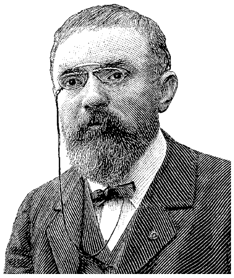

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Spectral gap properties for linear random walks and Pareto's asymptotics for affine stochastic recursions

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Abstract. Let $V = \mathbb{R}^d$ be the Euclidean d-dimensional space, μ (resp. λ) a probability measure on the linear (resp. affine) group $G = \mathrm{GL}(V)$ (resp. $H = \mathrm{Aff}(V)$) and assume that μ is the projection of λ on G. We study asymptotic properties of the iterated convolutions $\mu^n * \delta_v$ (resp. $\lambda^n * \delta_v$) if $v \in V$, i.e. asymptotics of the random walk on V defined by μ (resp. λ), if the subsemigroup $T \subset G$ (resp. $\Sigma \subset H$) generated by the support of μ (resp. λ) is "large." We show spectral gap properties for the convolution operator defined by μ on spaces of homogeneous functions of degree $s \geq 0$ on V, which satisfy Hölder type conditions. As a consequence of our analysis we get precise asymptotics for the potential kernel $\sum_0^\infty \mu^k * \delta_v$, which imply its asymptotic homogeneity. Under natural conditions the H-space V is a λ-boundary; then we use the above results and radial Fourier Analysis on $V \setminus \{0\}$ to show that the unique λ-stationary measure ρ on V is "homogeneous at infinity" with respect to dilations $v \to tv$ (for $t > 0$), with a tail measure depending essentially of μ and Σ. Our proofs are based on the simplicity of the dominant Lyapunov exponent for certain products of Markov-dependent random matrices, on the use of renewal theorems for "tame" Markov walks, and on the dynamical properties of a conditional λ-boundary dual to V.

Résumé. Soit V l'espace Euclidien de dimension d, μ (resp. λ) une probabilité sur le groupe linéaire (resp. affine) $G = \mathrm{GL}(V)$ (resp. $H = \mathrm{Aff}(V)$) et supposons que μ soit la projection de λ sur G. Nous étudions certaines propriétés asymptotiques des convolutions itérées de μ (resp. λ) appliquées à un vecteur non nul $v \in V$, c'est à dire de la marche aléatoire sur V définie par μ (resp. λ), si le semigroupe $T \subset G$ (resp. $\Sigma \subset H$) engendré par le support de μ (resp. λ) est « grand ». Nous montrons des propriétés d'isolation spectrale pour l'opérateur de convolution défini par μ sur des espaces de fonctions homogènes de degré $s \geq 0$ sur V, qui satisfont des conditions du type de Hölder. Comme conséquence de notre analyse nous obtenons des asymptotiques précises pour le noyau potentiel $\sum_0^\infty \mu^k * \delta_v$, qui impliquent son homogénéité à l'infini. Sous des conditions naturelles, le H-espace V est une λ-frontière ; nous utilisons alors les résultats précédents et l'analyse de Fourier radiale sur $V \setminus \{0\}$ afin de montrer que l'unique mesure λ-stationnaire est homogène à l'infini, par rapport aux dilatations $v \to tv$ (pour $t > 0$), avec une mesure de queue qui dépend essentiellement de μ et Σ. Nos preuves sont basées sur la simplicité de l'exposant de Lyapunov dominant de certains produits de matrices en dépendance markovienne, sur l'utilisation de théorèmes de renouvellement pour certaines marches markoviennes et sur les propriétés dynamiques d'une λ-frontière duale de V.

MSC: 60B50; 60J50

Keywords: Spectral gap; Renewal theorem; Pareto asymptotics; Random matrices; Affine random recursions

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Rescaled bipartite planar maps converge to the Brownian map

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Abstract. For every integer $n \geq 1$, we consider a random planar map $\mathcal{M}_n$ which is uniformly distributed over the class of all rooted bipartite planar maps with n edges. We prove that the vertex set of $\mathcal{M}_n$ equipped with the graph distance rescaled by the factor $(2n)^{-1/4}$ converges in distribution, in the Gromov–Hausdorff sense, to the Brownian map. This complements several recent results giving the convergence of various classes of random planar maps to the Brownian map.

