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WEAK CONVERGENCE RATES OF SPECTRAL GALERKIN APPROXIMATIONS FOR SPDES WITH NONLINEAR DIFFUSION COEFFICIENTS[1]

BY DANIEL CONUS*, ARNULF JENTZEN† AND RYAN KURNIAWAN†

Lehigh University and ETH Zürich†*

Strong convergence rates for (temporal, spatial, and noise) numerical approximations of semilinear stochastic evolution equations (SEEs) with smooth and regular nonlinearities are well understood in the scientific literature. Weak convergence rates for numerical approximations of such SEEs have been investigated for about two decades and are far away from being well understood: roughly speaking, no essentially sharp weak convergence rates are known for parabolic SEEs with nonlinear diffusion coefficient functions; see Remark 2.3 in [*Math. Comp.* **80** (2011) 89–117] for details. In this article, we solve the weak convergence problem emerged from Debussche's article in the case of spectral Galerkin approximations and establish essentially sharp weak convergence rates for spatial spectral Galerkin approximations of semilinear SEEs with nonlinear diffusion coefficient functions. Our solution to the weak convergence problem does not use Malliavin calculus. Rather, key ingredients in our solution to the weak convergence problem emerged from Debussche's article are the use of appropriately modified versions of the spatial Galerkin approximation processes and applications of a mild Itô-type formula for solutions and numerical approximations of semilinear SEEs. This article solves the weak convergence problem emerged from Debussche's article merely in the case of spatial spectral Galerkin approximations instead of other more complicated numerical approximations. Our method of proof extends, however, to a number of other kinds of spatial and temporal numerical approximations for semilinear SEEs.

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CHANGE-POINT DETECTION FOR LÉVY PROCESSES

BY JOSÉ E. FIGUEROA-LÓPEZ AND SVEINN ÓLAFSSON

Washington University in St. Louis and University of California, Santa Barbara

Since the work of Page in the 1950s, the problem of detecting an abrupt change in the distribution of stochastic processes has received a great deal of attention. In particular, a deep connection has been established between Lorden's minimax approach to change-point detection and the widely used CUSUM procedure, first for discrete-time processes, and subsequently for some of their continuous-time counterparts. However, results for processes with jumps are still scarce, while the practical importance of such processes has escalated since the turn of the century. In this work, we consider the problem of detecting a change in the distribution of continuous-time processes with independent and stationary increments, that is, Lévy processes, and our main result shows that CUSUM is indeed optimal in Lorden's sense. This is the most natural continuous-time analogue of the seminal work of Moustakides [*Ann. Statist.* **14** (1986) 1379–1387] for sequentially observed random variables that are assumed to be i.i.d. before and after the change-point. From a practical perspective, the approach we adopt is appealing as it consists in approximating the continuous-time problem by a suitable sequence of change-point problems with equispaced sampling points, and for which a CUSUM procedure is shown to be optimal.

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SUPER-REPLICATION WITH FIXED TRANSACTION COSTS

BY PETER BANK[*,1] AND YAN DOLINSKY[†,‡,1,2]

Technische Universität Berlin[*], *Hebrew University*[†] *and Monash University*[‡]

We study super-replication of contingent claims in markets with fixed transaction costs. This can be viewed as a stochastic impulse control problem with a terminal state constraint. The first result in this paper reveals that in reasonable continuous time financial market models the super-replication price is prohibitively costly and leads to trivial buy-and-hold strategies. Our second result derives nontrivial scaling limits of super-replication prices for binomial models with small fixed costs.

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FIRST-ORDER EULER SCHEME FOR SDES DRIVEN BY FRACTIONAL BROWNIAN MOTIONS: THE ROUGH CASE

BY YANGHUI LIU AND SAMY TINDEL[1]

Purdue University

In this article, we consider the so-called modified Euler scheme for stochastic differential equations (SDEs) driven by fractional Brownian motions (fBm) with Hurst parameter $\frac{1}{3} < H < \frac{1}{2}$. This is a first-order time-discrete numerical approximation scheme, and has been introduced in [*Ann. Appl. Probab.* **26** (2016) 1147–1207] recently in order to generalize the classical Euler scheme for Itô SDEs to the case $H > \frac{1}{2}$. The current contribution generalizes the modified Euler scheme to the rough case $\frac{1}{3} < H < \frac{1}{2}$. Namely, we show a convergence rate of order $n^{\frac{1}{2}-2H}$ for the scheme, and we argue that this rate is exact. We also derive a central limit theorem for the renormalized error of the scheme, thanks to some new techniques for asymptotics of weighted random sums. Our main idea is based on the following observation: the triple of processes obtained by considering the fBm, the scheme process and the normalized error process, can be lifted to a new rough path. In addition, the Hölder norm of this new rough path has an estimate which is independent of the step-size of the scheme.

