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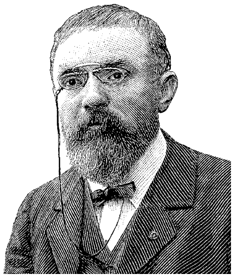

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ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

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Gaussian fluctuations for the classical XY model

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Abstract. We study the classical XY model in bounded domains of $\mathbb{Z}^d$ with Dirichlet boundary conditions. We prove that when the temperature goes to zero faster than a certain rate as the lattice spacing goes to zero, the fluctuation field converges to a Gaussian white noise. This and related results also apply to a large class of gradient field models.

Résumé. Nous étudions le modèle XY classique dans un domaine borné de $\mathbb{Z}^d$ avec condition de Dirichlet au bord. Nous prouvons que quand la température tend vers 0 suffisamment vite avec le pas du graphe, le champ des fluctuations converge vers le bruit blanc Gaussien. Ce résultat ainsi que les résultats associés s'appliquent aussi à une classe large de modèles de champs gradients.

MSC: Primary 60K35; 82B20; secondary 60F17; 60G60

Keywords: XY model; Spin-wave approximation; Gaussian free field; Gradient field models; Random walk representation; Central limit theorem

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Continuum percolation in high dimensions

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Abstract. Consider a Boolean model Σ in $\mathbb{R}^d$. The centers are given by a homogeneous Poisson point process with intensity λ and the radii of distinct balls are i.i.d. with common distribution ν. The critical covered volume is the proportion of space covered by Σ when the intensity λ is critical for percolation. We study the asymptotic behaviour, as d tends to infinity, of the critical covered volume. It appears that, in contrast to what happens in the constant radii case studied by Penrose, geometrical dependencies do not always vanish in high dimension.

Résumé. Considérons un modèle booléen Σ dans $\mathbb{R}^d$. Les centres des boules sont donnés par un processus ponctuel de Poisson homogène d'intensité λ, et les rayons par une suite de variables aléatoires indépendantes et identiquement distribuées de loi commune ν. Le volume critique recouvert est la proportion de l'espace recouverte par Σ quand on prend pour λ la valeur critique pour la percolation des boules. Nous étudions le comportement asymptotique, quand la dimension d tend vers $+\infty$, de ce volume critique recouvert. En particulier, nous montrons que contrairement à ce qui se passe dans le cas des boules de rayon constant étudié par Penrose, les dépendances liées à la géométrie ne disparaissent pas toujours en grande dimension.

MSC: 60K35; 82B43

Keywords: Percolation; Continuum percolation; Boolean model

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Convergence to equilibrium in the free Fokker–Planck equation with a double-well potential

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Abstract. We consider the one-dimensional free Fokker–Planck equation

$$\frac{\partial \mu_t}{\partial t} = \frac{\partial}{\partial x}\left[\mu_t \cdot \left(\frac{1}{2}V' - H\mu_t\right)\right],$$

where H denotes the Hilbert transform and V is a particular double-well quartic potential, namely $V(x) = \frac{1}{4}x^4 + \frac{c}{2}x^2$, with $c \geq -2$. We prove that the solution $(\mu_t)_{t\geq 0}$ of this PDE converges in Wasserstein distance of any order $p \geq 1$ to the equilibrium measure μ_V as t goes to infinity. This provides a first result of convergence for this equation in a non-convex setting. The proof involves free probability and complex analysis techniques.

Résumé. On considère l'équation de Fokker–Planck libre unidimensionnelle

$$\frac{\partial \mu_t}{\partial t} = \frac{\partial}{\partial x}\left[\mu_t \cdot \left(\frac{1}{2}V' - H\mu_t\right)\right],$$

où H désigne la transformée de Hilbert et V est un potentiel quartique à double puits particulier, à savoir $V(x) = \frac{1}{4}x^4 + \frac{c}{2}x^2$ avec $c \geq -2$. On démontre que la solution $(\mu_t)_{t\geq 0}$ de cette EDP converge pour une distance de Wasserstein d'ordre quelconque $p \geq 1$ vers la mesure d'équilibre μ_V quand t tend vers l'infini. Cela fournit un premier résultat de convergence pour cette équation dans un cadre non convexe. La démonstration fait intervenir les probabilités libres et l'analyse complexe.

