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STOCHASTIC REPRESENTATIONS FOR SOLUTIONS TO PARABOLIC DIRICHLET PROBLEMS FOR NONLOCAL BELLMAN EQUATIONS

BY RUOTING GONG, CHENCHEN MOU AND ANDRZEJ ŚWIĘCH

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We prove a stochastic representation formula for the viscosity solution of Dirichlet terminal-boundary value problem for a degenerate Hamilton–Jacobi–Bellman integro-partial differential equation in a bounded domain. We show that the unique viscosity solution is the value function of the associated stochastic optimal control problem. We also obtain the dynamic programming principle for the associated stochastic optimal control problem in a bounded domain.

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COMPUTATIONAL METHODS FOR MARTINGALE OPTIMAL TRANSPORT PROBLEMS[1]

BY GAOYUE GUO AND JAN OBŁÓJ

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We develop computational methods for solving the *martingale optimal transport* (MOT) problem—a version of the classical optimal transport with an additional martingale constraint on the transport's dynamics. We prove that a general, multi-step multi-dimensional, MOT problem can be approximated through a sequence of *linear programming* (LP) problems which result from a discretization of the marginal distributions combined with an appropriate relaxation of the martingale condition. Further, we establish two generic approaches for discretising probability distributions, suitable respectively for the cases when we can compute integrals against these distributions or when we can sample from them. These render our main result applicable and lead to an implementable numerical scheme for solving MOT problems. Finally, specialising to the one-step model on real line, we provide an estimate of the convergence rate which, to the best of our knowledge, is the first of its kind in the literature.

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ROBUST PRICING AND HEDGING AROUND THE GLOBE

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We consider the martingale optimal transport duality for càdlàg processes with given initial and terminal laws. Strong duality and existence of dual optimizers (robust semistatic superhedging strategies) are proved for a class of payoffs that includes American, Asian, Bermudan and European options with intermediate maturity. We exhibit an optimal superhedging strategy for which the static part solves an auxiliary problem and the dynamic part is given explicitly in terms of the static part.

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AFFINE PROCESSES BEYOND STOCHASTIC CONTINUITY

BY MARTIN KELLER-RESSEL*, THORSTEN SCHMIDT† AND ROBERT WARDENGA*

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In this paper, we study time-inhomogeneous affine processes beyond the common assumption of stochastic continuity. In this setting, times of jumps can be both inaccessible and predictable. To this end, we develop a general theory of finite dimensional affine semimartingales under very weak assumptions. We show that the corresponding semimartingale characteristics have affine form and that the conditional characteristic function can be represented with solutions to measure differential equations of Riccati type. We prove existence of affine Markov processes and affine semimartingales under mild conditions and elaborate on examples and applications including affine processes in discrete time.

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POINCARÉ AND LOGARITHMIC SOBOLEV CONSTANTS FOR METASTABLE MARKOV CHAINS VIA CAPACITARY INEQUALITIES

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We investigate the metastable behavior of reversible Markov chains on possibly countable infinite state spaces. Based on a new definition of metastable Markov processes, we compute precisely the mean transition time between metastable sets. Under additional size and regularity properties of metastable sets, we establish asymptotic sharp estimates on the Poincaré and logarithmic Sobolev constant. The main ingredient in the proof is a capacitary inequality along the lines of V. Maz'ya that relates regularity properties of harmonic functions and capacities. We exemplify the usefulness of this new definition in the context of the random field Curie–Weiss model, where metastability and the additional regularity assumptions are verifiable.

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A MARTINGALE APPROACH FOR FRACTIONAL BROWNIAN MOTIONS AND RELATED PATH DEPENDENT PDES

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In this paper, we study dynamic backward problems, with the computation of conditional expectations as a special objective, in a framework where the (forward) state process satisfies a Volterra type SDE, with fractional Brownian motion as a typical example. Such processes are neither Markov processes nor semimartingales, and most notably, they feature a certain time inconsistency which makes any direct application of Markovian ideas, such as flow properties, impossible without passing to a path-dependent framework. Our main result is a functional Itô formula, extending the seminal work of Dupire (*Quant. Finance* **19** (2019) 721–729) to our more general framework. In particular, unlike in (*Quant. Finance* **19** (2019) 721–729) where one needs only to consider the stopped paths, here we need to concatenate the observed path up to the current time with a certain smooth observable curve derived from the distribution of the future paths. This new feature is due to the time inconsistency involved in this paper. We then derive the path dependent PDEs for the backward problems. Finally, an application to option pricing and hedging in a financial market with rough volatility is presented.

