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THE SCALING LIMIT OF THE INTERFACE OF THE CONTINUOUS-SPACE SYMBIOTIC BRANCHING MODEL[1]

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The continuous-space symbiotic branching model describes the evolution of two interacting populations that can reproduce locally only in the simultaneous presence of each other. If started with complementary Heaviside initial conditions, the interface where both populations coexist remains compact. Together with a diffusive scaling property, this suggests the presence of an interesting scaling limit. Indeed, in the present paper, we show weak convergence of the diffusively rescaled populations as measure-valued processes in the Skorokhod, respectively the Meyer–Zheng, topology (for suitable parameter ranges). The limit can be characterized as the unique solution to a martingale problem and satisfies a "separation of types" property. This provides an important step toward an understanding of the scaling limit for the interface. As a corollary, we obtain an estimate on the moments of the width of an approximate interface.

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Key words and phrases. Symbiotic branching model, mutually catalytic branching, stepping stone model, rescaled interface, moment duality, Meyer–Zheng topology.

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A NONCOMMUTATIVE MARTINGALE CONVEXITY INEQUALITY[1]

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Let $\mathcal{M}$ be a von Neumann algebra equipped with a faithful semifinite normal weight ϕ and $\mathcal{N}$ be a von Neumann subalgebra of $\mathcal{M}$ such that the restriction of ϕ to $\mathcal{N}$ is semifinite and such that $\mathcal{N}$ is invariant by the modular group of ϕ. Let $\mathcal{E}$ be the weight preserving conditional expectation from $\mathcal{M}$ onto $\mathcal{N}$. We prove the following inequality:

$$\|x\|_p^2 \geq \|\mathcal{E}(x)\|_p^2 + (p-1)\|x - \mathcal{E}(x)\|_p^2, \qquad x \in L_p(\mathcal{M}), 1 < p \leq 2,$$

which extends the celebrated Ball–Carlen–Lieb convexity inequality. As an application we show that there exists $\varepsilon_0 > 0$ such that for any free group $\mathbb{F}_n$ and any $q \geq 4 - \varepsilon_0$,

$$\|P_t\|_{2\to q} \leq 1 \quad \Leftrightarrow \quad t \geq \log\sqrt{q-1},$$

where (P_t) is the Poisson semigroup defined by the natural length function of $\mathbb{F}_n$.

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Key words and phrases. Noncommutative L_p-spaces, martingale convexity inequality, hypercontractivity, free groups.

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STUCK WALKS: A CONJECTURE OF ERSCHLER, TÓTH AND WERNER

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In this paper, we work on a class of self-interacting nearest neighbor random walks, introduced in [*Probab. Theory Related Fields* **154** (2012) 149–163], for which there is competition between repulsion of neighboring edges and attraction of next-to-neighboring edges. Erschler, Tóth and Werner proved in [*Probab. Theory Related Fields* **154** (2012) 149–163] that, for any $L \geq 1$, if the parameter α belongs to a certain interval (α_{L+1}, α_L), then such random walks localize on $L+2$ sites with positive probability. They also conjectured that this is the almost sure behavior. We prove this conjecture partially, stating that the walk localizes on $L+2$ or $L+3$ sites almost surely, under the same assumptions. We also prove that, if $\alpha \in (1, +\infty) = (\alpha_2, \alpha_1)$, then the walk localizes a.s. on 3 sites.

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Key words and phrases. Stuck walks, reinforced random walks, localization, Rubin, time-line construction, martingale.

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GENERALIZATION OF THE NUALART–PECCATI CRITERION

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The celebrated Nualart–Peccati criterion [*Ann. Probab.* **33** (2005) 177–193] ensures the convergence in distribution toward a standard Gaussian random variable N of a given sequence $\{X_n\}_{n\geq 1}$ of multiple Wiener–Itô integrals of fixed order, if $\mathbb{E}[X_n^2] \to 1$ and $\mathbb{E}[X_n^4] \to \mathbb{E}[N^4] = 3$. Since its appearance in 2005, the natural question of ascertaining which other moments can replace the fourth moment in the above criterion has remained entirely open. Based on the technique recently introduced in [*J. Funct. Anal.* **266** (2014) 2341–2359], we settle this problem and establish that the convergence of *any even moment*, greater than four, to the corresponding moment of the standard Gaussian distribution, guarantees the central convergence. As a byproduct, we provide many new moment inequalities for multiple Wiener–Itô integrals. For instance, if X is a normalized multiple Wiener–Itô integral of order greater than one,

$$\forall k \geq 2, \qquad \mathbb{E}\big[X^{2k}\big] > \mathbb{E}\big[N^{2k}\big] = (2k-1)!!.$$