MSC: 35B40; 46L54; 60B20

Keywords: Fokker–Planck equation; Granular media equation; Long-time behaviour; Double-well potential; Free probability; Equilibrium measure; Random matrices

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Percolation and isoperimetry on roughly transitive graphs

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Abstract. In this paper we study percolation on a roughly transitive graph G with polynomial growth and isoperimetric dimension larger than one. For these graphs we are able to prove that $p_c < 1$, or in other words, that there exists a percolation phase. The main results of the article work for both dependent and independent percolation processes, since they are based on a quite robust renormalization technique. When G is transitive, the fact that $p_c < 1$ was already known before. But even in that case our proof yields some new results and it is entirely probabilistic, not involving the use of Gromov's theorem on groups of polynomial growth. We finish the paper giving some examples of dependent percolation for which our results apply.

Résumé. Dans cet article, nous étudions la percolation sur un graphe grossièrement transitif G à croissance polynomiale et de dimension isopérimétrique plus grande que 1. Pour ces graphes, nous prouvons que $p_c < 1$ ou, en d'autres termes, nous prouvons qu'il existe une phase de percolation. Les résultats principaux de l'article sont valables à la fois pour les processus de percolation dépendants ou indépendants, car ils s'appuient sur des arguments de renormalisation assez robustes. Quand G est transitif, le fait que $p_c < 1$ était déjà connu. Mais même dans ce cas notre preuve donne des résultats nouveaux et est entièrement probabiliste, évitant l'utilisation du théorème de Gromov sur les groupes à croissance polynomiale. Nous concluons l'article par quelques exemples de percolation dépendante pour lesquels nos résultats s'appliquent.

MSC: 60K35; 82B43; 05C10

Keywords: Percolation; Isoperimetric inequalities; Roughly transitive graphs; Dependent percolation; Decoupling inequalities

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Multi-arm incipient infinite clusters in 2D: Scaling limits and winding numbers

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Abstract. We study the alternating k-arm incipient infinite cluster (IIC) of site percolation on the triangular lattice $\mathbb{T}$. Using Camia and Newman's result that the scaling limit of critical site percolation on $\mathbb{T}$ is CLE_6, we prove the existence of the scaling limit of the k-arm IIC for $k = 1, 2, 4$. Conditioned on the event that there are open and closed arms connecting the origin to $\partial\mathbb{D}_R$, we show that the winding number variance of the arms is $(3/2 + o(1)) \log R$ as $R \to \infty$, which confirms a prediction of Wieland and Wilson [*Phys. Rev. E* **68** (2003) 056101]. Our proof uses two-sided radial SLE_6 and coupling argument. Using this result we get an explicit form for the CLT of the winding numbers, and get analogous result for the 2-arm IIC, thus improving our earlier result.

Résumé. Nous étudions le cluster infini conditionné (IIC) à k-bras alternants pour la percolation par site sur le réseau triangulaire $\mathbb{T}$. En utilisant le résultat de Camia et Newman montrant que la limite d'échelle de la percolation par site sur $\mathbb{T}$ est le CLE_6, nous prouvons l'existence de la limite d'échelle de l'IIC à k bras pour $k = 1, 2, 4$. Conditionnellement à l'événement qu'il y ait un bras ouvert et un bras fermé connectant l'origine à $\partial\mathbb{D}_R$, nous montrons que la variance du nombre d'enroulements est $(3/2 + o(1)) \log R$ quand $R \to \infty$, ce qui confirme la prédiction de Wieland et Wilson [*Phys. Rev. E* **68** (2003) 056101]. Notre preuve utilise le SLE_6 radial à deux côtés ainsi que des arguments de couplage. En utilisant ce résultat, nous obtenons une forme explicite pour le CLT sur le nombre d'enroulements, et obtenons des résultats analogues pour le IIC à deux bras, améliorant ainsi notre résultat précédent.

MSC: 60K35; 82B43

Keywords: Percolation; Scaling limit; SLE; CLE; Incipient infinite cluster; Winding number

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Brownian motion and random walk above quenched random wall

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Abstract. We study the persistence exponent for the first passage time of a random walk below the trajectory of another random walk. More precisely, let $\{B_n\}$ and $\{W_n\}$ be two centered, weakly dependent random walks. We establish that $\mathbb{P}(\forall_{n\le N} B_n \ge W_n|W) = N^{-\gamma+o(1)}$ for a non-random $\gamma \ge 1/2$. In the classical setting, $W_n \equiv 0$, it is well-known that $\gamma = 1/2$. We prove that for any non-trivial W one has $\gamma > 1/2$ and the exponent γ depends only on $\operatorname{Var}(B_1)/\operatorname{Var}(W_1)$. Our result holds also in the continuous setting, when B and W are independent and possibly perturbed Brownian motions or Ornstein–Uhlenbeck processes. In the latter case the probability decays at exponential rate.