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CROSSING A FITNESS VALLEY AS A METASTABLE TRANSITION IN A STOCHASTIC POPULATION MODEL[1]

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We consider a stochastic model of population dynamics where each individual is characterised by a trait in $\{0, 1, \dots, L\}$ and has a natural reproduction rate, a logistic death rate due to age or competition and a probability of mutation towards neighbouring traits at each reproduction event. We choose parameters such that the induced fitness landscape exhibits a valley: mutant individuals with negative fitness have to be created in order for the population to reach a trait with positive fitness. We focus on the limit of large population and rare mutations at several speeds. In particular, when the mutation rate is low enough, metastability occurs: the exit time of the valley is an exponentially distributed random variable.

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NONPARAMETRIC SPOT VOLATILITY FROM OPTIONS[1]

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We propose a nonparametric estimator of spot volatility from noisy short-dated option data. The estimator is based on forming portfolios of options with different strikes that replicate the (risk-neutral) conditional characteristic function of the underlying price in a model-free way. The separation of volatility from jumps is done by making use of the dominant role of the volatility in the conditional characteristic function over short time intervals and for large values of the characteristic exponent. The latter is chosen in an adaptive way in order to account for the time-varying volatility. We show that the volatility estimator is near rate-optimal in minimax sense. We further derive a feasible joint central limit theorem for the proposed option-based volatility estimator and existing high-frequency return-based volatility estimators. The limit distribution is mixed Gaussian reflecting the time-varying precision in the volatility recovery.

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RIGHT MARKER SPEEDS OF SOLUTIONS TO THE KPP EQUATION WITH NOISE[1]

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We consider the one-dimensional KPP-equation driven by space–time white noise. We show that for all parameters above the critical value for survival, there exist stochastic wavelike solutions which travel with a deterministic positive linear speed. We further give a sufficient condition on the initial condition of a solution to attain this speed. Our approach is in the spirit of corresponding results for the nearest-neighbor contact process respectively oriented percolation. Here, the main difficulty arises from the moderate size of the parameter and the long range interaction. Stopping times and averaging techniques are used to overcome this difficulty.

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LARGE TOURNAMENT GAMES

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We consider a stochastic tournament game in which each player is rewarded based on her rank in terms of the completion time of her own task and is subject to cost of effort. When players are homogeneous and the rewards are purely rank dependent, the equilibrium has a surprisingly explicit characterization, which allows us to conduct comparative statics and obtain explicit solution to several optimal reward design problems. In the general case when the players are heterogenous and payoffs are not purely rank dependent, we prove the existence, uniqueness and stability of the Nash equilibrium of the associated mean field game, and the existence of an approximate Nash equilibrium of the finite-player game.

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LOCALIZATION OF THE GAUSSIAN MULTIPLICATIVE CHAOS IN THE WIENER SPACE AND THE STOCHASTIC HEAT EQUATION IN STRONG DISORDER

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We consider a *Gaussian multiplicative chaos* (GMC) measure on the classical Wiener space driven by a smoothened (Gaussian) space-time white noise. For $d \geq 3$, it was shown in (*Electron. Commun. Probab.* **21** (2016) 61) that for small noise intensity, the total mass of the GMC converges to a strictly positive random variable, while larger disorder strength (i.e., low temperature) forces the total mass to lose uniform integrability, eventually producing a vanishing limit. Inspired by strong localization phenomena for log-correlated Gaussian fields and Gaussian multiplicative chaos in the finite dimensional Euclidean spaces (*Ann. Appl. Probab.* **26** (2016) 643–690; *Adv. Math.* **330** (2018) 589–687), and related results for discrete directed polymers (*Probab. Theory Related Fields* **138** (2007) 391–410; Bates and Chatterjee (2016)), we study the endpoint distribution of a Brownian path under the *renormalized* GMC measure in this setting. We show that in the low temperature regime, the energy landscape of the system freezes and enters the so-called *glassy phase* as the entire mass of the Cesàro average of the endpoint GMC distribution stays localized in few spatial islands, forcing the endpoint GMC to be *asymptotically purely atomic* (*Probab. Theory Related Fields* **138** (2007) 391–410). The method of our proof is based on the translation-invariant compactification introduced in (*Ann. Probab.* **44** (2016) 3934–3964) and a fixed point approach related to the cavity method from spin glasses recently used in (Bates and Chatterjee (2016)) in the context of the directed polymer model in the lattice.

