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RECURRENCE OF TWO-DIMENSIONAL QUEUEING PROCESSES, AND RANDOM WALK EXIT TIMES FROM THE QUADRANT

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Let $X = (X_1, X_2)$ be a two-dimensional random variable and $X(n)$, $n \in \mathbb{N}$, a sequence of i.i.d. copies of X. The associated random walk is $S(n) = X(1) + \cdots + X(n)$. The corresponding absorbed-reflected walk $W(n)$, $n \in \mathbb{N}$, in the first quadrant is given by $W(0) = x \in \mathbb{R}_+^2$ and $W(n) = \max\{0, W(n-1) - X(n)\}$, where the maximum is taken coordinate-wise. This is often called the Lindley process and models the waiting times in a two-server queue. We characterize recurrence of this process, assuming suitable, rather mild moment conditions on X. It turns out that this is directly related with the tail asymptotics of the exit time of the random walk $x + S(n)$ from the quadrant, so that the main part of this paper is devoted to an analysis of that exit time in relation with the drift vector, that is, the expectation of X.

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SUBMODULAR MEAN FIELD GAMES: EXISTENCE AND APPROXIMATION OF SOLUTIONS

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We study mean field games with scalar Itô-type dynamics and costs that are *submodular* with respect to a suitable order relation on the state and measure space. The submodularity assumption has a number of interesting consequences. First, it allows us to prove existence of solutions via an application of Tarski's fixed point theorem, covering cases with discontinuous dependence on the measure variable. Second, it ensures that the set of solutions enjoys a lattice structure: in particular, there exist minimal and maximal solutions. Third, it guarantees that those two solutions can be obtained through a simple learning procedure based on the iterations of the best-response-map. The mean field game is first defined over ordinary stochastic controls, then extended to relaxed controls. Our approach also allows us to prove existence of a strong solution for a class of submodular mean field games with common noise, where the representative player at equilibrium interacts with the (conditional) mean of its state's distribution.

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A SIMPLE FOURIER ANALYTIC PROOF OF THE AKT OPTIMAL MATCHING THEOREM

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We present a short and elementary proof of the Ajtai–Komlós–Tusnády (AKT) optimal matching theorem in dimension 2 via Fourier analysis and a smoothing argument. The upper bound applies to more general families of samples, including dependent variables, of interest in the study of rates of convergence for empirical measures. Following the recent pde approach by L. Ambrosio, F. Stra and D. Trevisan, we also adapt a simple proof of the lower bound.

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BREAKING A CHAIN OF INTERACTING BROWNIAN PARTICLES

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We investigate the behaviour of a finite chain of Brownian particles, interacting through a pairwise linear force, with one end of the chain fixed and the other end pulled away at slow speed, in the limit of slow speed and small Brownian noise.

We study the instant when the chain "breaks," that is, the distance between two neighbouring particles becomes larger than a certain threshold. There are three different regimes depending on the relation between the speed of pulling and the Brownian noise. We provide weak limit theorems for the break time and the break position for each regime.

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RANDOMIZED HAMILTONIAN MONTE CARLO AS SCALING LIMIT OF THE BOUNCY PARTICLE SAMPLER AND DIMENSION-FREE CONVERGENCE RATES

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The bouncy particle sampler is a Markov chain Monte Carlo method based on a nonreversible piecewise deterministic Markov process. In this scheme, a particle explores the state space of interest by evolving according to a linear dynamics which is altered by bouncing on the hyperplane perpendicular to the gradient of the negative log-target density at the arrival times of an inhomogeneous poisson process (PP) and by randomly perturbing its velocity at the arrival times of a homogeneous PP. Under regularity conditions, we show here that the process corresponding to the first component of the particle and its corresponding velocity converges weakly towards a randomized Hamiltonian Monte Carlo (RHMC) process as the dimension of the ambient space goes to infinity. RHMC is another piecewise deterministic nonreversible Markov process where a Hamiltonian dynamics is altered at the arrival times of a homogeneous PP by randomly perturbing the momentum component. We then establish dimension-free convergence rates for RHMC for strongly log-concave targets with bounded Hessians using coupling ideas and hypocoercivity techniques. We use our understanding of the mixing properties of the limiting RHMC process to choose the refreshment rate parameter of BPS. This results in significantly better performance in our simulation study than previously suggested guidelines.

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THE PLANTED MATCHING PROBLEM: PHASE TRANSITIONS AND EXACT RESULTS

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We study the problem of recovering a planted matching in randomly weighted complete bipartite graphs $K_{n,n}$. For some unknown perfect matching M^*, the weight of an edge is drawn from one distribution P if $e \in M^*$ and another distribution Q if $e \notin M^*$. Our goal is to infer M^*, exactly or approximately, from the edge weights. In this paper we take $P = \exp(\lambda)$ and $Q = \exp(1/n)$, in which case the maximum-likelihood estimator of M^* is the minimum-weight matching $M_{\min}$. We obtain precise results on the overlap between M^* and $M_{\min}$, that is, the fraction of edges they have in common. For $\lambda \geq 4$ we have almost perfect recovery, with overlap $1 - o(1)$ with high probability. For $\lambda < 4$ the expected overlap is an explicit function $\alpha(\lambda) < 1$: we compute it by generalizing Aldous' celebrated proof of the $\zeta(2)$ conjecture for the unplanted model, using local weak convergence to relate $K_{n,n}$ to a type of weighted infinite tree, and then deriving a system of differential equations from a message-passing algorithm on this tree.