Résumé. Pour tout entier n strictement positif, on considère une carte planaire aléatoire $\mathcal{M}_n$ de loi uniforme sur l'ensemble des cartes biparties enracinées à n arêtes. On montre que l'ensemble des sommets de $\mathcal{M}_n$ muni de la distance de graphe renormalisée par $(2n)^{-1/4}$ converge en loi au sens de Gromov–Hausdorff vers la carte brownienne. Ce travail complète une série de résultats de convergence de différents modèles de cartes aléatoires vers la carte brownienne.

MSC: Primary 60D05; 60F17; secondary 05C80

Keywords: Brownian map; Planar map; Graph distance; Bipartite map; Scaling limit; Gromov–Hausdorff convergence; Two-type Galton–Watson tree

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Random infinite squarings of rectangles

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Abstract. A recent publication (*Electron. Commun. Probab.* **19** (2014) 1–12) introduced a growth procedure for planar maps, whose almost sure limit is "the uniform infinite 3-connected planar map." A classical construction of Brooks, Smith, Stone and Tutte (*Duke Math. J.* **7** (1940) 312–340) associates a squaring of a rectangle (i.e. a tiling of a rectangle by squares) to any finite, edge-rooted planar map with non-separating root edge. We use this construction together with the map growth procedure to define a growing sequence of squarings of rectangles. We prove that the sequence of squarings converges to an almost sure limit: a random infinite squaring of a finite rectangle. This provides a canonical planar embedding of the uniform infinite 3-connected planar map. We also show that the limiting random squaring almost surely has a unique point of accumulation.

Résumé. Un papier récément publié (*Electron. Commun. Probab.* **19** (2014) 1–12) introduit une procédure pour générer une suite de cartes aléatoires qui a presque sûrement comme limite la « carte 3-connexe infinie uniforme » . La construction classique de Brooks, Smith, Stone et Tutte (*Duke Math. J.* **7** (1940) 312–340) associe à chaque carte finie, avec une arête racine qui n'est pas un isthme, un rectangle pavé par des carrés. Nous utilisons ces deux procédures afin de générer une suite aléatoire de rectangles pavé par des carrés. Nous démontrons que cette suite a presque surement une limite qui est un rectangle aléatoire infiniment pavés par des carrés, et que cet objet a presque surement un seul point d'accumulation. Ceci fournit un plongement canonique de la carte 3-connexe infinie uniforme dans le plan.

MSC: 60C05; 05C81; 05C10; 60F15; 05C62

Keywords: Random maps; Graph limits; Square tilings

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Geodesics in Brownian surfaces (Brownian maps)

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Abstract. We define a class a metric spaces we call *Brownian surfaces*, arising as the scaling limits of random maps on general orientable surfaces with a boundary and we study the geodesics from a uniformly chosen random point. These metric spaces generalize the well-known *Brownian map* and our results generalize the properties shown by Le Gall on geodesics in the latter space. We use a different approach based on two ingredients: we first study typical geodesics and then all geodesics by an "entrapment" strategy. In particular, we give geometrical characterizations of some subsets of interest, in terms of geodesics, boundary points and concatenations of geodesics forming a loop that is not homotopic to 0.

Résumé. On définit une classe d'espaces métriques aléatoires que nous appelons *surfaces browniennes* : ces objets apparaissent comme limites d'échelle de cartes aléatoires sur des surfaces orientables à bord générales. Dans un second temps, on étudie les géodésiques émanant d'un point choisi uniformément au hasard. Les surfaces browniennes généralisent la fameuse *carte brownienne* et nos résultats généralisent les propriétés obtenues par Le Gall sur les géodésiques dans cet espace. On utilise une approche différente reposant sur deux ingrédients : on étudie d'abord les géodésiques aux points typiques et on attrape ensuite les autres points en les « encerclant » par de telles géodésiques. En particulier, on obtient des caractérisations géométriques de certains sous-ensembles d'intérêt en termes de géodésiques, points du bord et concaténations des géodésiques formant une boucle non homotope à 0.

