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Articles—*Continued from front cover*

Addendum

Corrigendum

Addendum and corrigendum

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RECONSTRUCTION OF A MULTIDIMENSIONAL SCENERY WITH A BRANCHING RANDOM WALK

BY HEINRICH MATZINGER, ANGELICA PACHON[1] AND SERGUEI POPOV

Georgia Institute of Technology, University of Turin and University of Campinas

We consider a d-dimensional scenery seen along a simple symmetric branching random walk, where at each time each particle gives the color record it observes. We show that up to equivalence the scenery can be reconstructed a.s. from the color record of all particles. To do so, we assume that the scenery has at least $2d + 1$ colors which are i.i.d. with uniform probability. This is an improvement in comparison to Popov and Pachon [*Stochastics* **83** (2011) 107–116], where at each time the particles needed to see a window around their current position, and in Löwe and Matzinger [*Ann. Appl. Probab.* **12** (2002) 1322–1347], where the reconstruction is done for $d = 2$ with a single particle instead of a branching random walk, but millions of colors are necessary.

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Key words and phrases. Random walk, branching random walk, reconstruction algorithm.

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OPTIMAL SKOROKHOD EMBEDDING GIVEN FULL MARGINALS AND AZÉMA–YOR PEACOCKS[1]

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We consider the optimal Skorokhod embedding problem (SEP) given full marginals over the time interval [0, 1]. The problem is related to the study of extremal martingales associated with a peacock ("process increasing in convex order," by Hirsch, Profeta, Roynette and Yor [*Peacocks and Associated Martingales, with Explicit Constructions* (2011), Springer, Milan]). A general duality result is obtained by convergence techniques. We then study the case where the reward function depends on the maximum of the embedding process, which is the limit of the martingale transport problem studied in Henry-Labordère, Obłój, Spoida and Touzi [*Ann. Appl. Probab.* **26** (2016) 1–44]. Under technical conditions, we then characterize the optimal value and the solution to the dual problem. In particular, the optimal embedding corresponds to the Madan and Yor [*Bernoulli* **8** (2002) 509–536] peacock under their "increasing mean residual value" condition. We also discuss the associated martingale inequality.

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Key words and phrases. Skorokhod embedding problem, peacocks, martingale inequality, martingale transport problem, maximum of martingale given marginals.

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CHI-SQUARE APPROXIMATION BY STEIN'S METHOD WITH APPLICATION TO PEARSON'S STATISTIC

BY ROBERT E. GAUNT[1], ALASTAIR M. PICKETT[2] AND GESINE REINERT[3]

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This paper concerns the development of Stein's method for chi-square approximation and its application to problems in statistics. New bounds for the derivatives of the solution of the gamma Stein equation are obtained. These bounds involve both the shape parameter and the order of the derivative. Subsequently, Stein's method for chi-square approximation is applied to bound the distributional distance between Pearson's statistic and its limiting chi-square distribution, measured using smooth test functions. In combination with the use of symmetry arguments, Stein's method yields explicit bounds on this distributional distance of order n^{-1}.

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MSC2010 subject classifications. 60F05, 62G10, 62G20.

Key words and phrases. Stein's method, chi-square approximation, Pearson's statistic, rate of convergence.

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ON THE CONNECTION BETWEEN SYMMETRIC N-PLAYER GAMES AND MEAN FIELD GAMES[1]

BY MARKUS FISCHER

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Mean field games are limit models for symmetric N-player games with interaction of mean field type as $N \to \infty$. The limit relation is often understood in the sense that a solution of a mean field game allows to construct approximate Nash equilibria for the corresponding N-player games. The opposite direction is of interest, too: When do sequences of Nash equilibria converge to solutions of an associated mean field game? In this direction, rigorous results are mostly available for stationary problems with ergodic costs. Here, we identify limit points of sequences of certain approximate Nash equilibria as solutions to mean field games for problems with Itô-type dynamics and costs over a finite time horizon. Limits are studied through weak convergence of associated normalized occupation measures and identified using a probabilistic notion of solution for mean field games.

