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NONPARAMETRIC ESTIMATION FOR LINEAR SPDES FROM LOCAL MEASUREMENTS

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The coefficient function of the leading differential operator is estimated from observations of a linear stochastic partial differential equation (SPDE). The estimation is based on continuous time observations which are localised in space. For the asymptotic regime with fixed time horizon and with the spatial resolution of the observations tending to zero, we provide rate-optimal estimators and establish scaling limits of the deterministic PDE and of the SPDE on growing domains. The estimators are robust to lower order perturbations of the underlying differential operator and achieve the parametric rate even in the nonparametric setup with a spatially varying coefficient. A numerical example illustrates the main results.

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CRANK–NICOLSON SCHEME FOR STOCHASTIC DIFFERENTIAL EQUATIONS DRIVEN BY FRACTIONAL BROWNIAN MOTIONS

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We study the Crank–Nicolson scheme for stochastic differential equations (SDEs) driven by a multidimensional fractional Brownian motion with Hurst parameter $H > 1/2$. It is well known that for ordinary differential equations with proper conditions on the regularity of the coefficients, the Crank–Nicolson scheme achieves a convergence rate of n^{-2}, regardless of the dimension. In this paper we show that, due to the interactions between the driving processes, the corresponding Crank–Nicolson scheme for m-dimensional SDEs has a slower rate than for one-dimensional SDEs. Precisely, we shall prove that when the fBm is one-dimensional and when the drift term is zero, the Crank–Nicolson scheme achieves the convergence rate n^{-2H}, and when the drift term is nonzero, the exact rate turns out to be $n^{-\frac{1}{2}-H}$. In the general multidimensional case the exact rate equals $n^{\frac{1}{2}-2H}$. In all these cases the asymptotic error is proved to satisfy some linear SDE. We also consider the degenerated cases when the asymptotic error equals zero.

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THE INTERCHANGE PROCESS ON HIGH-DIMENSIONAL PRODUCTS

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We resolve a long-standing conjecture of Wilson (*Ann. Appl. Probab.* **14** (2004) 274–325), reiterated by Oliveira (2016), asserting that the mixing time of the interchange process with unit edge rates on the n-dimensional hypercube is of order n. This follows from a sharp inequality established at the level of Dirichlet forms, from which we also deduce that macroscopic cycles emerge in constant time, and that the log-Sobolev constant of the exclusion process is of order 1. Beyond the hypercube, our results apply to cartesian products of arbitrary graphs of fixed size, shedding light on a broad conjecture of Oliveira (*Ann. Probab.* **41** (2013) 871–913).

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THE SLOW BOND RANDOM WALK AND THE SNAPPING OUT BROWNIAN MOTION

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We consider the continuous time symmetric random walk with a slow bond on $\mathbb{Z}$, which rates are equal to $1/2$ for all bonds, except for the bond of vertices $\{-1, 0\}$, which associated rate is given by $\alpha n^{-\beta}/2$, where $\alpha > 0$ and $\beta \in [0, \infty]$ are the parameters of the model. We prove here a functional central limit theorem for the random walk with a slow bond: if $\beta \in [0, 1)$, then it converges to the usual Brownian motion. If $\beta \in (1, \infty]$, then it converges to the reflected Brownian motion. And at the critical value $\beta = 1$, it converges to the *snapping out Brownian motion* (SNOB) of parameter $\kappa = 2\alpha$, which is a Brownian type-process recently constructed by A. Lejay in *Ann. Appl. Probab.* **26** (2016) 1727–1742. We also provide Berry–Esseen estimates in the dual bounded Lipschitz metric for the weak convergence of one-dimensional distributions, which we believe to be sharp.

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THE RANDOM CONNECTION MODEL AND FUNCTIONS OF EDGE-MARKED POISSON PROCESSES: SECOND ORDER PROPERTIES AND NORMAL APPROXIMATION

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The random connection model is a random graph whose vertices are given by the points of a Poisson process and whose edges are obtained by randomly connecting pairs of Poisson points in a position dependent but independent way. We study first and second order properties of the numbers of components isomorphic to given finite connected graphs. For increasing observation windows in an Euclidean setting we prove qualitative multivariate and quantitative univariate central limit theorems for these component counts as well as a qualitative central limit theorem for the total number of finite components. To this end we first derive general results for functions of edge marked Poisson processes, which we believe to be of independent interest.