MSC: 60F17; 60C05; 60D05; 57N05; 05C80; 05C12

Keywords: Brownian surfaces; Brownian map; Geodesics; Random maps; Scaling limits; Gromov–Hausdorff topology; Random metric spaces; Bijections

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Fleming–Viot selects the minimal quasi-stationary distribution: The Galton–Watson case

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Abstract. Consider N particles moving independently, each one according to a subcritical continuous-time Galton–Watson process unless it hits 0, at which time it jumps instantaneously to the position of one of the other particles chosen uniformly at random. The resulting dynamics is called Fleming–Viot process. We show that for each N there exists a unique invariant measure for the Fleming–Viot process, and that its stationary empirical distribution converges, as N goes to infinity, to the minimal quasi-stationary distribution of the Galton–Watson process conditioned on non-extinction.

Résumé. Nous considérons N particules indépendantes. Chaque particule suit l'évolution d'un processus de Galton–Watson sous-critique jusqu'au moment où elle touche 0. À cet instant, cette particule choisit uniformément au hasard la position d'une des autres particules et y saute. Ce processus est appelé Fleming–Viot. Nous montrons que pour chaque entier N, il existe une unique mesure invariante pour le processus de Fleming–Viot, et que la mesure empirique stationnaire converge vers la loi quasi-stationnaire minimale d'un processus de Galton–Watson conditionné à ne pas mourir.

MSC: Primary 60K35; secondary 60J25

Keywords: Quasi-stationary distributions; Fleming–Viot processes; Galton–Watson processes; Selection principle

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Evolution of the ABC model among the segregated configurations in the zero-temperature limit

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Abstract. We consider the ABC model on a ring in a strongly asymmetric regime. The main result asserts that the particles almost always form three pure domains (one of each species) and that this segregated shape evolves, in a proper time scale, as a Brownian motion on the circle, which may have a drift. This is, to our knowledge, the first proof of a zero-temperature limit for a non-reversible dynamics whose invariant measure is not explicitly known.

Résumé. Nous considérons le modèle ABC sur un anneau dans un régime fortement asymétrique. Le résultat principal affirme que les particules forment presque toujours trois domaines purs (un pour chaque espèce) et que cette forme ségréguée évolue, dans une échelle temporelle appropriée, comme un mouvement brownien sur le cercle, avec éventuellement une dérive. Il s'agit, à notre connaissance, de la première preuve d'une limite à température nulle pour une dynamique non-réversible dont la mesure invariante n'est pas explicitement connue.

MSC: 60K35; 82C20; 82C22

Keywords: Metastability; Tunneling; Scaling limits; ABC model; Brownian motion; Convergence to diffusions

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The quenched limiting distributions of a charged-polymer model[1]

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Abstract. The limit distributions of the charged-polymer Hamiltonian of Kantor and Kardar [Bernoulli case] and Derrida, Griffiths and Higgs [Gaussian case] are considered. Two sources of randomness enter in the definition: a random field $q = (q_i)_{i\geq 1}$ of i.i.d. random variables, which is called the random *charges*, and a random walk $S = (S_n)_{n\in\mathbb{N}}$ evolving in $\mathbb{Z}^d$, independent of the charges. The energy or Hamiltonian $K = (K_n)_{n\geq 2}$ is then defined as

$$K_n := \sum_{1\leq i<j\leq n} q_i q_j \mathbf{1}_{\{S_i=S_j\}}.$$

The law of K under the joint law of q and S is called "annealed," and the conditional law given q is called "quenched." Recently, strong approximations under the annealed law were proved for K. In this paper we consider the limit distributions of K under the quenched law.