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FINITE-LENGTH ANALYSIS ON TAIL PROBABILITY FOR MARKOV CHAIN AND APPLICATION TO SIMPLE HYPOTHESIS TESTING

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Using terminologies of information geometry, we derive upper and lower bounds of the tail probability of the sample mean for the Markov chain with finite state space. Employing these bounds, we obtain upper and lower bounds of the minimum error probability of the type-2 error under the exponential constraint for the error probability of the type-1 error in a simple hypothesis testing for a finite-length Markov chain, which yields the Hoeffding-type bound. For these derivations, we derive upper and lower bounds of cumulant generating function for Markov chain with finite state space. As a byproduct, we obtain another simple proof of central limit theorem for Markov chain with finite state space.

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A PIECEWISE DETERMINISTIC SCALING LIMIT OF LIFTED METROPOLIS–HASTINGS IN THE CURIE–WEISS MODEL[1]

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In Turitsyn, Chertkov and Vucelja [*Phys. D* **240** (2011) 410–414] a nonreversible Markov Chain Monte Carlo (MCMC) method on an augmented state space was introduced, here referred to as Lifted Metropolis–Hastings (LMH). A scaling limit of the magnetization process in the Curie–Weiss model is derived for LMH, as well as for Metropolis–Hastings (MH). The required jump rate in the high (supercritical) temperature regime equals $n^{1/2}$ for LMH, which should be compared to n for MH. At the critical temperature, the required jump rate equals $n^{3/4}$ for LMH and $n^{3/2}$ for MH, in agreement with experimental results of Turitsyn, Chertkov and Vucelja (2011). The scaling limit of LMH turns out to be a nonreversible piecewise deterministic exponentially ergodic "zig-zag" Markov process.

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EQUIVALENCE OF ENSEMBLES FOR LARGE VEHICLE-SHARING MODELS

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For a class of large closed Jackson networks submitted to capacity constraints, asymptotic independence of the nodes in normal traffic phase is proved at stationarity under mild assumptions, using a local limit theorem. The limiting distributions of the queues are explicit. In the Statistical Mechanics terminology, the equivalence of ensembles—canonical and grand canonical—is proved for specific marginals. The framework includes the case of networks with two types of nodes: single server/finite capacity nodes and infinite servers/infinite capacity nodes, that can be taken as basic models for bike-sharing systems. The effect of local saturation is modeled by generalized blocking and rerouting procedures, under which the stationary state is proved to have product-form. The grand canonical approximation can then be used for adjusting the total number of bikes and the capacities of the stations to the expected demand.

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THE ROUNDING OF THE PHASE TRANSITION FOR DISORDERED PINNING WITH STRETCHED EXPONENTIAL TAILS[1]

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The presence of frozen-in or quenched disorder in a system can often modify the nature of its phase transition. A particular instance of this phenomenon is the so-called rounding effect: it has been shown in many cases that the free energy curve of the disordered system at its critical point is smoother than that of the homogeneous one. In particular some disordered systems do not allow first-order transitions. We study this phenomenon for the pinning of a renewal with stretched-exponential tails on a defect line (the distribution K of the renewal increments satisfies $K(n) \sim c_K \exp(-n^{\zeta})$, $\zeta \in (0,1)$) which has a first order transition when disorder is not present. We show that the critical behavior of the disordered system depends on the value of ζ: when $\zeta > 1/2$ the transition remains of first order, whereas the free energy diagram is smoothed for $\zeta \leq 1/2$. Furthermore we show that the rounding effect is getting stronger when ζ diminishes.

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SCALING LIMIT OF THE CORRECTOR IN STOCHASTIC HOMOGENIZATION

BY JEAN-CHRISTOPHE MOURRAT AND JAMES NOLEN[1]

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In the homogenization of divergence-form equations with random coefficients, a central role is played by the corrector. We focus on a discrete space setting and on dimension 3 and more. Under a minor smoothness assumption on the law of the random coefficients, we identify the scaling limit of the corrector, which is akin to a Gaussian free field. This completes the argument started in [*Ann. Probab.* **44** (2016) 3207–3233].