Résumé. On s'intéresse à l'exposant de persistance du temps de premier passage d'une marche aléatoire en-dessous de la trajectoire d'une autre marche aléatoire. Plus précisément, étant données deux marches aléatoires $\{B_n\}$ et $\{W_n\}$, centrées et faiblement corrélées, on établit que $\mathbb{P}(\forall_{n\le N} B_n \ge W_n|W) = N^{-\gamma+o(1)}$ pour un certain exposant $\gamma \ge 1/2$ déterministe. Il est bien connu que lorsque $W_n \equiv 0$, on a $\gamma = 1/2$. On prouve ici que lorsque W n'est pas la marche nulle, alors $\gamma > 1/2$, et dépend seulement du rapport $\operatorname{Var}(B_1)/\operatorname{Var}(W_1)$. Notre résultat est également valable en temps continu, lorsque B et W sont des mouvements browniens ou des processus d'Ornstein–Uhlenbeck indépendants. Dans ce dernier cas cependant, la queue du temps de premier passage décroit à taux exponentiel.

MSC: Primary 60J65; secondary 60G10; 60G15; 60G50; 60K37

Keywords: Brownian motion; Persistence exponent; Quenched environment; First passage time; Ornstein–Uhlenbeck process

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Mesoscopic central limit theorem for general β-ensembles

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Abstract. We prove that the linear statistics of eigenvalues of β-log gases satisfying the one-cut and off-critical assumption with a potential $V \in C^7(\mathbb{R})$ satisfy a central limit theorem at all mesoscopic scales $\alpha \in (0; 1)$. We prove this for compactly supported test functions $f \in C^6(\mathbb{R})$ using loop equations at all orders along with rigidity estimates.

Résumé. Nous prouvons que les statistiques linéaires du β-gaz de Coulomb confiné par un potentiel $V \in C^7(\mathbb{R})$ et avec une mesure d'équilibre non critique à support connexe satisfont un théorème central limite à toutes les échelles mésoscopiques $\alpha \in (0; 1)$. Nous prouvons ce résultat pour toute fonction test $f \in C^6(\mathbb{R})$ à support compact en utilisant les équations de boucles et des estimées de rigidité.

MSC: 60B20; 60F05

Keywords: Random matrices; Central Limit Theorems; Mesoscopic Statistics; Log-Gasses

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Scaling limits of stochastic processes associated with resistance forms

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Abstract. We establish that if a sequence of spaces equipped with resistance metrics and measures converge with respect to the Gromov–Hausdorff-vague topology, and a certain non-explosion condition is satisfied, then the associated stochastic processes also converge. This result generalises previous work on trees, fractals, and various models of random graphs. We further conjecture that it will be applicable to the random walk on the incipient infinite cluster of critical bond percolation on the high-dimensional integer lattice.

Résumé. Nous établissons que si une suite d'espaces équipée des métriques de résistance et de mesures converge par rapport à la topologie de Gromov–Hausdorff-vague, et qu'une certaine condition de non explosion est satisfaite, alors les processus stochastiques associés convergent également. Ces résultats généralisent des travaux précédents sur les arbres, fractals et divers modèles de graphes aléatoires. De plus nous conjecturons que cela devrait s'appliquer à la marche aléatoire sur l'amas de percolation par arêtes au point critique conditionné à être infini sur les réseaux entiers de grande dimension.

MSC: 60J25; 28A80; 60J35; 60J45

Keywords: Fractal; Gromov–Hausdorff-vague topology; Random graph; Resistance form; Resolvent kernel; Tree

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Global well-posedness of complex Ginzburg–Landau equation with a space–time white noise[1]

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Abstract. We show the global-in-time well-posedness of the complex Ginzburg–Landau (CGL) equation with a space–time white noise on the 3-dimensional torus. Our method is based on Mourrat and Weber (Global well-posedness of the dynamic Φ^4_3 model on the torus), where Mourrat and Weber showed the global well-posedness for the dynamical Φ^4_3 model. We prove a priori L^{2p} estimate for the paracontrolled solution as in the deterministic case [*Phys. D* **71** (1994) 285–318].

