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ASYMPTOTIC BIAS OF STOCHASTIC GRADIENT SEARCH

BY VLADISLAV B. TADIĆ AND ARNAUD DOUCET[1]

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The asymptotic behavior of the stochastic gradient algorithm using biased gradient estimates is analyzed. Relying on arguments based on dynamic system theory (chain-recurrence) and differential geometry (Yomdin theorem and Lojasiewicz inequalities), upper bounds on the asymptotic bias of this algorithm are derived. The results hold under mild conditions and cover a broad class of algorithms used in machine learning, signal processing and statistics.

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UNBIASED SIMULATION OF STOCHASTIC DIFFERENTIAL EQUATIONS

BY PIERRE HENRY-LABORDÈRE, XIAOLU TAN[1] AND NIZAR TOUZI[1]

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We propose an unbiased Monte 3 estimator for $\mathbb{E}[g(X_{t_1}, \dots, X_{t_n})]$, where X is a diffusion process defined by a multidimensional stochastic differential equation (SDE). The main idea is to start instead from a well-chosen simulatable SDE whose coefficients are updated at independent exponential times. Such a simulatable process can be viewed as a regime-switching SDE, or as a branching diffusion process with one single living particle at all times. In order to compensate for the change of the coefficients of the SDE, our main representation result relies on the automatic differentiation technique induced by the Bismut–Elworthy–Li formula from Malliavin calculus, as exploited by Fournié et al. [*Finance Stoch.* **3** (1999) 391–412] for the simulation of the Greeks in financial applications. In particular, this algorithm can be considered as a variation of the (infinite variance) estimator obtained in Bally and Kohatsu-Higa [*Ann. Appl. Probab.* **25** (2015) 3095–3138, Section 6.1] as an application of the parametrix method.

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ON DYNAMIC DEVIATION MEASURES AND CONTINUOUS-TIME PORTFOLIO OPTIMIZATION

BY MARTIJN PISTORIUS[1] AND MITJA STADJE[2]

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In this paper, we propose the notion of *dynamic deviation measure*, as a dynamic time-consistent extension of the (static) notion of deviation measure. To achieve time-consistency, we require that a dynamic deviation measures satisfies a generalised conditional variance formula. We show that, under a domination condition, dynamic deviation measures are characterised as the solutions to a certain class of stochastic differential equations. We establish for any dynamic deviation measure an integral representation, and derive a dual characterisation result in terms of *additively* m*-stable* dual sets. Using this notion of dynamic deviation measure, we formulate a dynamic mean-deviation portfolio optimization problem in a jump-diffusion setting and identify a subgame-perfect Nash equilibrium strategy that is linear as function of wealth by deriving and solving an associated extended HJB equation.

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ON THE INSTABILITY OF MATCHING QUEUES

BY PASCAL MOYAL AND OHAD PERRY[1]

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A matching queue is described via a graph, an arrival process and a matching policy. Specifically, to each node in the graph there is a corresponding arrival process of items, which can either be queued or matched with queued items in neighboring nodes. The matching policy specifies how items are matched whenever more than one matching is possible. Given the matching graph and the matching policy, the stability region of the system is the set of intensities of the arrival processes rendering the underlying Markov process positive recurrent. In a recent paper, a condition on the arrival intensities, which was named NCOND, was shown to be necessary for the stability of a matching queue. That condition can be thought of as an analogue to the usual traffic condition for traditional queueing networks, and it is thus natural to study whether it is also sufficient. In this paper, we show that this is not the case in general. Specifically, we prove that, except for a particular class of graphs, there always exists a matching policy rendering the stability region strictly smaller than the set of arrival intensities satisfying NCOND. Our proof combines graph- and queueing-theoretic techniques: After showing explicitly, via fluid-limit arguments that the stability regions of two basic models is strictly included in NCOND, we generalize this result to any graph inducing either one of those two basic graphs.

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DYNAMIC APPROACHES FOR SOME TIME-INCONSISTENT OPTIMIZATION PROBLEMS

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In this paper, we investigate possible approaches to study general time-inconsistent optimization problems without assuming the existence of optimal strategy. This leads immediately to the need to refine the concept of time consistency as well as any method that is based on Pontryagin's maximum principle. The fundamental obstacle is the dilemma of having to invoke the *Dynamic Programming Principle* (DPP) in a time-inconsistent setting, which is contradictory in nature. The main contribution of this work is the introduction of the idea of the "dynamic utility" under which the original time-inconsistent problem (under the fixed utility) becomes a time-consistent one. As a benchmark model, we shall consider a stochastic controlled problem with multidimensional backward SDE dynamics, which covers many existing time-inconsistent problems in the literature as special cases; and we argue that the time inconsistency is essentially equivalent to the lack of *comparison principle*. We shall propose three approaches aiming at reviving the DPP in this setting: the duality approach, the dynamic utility approach and the master equation approach. Unlike the game approach in many existing works in continuous time models, all our approaches produce the same value as the original static problem.