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OPTIMAL CONSUMPTION UNDER HABIT FORMATION IN MARKETS WITH TRANSACTION COSTS AND RANDOM ENDOWMENTS

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This paper studies the optimal consumption via the habit formation preference in markets with transaction costs and unbounded random endowments. To model the proportional transaction costs, we adopt the Kabanov's multi-asset framework with a cash account. At the terminal time T, the investor can receive unbounded random endowments for which we propose a new definition of acceptable portfolios based on the strictly consistent price system (SCPS). We prove a type of super-hedging theorem using the acceptable portfolios which enables us to obtain the consumption budget constraint condition under market frictions. Following similar ideas in [*Ann. Appl. Probab.* **25** (2015) 1383–1419] with the path dependence reduction and the embedding approach, we obtain the existence and uniqueness of the optimal consumption using some auxiliary processes and the duality analysis. As an application of the duality theory, the market isomorphism with special discounting factors is also discussed in the sense that the original optimal consumption with habit formation is equivalent to the standard optimal consumption problem without the habits impact, however, in a modified isomorphic market model.

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Key words and phrases. Proportional transaction costs, unbounded random endowments, acceptable portfolios, consumption budget constraint, consumption habit formation, convex duality, market isomorphism.

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QUICKEST DETECTION PROBLEMS FOR BESSEL PROCESSES[1]

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Consider the motion of a Brownian particle that initially takes place in a two-dimensional plane and then after some random/unobservable time continues in the three-dimensional space. Given that only the distance of the particle to the origin is being observed, the problem is to detect the time at which the particle departs from the plane as accurately as possible. We solve this problem in the most uncertain scenario when the random/unobservable time is (i) exponentially distributed and (ii) independent from the initial motion of the particle in the plane. The solution is expressed in terms of a stopping time that minimises the probability of a false early detection and the expected delay of a missed late detection.

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THE ASYMPTOTIC VARIANCE OF THE GIANT COMPONENT OF CONFIGURATION MODEL RANDOM GRAPHS

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For a supercritical configuration model random graph, it is well known that, subject to mild conditions, there exists a unique giant component, whose size R_n is $O(n)$, where n is the total number of vertices in the random graph. Moreover, there exists $0 < \rho \leq 1$ such that $R_n/n \xrightarrow{p} \rho$ as $n \to \infty$. We show that for a sequence of *well behaved* configuration model random graphs with a deterministic degree sequence satisfying $0 < \rho < 1$; there exists $\sigma^2 > 0$, such that $\text{var}(\sqrt{n}(R_n/n - \rho)) \to \sigma^2$ as $n \to \infty$. Moreover, an explicit, easy to compute, formula is given for σ^2. This provides a key stepping stone for computing the asymptotic variance of the size of the giant component for more general random graphs.

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FROM STOCHASTIC, INDIVIDUAL-BASED MODELS TO THE CANONICAL EQUATION OF ADAPTIVE DYNAMICS IN ONE STEP

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We consider a model for Darwinian evolution in an asexual population with a large but nonconstant populations size characterized by a natural birth rate, a logistic death rate modeling competition and a probability of mutation at each birth event. In the present paper, we study the long-term behavior of the system in the limit of large population ($K \to \infty$) size, rare mutations ($u \to 0$) and small mutational effects ($\sigma \to 0$), proving convergence to the canonical equation of adaptive dynamics (CEAD). In contrast to earlier works, for example, by Champagnat and Méléard, we take the three limits simultaneously, that is, $u = u_K$ and $\sigma = \sigma_K$, tend to zero with K, subject to conditions that ensure that the time-scale of birth and death events remains separated from that of successful mutational events. This slows down the dynamics of the microscopic system and leads to serious technical difficulties that require the use of completely different methods. In particular, we cannot use the law of large numbers on the diverging time needed for fixation to approximate the stochastic system with the corresponding deterministic one. To solve this problem, we develop a "stochastic Euler scheme" based on coupling arguments that allows to control the time evolution of the stochastic system over time-scales that diverge with K.