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DECOMPOSITIONS OF LOG-CORRELATED FIELDS WITH APPLICATIONS

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In this article, we establish novel decompositions of Gaussian fields taking values in suitable spaces of generalized functions, and then use these decompositions to prove results about Gaussian multiplicative chaos.

We prove two decomposition theorems. The first one is a global one and says that if the difference between the covariance kernels of two Gaussian fields, taking values in some Sobolev space, has suitable Sobolev regularity, then these fields differ by a Hölder continuous Gaussian process. Our second decomposition theorem is more specialized and is in the setting of Gaussian fields whose covariance kernel has a logarithmic singularity on the diagonal—or log-correlated Gaussian fields. The theorem states that any log-correlated Gaussian field X can be decomposed locally into a sum of a Hölder continuous function and an independent almost $\star$-scale invariant field (a special class of stationary log-correlated fields with 'cone-like' white noise representations). This decomposition holds whenever the term g in the covariance kernel $C_X(x, y) = \log(1/|x-y|) + g(x, y)$ has locally $H^{d+\varepsilon}$ Sobolev smoothness.

We use these decompositions to extend several results that have been known basically only for $\star$-scale invariant fields to general log-correlated fields. These include the existence of critical multiplicative chaos, analytic continuation of the subcritical chaos in the so-called inverse temperature parameter β, as well as generalised Onsager-type covariance inequalities which play a role in the study of imaginary multiplicative chaos.

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ZERO TEMPERATURE LIMIT FOR THE BROWNIAN DIRECTED POLYMER AMONG POISSONIAN DISASTERS

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We study a continuum model of directed polymer in random environment. The law of the polymer is defined as the Brownian motion conditioned to survive among space-time Poissonian disasters. This model is well studied in the positive temperature regime. However, at zero-temperature, even the existence of the free energy has not been proved. In this article, we show that the free energy exists and is continuous at zero-temperature.

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CUTOFF FOR THE CYCLIC ADJACENT TRANSPOSITION SHUFFLE

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We study the cyclic adjacent transposition (CAT) shuffle of n cards, which is a systematic scan version of the random adjacent transposition (AT) card shuffle. In this paper, we prove that the CAT shuffle exhibits cutoff at $\frac{n^3}{2\pi^2}\log n$, which concludes that it is twice as fast as the AT shuffle. This is the first verification of cutoff phenomenon for a time-inhomogeneous card shuffle.

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Key words and phrases. Markov chains, mixing time, adjacent transpositions, time inhomogeneous card shuffles, cutoff phenomenon.

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CONTINUOUS-TIME DUALITY FOR SUPERREPLICATION WITH TRANSIENT PRICE IMPACT

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We establish a superreplication duality in a continuous-time financial model as in (Bank and Voß (2018)) where an investor's trades adversely affect bid- and ask-prices for a risky asset and where market resilience drives the resulting spread back towards zero at an exponential rate. Similar to the literature on models with a constant spread (cf., e.g., *Math. Finance* **6** (1996) 133–165; *Ann. Appl. Probab.* **20** (2010) 1341–1358; *Ann. Appl. Probab.* **27** (2017) 1414–1451), our dual description of superreplication prices involves the construction of suitable absolutely continuous measures with martingales close to the unaffected reference price. A novel feature in our duality is a liquidity weighted L^2-norm that enters as a measurement of this closeness and that accounts for strategy dependent spreads. As applications, we establish optimality of buy-and-hold strategies for the superreplication of call options and we prove a verification theorem for utility maximizing investment strategies.