Résumé. Les lois limites de l'hamiltonien dans le modèle de polymère chargé introduit par Kantor et Kardar dans le cas Bernoulli et par Derrida, Griffiths et Higgs dans le cas gaussien sont considérées. Deux aléas interviennent dans la définition : un champ aléatoire $q = (q_i)_{i\geq 1}$ de variables aléatoires i.i.d., appelées *charges* et une marche aléatoire $S = (S_n)_{n\in\mathbb{N}}$ dans $\mathbb{Z}^d$, indépendante des charges. L'énergie ou hamiltonien $K = (K_n)_{n\geq 2}$ est définie par

$$K_n := \sum_{1\leq i<j\leq n} q_i q_j \mathbf{1}_{\{S_i=S_j\}}.$$

La loi de K sous la loi conjointe de q et S est appelée « annealed » et la loi conditionnelle sachant q est appelée « quenched ». Récemment, des approximations fortes sous la loi annealed ont été prouvées pour K. Dans ce papier, nous considérons les lois limites de K sous la loi quenched.

MSC: 60G50; 60K35; 60F05

Keywords: Random walk; Polymer model; Self-intersection local time; Limit theorems; Law of the iterated logarithm; Martingale

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A three-series theorem on Lie groups

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Abstract. We obtain a necessary and sufficient condition for the convergence of independent products on Lie groups, as a natural extension of Kolmogorov's three-series theorem. Application to independent random matrices is discussed.

Résumé. Nous obtenons une condition nécessaire et suffisante pour la convergence de produits indépendants sur des groupes de Lie, comme extension naturelle du théorème des trois séries de Kolmogorov. Une application à des matrices aléatoires indépendantes est discutée.

MSC: 60B15

Keywords: Lie groups; Three-series theorem

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Construction and analysis of a sticky reflected distorted Brownian motion

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Abstract. We give a Dirichlet form approach for the construction of a distorted Brownian motion in $E := [0, \infty)^n$, $n \in \mathbb{N}$, where the behavior on the boundary is determined by the competing effects of reflection from and pinning at the boundary (sticky boundary behavior). In providing a Skorokhod decomposition of the constructed process we are able to justify that the stochastic process is solving the underlying stochastic differential equation weakly for quasi every starting point with respect to the associated Dirichlet form. That the boundary behavior of the constructed process indeed is sticky, we obtain by proving ergodicity of the constructed process. Therefore, we are able to show that the occupation time on specified parts of the boundary is positive. In particular, our considerations enable us to construct a dynamical wetting model (also known as Ginzburg–Landau dynamics) on a bounded set $D_N \subset \mathbb{Z}^d$ under mild assumptions on the underlying pair interaction potential in all dimensions $d \in \mathbb{N}$. In dimension $d = 2$ this model describes the motion of an interface resulting from wetting of a solid surface by a fluid.

Résumé. Nous construisons un mouvement brownien tordu dans $E := [0, \infty)^n$, $n \in \mathbb{N}$, en utilisant des méthodes de la théorie des formes de Dirichlet alors que le comportement à la frontière est déterminé par les effets concurrents de la réflexion de la frontière et l'ancrage à la frontière (comportement adhésif sur la frontière de E). En fournissant une décomposition de Skorokhod du processus construit nous pouvons justifier que le processus stochastique est une solution faible de l'équation différentielle stochastique fondamentale pour quasi tous les points de départ par rapport à la forme de Dirichlet associée. En démontrant l'ergodicité du processus construit, nous obtenons que le comportement sur la frontière du processus est en effet adhésif. En conséquence, il est possible de démontrer que le séjour sur des parties fixées de la frontière de E est positif. En particulier, nos considérations nous permettent de construire un modèle dynamique d'humectage (ausssi connu comme dynamique de Ginzburg–Landau) sur un ensemble borné $D_N \subset \mathbb{Z}^d$, $d \in \mathbb{N}$. Le potentiel qui détermine l'interaction des variables adjacentes est soumis à des conditions peu restrictives en toute dimension $d \in \mathbb{N}$. En dimension $d = 2$, ce modèle décrit le mouvement d'une interface résultant de l'humectage d'une surface solide par un fluide.