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MALLIAVIN CALCULUS APPROACH TO LONG EXIT TIMES FROM AN UNSTABLE EQUILIBRIUM

BY YURI BAKHTIN[1] AND ZSOLT PAJOR-GYULAI

New York University

For a one-dimensional smooth vector field in a neighborhood of an unstable equilibrium, we consider the associated dynamics perturbed by small noise. Using Malliavin calculus tools, we obtain precise vanishing noise asymptotics for the tail of the exit time and for the exit distribution conditioned on atypically long exits.

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OPTIMAL MEAN-BASED ALGORITHMS FOR TRACE RECONSTRUCTION[1,2]

BY ANINDYA DE, RYAN O'DONNELL AND ROCCO A. SERVEDIO

Northwestern University, Carnegie Mellon University and Columbia University

In the *(deletion-channel) trace reconstruction problem*, there is an unknown n-bit *source string* x. An algorithm is given access to independent *traces* of x, where a trace is formed by deleting each bit of x independently with probability δ. The goal of the algorithm is to recover x exactly (with high probability), while minimizing samples (number of traces) and running time.

Previously, the best known algorithm for the trace reconstruction problem was due to Holenstein et al. [in *Proceedings of the Nineteenth Annual ACM-SIAM Symposium on Discrete Algorithms* 389–398 (2008) ACM]; it uses $\exp(\widetilde{O}(n^{1/2}))$ samples and running time for any fixed $0 < \delta < 1$. It is also what we call a "mean-based algorithm," meaning that it only uses the empirical means of the individual bits of the traces. Holenstein et al. also gave a lower bound, showing that any mean-based algorithm must use at least $n^{\widetilde{\Omega}(\log n)}$ samples.

In this paper, we improve both of these results, obtaining matching upper and lower bounds for mean-based trace reconstruction. For any constant deletion rate $0 < \delta < 1$, we give a mean-based algorithm that uses $\exp(O(n^{1/3}))$ time and traces; we also prove that any mean-based algorithm must use at least $\exp(\Omega(n^{1/3}))$ traces. In fact, we obtain matching upper and lower bounds even for δ subconstant and $\rho = 1 - \delta$ subconstant: when $(\log^3 n)/n \ll \delta \le 1/2$ the bound is $\exp(-\Theta(\delta n)^{1/3})$, and when $1/\sqrt{n} \ll \rho \le 1/2$ the bound is $\exp(-\Theta(n/\rho)^{1/3})$.

Our proofs involve estimates for the maxima of Littlewood polynomials on complex disks. We show that these techniques can also be used to perform trace reconstruction with random insertions and bit-flips in addition to deletions. We also find a surprising result: for deletion probabilities $\delta > 1/2$, the presence of insertions can actually *help* with trace reconstruction.

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Key words and phrases. Trace reconstruction, deletion channel, Littlewood polynomials.

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A SHAPE THEOREM FOR THE SCALING LIMIT OF THE IPDSAW AT CRITICALITY

BY PHILIPPE CARMONA AND NICOLAS PÉTRÉLIS

Université de Nantes

In this paper we give a complete characterization of the scaling limit of the critical Interacting Partially Directed Self-Avoiding Walk (IPDSAW) introduced in Zwanzig and Lauritzen [*J. Chem. Phys.* **48** (1968) 3351]. As the system size $L \in \mathbb{N}$ diverges, we prove that the set of occupied sites, rescaled horizontally by $L^{2/3}$ and vertically by $L^{1/3}$ converges in law for the Hausdorff distance toward a nontrivial random set. This limiting set is built with a Brownian motion B conditioned to come back at the origin at a_1 the time at which its geometric area reaches 1. The modulus of B up to a_1 gives the height of the limiting set, while its center of mass process is an independent Brownian motion.

Obtaining the shape theorem requires to derive a functional central limit theorem for the excursion of a random walk with Laplace symmetric increments conditioned on sweeping a prescribed geometric area. This result is proven in a companion paper Carmona and Pétrélis (2017).

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Key words and phrases. Polymer collapse, phase transition, shape theorem, Brownian motion, Local limit theorem.