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LOGARITHMIC TAILS OF SUMS OF PRODUCTS OF POSITIVE RANDOM VARIABLES BOUNDED BY ONE

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In this paper, we show under weak assumptions that for $R \stackrel{d}{=} 1 + M_1 + M_1 M_2 + \cdots$, where $\mathbb{P}(M \in [0, 1]) = 1$ and M_i are independent copies of M, we have $\ln \mathbb{P}(R > x) \sim Cx \ln \mathbb{P}(M > 1 - 1/x)$ as $x \to \infty$. The constant C is given explicitly and its value depends on the rate of convergence of $\ln \mathbb{P}(M > 1 - 1/x)$. Random variable R satisfies the stochastic equation $R \stackrel{d}{=} 1 + MR$ with M and R independent, thus this result fits into the study of tails of iterated random equations, or more specifically, perpetuities.

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INVARIANCE PRINCIPLES FOR OPERATOR-SCALING GAUSSIAN RANDOM FIELDS

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Recently, Hammond and Sheffield [*Probab. Theory Related Fields* **157** (2013) 691–719] introduced a model of correlated one-dimensional random walks that scale to fractional Brownian motions with long-range dependence. In this paper, we consider a natural generalization of this model to dimension $d \geq 2$. We define a $\mathbb{Z}^d$-indexed random field with dependence relations governed by an underlying random graph with vertices $\mathbb{Z}^d$, and we study the scaling limits of the partial sums of the random field over rectangular sets. An interesting phenomenon appears: depending on how fast the rectangular sets increase along different directions, different random fields arise in the limit. In particular, there is a critical regime where the limit random field is operator-scaling and inherits the full dependence structure of the discrete model, whereas in other regimes the limit random fields have at least one direction that has either invariant or independent increments, no longer reflecting the dependence structure in the discrete model. The limit random fields form a general class of operator-scaling Gaussian random fields. Their increments and path properties are investigated.

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EVOLVING VOTER MODEL ON DENSE RANDOM GRAPHS

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In this paper, we examine a variant of the voter model on a dynamically changing network where agents have the option of changing their friends rather than changing their opinions. We analyse, in the context of dense random graphs, two models considered in Durrett et al. [*Proc. Natl. Acad. Sci. USA* **109** (2012) 3682–3687]. When an edge with two agents holding different opinion is updated, with probability $\frac{\beta}{n}$, one agent performs a voter model step and changes its opinion to copy the other, and with probability $1-\frac{\beta}{n}$, the edge between them is broken and reconnected to a new agent chosen randomly from (i) the whole network (rewire-to-random model) or, (ii) the agents having the same opinion (rewire-to-same model). We rigorously establish in both the models, the time for this dynamics to terminate exhibits a phase transition in the model parameter β. For β sufficiently small, with high probability the network rapidly splits into two disconnected communities with opposing opinions, whereas for β large enough the dynamics runs for longer and the density of opinion changes significantly before the process stops. In the rewire-to-random model, we show that a positive fraction of both opinions survive with high probability.

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ADDENDUM TO "OPTIMAL STOPPING UNDER MODEL UNCERTAINTY: RANDOMIZED STOPPING TIMES APPROACH"

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CORRIGENDUM

WEAK APPROXIMATIONS FOR WIENER FUNCTIONALS [*Ann. Appl. Probab.* (2013) 23 1660–1691]

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ADDENDUM AND CORRIGENDUM TO "RANDOMIZED URN MODELS REVISITED USING STOCHASTIC APPROXIMATION"

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This is a short addendum and corrigendum to the paper "Randomized Urn Models revisited using Stochastic Approximation" published in *Annals of Applied Probability*.

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