MSC: 60K35; 60J50; 60J55; 82C41

Keywords: Interacting sticky reflected distorted Brownian motion; Skorokhod decomposition; Wentzell boundary condition; Interface models

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Berry–Esseen bounds and multivariate limit theorems for functionals of Rademacher sequences

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Abstract. Berry–Esseen bounds for non-linear functionals of infinite Rademacher sequences are derived by means of the Malliavin–Stein method. Moreover, multivariate extensions for vectors of Rademacher functionals are shown. The results establish a connection to small ball probabilities and shed new light onto the relation between central limit theorems on the Rademacher chaos and norms of contraction operators. Applications concern infinite weighted 2-runs, a combinatorial central limit theorem and traces of Bernoulli random matrices.

Résumé. Nous dérivons des estimations de type Berry–Esseen pour des fonctionnelles non-linéaires de suites infinies de Rademacher par la méthode de Malliavin–Stein. De plus, nous prouvons des extensions multivariées pour des vecteurs de fonctionnelles de Rademacher. Ces résultats établissent une connexion avec les probabilités de petites boules et apportent un nouvel éclairage sur la relation entre des théorèmes centraux limites sur le chaos de Rademacher et les normes d'opérateurs de contraction. Des applications concernent des succès consécutifs pondérés, un théorème central limite combinatoire et des traces de matrices de Bernoulli aléatoires.

MSC: 60F05; 60G50; 60B20; 60H07

Keywords: Berry–Esseen bound; Central limit theorem; Malliavin calculus; Normal approximation; Rademacher chaos; Stein's method

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CLT for the zeros of classical random trigonometric polynomials

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Abstract. We prove a Central Limit Theorem for the number of zeros of random trigonometric polynomials of the form $K^{-1/2}\sum_{n=1}^{K} a_n \cos(nt)$, being $(a_n)_n$ independent standard Gaussian random variables. In particular we show that the variance is equivalent to $V^2 K\pi$, $0 < V^2 < \infty$, as $K \to \infty$. This last result was recently proved by Su and Shao in (*Sci. China Math.* **55** (2012) 2347–2366). Our approach is based on the Hermite/Wiener Chaos decomposition for square-integrable functionals of a Gaussian process and on Rice Formula for zero counting.

Résumé. Nous montrons un Théorème de la Limite Central pour le nombre de racines d'un polynôme trigonométrique aléatoire de la forme $K^{-1/2}\sum_{n=1}^{K} a_n \cos(nt)$, ici les a_n sont des variables aléatoires Gaussiennes standard et indépendantes. En particulier, nos démontrons que la variance asymptotique du nombre de racines est équivalent à $V^2 K\pi$, pour une certaine constante $V > 0$, lorsque $K \to \infty$. Ce dernier résultat a été récemment démontré par Su and Shao dans (*Sci. China Math.* **55** (2012) 2347–2366). Notre approche utilise la décomposition dans le chaos d'Itô–Wiener d'une fonctionnelle non linéaire de carré intégrable et la formule de Rice.

MSC: 60G15

Keywords: Classical trigonometric polynomials; Random cosines polynomials; Number of zeroes; CLT; Wiener Chaos

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Universality and Borel summability of arbitrary quartic tensor models

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Abstract. We extend the study of *melonic* quartic tensor models to models with arbitrary quartic interactions. This extension requires a new version of the loop vertex expansion using several species of intermediate fields and iterated Cauchy–Schwarz inequalities. Borel summability is proven, uniformly as the tensor size N becomes large. Every cumulant is written as a sum of explicitly calculated terms plus a remainder, suppressed in $1/N$. Together with the existence of the large N limit of the second cumulant, this proves that the corresponding sequence of probability measures is uniformly bounded and obeys the tensorial universality theorem.

Résumé. Nous étendons l'étude de modèles de tenseurs quartiques meloniques aux modèles avec des interactions quartiques arbitraires. Cette extension nécessite une nouvelle version du développement en vertex à boucles à l'aide de plusieurs nouveaux champs intermédiaires ainsi que l'utilisation répétée d'inégalités de Cauchy–Schwarz. La sommabilité de Borel est prouvée uniformément dans la taille N du tenseur. Chaque cumulant est écrit comme une somme de termes explicitement calculés plus un reste supprimé à grand N. L'existence d'une limite finie à grand N du second cumulant est établie et l'on démontre que la suite correspondante de mesures de probabilité est uniformément bornée en N et obéit bien au théorème tensoriel d'universalité comme dans le cas melonique.