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NORMAL APPROXIMATION FOR STABILIZING FUNCTIONALS[1]

BY RAPHAËL LACHIÈZE-REY*, MATTHIAS SCHULTE† AND J. E. YUKICH‡

Université Paris Descartes, Sorbonne Paris Cité, Universität Bern†
and Lehigh University‡*

We establish presumably optimal rates of normal convergence with respect to the Kolmogorov distance for a large class of geometric functionals of marked Poisson and binomial point processes on general metric spaces. The rates are valid whenever the geometric functional is expressible as a sum of exponentially stabilizing score functions satisfying a moment condition. By incorporating stabilization methods into the Malliavin–Stein theory, we obtain rates of normal approximation for sums of stabilizing score functions which either improve upon existing rates or are the first of their kind.

Our general rates hold for functionals of marked input on spaces more general than full-dimensional subsets of $\mathbb{R}^d$, including m-dimensional Riemannian manifolds, $m \leq d$. We use the general results to deduce improved and new rates of normal convergence for several functionals in stochastic geometry, including those whose variances re-scale as the volume or the surface area of an underlying set. In particular, we improve upon rates of normal convergence for the k-face and ith intrinsic volume functionals of the convex hull of Poisson and binomial random samples in a smooth convex body in dimension $d \geq 2$. We also provide improved rates of normal convergence for statistics of nearest neighbors graphs and high-dimensional data sets, the number of maximal points in a random sample, estimators of surface area and volume arising in set approximation via Voronoi tessellations, and clique counts in generalized random geometric graphs.

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ERGODICITY OF AN SPDE ASSOCIATED WITH A MANY-SERVER QUEUE

BY REZA AGHAJANI[1] AND KAVITA RAMANAN[2]

University of California, San Diego and Brown University

We consider the so-called GI/GI/N queueing network in which a stream of jobs with independent and identically distributed service times arrive according to a renewal process to a common queue served by N identical servers in a first-come-first-serve manner. We introduce a two-component infinite-dimensional Markov process that serves as a diffusion model for this network, in the regime where the number of servers goes to infinity and the load on the network scales as $1 - \beta N^{-1/2} + o(N^{-1/2})$ for some $\beta > 0$. Under suitable assumptions, we characterize this process as the unique solution to a pair of stochastic evolution equations comprised of a real-valued Itô equation and a stochastic partial differential equation on the positive half line, which are coupled together by a nonlinear boundary condition. We construct an asymptotic (equivalent) coupling to show that this Markov process has a unique invariant distribution. This invariant distribution is shown in a companion paper [Aghajani and Ramanan (2016)] to be the limit of the sequence of suitably scaled and centered stationary distributions of the GI/GI/N network, thus resolving (for a large class service distributions) an open problem raised by Halfin and Whitt in [*Oper. Res.* **29** (1981) 567–588]. The methods introduced here are more generally applicable for the analysis of a broader class of networks.

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CENTRAL LIMIT THEOREMS IN THE CONFIGURATION MODEL

BY A. D. BARBOUR[1] AND ADRIAN RÖLLIN[2]

Universität Zürich and National University of Singapore

We prove a general normal approximation theorem for local graph statistics in the configuration model, together with an explicit bound on the error in the approximation with respect to the Wasserstein metric. Such statistics take the form $T := \sum_{v \in V} H_v$, where V is the vertex set, and H_v depends on a neighbourhood in the graph around v of size at most ℓ. The error bound is expressed in terms of ℓ, $|V|$, an almost sure bound on H_v, the maximum vertex degree $d_{\max}$ and the variance of T. Under suitable assumptions on the convergence of the empirical degree distributions to a limiting distribution, we deduce that the size of the giant component in the configuration model has asymptotically Gaussian fluctuations.

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ERGODICITY OF A LÉVY-DRIVEN SDE ARISING FROM MULTICLASS MANY-SERVER QUEUES[1]

BY ARI ARAPOSTATHIS, GUODONG PANG AND NIKOLA SANDRIĆ

University of Texas at Austin, Pennsylvania State University and University of Zagreb

We study the ergodic properties of a class of multidimensional piecewise Ornstein–Uhlenbeck processes with jumps, which contains the limit of the queueing processes arising in multiclass many-server queues with heavy-tailed arrivals and/or asymptotically negligible service interruptions in the Halfin–Whitt regime as special cases. In these queueing models, the Itô equations have a piecewise linear drift, and are driven by either (1) a Brownian motion and a pure-jump Lévy process, or (2) an anisotropic Lévy process with independent one-dimensional symmetric α-stable components or (3) an anisotropic Lévy process as in (2) and a pure-jump Lévy process. We also study the class of models driven by a subordinate Brownian motion, which contains an isotropic (or rotationally invariant) α-stable Lévy process as a special case. We identify conditions on the parameters in the drift, the Lévy measure and/or covariance function which result in subexponential and/or exponential ergodicity. We show that these assumptions are sharp, and we identify some key necessary conditions for the process to be ergodic. In addition, we show that for the queueing models described above with no abandonment, the rate of convergence is polynomial, and we provide a sharp quantitative characterization of the rate via matching upper and lower bounds.