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THE MAJORITY VOTE PROCESS AND OTHER CONSENSUS PROCESSES ON TREES

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The majority vote process was one of the first interacting particle systems to be investigated. It can be described briefly as follows. There are two possible opinions at each site of a graph G. At rate $1 - \varepsilon$, the opinion at a site aligns with the majority opinion at its neighboring sites and, at rate ε, the opinion at a site is randomized due to noise, where $\varepsilon \in [0, 1]$ is a parameter.

Despite the simple dynamics of the majority vote process, its equilibrium behavior is difficult to analyze when the noise rate is small but positive. In particular, when the underlying graph is $G = \mathbb{Z}^n$ with $n \geq 2$, it is not known whether the process possesses more than one equilibrium. This is surprising, especially in light of the close analogy between this model and the stochastic Ising model, where much more is known.

Here, we study the majority vote process on the infinite tree $\mathbb{T}_d$ with vertex degree d. For $d \geq 5$ and small noise, we show that there are uncountably many mutually singular equilibria, with convergence to such an equilibrium occurring exponentially quickly from nearby initial states.

Our methods are quite flexible and extend to a broader class of models, *consensus processes*. This class includes the stochastic Ising model and other processes in which the dynamics at a site depend on the number of neighbors holding a given opinion. All of our proofs are carried out in this broader context.

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ABSENCE OF WARM PERCOLATION IN THE VERY STRONG REINFORCEMENT REGIME

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We study a class of reinforcement models involving a Poisson process on the vertices of certain infinite graphs G. When a vertex fires, one of the edges incident to that vertex is selected. The edge selection is biased towards edges that have been selected many times previously, and a parameter α governs the strength of this bias.

We show that for various graphs (including all graphs of bounded degree), if $\alpha \gg 1$ (the very strong reinforcement regime) then the random subgraph consisting of edges that are ever selected by this process does not percolate (all connected components are finite).

Combined with results appearing in a companion paper, this proves that on these graphs, with α sufficiently large, all connected components are in fact trees. If the Poisson firing rates are constant over the vertices, then these trees are of diameter at most 3.

The proof of nonpercolation relies on coupling with a percolation-type model that may be of interest in its own right.

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POLLUTED BOOTSTRAP PERCOLATION IN THREE DIMENSIONS

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In the polluted bootstrap percolation model, vertices of the cubic lattice $\mathbb{Z}^3$ are independently declared initially occupied with probability p or closed with probability q, where $p + q \leq 1$. Under the standard (respectively, modified) bootstrap rule, a vertex becomes occupied at a subsequent step if it is not closed and it has at least 3 occupied neighbors (respectively, an occupied neighbor in each coordinate). We study the final density of occupied vertices as $p, q \to 0$. We show that this density converges to 1 if $q \ll p^3(\log p^{-1})^{-3}$ for both standard and modified rules. Our principal result is a complementary bound with a matching power for the modified model: there exists C such that the final density converges to 0 if $q > Cp^3$. For the standard model, we establish convergence to 0 under the stronger condition $q > Cp^2$.

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CRAMÉR-TYPE MODERATE DEVIATION THEOREMS FOR NONNORMAL APPROXIMATION

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A Cramér-type moderate deviation theorem quantifies the relative error of the tail probability approximation. It provides a criterion whether the limiting tail probability can be used to estimate the tail probability under study. Chen, Fang and Shao (2013) obtained a general Cramér-type moderate result using Stein's method when the limiting was a normal distribution. In this paper, Cramér-type moderate deviation theorems are established for nonnormal approximation under a general Stein identity, which is satisfied via the exchangeable pair approach and Stein's coupling. In particular, a Cramér-type moderate deviation theorem is obtained for the general Curie–Weiss model and the imitative monomer-dimer mean-field model.

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FLUCTUATIONS OF THE ARCTIC CURVE IN THE TILINGS OF THE AZTEC DIAMOND ON RESTRICTED DOMAINS

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We consider uniform random domino tilings of the restricted Aztec diamond which is obtained by cutting off an upper triangular part of the Aztec diamond by a horizontal line. The restriction line asymptotically touches the arctic circle that is the limit shape of the north polar region in the unrestricted model. We prove that the rescaled boundary of the north polar region in the restricted domain converges to the Airy$_2$ process conditioned to stay below a parabola with explicit continuous statistics and the finite dimensional distribution kernels. The limit is the hard-edge tacnode process which was first discovered in the framework of nonintersecting Brownian bridges. The proof relies on a random walk representation of the correlation kernel of the nonintersecting line ensemble which corresponds to a random tiling.