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GENERAL EDGEWORTH EXPANSIONS WITH APPLICATIONS TO PROFILES OF RANDOM TREES

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We prove an asymptotic Edgeworth expansion for the profiles of certain random trees including binary search trees, random recursive trees and plane-oriented random trees, as the size of the tree goes to infinity. All these models can be seen as special cases of the one-split branching random walk for which we also provide an Edgeworth expansion. These expansions lead to new results on mode, width and occupation numbers of the trees, settling several open problems raised in Devroye and Hwang [*Ann. Appl. Probab.* **16** (2006) 886–918], Fuchs, Hwang and Neininger [*Algorithmica* **46** (2006) 367–407], and Drmota and Hwang [*Adv. in Appl. Probab.* **37** (2005) 321–341]. The aforementioned results are special cases and corollaries of a general theorem: an Edgeworth expansion for an arbitrary sequence of random or deterministic functions $\mathbb{L}_n : \mathbb{Z} \to \mathbb{R}$ which converges in the mod-ϕ-sense. Applications to Stirling numbers of the first kind will be given in a separate paper.

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THE DIVIDEND PROBLEM WITH A FINITE HORIZON

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We characterise the value function of the optimal dividend problem with a finite time horizon as the unique classical solution of a suitable Hamilton–Jacobi–Bellman equation. The optimal dividend strategy is realised by a Skorokhod reflection of the fund's value at a time-dependent optimal boundary. Our results are obtained by establishing for the first time a new connection between singular control problems with an absorbing boundary and optimal stopping problems on a diffusion reflected at 0 and created at a rate proportional to its local time.

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LARGE DEVIATIONS FOR THE EXCLUSION PROCESS WITH A SLOW BOND

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We consider the one-dimensional symmetric simple exclusion process with a slow bond. In this model, whilst all the transition rates are equal to one, a particular bond, the *slow bond*, has associated transition rate of value N^{-1}, where N is the scaling parameter. This model has been considered in previous works on the subject of hydrodynamic limit and fluctuations. In this paper, assuming uniqueness for weak solutions of hydrodynamic equation associated to the perturbed process, we obtain dynamical large deviations estimates in the diffusive scaling. The main challenge here is the fact that the presence of the slow bond gives rise to Robin's boundary conditions in the *continuum*, substantially complicating the large deviations scenario.

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OPTIMAL DIVIDEND AND INVESTMENT PROBLEMS UNDER SPARRE ANDERSEN MODEL

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In this paper, we study a class of optimal dividend and investment problems assuming that the underlying reserve process follows the Sparre Andersen model, that is, the claim frequency is a "renewal" process, rather than a standard compound Poisson process. The main feature of such problems is that the underlying reserve dynamics, even in its simplest form, is no longer Markovian. By using the *backward Markovization* technique, we recast the problem in a Markovian framework with expanded dimension representing the time elapsed after the last claim, with which we investigate the regularity of the value function, and validate the dynamic programming principle. Furthermore, we show that the value function is the unique *constrained viscosity solution* to the associated HJB equation on a cylindrical domain on which the problem is well defined.

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IMPROVED FRÉCHET–HOEFFDING BOUNDS ON *d*-COPULAS AND APPLICATIONS IN MODEL-FREE FINANCE

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We derive upper and lower bounds on the expectation of $f(\mathbf{S})$ under dependence uncertainty, that is, when the marginal distributions of the random vector $\mathbf{S} = (S_1, \dots, S_d)$ are known but their dependence structure is partially unknown. We solve the problem by providing improved Fréchet–Hoeffding bounds on the copula of $\mathbf{S}$ that account for additional information. In particular, we derive bounds when the values of the copula are given on a compact subset of $[0, 1]^d$, the value of a functional of the copula is prescribed or different types of information are available on the lower dimensional marginals of the copula. We then show that, in contrast to the two-dimensional case, the bounds are quasi-copulas but fail to be copulas if $d > 2$. Thus, in order to translate the improved Fréchet–Hoeffding bounds into bounds on the expectation of $f(\mathbf{S})$, we develop an alternative representation of multivariate integrals with respect to copulas that admits also quasi-copulas as integrators. By means of this representation, we provide an integral characterization of orthant orders on the set of quasi-copulas which relates the improved Fréchet–Hoeffding bounds to bounds on the expectation of $f(\mathbf{S})$. Finally, we apply these results to compute model-free bounds on the prices of multi-asset options that take partial information on the dependence structure into account, such as correlations or market prices of other traded derivatives. The numerical results show that the additional information leads to a significant improvement of the option price bounds compared to the situation where only the marginal distributions are known.