Résumé. Nous montrons que l'équation de Ginzburg–Landau complexe (CGL) sur le tore de dimension 3 avec un bruit blanc en espace-temps est bien posée et admet une solution globale en temps. Notre méthode prend son origine dans Mourrat et Weber (Global well-posedness of the dynamic Φ^4_3 model on the torus), où Mourrat et Weber montrent ce caractère bien posé global pour le modèle Φ^4_3 dynamique. Nous établissons une estimée L^{2p} a priori pour la solution paracontrôlée, comme dans le cas déterministe [*Phys. D* **71** (1994) 285–318].

MSC: 60H15; 82C28

Keywords: Complex Ginzburg–Landau equation; Paracontrolled calculus

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Local large deviations principle for occupation measures of the stochastic damped nonlinear wave equation

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Abstract. We consider the damped nonlinear wave (NLW) equation driven by a noise which is white in time and colored in space. Assuming that the noise is non-degenerate in all Fourier modes, we establish a large deviations principle (LDP) for the occupation measures of the trajectories. The lower bound in the LDP is of a local type, which is related to the weakly dissipative nature of the equation and is a novelty in the context of randomly forced PDE's. The proof is based on an extension of methods developed in (*Comm. Pure Appl. Math.* **68** (12) (2015) 2108–2143) and (Large deviations and mixing for dissipative PDE's with unbounded random kicks (2014) Preprint) in the case of kick forced dissipative PDE's with parabolic regularization property such as, for example, the Navier–Stokes system and the complex Ginzburg–Landau equations. We also show that a high concentration towards the stationary measure is impossible, by proving that the rate function that governs the LDP cannot have the trivial form (i.e., vanish on the stationary measure and be infinite elsewhere).

Résumé. Nous considérons l'équation des ondes non linéaire avec un bruit qui est blanc en temps et coloré en espace. Sous l'hypothèse que le bruit est non dégénéré, nous établissons un principe de grandes déviations (PGD) pour la famille de mesures d'occupation des trajectoires. La borne inférieure dans le PGD est d'un type local, qui est lié à la nature faiblement dissipative de l'équation. La preuve est basée sur une généralisation des méthodes développées dans (*Comm. Pure Appl. Math.* **68** (12) (2015) 2108–2143) et (Large deviations and mixing for dissipative PDE's with unbounded random kicks (2014) Preprint) pour des EDP paraboliques, comme les équations de Navier–Stokes ou de Ginzburg–Landau complexe, perturbées par une force aléatoire discrète en temps. Nous montrons également que la fonction de taux du PGD n'est pas triviale, ce qui implique qu'une forte concentration vers la mesure stationnaire est impossible.

MSC: 35L70; 35R60; 60B12; 60F10

Keywords: Nonlinear wave equation; White in time noise; Large deviations principle; Coupling method

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Multifractality of jump diffusion processes[1]

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Abstract. We study the local regularity and multifractal nature of the sample paths of jump diffusion processes, which are solutions to a class of stochastic differential equations with jumps. This article extends the recent work of Barral *et al.* who constructed a pure jump monotone Markov process with random multifractal spectrum. The class of processes studied here is much larger and exhibits novel features on the extreme values of the spectrum. This class includes Bass' stable-like processes and non-degenerate stable-driven SDEs.

Résumé. Nous étudions la régularité locale et la nature multifractale des trajectoires de diffusion à sauts, qui sont solutions d'une classe d'équations stochastiques à sauts. Cet article prolonge et étend substantiellement le travail récent de Barral *et al.* qui ont construit un processus de Markov de sauts purs avec un spectre multifractal aléatoire. La classe considérée est beaucoup plus large et présente de nouveaux phénomènes multifractals notamment sur les valeurs extrêmes du spectre. Cette classe comprend les processus de type stable au sens de Bass et des EDS non dégénérées guidées par un processus stable.

MSC: 60H10; 60J25; 60J75; 28A80; 28A78

Keywords: Jump diffusions; Markov processes; Stochastic differential equations; Hausdorff dimensions; Multifractals

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A characterization of a class of convex log-Sobolev inequalities on the real line

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Abstract. We give a sufficient and necessary condition for a probability measure μ on the real line to satisfy the logarithmic Sobolev inequality for convex functions. The condition is expressed in terms of the unique left-continuous and non-decreasing map transporting the symmetric exponential measure onto μ. The main tool in the proof is the theory of weak transport costs.