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Keywords: Random tensors; Borel summability

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Fisher information and the fourth moment theorem

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Abstract. Using a representation of the score function by means of the divergence operator, we exhibit a sufficient condition, in terms of the negative moments of the norm of the Malliavin derivative under which, convergence in Fisher information to the standard Gaussian of sequences belonging to a given Wiener chaos is actually equivalent to convergence of only the fourth moment. Thus, our result may be considered as a further building block associated to the recent but already rich literature dedicated to the Fourth Moment Theorem of Nualart and Peccati (*Ann. Probab.* **33** (2005) 177–193). To illustrate the power of our approach, we prove a local limit theorem together with some rates of convergence for the normal convergence of a standardized version of the quadratic variation of the fractional Brownian motion.

Résumé. À l'aide d'une représentation de la fonction score au moyen de l'opérateur de divergence, nous mettons en évidence une condition suffisante, exprimée en terme de moments négatifs de la norme de la dérivée de Malliavin, sous laquelle la convergence au sens de l'information de Fisher vers la loi gaussienne d'une suite d'éléments appartenant à un chaos de Wiener fixé se trouve être équivalente à la simple convergence du moment quatrième. Nos résultats peuvent être vus comme une nouvelle pierre apportée à l'édification de la récente mais déjà riche littérature dédiée au théorème du moment quatrième de Nualart and Peccati (*Ann. Probab.* **33** (2005) 177–193). Pour illustrer notre approche, nous prouvons un théorème de la limite locale, avec calcul de la vitesse de convergence associée, pour la convergence normale d'une version renormalisée de la variation quadratique du mouvement brownien fractionnaire.

MSC: 60H07; 94A17; 60G22

Keywords: Fisher information; Total variation distance; Relative entropy; Fourth moment theorem; Fractional Brownian motion; Malliavin calculus

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Dual representation of minimal supersolutions of convex BSDEs

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Abstract. We give a dual representation of minimal supersolutions of BSDEs with non-bounded, but integrable terminal conditions and under weak requirements on the generator which is allowed to depend on the value process of the equation. Conversely, we show that any dynamic risk measure satisfying such a dual representation stems from a BSDE. We also give a condition under which a supersolution of a BSDE is even a solution.

Résumé. Nous donnons une représentation duale des sur-solutions minimales d'équations différentielles stochastiques rétrogrades avec des conditions terminales intégrables mais non nécessairement bornées, et de faibles hypotheses sur le générateur qui peut de plus dépendre de la valeur processus de l'équation même. Réciproquement, nous montrons que toute mesure de risque dynamique satisfaisant une telle représentation duale provient d'une EDSR. Nous donnons aussi une condition sous laquelle une sur-solution d'EDSR est en fait une solution.

MSC: 60H20; 60H30

Keywords: Convex duality; Supersolutions of BSDEs; Cash-subadditive risk measures

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Strong Feller properties for degenerate SDEs with jumps

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Abstract. Under full Hörmander's conditions, we prove the strong Feller property of the semigroup determined by an SDE driven by additive subordinate Brownian motions, where the drift is allowed to be arbitrary growth. For this, we extend a criterion due to Malicet and Poly (*J. Funct. Anal.* **264** (2013) 2077–2096) and Bally and Caramellino (*Electron. J. Probab.* **19** (2014) 1–33) about the convergence of the laws of Wiener functionals in total variation. Moreover, the example of a chain of coupled oscillators is verified.

Résumé. Sous des conditions de Hörmander fortes, nous prouvons la propriété forte de Feller pour le semi-groupe déterminé par une SDE dirigée par des mouvements browniens subordonnés additifs, où la dérive est autorisée à être arbitrairement croissante. Pour cela, nous étendons un critère dû à Malicet et Poly (*J. Funct. Anal.* **264** (2013) 2077–2096) et à Bally et Caramellino (*Electron. J. Probab.* **19** (2014) 1–33) sur la convergence, en variation totale, des lois de fonctionnelles de Wiener. Ce résultat couvre le cas d'une chaîne d'oscillateurs couplés.