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ON THE PROBABILITY THAT SELF-AVOIDING WALK ENDS AT A GIVEN POINT

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We prove two results on the delocalization of the endpoint of a uniform self-avoiding walk on $\mathbb{Z}^d$ for $d \geq 2$. We show that the probability that a walk of length n ends at a point x tends to 0 as n tends to infinity, uniformly in x. Also, when x is fixed, with $\|x\| = 1$, this probability decreases faster than $n^{-1/4+\varepsilon}$ for any $\varepsilon > 0$. This provides a bound on the probability that a self-avoiding walk is a polygon.

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EXTREMES OF A CLASS OF NONHOMOGENEOUS GAUSSIAN RANDOM FIELDS

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This contribution establishes exact tail asymptotics of $\sup_{(s,t)\in\mathbf{E}} X(s,t)$ for a large class of nonhomogeneous Gaussian random fields X on a bounded convex set $\mathbf{E} \subset \mathbb{R}^2$, with variance function that attains its maximum on a segment on $\mathbf{E}$. These findings extend the classical results for homogeneous Gaussian random fields and Gaussian random fields with unique maximum point of the variance. Applications of our result include the derivation of the exact tail asymptotics of the Shepp statistics for stationary Gaussian processes, Brownian bridge and fractional Brownian motion as well as the exact tail asymptotic expansion for the maximum loss and span of stationary Gaussian processes.

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EXTREME NESTING IN THE CONFORMAL LOOP ENSEMBLE

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The conformal loop ensemble CLE_κ with parameter $8/3 < \kappa < 8$ is the canonical conformally invariant measure on countably infinite collections of noncrossing loops in a simply connected domain. Given κ and ν, we compute the almost-sure Hausdorff dimension of the set of points z for which the number of CLE loops surrounding the disk of radius ε centered at z has asymptotic growth $\nu \log(1/\varepsilon)$ as $\varepsilon \to 0$. By extending these results to a setting in which the loops are given i.i.d. weights, we give a CLE-based treatment of the extremes of the Gaussian free field.

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NOISE-STABILITY AND CENTRAL LIMIT THEOREMS FOR EFFECTIVE RESISTANCE OF RANDOM ELECTRIC NETWORKS[1]

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We investigate the (generalized) Walsh decomposition of point-to-point effective resistances on countable random electric networks with i.i.d. resistances. We show that it is concentrated on low levels, and thus point-to-point effective resistances are uniformly stable to noise. For graphs that satisfy some homogeneity property, we show in addition that it is concentrated on sets of small diameter. As a consequence, we compute the right order of the variance and prove a central limit theorem for the effective resistance through the discrete torus of side length n in $\mathbb{Z}^d$, when n goes to infinity.

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Key words and phrases. Effective resistance, conductance, noise sensitivity and stability, Efron–Stein inequality, generalized Walsh decomposition, central limit theorem, stochastic homogenization.

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ZERO-SUM REPEATED GAMES: COUNTEREXAMPLES TO THE EXISTENCE OF THE ASYMPTOTIC VALUE AND THE CONJECTURE $\mathbf{maxmin} = \lim v_n$

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Mertens [In *Proceedings of the International Congress of Mathematicians* (*Berkeley*, *Calif.*, 1986) (1987) 1528–1577 Amer. Math. Soc.] proposed two general conjectures about repeated games: the first one is that, in any two-person zero-sum repeated game, the asymptotic value exists, and the second one is that, when Player 1 is more informed than Player 2, in the long run Player 1 is able to guarantee the asymptotic value. We disprove these two long-standing conjectures by providing an example of a zero-sum repeated game with public signals and perfect observation of the actions, where the value of the λ-discounted game does not converge when λ goes to 0. The aforementioned example involves seven states, two actions and two signals for each player. Remarkably, players observe the payoffs, and play in turn.