As a consequence, we obtain dimension-free concentration bounds for the lower and upper tails of convex functions of independent random variables which satisfy the convex log-Sobolev inequality.

Résumé. Nous proposons une condition nécessaire et suffisante pour qu'une mesure de probabilité μ sur la droite réelle satisfasse une condition de Sobolev logarithmique sur les fonctions convexes. Cette condition est exprimée en termes de l'unique plan de transport optimal croissant et continu à gauche entre la mesure exponentielle symétrique et la mesure μ. L'outil principal vient de la théorie du transport faible.

Comme conséquence, nous obtenons un résultat de concentration adimensionnelle sur les estimées de queue de fonctions convexes de variables aléatoires indépendantes, lié à l'inégalité de Sobolev logarithmique convexe.

MSC: Primary 60E15; secondary 26A51; 26D10

Keywords: Concentration of measure; Convex functions; Log-Sobolev inequality; Weak transport-entropy inequalities

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Isoperimetry in supercritical bond percolation in dimensions three and higher

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Abstract. We study the isoperimetric subgraphs of the infinite cluster $\mathbf{C}_\infty$ for supercritical bond percolation on $\mathbb{Z}^d$ with $d \geq 3$. Specifically, we consider subgraphs of $\mathbf{C}_\infty \cap [-n,n]^d$ having minimal open edge boundary to volume ratio. We prove a shape theorem for these subgraphs: when suitably rescaled, they converge almost surely to a translate of a deterministic shape. This deterministic shape is itself an isoperimetric set for a norm we construct. As a corollary, we obtain sharp asymptotics on a natural modification of the Cheeger constant for $\mathbf{C}_\infty \cap [-n,n]^d$, settling a conjecture of Benjamini for the version of the Cheeger constant defined here.

Résumé. Nous étudions les sous-graphes isopérimétriques du cluster infini $\mathbf{C}_\infty$ pour la percolation par arêtes surcritique sur $\mathbb{Z}^d$ avec $d \geq 3$. Plus précisément, nous considérons les sous-graphes de $\mathbf{C}_\infty \cap [-n,n]^d$ qui ont une frontière ouverte minimale par rapport au volume. Nous prouvons un théorème de forme pour ces sous-graphes: convenablement normalisés, ils convergent presque surement vers une translation d'une forme limite déterministe. Cette forme déterministe est elle aussi un ensemble isopérimétrique pour une norme que nous définissons. Comme corollaire, nous obtenons une estimée précise sur une modification naturelle de la constante de Cheeger pour $\mathbf{C}_\infty \cap [-n,n]^d$, résolvant ainsi une conjecture de Benjamini pour cette version de la constante de Cheeger.

MSC: 60K35; 82B43; 52B60

Keywords: Percolation; Limit shapes; Isoperimetry; Cheeger constant

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Classical and quantum part of the environment for quantum Langevin equations[1]

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Abstract. Among quantum Langevin equations describing the unitary time evolution of a quantum system in contact with a quantum bath, we completely characterize those equations which are actually driven by classical noises. The characterization is purely algebraic, in terms of the coefficients of the equation. In a second part, we consider general quantum Langevin equations and we prove that they can always be split into a maximal part driven by classical noises and a purely quantum one.

Résumé. Parmi les équations de Langevin quantiques qui modélisent l'évolution temporelle d'un système quantique en contact avec un bain thermique, nous caractérisons celles où les bruits sont en réalité classiques. Cette caractérisation est purement algébrique et s'exprime en termes des coefficients de l'équation. Dans un second temps, nous nous intéressons à des équations de Langevin générales et prouvons qu'elles peuvent toujours se décomposer en une partie maximale dirigée par des bruits classiques et une partie purement quantique.

MSC: 81S25; 60H05

Keywords: Quantum stochastic calculus; Commutative quantum processes; Classical and quantum noises

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How can a clairvoyant particle escape the exclusion process?