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ON ONE-DIMENSIONAL RICCATI DIFFUSIONS

BY A. N. BISHOP[*,†,1], P. DEL MORAL[‡,§,2], K. KAMATANI[¶] AND B. RÉMILLARD[‖,3]

University of Technology Sydney (UTS), CSIRO†, INRIA‡, UNSW§, Osaka University¶ and HEC Montréal‖*

This article is concerned with the fluctuation analysis and the stability properties of a class of one-dimensional Riccati diffusions. These one-dimensional stochastic differential equations exhibit a quadratic drift function and a non-Lipschitz continuous diffusion function. We present a novel approach, combining tangent process techniques, Feynman–Kac path integration and exponential change of measures, to derive sharp exponential decays to equilibrium. We also provide uniform estimates with respect to the time horizon, quantifying with some precision the fluctuations of these diffusions around a limiting deterministic Riccati differential equation. These results provide a stronger and almost sure version of the conventional central limit theorem. We illustrate these results in the context of ensemble Kalman–Bucy filtering. To the best of our knowledge, the exponential stability and the fluctuation analysis developed in this work are the first results of this kind for this class of nonlinear diffusions.

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ON POISSON APPROXIMATIONS FOR THE EWENS SAMPLING FORMULA WHEN THE MUTATION PARAMETER GROWS WITH THE SAMPLE SIZE

BY KOJI TSUKUDA

The University of Tokyo

The Ewens sampling formula was first introduced in the context of population genetics by Warren John Ewens in 1972, and has appeared in a lot of other scientific fields. There are abundant approximation results associated with the Ewens sampling formula especially when one of the parameters, the sample size n or the mutation parameter θ which denotes the scaled mutation rate, tends to infinity while the other is fixed. By contrast, the case that θ grows with n has been considered in a relatively small number of works, although this asymptotic setup is also natural. In this paper, when θ grows with n, we advance the study concerning the asymptotic properties of the total number of alleles and of the component counts in the allelic partition assuming the Ewens sampling formula, from the viewpoint of Poisson approximations. Specifically, the main contributions of this paper are deriving Poisson approximations of the total number of alleles, an independent process approximation of small component counts, and functional central limit theorems, under the asymptotic regime that both n and θ tend to infinity.

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THE CRITICAL GREEDY SERVER ON THE INTEGERS IS RECURRENT

BY JAMES R. CRUISE[1] AND ANDREW R. WADE

Heriot-Watt University and Durham University

Each site of $\mathbb{Z}$ hosts a queue with arrival rate λ. A single server, starting at the origin, serves its current queue at rate μ until that queue is empty, and then moves to the longest neighbouring queue. In the critical case $\lambda = \mu$, we show that the server returns to every site infinitely often. We also give a sharp iterated logarithm result for the server's position. Important ingredients in the proofs are that the times between successive queues being emptied exhibit doubly exponential growth, and that the probability that the server changes its direction is asymptotically equal to $1/4$.

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JOIN-THE-SHORTEST QUEUE DIFFUSION LIMIT IN HALFIN–WHITT REGIME: TAIL ASYMPTOTICS AND SCALING OF EXTREMA

BY SAYAN BANERJEE AND DEBANKUR MUKHERJEE[1]

University of North Carolina, Chapel Hill and Brown University

Consider a system of N parallel single-server queues with unit-ex ponential service time distribution and a single dispatcher where tasks arrive as a Poisson process of rate $\lambda(N)$. When a task arrives, the dispatcher assigns it to one of the servers according to the Join-the-Shortest Queue (JSQ) policy. Eschenfeldt and Gamarnik (*Math. Oper. Res.* **43** (2018) 867–886) established that in the Halfin–Whitt regime where $(N - \lambda(N))/\sqrt{N} \to \beta > 0$ as $N \to \infty$, appropriately scaled occupancy measure of the system under the JSQ policy converges weakly on any finite time interval to a certain diffusion process as $N \to \infty$. Recently, it was further established by Braverman (2018) that the convergence result extends to the steady state as well, that is, stationary occupancy measure of the system converges weakly to the steady state of the diffusion process as $N \to \infty$, proving the interchange of limits result.