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NONINTERSECTING BROWNIAN MOTIONS ON THE UNIT CIRCLE

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We consider an ensemble of n nonintersecting Brownian particles on the unit circle with diffusion parameter $n^{-1/2}$, which are conditioned to begin at the same point and to return to that point after time T, but otherwise not to intersect. There is a critical value of T which separates the subcritical case, in which it is vanishingly unlikely that the particles wrap around the circle, and the supercritical case, in which particles may wrap around the circle. In this paper, we show that in the subcritical and critical cases the probability that the total winding number is zero is almost surely 1 as $n \to \infty$, and in the supercritical case that the distribution of the total winding number converges to the discrete normal distribution. We also give a streamlined approach to identifying the Pearcey and tacnode processes in scaling limits. The formula of the tacnode correlation kernel is new and involves a solution to a Lax system for the Painlevé II equation of size 2×2. The proofs are based on the determinantal structure of the ensemble, asymptotic results for the related system of discrete Gaussian orthogonal polynomials, and a formulation of the correlation kernel in terms of a double contour integral.

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Key words and phrases. Nonintersecting Brownian motions, determinantal process, discrete orthogonal polynomial, tacnode process, Pearcey process, Riemann–Hilbert problem, double contour integral formula.

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VISCOSITY SOLUTIONS OF FULLY NONLINEAR PARABOLIC PATH DEPENDENT PDES: PART I

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The main objective of this paper and the accompanying one [Viscosity solutions of fully nonlinear parabolic path dependent PDEs: Part II (2012) Preprint] is to provide a notion of viscosity solutions for fully nonlinear parabolic path-dependent PDEs. Our definition extends our previous work [*Ann. Probab.* (2014) **42** 204–236], focused on the semilinear case, and is crucially based on the nonlinear optimal stopping problem analyzed in [*Stochastic Process. Appl.* (2014) **124** 3277–3311]. We prove that our notion of viscosity solutions is consistent with the corresponding notion of classical solutions, and satisfies a stability property and a partial comparison result. The latter is a key step for the well-posedness results established in [Viscosity solutions of fully nonlinear parabolic path dependent PDEs: Part II (2012) Preprint]. We also show that the value processes of path-dependent stochastic control problems are viscosity solutions of the corresponding path-dependent dynamic programming equations.

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COMPENSATED FRAGMENTATION PROCESSES AND LIMITS OF DILATED FRAGMENTATIONS

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A new class of fragmentation-type random processes is introduced, in which, roughly speaking, the accumulation of small dislocations which would instantaneously shatter the mass into dust, is compensated by an adequate dilation of the components. An important feature of these compensated fragmentations is that the dislocation measure ν which governs their evolutions has only to fulfill the integral condition $\int_{\mathcal{P}}(1-p_1)^2\nu(\mathrm{d}\mathbf{p})<\infty$, where $\mathbf{p}=(p_1,\dots)$ denotes a generic mass-partition. This is weaker than the necessary and sufficient condition $\int_{\mathcal{P}}(1-p_1)\nu(\mathrm{d}\mathbf{p})<\infty$ for ν to be the dislocation measure of a homogeneous fragmentation. Our main results show that such compensated fragmentations naturally arise as limits of homogeneous dilated fragmentations, and bear close connections to spectrally negative Lévy processes.

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MSC2010 subject classifications. 60F17, 60G51, 60G80.

Key words and phrases. Homogeneous fragmentation, dilation, compensation, dislocation measure.

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A POISSON ALLOCATION OF OPTIMAL TAIL

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The allocation problem for a d-dimensional Poisson point process is to find a way to partition the space to parts of equal size, and to assign the parts to the configuration points in a measurable, "deterministic" (equivariant) way. The goal is to make the diameter R of the part assigned to a configuration point have fast decay. We present an algorithm for $d \geq 3$ that achieves an $O(\exp(-cR^d))$ tail, which is optimal up to c. This improves the best previously known allocation rule, the gravitational allocation, which has an $\exp(-R^{1+o(1)})$ tail. The construction is based on the Ajtai–Komlós–Tusnády algorithm and uses the Gale–Shapley–Hoffman–Holroyd–Peres stable marriage scheme (as applied to allocation problems).

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CENTRAL LIMIT THEOREM FOR LINEAR GROUPS

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We prove a central limit theorem for random walks with finite variance on linear groups.