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Abstract. We study a detection problem in the following setting: On the one-dimensional integer lattice, at time zero, place detectors on each site independently with probability $\rho \in [0, 1)$ and let they evolve as a simple symmetric exclusion process. At time zero, place a target at the origin. The target moves only at integer times, and can move to any site that is within distance R from its current position. Assume also that the target can predict the future movement of all detectors. We prove that, for R large enough (depending on the value of ρ) it is possible for the target to avoid detection forever with positive probability. The proof of this result uses two ingredients of independent interest. First we establish a renormalisation scheme that can be used to prove percolation for dependent oriented models under a certain decoupling condition. This result is general and does not rely on the specifities of the model. As an application, we prove our main theorem for different dynamics, such as independent random walks and independent renewal chains. We also prove existence of oriented percolation for random interlacements and for its vacant set for large dimensions. The second step of the proof is a space–time decoupling for the exclusion process.

Résumé. Nous étudions un problème de détection dans le cadre suivant : sur le réseau entier unidimensionnel, au temps 0, on place des détecteurs sur chaque site indépendamment avec probabilité $\rho \in [0, 1)$ et on les laisse évoluer suivant un processus d'exclusion simple symétrique. Au temps 0, on place une cible à l'origine. Cette cible bouge seulement aux temps entiers, et peut se déplacer en tous les sites jusqu'à une distance R de sa position courante. Supposons aussi que la cible connait le déplacement futur de tous les détecteurs. Nous prouvons que, pour R assez grand (en fonction de la valeur ρ), il est possible pour la cible d'éviter la détection en tout temps avec probabilité positive. La preuve de ce résultat utilise deux ingrédients d'intérêt indépendant. Premièrement, nous établissons un argument de renormalisation qui peut être utilisé pour prouver la percolation pour des modèles dépendants orientés sous une certaine condition de découplage. Le résultat est général et ne s'appuie pas sur les propriétés spécifiques du modèle. Comme application, nous démontrons notre résultat pour différentes dynamiques, comme les marches aléatoires indépendantes et les chaînes de renouvellement indépendantes. Nous montrons aussi l'existence de la percolation orientée pour les entrelacs aléatoires et pour l'ensemble vacant en grandes dimensions. La deuxième étape de la preuve est basée sur un découplage espace-temps pour le processus d'exclusion.

MSC: Primary 60K37; secondary 60K35; 82B43; 82C22

Keywords: Target detection; Oriented percolation; Exclusion process decoupling

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The geometry of a critical percolation cluster on the UIPT

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Abstract. We consider a critical Bernoulli site percolation on the uniform infinite planar triangulation. We study the tail distributions of the peeling time, perimeter, and volume of the hull of a critical cluster. The exponents obtained here differ by a factor 2 from those computed previously by Angel and Curien [*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 405–431] in the case of critical site percolation on the uniform infinite *half-plane* triangulation.

Résumé. Nous examinons le modèle de percolation de Bernoulli par sites critique sur la triangulation infinie uniforme du plan. Nous étudions les queues de distribution du temps d'exploration, du périmètre et du volume de l'enveloppe d'une composante connexe. Les exposants obtenus diffèrent d'un facteur 2 de ceux calculés auparavant par Angel et Curien [*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 405–431] dans le cas de la percolation critique par site sur la triangulation uniforme du *demi-plan*.

MSC: 05C80; 60K35

Keywords: Random planar triangulation; Percolation; Critical exponents

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On the large deviations of traces of random matrices

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Abstract. We present large deviations principles for the moments of the empirical spectral measure of Wigner matrices and empirical measure of β-ensembles in three cases: the case of β-ensembles associated with a convex potential with polynomial growth, the case of Gaussian Wigner matrices, and the case of Wigner matrices without Gaussian tails, that is Wigner matrices whose entries have tail distributions decreasing as e^{-ct^α}, for some constant $c > 0$ and with $\alpha \in (0, 2)$.

Résumé. Nous proposons des principes de grandes déviations pour les moments de la mesure spectrale empirique de matrices de Wigner et de la mesure empirique de β-ensembles dans trois cas : celui des β-ensembles associés à un potentiel convexe à croissance polynomiale, le cas des matrices de Wigner Gaussiennes, et le cas des matrices de Wigner sans queues Gaussiennes, c'est-à-dire dont les entrées ont une queue de distribution ayant le même comportement que e^{-ct^α}, pour une certaine constante $c > 0$ et $\alpha \in (0, 2)$.