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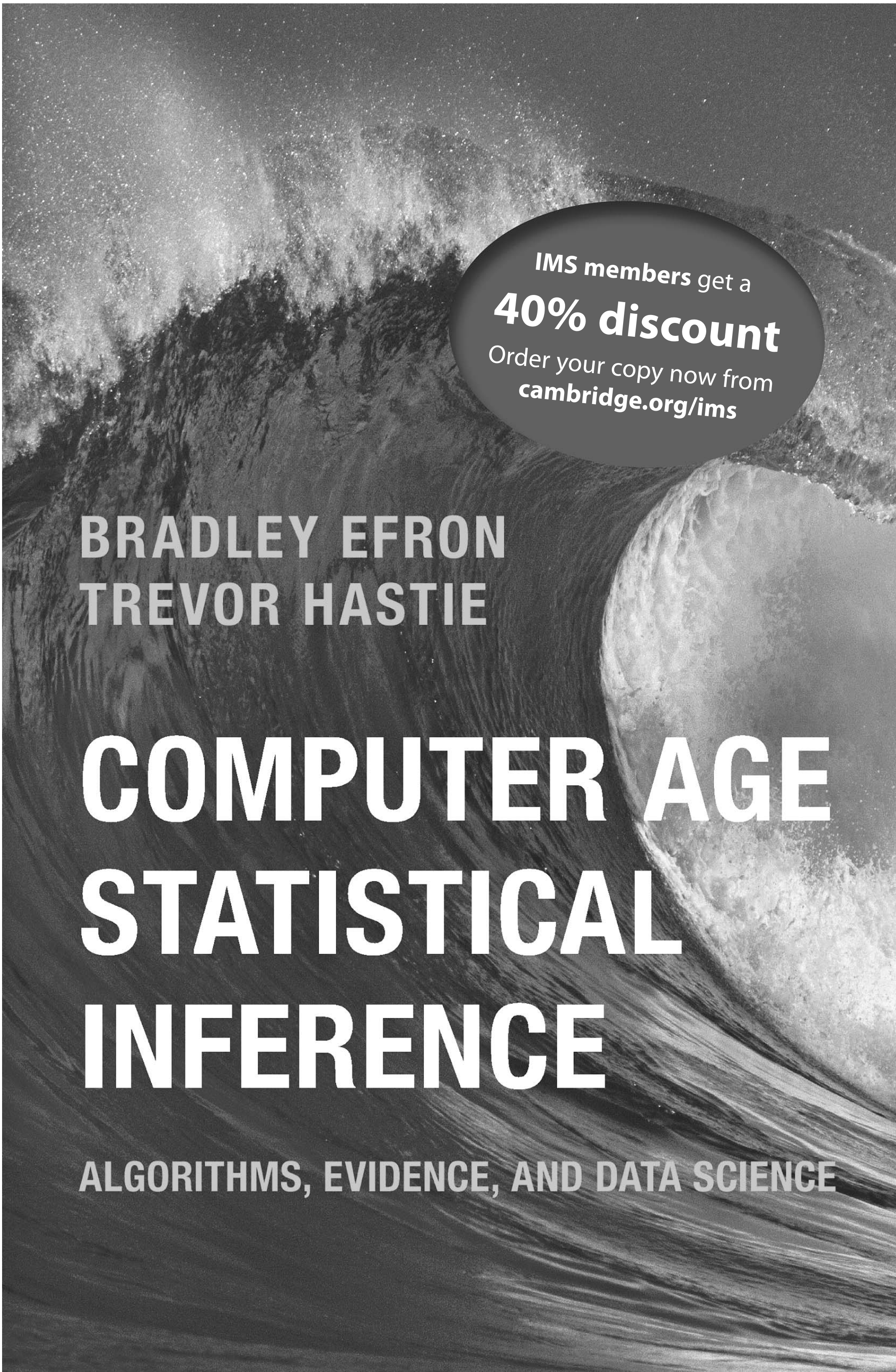
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