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ASYMPTOTIC LYAPUNOV EXPONENTS FOR LARGE RANDOM MATRICES

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Suppose that $A_1, \dots, A_N$ are independent random matrices of size n whose entries are i.i.d. copies of a random variable ξ of mean zero and variance one. It is known from the late 1980s that when ξ is Gaussian then $N^{-1} \log \|A_N \dots A_1\|$ converges to $\log \sqrt{n}$ as $N \to \infty$. We will establish similar results for more general matrices with explicit rate of convergence. Our method relies on a simple interplay between additive structures and growth of matrices.

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ROBUST BOUNDS IN MULTIVARIATE EXTREMES

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Extreme value theory provides an asymptotically justified framework for estimation of exceedance probabilities in regions where few or no observations are available. For multivariate tail estimation, the strength of extremal dependence is crucial and it is typically modeled by a parametric family of spectral distributions. In this work, we provide asymptotic bounds on exceedance probabilities that are robust against misspecification of the extremal dependence model. They arise from optimizing the statistic of interest over all dependence models within some neighborhood of the reference model. A certain relaxation of these bounds yields surprisingly simple and explicit expressions, which we propose to use in applications. We show the effectiveness of the robust approach compared to classical confidence bounds when the model is misspecified. The results are further applied to quantify the effect of model uncertainty on the Value-at-Risk of a financial portfolio.

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FINANCIAL MARKETS WITH A LARGE TRADER[1]

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We construct a large trader model using tools from nonlinear stochastic integration theory and an impact function. It encompasses many well-known models from the literature. In particular, the model allows price changes to depend on the size as well as on the speed and timing of the large trader's transactions. Moreover, a volume impact limit order book can be studied in this framework. Relaxing a condition about existence of a universal martingale measure governing all resulting small trader models, we can show absence of arbitrage for the small trader under mild conditions. Furthermore, a case study on utility maximization from terminal wealth highlights new phenomena that can arise in our framework. Finally, an outlook on further research provides insights on (no) arbitrage opportunities for the large trader and how different levels of information may affect our analysis.

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SYNCHRONIZATION OF REINFORCED STOCHASTIC PROCESSES WITH A NETWORK-BASED INTERACTION

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Randomly evolving systems composed by elements which interact among each other have always been of great interest in several scientific fields. This work deals with the *synchronization* phenomenon that could be roughly defined as the tendency of different components to adopt a common behavior. We continue the study of a model of *interacting stochastic processes with reinforcement* that recently has been introduced in [Crimaldi et al. (2016)]. Generally speaking, by reinforcement we mean any mechanism for which the probability that a given event occurs has an increasing dependence on the number of times that events of the same type occurred in the past. The particularity of systems of such interacting stochastic processes is that synchronization is induced *along time* by the reinforcement mechanism itself and does not require a large-scale limit. We focus on the relationship between the topology of the network of the interactions and the long-time synchronization phenomenon. After proving the almost sure synchronization, we provide some CLTs in the sense of stable convergence that establish the convergence rates and the asymptotic distributions for both convergence to the common limit and synchronization. The obtained results lead to the construction of asymptotic confidence intervals for the limit random variable and of statistical tests to make *inference on the topology of the network*.

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THE WIDOM–ROWLINSON MODEL UNDER SPIN FLIP: IMMEDIATE LOSS AND SHARP RECOVERY OF QUASILOCALITY[1]

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We consider the continuum Widom–Rowlinson model under independent spin-flip dynamics and investigate whether and when the time-evolved point process has an (almost) quasilocal specification (Gibbs-property of the time-evolved measure). Our study provides a first analysis of a Gibbs–non-Gibbs transition for point particles in Euclidean space. We find a picture of loss and recovery, in which even more regularity is lost faster than it is for time-evolved spin models on lattices.

We show immediate loss of quasilocality in the percolation regime, with full measure of discontinuity points for any specification. For the color-asymmetric percolating model, there is a transition from this non-almost-sure quasilocal regime back to an everywhere Gibbsian regime. At the sharp reentrance time $t_G > 0$, the model is a.s. quasilocal. For the color-symmetric model, there is no reentrance. On the constructive side, for all $t > t_G$, we provide everywhere quasilocal specifications for the time-evolved measures and give precise exponential estimates on the influence of boundary condition.