MSC: Primary 60B20; secondary 60F10

Keywords: Large deviations; Wigner matrices; β-ensembles

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Transporting random measures on the line and embedding excursions into Brownian motion

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Abstract. We consider two jointly stationary and ergodic random measures ξ and η on the real line $\mathbb{R}$ with equal intensities. An allocation is an equivariant random mapping from $\mathbb{R}$ to $\mathbb{R}$. We give sufficient and partially necessary conditions for the existence of allocations transporting ξ to η. An important ingredient of our approach is a transport kernel balancing ξ and η, provided these random measures are mutually singular. In the second part of the paper, we apply this result to the path decomposition of a two-sided Brownian motion into three independent pieces: a time reversed Brownian motion on $(-\infty, 0]$, an excursion distributed according to a conditional Itô measure and a Brownian motion starting after this excursion. An analogous result holds for Bismut's excursion measure.

Résumé. On considère deux mesures aléatoires conjointement stationnaires et ergodiques ξ et η sur la droite réelle $\mathbb{R}$ et d'intensités égales. Une allocation est une carte aléatoire équivariante de $\mathbb{R}$ dans $\mathbb{R}$. On donne des conditions suffisantes et partiellement nécessaires pour l'existence d'allocations transportant ξ sur η. Un ingrédient important de notre approche est un noyau de transport équilibrant ξ et η, sous la condition que ces mesures aléatoires sont mutuellement singulières. Dans la deuxième partie de cet article, on applique ce résultat à la décomposition des trajectoires d'un mouvement brownien symétrique en trois parties indépendantes: un mouvement brownien renversé dans le temps sur $(-\infty, 0]$, une excursion distribuée selon une mesure conditionnelle d'Itó, et un mouvement brownien après cette excursion. Un résultat analogue est valable pour la measure d'excursion de Bismut.

MSC: Primary 60G57; 60G55; secondary 60G60

Keywords: Stationary random measure; Point process; Allocation; Invariant transport; Palm measure; Shift-coupling; Brownian motion; Excursion theory

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A temporal central limit theorem for real-valued cocycles over rotations

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Abstract. We consider deterministic random walks on the real line driven by irrational rotations, or equivalently, skew product extensions of a rotation by α where the skewing cocycle is a piecewise constant mean zero function with a jump by one at a point β. When α is badly approximable and β is badly approximable with respect to α, we prove a *Temporal Central Limit theorem* (in the terminology recently introduced by D. Dolgopyat and O. Sarig), namely we show that for any fixed initial point, the *occupancy random variables*, suitably rescaled, converge to a Gaussian random variable. This result generalizes and extends a theorem by J. Beck for the special case when α is quadratic irrational, β is rational and the initial point is the origin, recently reproved and then generalized to cover any initial point using geometric renormalization arguments by Avila–Dolgopyat–Duryev–Sarig (*Israel J. Math.* **207** (2015) 653–717) and Dolgopyat–Sarig (*J. Stat. Phys.* **166** (2017) 680–713). We also use renormalization, but in order to treat irrational values of β, instead of geometric arguments, we use the renormalization associated to the continued fraction algorithm and dynamical Ostrowski expansions. This yields a suitable symbolic coding framework which allows us to reduce the main result to a CLT for non homogeneous Markov chains.

Résumé. On considère des marches aléatoires sur la droite réelle, engendrés par des rotations irrationnelles, ou, de manière équivalente, des produits croisés d'une rotation par un nombre réel α, dont le cocycle est une fonction constante par morceaux de moyenne nulle admettant un saut de un à une singularité β. Si α est mal approché par des rationnels et β n'est pas bien approché par l'orbite de α, nous démontrons une version temporelle du Théorème de la Limite Centrale (ou un *Temporal Central Limit theorem* dans la terminologie qui a été introduite récemment par D. Dolgopyat et O. Sarig). Plus précisément, nous montrons que, pour chaque point initial fixé, les *variables aléatoires d'occupation*, proprement renormalisées, tendent vers une variable aléatoire de loi normale. Ce résultat généralise un théorème de J. Beck dans le cas sparticulier où α est un nombre irrationnel quadratique, β est un nombre rationnel et le point initial est l'origine. Ce résultat de Beck a été montré avec de nouvelles méthodes et étendu par Avila–Dolgopyat–Duryev–Sarig (*Israel J. Math.* **207** (2015) 653–717) et Dolgopyat–Sarig (*J. Stat. Phys.* **166** (2017) 680–713) à l'aide d'une renormalisation géométrique. Dans ce papier, nous utilisons aussi la renormalisation, mais, au lieu d'avoir recours à un argument géométrique, nous proposons d'utiliser l'algorithme de fraction continue avec une version dynamique de l'expansion de Ostrowski. Cela nous donne un codage symbolique qui nous permet de réduire le résultat principal à un théorème de la limite centrale pour de chaînes de Markov non-homogènes.