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A STOCHASTIC TARGET APPROACH TO RICCI FLOW ON SURFACES

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We develop a stochastic target representation for Ricci flow and normalized Ricci flow on smooth, compact surfaces, analogous to Soner and Touzi's representation of mean curvature flow. We prove a verification/uniqueness theorem, and then consider geometric consequences of this stochastic representation.

Based on this stochastic approach, we give a proof that, for surfaces of nonpositive Euler characteristic, the normalized Ricci flow converges to a constant curvature metric exponentially quickly in every C^k-norm. In the case of C^0 and C^1-convergence, we achieve this by coupling two particles. To get C^2-convergence (in particular, convergence of the curvature), we use a coupling of three particles. This triple coupling is developed here only for the case of constant curvature metrics on surfaces, though we suspect that some variants of this idea are applicable in other situations and therefore be of independent interest. Finally, for $k \geq 3$, the C^k-convergence follows relatively easily using induction and coupling of two particles.

None of these techniques appear in the Ricci flow literature and thus provide an alternative approach to the field.

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MIXING TIME AND CUTOFF FOR THE ADJACENT TRANSPOSITION SHUFFLE AND THE SIMPLE EXCLUSION

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In this paper, we investigate the mixing time of the adjacent transposition shuffle for a deck of N cards. We prove that around time $N^2 \log N/(2\pi^2)$, the total variation distance to equilibrium of the deck distribution drops abruptly from 1 to 0, and that the separation distance has a similar behavior but with a transition occurring at time $(N^2 \log N)/\pi^2$. This solves a conjecture formulated by David Wilson. We present also similar results for the exclusion process on a segment of length N with k particles.

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INTERMITTENCY FOR THE WAVE AND HEAT EQUATIONS WITH FRACTIONAL NOISE IN TIME

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In this article, we consider the stochastic wave and heat equations driven by a Gaussian noise which is spatially homogeneous and behaves in time like a fractional Brownian motion with Hurst index $H > 1/2$. The solutions of these equations are interpreted in the Skorohod sense. Using Malliavin calculus techniques, we obtain an upper bound for the moments of order $p \geq 2$ of the solution. In the case of the wave equation, we derive a Feynman–Kac-type formula for the second moment of the solution, based on the points of a planar Poisson process. This is an extension of the formula given by Dalang, Mueller and Tribe [*Trans. Amer. Math. Soc.* **360** (2008) 4681–4703], in the case $H = 1/2$, and allows us to obtain a lower bound for the second moment of the solution. These results suggest that the moments of the solution grow much faster in the case of the fractional noise in time than in the case of the white noise in time.

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Key words and phrases. Stochastic heat and wave equations, spatially homogeneous noise, fractional Brownian motion, Malliavin calculus, intermittency.

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SPATIAL ASYMPTOTICS FOR THE PARABOLIC ANDERSON MODELS WITH GENERALIZED TIME–SPACE GAUSSIAN NOISE[1]

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Partially motivated by the recent papers of Conus, Joseph and Khoshnevisan [*Ann. Probab.* **41** (2013) 2225–2260] and Conus et al. [*Probab. Theory Related Fields* **156** (2013) 483–533], this work is concerned with the precise spatial asymptotic behavior for the parabolic Anderson equation

$$\begin{cases} \dfrac{\partial u}{\partial t}(t,x) = \dfrac{1}{2}\Delta u(t,x) + V(t,x)u(t,x), \\ u(0,x) = u_0(x), \end{cases}$$

where the homogeneous generalized Gaussian noise $V(t,x)$ is, among other forms, white or fractional white in time and space. Associated with the Cole–Hopf solution to the KPZ equation, in particular, the precise asymptotic form

$$\lim_{R\to\infty} (\log R)^{-2/3} \log \max_{|x|\le R} u(t,x) = \frac{3}{4}\sqrt[3]{\frac{2t}{3}} \qquad \text{a.s.}$$

is obtained for the parabolic Anderson model $\partial_t u = \frac{1}{2}\partial_{xx}^2 u + \dot{W}u$ with the $(1+1)$-white noise $\dot{W}(t,x)$. In addition, some links between time and space asymptotics for the parabolic Anderson equation are also pursued.

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ERRATUM TO "SHARP METASTABILITY THRESHOLD FOR AN ANISOTROPIC BOOTSTRAP PERCOLATION MODEL"

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We provide an Erratum, correcting how our main result generalises and correct some steps in the proof.

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