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ON THE UNIQUE CROSSING CONJECTURE OF DIACONIS AND PERLMAN ON CONVOLUTIONS OF GAMMA RANDOM VARIABLES

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Diaconis and Perlman [In *Topics in Statistical Dependence (Somerset, PA, 1987)* (1990) 147–166, IMS] conjecture that the distribution functions of two weighted sums of i.i.d. gamma random variables cross exactly once if one weight vector majorizes the other. We disprove this conjecture when the shape parameter of the gamma variates is $\alpha < 1$ and prove it when $\alpha \geq 1$.

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Corrigendum

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Erratum

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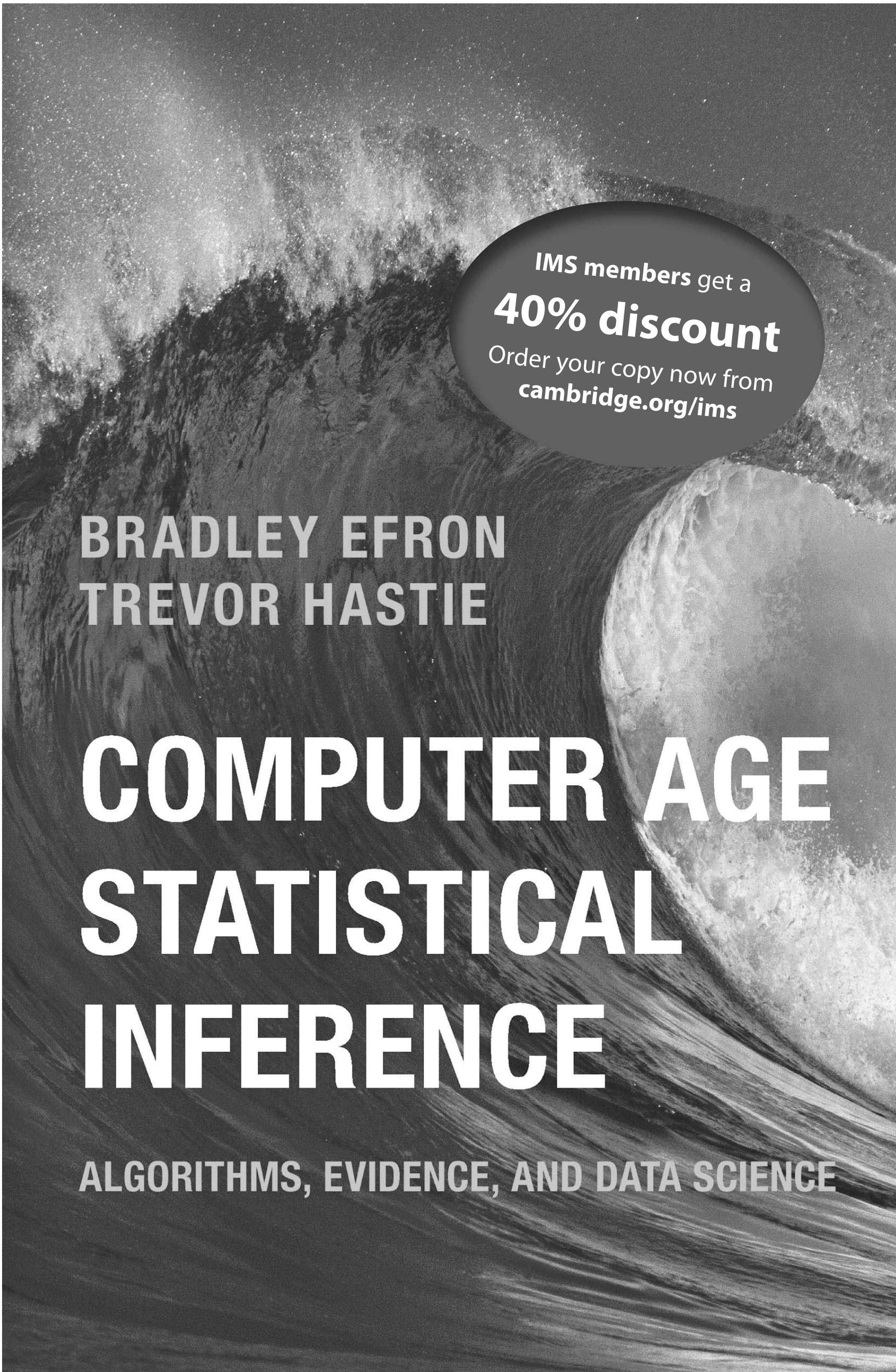
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