MSC: Primary 37A50; 37E10; secondary 11K06

Keywords: Limit theorems for dynamical systems; Single orbit dynamics; Skew-products over irrational rotations; Discrepancy; Renormalization; Ostrowski expansion

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Kinetically constrained lattice gases: Tagged particle diffusion[1]

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Abstract. Kinetically constrained lattice gases (KCLG) are interacting particle systems on the integer lattice $\mathbb{Z}^d$ with hard core exclusion and Kawasaki type dynamics. Their peculiarity is that jumps are allowed only if the configuration satisfies a constraint which asks for enough empty sites in a certain local neighborhood. KCLG have been introduced and extensively studied in physics literature as models of glassy dynamics. We focus on the most studied class of KCLG, the Kob Andersen (KA) models. We analyze the behavior of a tracer (i.e. a tagged particle) at equilibrium. We prove that for all dimensions $d \geq 2$ and for any equilibrium particle density, under diffusive rescaling the motion of the tracer converges to a d-dimensional Brownian motion with non-degenerate diffusion matrix. Therefore we disprove the occurrence of a diffusive/non diffusive transition which had been conjectured in physics literature. Our technique is flexible enough and can be extended to analyse the tracer behavior for other choices of constraints.

Résumé. Les gaz réticulaires avec contrainte cinétique (KCLG) sont des systèmes de particules en interaction sur le réseau $\mathbb{Z}^d$ avec au plus une particule par site et une dynamique de type Kawasaki. Leur particularité est que les sauts ne sont autorisés que si la configuration satisfait une contrainte exigeant suffisamment de sites vides dans un certain voisinage local. Les KCLG ont été introduits et massivement étudiés dans la littérature physique comme modèles pour les systèmes vitreux. Nous nous concentrons sur l'une des classes de KCLG les plus étudiées : les modèles de Kob–Andersen (KA). Nous analysons le comportement d'un traceur (c.-à-d. une particule marquée) à l'équilibre. Pour toute dimension $d \geq 2$ et toute densité de particules, nous montrons qu'à l'échelle diffusive la trajectoire du traceur converge vers un mouvement brownien d-dimensionnel avec matrice de diffusion non dégénérée. Par conséquent nous démontrons la non-existence d'une transition (qui avait été conjecturée dans la littérature physique) entre un régime diffusif et un régime non diffusif. Notre technique est assez flexible et peut être étendue pour analyser le comportement du traceur sous d'autres choix de contraintes.

MSC: 60K35; 60J27

Keywords: Kawasaki dynamics; Tagged particle; Kinetically constrained models

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Location of the path supremum for self-similar processes with stationary increments

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Abstract. In this paper we consider the distribution of the location of the path supremum in a fixed interval for self-similar processes with stationary increments. A point process is constructed and its relation to the distribution of the location of the path supremum is studied. Using this framework, we show that the distribution has a spectral-type representation, in the sense that it is always a mixture of a special group of absolutely continuous distributions, plus point masses on the two boundaries. An upper bound for the value of the density function is established. We further discuss self-similar Lévy processes as an example. Most of the results in this paper can be generalized to a group of random locations, including the location of the largest jump, etc.

Résumé. Dans cet article, nous considérons la distribution de la position du supremum de la trajectoire d'un processus auto-similaire à accroissements stationnaires dans un intervalle fixé. Un processus ponctuel est construit et sa relation avec la distribution de la position du supremum est étudiée. Dans ce cadre, nous montrons que cette distribution a une représentation de type spectral, dans le sens où il s'agit toujours d'un mélange d'un groupe particulier de distributions absolument continues et de masses ponctuelles aux bords de l'intervalle. Une borne supérieure pour la valeur de la fonction de densité est obtenue. De plus, à titre d'exemple, nous discutons des processus de Lévy auto-similaires. La plupart des résultats de cet article peuvent être généralisés à un groupe de positions aléatoires, y compris la position du plus grand saut, etc.

MSC: Primary 60G18; secondary 60G55; 60G10

Keywords: Self-similar processes; Stationary increment processes; Random locations

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