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GUE × GUE LIMIT LAW AT HARD SHOCKS IN ASEP

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We consider the asymmetric simple exclusion process (ASEP) on $\mathbb{Z}$ with initial data such that in the large time particle density $\rho(\cdot)$ a discontinuity (shock) at the origin is created. At the shock, the value of ρ jumps from zero to one, but $\rho(-\varepsilon), 1-\rho(\varepsilon) > 0$ for any $\varepsilon > 0$. We are interested in the rescaled position of a tagged particle which enters the shock with positive probability. We show that, inside the shock region, the particle position has the KPZ-typical $1/3$ fluctuations, a $F_{\mathrm{GUE}} \times F_{\mathrm{GUE}}$ limit law and a degenerated correlation length. Outside the shock region, the particle fluctuates as if there was no shock. Our arguments are mostly probabilistic, in particular, the mixing times of countable state space ASEPs are instrumental to study the fluctuations at shocks.

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BAND-LIMITED MIMICRY OF POINT PROCESSES BY POINT PROCESSES SUPPORTED ON A LATTICE

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We say that one point process on the line $\mathbb{R}$ mimics another at a bandwidth B if for each $n \geq 1$ the two point processes have n-level correlation functions that agree when integrated against all band-limited test functions on bandwidth $[-B, B]$. This paper asks the question of for what values a and B can a given point process on the real line be mimicked at bandwidth B by a point process supported on the lattice $a\mathbb{Z}$. For Poisson point processes we give a complete answer for allowed parameter ranges (a, B), and for the sine process we give existence and nonexistence regions for parameter ranges. The results for the sine process have an application to the alternative hypothesis regarding the scaled spacing of zeros of the Riemann zeta function, given in a companion paper.

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OPTIMAL CORRECTOR ESTIMATES ON PERCOLATION CLUSTER

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We prove optimal quantitative estimates on the first-order correctors on supercritical percolation clusters: we show that they are bounded in dimension larger than 3 and have logarithmic growth in dimension 2 in the sense of stretched exponential moments. The main ingredients are a renormalization scheme of the supercritical percolation cluster, following the works of Pisztora (*Probab. Theory Related Fields* **104** (1996) 427–466); large-scale regularity estimates developed by Armstrong and the author in (*Comm. Pure Appl. Math.* **71** (2018) 1717–1849); and a nonlinear concentration inequality of the Efron–Stein type which is used to transfer quantitative information from the environment to the correctors.

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A NEW MCKEAN–VLASOV STOCHASTIC INTERPRETATION OF THE PARABOLIC-PARABOLIC KELLER–SEGEL MODEL: THE TWO-DIMENSIONAL CASE

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Recently, we proposed a new stochastic interpretation of the parabolic-parabolic Keller–Segel system without cut-off via a McKean–Vlasov stochastic process. The process was defined through an original type of interaction kernel which involved, in a singular way, all its past time marginal distributions. In the present paper, we study this McKean–Vlasov representation in the two-dimensional case. In this setting, there exists a possibility of a blow-up in finite time for the Keller–Segel system if some parameters of the model are large. Indeed, we prove the global in time well-posedness of the McKean–Vlasov process under some constraints involving a parameter of the model and the initial datum. Under these constraints, we also prove the global well-posedness for the Keller–Segel model in the plane.

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INDEFINITE STOCHASTIC LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEMS WITH RANDOM COEFFICIENTS: CLOSED-LOOP REPRESENTATION OF OPEN-LOOP OPTIMAL CONTROLS

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This paper is concerned with a stochastic linear-quadratic optimal control problem in a finite time horizon, where the coefficients of the control system are allowed to be random, and the weighting matrices in the cost functional are allowed to be random and indefinite. It is shown, with a Hilbert space approach, that for the existence of an open-loop optimal control, the convexity of the cost functional (with respect to the control) is necessary; and the uniform convexity, which is slightly stronger, turns out to be sufficient, which also leads to the unique solvability of the associated stochastic Riccati equation. Further, it is shown that the open-loop optimal control admits a closed-loop representation. In addition, some sufficient conditions are obtained for the uniform convexity of the cost functional, which are strictly more general than the classical conditions that the weighting matrix-valued processes are positive (semi-) definite.

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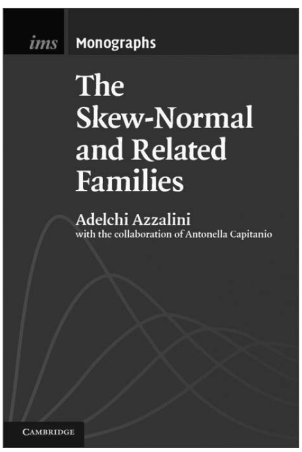

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Adelchi Azzalini
in collaboration with Antonella Capitanio

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