Keywords: Strong Feller property; SDE; Malliavin's calculus; Cylindrical α-stable process; Hörmander's condition

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Functional inequalities for convolution probability measures

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Abstract. Let μ and ν be two probability measures on $\mathbb{R}^d$, where $\mu(\mathrm{d}x) = \frac{\mathrm{e}^{-V(x)}\,\mathrm{d}x}{\int_{\mathbb{R}^d}\mathrm{e}^{-V(x)}\,\mathrm{d}x}$ for some $V \in C^1(\mathbb{R}^d)$. Explicit sufficient conditions on V and ν are presented such that $\mu * \nu$ satisfies the log-Sobolev, Poincaré and super Poincaré inequalities. In particular, if $V(x) = \lambda|x|^2$ for some $\lambda > 0$ and $\nu(\mathrm{e}^{\lambda\theta|\cdot|^2}) < \infty$ for some $\theta > 1$, then $\mu * \nu$ satisfies the log-Sobolev inequality. This improves and extends the recent results on the log-Sobolev inequality derived in (*J. Funct. Anal.* **265** (2013) 1064–1083) for convolutions of the Gaussian measure and compactly supported probability measures. On the other hand, it is well known that the log-Sobolev inequality for $\mu * \nu$ implies $\nu(\mathrm{e}^{\varepsilon|\cdot|^2}) < \infty$ for some $\varepsilon > 0$.

Résumé. Soit μ et ν deux mesures de probabilité sur $\mathbb{R}^d$, où $\mu(\mathrm{d}x) = \frac{\mathrm{e}^{-V(x)}\,\mathrm{d}x}{\int_{\mathbb{R}^d}\mathrm{e}^{-V(x)}\,\mathrm{d}x}$ avec $V \in C^1(\mathbb{R}^d)$. Des conditions explicites suffisantes sur V et ν sont présentées telles que $\mu * \nu$ satisfait des inégalités de Sobolev logarithmique, de Poincaré et de super-Poincaré. En particulier, si $V(x) = \lambda|x|^2$ pour quelque $\lambda > 0$ et $\nu(\mathrm{e}^{\lambda\theta|\cdot|^2}) < \infty$ avec $\theta > 1$, alors $\mu * \nu$ satisfait l'inégalité de Sobolev logarithmique. Cela améliore et étend des résultats récents sur l'inégalité de Sobolev logarithmique obtenus dans (*J. Funct. Anal.* **265** (2013) 1064–1083) pour des convolutions de la mesure de Gauss et des mesures de probabilité à support compact. D'autre part, il est bien connu que l'inégalité de Sobolev logarithmique pour $\mu * \nu$ implique $\nu(\mathrm{e}^{\varepsilon|\cdot|^2}) < \infty$ pour quelque $\varepsilon > 0$.

MSC: 60J75; 47G20; 60G52

Keywords: Log-Sobolev inequality; Poincaré inequality; Super Poincaré inequality; Convolution

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Pathwise solvability of stochastic integral equations with generalized drift and non-smooth dispersion functions

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Abstract. We study one-dimensional stochastic integral equations with non-smooth dispersion coëfficients, and with drift components that are not restricted to be absolutely continuous with respect to Lebesgue measure. In the spirit of Lamperti, Doss and Sussmann, we relate solutions of such equations to solutions of certain ordinary integral equations, indexed by a generic element of the underlying probability space. This relation allows us to solve the stochastic integral equations in a pathwise sense.

Résumé. Nous étudions des équations intégrales stochastiques unidimensionnelles avec coefficient de diffusion non-régulier, et avec termes de dérive non nécessairement absolument continus par rapport à la mesure de Lebesgue. En s'inspirant de Lamperti, Doss et Sussmann, la résolution de ces équations se ramène à la résolution de certaines équations intégrales ordinaires, paramétrées par un élément ω variant dans l'espace de probabilité de base. Ce lien nous permet de résoudre les équations intégrales stochastiques d'une façon trajectorielle.