In this paper, we perform a detailed analysis of the steady state of the above diffusion process. Specifically, we establish precise tail-asymptotics of the stationary distribution and scaling of extrema of the process on large time interval. Our results imply that the asymptotic steady-state scaled number of servers with queue length two or larger exhibits an exponential tail, whereas that for the number of idle servers turns out to be Gaussian. From the methodological point of view, the diffusion process under consideration goes beyond the state-of-the-art techniques in the study of the steady state of diffusion processes. Lack of any closed-form expression for the steady state and intricate interdependency of the process dynamics on its local times make the analysis significantly challenging. We develop a technique involving the theory of regenerative processes that provides a tractable form for the stationary measure, and in conjunction with several sharp hitting time estimates, acts as a key vehicle in establishing the results. The technique and the intermediate results might be of independent interest, and can possibly be used in understanding the bulk behavior of the process.

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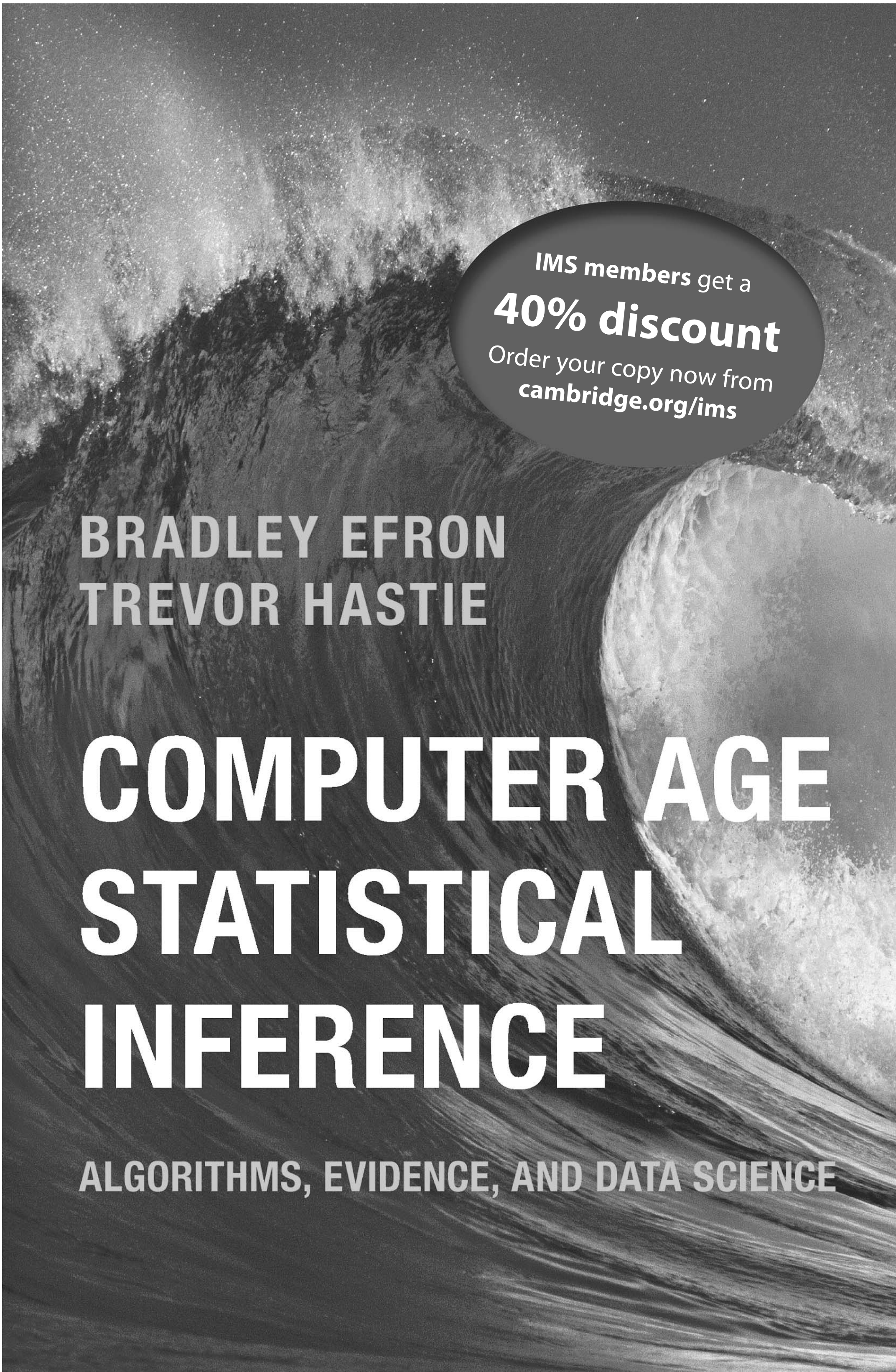
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