admissibility conditions for location models when the prior is Lebesgue measure and for the p-dimensional multivariate Normal distribution with unknown mean vector θ and a prior of the form $\nu(\|\theta\|^2)\,\mathrm{d}\theta$.

Résumé. Nous considérons l'évaluation de lois a priori impropres dans un cadre Bayésien formel en fonction des conséquences de leur utilisation. Soit Φ une classe de fonctions sur l'espace des paramètres, que l'on cherche à estimer sous une fonction de perte quadratique. Si l'estimateur Bayésien de toute fonction dans Φ est admissible, alors la loi a priori est fortement admissible par rapport à Φ. La méthode d'Eaton pour établir l'admissibilité forte est basée sur l'étude des propriétés de stabilité d'une certaine chaîne de Markov associé au cadre inférentiel. Dans des travaux précédents, nous considérions une nouvelle chaîne de Markov qui permet d'unifier et de généraliser les approches existantes tout en élargissant simultanément son champ d'application. Nous utilisons cette théorie générale pour étudier des conditions d'admissibilité forte pour des modéles à paramètre de position, une loi a priori donnée par la mesure de Lebesgue et la loi normale multivariée de dimension p et moyenne θ, et une loi a priori de la forme $\nu(\|\theta\|^2)\,\mathrm{d}\theta$.

MSC: Primary 62C15; secondary 60J05

Keywords: Admissibility; Improper prior distribution; Symmetric Markov chain; Recurrence; Dirichlet form; Formal Bayes rule

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Estimator selection in the Gaussian setting

Yannick Baraud[a], Christophe Giraud[b] and Sylvie Huet[c]

[a]*Laboratoire J. A. Dieudonné UMR CNRS 7351, Université de Nice Sophia-Antipolis, Parc Valrose, 06108 Nice cedex 02, France. E-mail: baraud@unice.fr*
[b]*CMAP, UMR CNRS 7641, Ecole Polytechnique, Route de Saclay, 91128 Palaiseau Cedex, France. E-mail: christophe.giraud@polytechnique.edu*
[c]*UR341 MIA, INRA, F-78350 Jouy-en-Josas, France. E-mail: sylvie.huet@jouy.inra.fr*

Abstract. We consider the problem of estimating the mean f of a Gaussian vector Y with independent components of common unknown variance σ^2. Our estimation procedure is based on estimator selection. More precisely, we start with an arbitrary and possibly infinite collection $\mathbb{F}$ of estimators of f based on Y and, with the same data Y, aim at selecting an estimator among $\mathbb{F}$ with the smallest Euclidean risk. No assumptions on the estimators are made and their dependencies with respect to Y may be unknown. We establish a non-asymptotic risk bound for the selected estimator and derive oracle-type inequalities when $\mathbb{F}$ consists of linear estimators. As particular cases, our approach allows to handle the problems of aggregation, model selection as well as those of choosing a window and a kernel for estimating a regression function, or tuning the parameter involved in a penalized criterion. In all theses cases but aggregation, the method can be easily implemented. For illustration, we carry out two simulation studies. One aims at comparing our procedure to cross-validation for choosing a tuning parameter. The other shows how to implement our approach to solve the problem of variable selection in practice.

Résumé. Nous présentons une nouvelle procédure de sélection d'estimateurs pour estimer l'espérance f d'un vecteur Y de n variables gaussiennes indépendantes dont la variance est inconnue. Nous proposons de choisir un estimateur de f, dont l'objectif est de minimiser le risque l_2, dans une collection arbitraire et éventuellement infinie $\mathbb{F}$ d'estimateurs. La procédure de choix ainsi que la collection $\mathbb{F}$ ne dépendent que des seules observations Y. Nous calculons une borne de risque, non asymptotique, ne nécessitant aucune hypothèse sur les estimateurs dans $\mathbb{F}$, ni la connaissance de leur dépendance en Y. Nous calculons des inégalités de type "oracle" quand $\mathbb{F}$ est une collection d'estimateurs linéaires. Nous considérons plusieurs cas particuliers : estimation par aggrégation, estimation par sélection de modèles, choix d'une fenêtre et du paramètre de lissage en régression fonctionnelle, choix du paramètre de régularisation dans un critère pénalisé. Pour tous ces cas particuliers, sauf pour les méthodes d'aggrégation, la méthode est très facile à programmer. A titre d'illustration nous montrons des résultats de simulations avec deux objectifs : comparer notre méthode à la procédure de cross-validation, montrer comment la mettre en œuvre dans le cadre de la sélection de variables.

MSC: 62J05; 62J07; 62G08

Keywords: Estimator selection; Model selection; Variable selection; Linear estimator; Kernel estimator; Ridge regression; Lasso; Elastic net; Random forest; PLS1 regression

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