MSC: 34A99; 45J05; 60G48; 60H10

Keywords: Stochastic integral equation; Ordinary integral equation; Pathwise solvability; Existence; Uniqueness; Generalized drift; Wong–Zakai approximation; Support theorem; Comparison theorem; Stratonovich integral

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Adaptive pointwise estimation of conditional density function

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Abstract. In this paper we consider the problem of estimating f, the conditional density of Y given X, by using an independent sample distributed as (X, Y) in the multivariate setting. We consider the estimation of $f(x, \cdot)$ where x is a fixed point. We define two different procedures of estimation, the first one using kernel rules, the second one inspired from projection methods. Both adaptive estimators are tuned by using the Goldenshluger and Lepski methodology. After deriving lower bounds, we show that these procedures satisfy oracle inequalities and are optimal from the minimax point of view on anisotropic Hölder balls. Furthermore, our results allow us to measure precisely the influence of $\mathrm{f}_X(x)$ on rates of convergence, where f_X is the density of X. Finally, some simulations illustrate the good behavior of our tuned estimates in practice.

Résumé. Dans cet article, nous considérons le problème de l'estimation de f, la densité conditionnelle de Y sachant X, en utilisant un échantillon de même loi que (X, Y), dans le cadre multivarié. On considère l'estimation de $f(x, \cdot)$ où x est un point fixé. Nous définissons deux procédures d'estimation différentes, la première utilisant des estimateurs à noyau, alors que la seconde s'inspire des méthodes de projection. Les deux procédures adaptatives sont calibrées en utilisant la méthodologie proposée par Goldenshulger et Lepski. Une fois obtenu le calcul des bornes inférieures du risque, nous montrons que ces procédures satisfont des inégalités oracles et sont optimales du point de vue minimax sur les boules de Hölder anisotropes. De plus, nos résultats nous permettent de mesurer précisément l'influence de $\mathrm{f}_X(x)$ sur les vitesses convergence, où f_X est la densité de X. Finalement, des simulations numériques illustrent le bon comportement de nos procédures calibrées en pratique.

MSC: 62G05; 62G20

Keywords: Conditional density; Adaptive estimation; Kernel rules; Projection estimates; Oracle inequality; Minimax rates; Anisotropic Hölder spaces

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Oracle inequalities for the Lasso in the high-dimensional Aalen multiplicative intensity model

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Abstract. In a general counting process setting, we consider the problem of obtaining a prognostic on the survival time adjusted on covariates in high-dimension. Towards this end, we construct an estimator of the whole conditional intensity. We estimate it by the best Cox proportional hazards model given two dictionaries of functions. The first dictionary is used to construct an approximation of the logarithm of the baseline hazard function and the second to approximate the relative risk. We introduce a new data-driven weighted Lasso procedure to estimate the unknown parameters of the best Cox model approximating the intensity. We provide non-asymptotic oracle inequalities for our procedure in terms of an appropriate empirical Kullback divergence. Our results rely on an empirical Bernstein's inequality for martingales with jumps and properties of modified self-concordant functions.

Résumé. Dans le cadre général d'un processus de comptage, nous intéressons à la façon d'obtenir un pronostic sur la durée de survie en fonction des covariables en grande dimension. Pour ce faire, nous construisons un estimateur de l'intensité conditionnelle. Nous l'estimons par le meilleur modèle de Cox étant donné deux dictionnaires de fonctions. Le premier dictionnaire est utilisé pour construire le logarithme du risque de base et le second, pour approximer le risque relatif. Nous introduisons une nouvelle procédure Lasso pondéré avec une pondération basée sur les données pour estimer les paramètres inconnus du meilleur modèle de Cox approximant l'intensité. Nous établissons une inégalité oracle non-asymptotique en divergence de Kullback empirique, qui est la fonction de perte la plus appropriée à notre procédure. Nos résultats reposent sur une inégalité de Bernstein pour les martingales à sauts et sur des propriétés des fonctions self-concordantes.

MSC: 62N02; 62G05; 62G08; 60E15

Keywords: Survival analysis; Right-censored data; Intensity; Cox proportional hazards model; Semiparametric model; Non-parametric model; High-dimensional covariates; Lasso; Non-asymptotic oracle inequalities; Empirical Bernstein's inequality

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