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LIMIT THEORY OF COMBINATORIAL OPTIMIZATION FOR RANDOM GEOMETRIC GRAPHS

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In the random geometric graph $G(n, r_n)$, n vertices are placed randomly in Euclidean d-space and edges are added between any pair of vertices distant at most r_n from each other. We establish strong laws of large numbers (LLNs) for a large class of graph parameters, evaluated for $G(n, r_n)$ in the thermodynamic limit with $nr_n^d =$ const., and also in the dense limit with $nr_n^d \to \infty$, $r_n \to 0$. Examples include domination number, independence number, clique-covering number, eternal domination number and triangle packing number. The general theory is based on certain subadditivity and superadditivity properties, and also yields LLNs for other functionals such as the minimum weight for the traveling salesman, spanning tree, matching, bipartite matching and bipartite traveling salesman problems, for a general class of weight functions with at most polynomial growth of order $d - \varepsilon$, under thermodynamic scaling of the distance parameter.

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RECONSTRUCTING TREES FROM TRACES

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We study the problem of learning a node-labeled tree given independent traces from an appropriately defined deletion channel. This problem, tree trace reconstruction, generalizes string trace reconstruction, which corresponds to the tree being a path. For many classes of trees, including complete trees and spiders, we provide algorithms that reconstruct the labels using only a polynomial number of traces. This exhibits a stark contrast to known results on string trace reconstruction, which require exponentially many traces, and where a central open problem is to determine whether a polynomial number of traces suffice. Our techniques combine novel combinatorial and complex analytic methods.

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LARGE DEVIATIONS FOR THE EMPIRICAL MEASURE OF THE ZIG-ZAG PROCESS

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The zig-zag process is a piecewise deterministic Markov process in position and velocity space. The process can be designed to have an arbitrary Gibbs type marginal probability density for its position coordinate, which makes it suitable for Monte Carlo simulation of continuous probability distributions. An important question in assessing the efficiency of this method is how fast the empirical measure converges to the stationary distribution of the process. In this paper we provide a partial answer to this question by characterizing the large deviations of the empirical measure from the stationary distribution. Based on the Feng–Kurtz approach, we develop an abstract framework aimed at encompassing piecewise deterministic Markov processes in position-velocity space. We derive explicit conditions for the zig-zag process to allow the Donsker–Varadhan variational formulation of the rate function, both for a compact setting (the torus) and one-dimensional Euclidean space. Finally we derive an explicit expression for the Donsker–Varadhan functional for the case of a compact state space and use this form of the rate function to address a key question concerning the optimal choice of the switching rate of the zig-zag process.

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REGENERATIVE PROPERTIES OF THE LINEAR HAWKES PROCESS WITH UNBOUNDED MEMORY

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Regenerative properties of the linear Hawkes process are proved under minimal assumptions on the transfer function, which may have unbounded support. For this, an original construction of the Hawkes process as a functional of a Poisson point process is derived from the immigration-birth representation, and the independence properties of the Poisson point process are exploited to exhibit regeneration times which are anticipative and not even measurable w.r.t. the Hawkes process. The regeneration time is interpreted as the renewal time at zero of an $M/G/\infty$ queue, which yields a formula for its Laplace transform. When the transfer function has exponential moments, we stochastically dominate the cluster length by exponential random variables with computable parameters. This provides explicit bounds on the Laplace transform of the regeneration time in terms of simple integrals or of special functions, which yields an explicit negative upper-bound on its abscissa of convergence. The power of the regenerative properties is showcased by being applied to long-time asymptotic results for a class of sliding window statistical estimators, using coupling and sample-path decomposition techniques.

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UNIVERSALITY FOR LANGEVIN-LIKE SPIN GLASS DYNAMICS

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We study dynamics for asymmetric spin glass models, proposed by Hertz et al. and Sompolinsky et al. in the 1980's in the context of neural networks: particles evolve via a modified Langevin dynamics for the Sherrington–Kirkpatrick model with soft spins, whereby the disorder is i.i.d. standard Gaussian rather than symmetric. Ben Arous and Guionnet (*Probab. Theory Related Fields* **102** (1995) 455–509), followed by Guionnet (*Probab. Theory Related Fields* **109** (1997) 183–215), proved for Gaussian interactions that as the number of particles grows, the short-term empirical law of this dynamics converges a.s. to a nonrandom law $\mu_\star$ of a "self-consistent single spin dynamics," as predicted by physicists. Here we obtain universality of this fact: For asymmetric disorder given by i.i.d. variables of zero mean, unit variance and exponential or better tail decay, at every temperature, the empirical law of sample paths of the Langevin-like dynamics in a fixed time interval has the same a.s. limit $\mu_\star$.

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PALM THEORY, RANDOM MEASURES AND STEIN COUPLINGS

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We establish a general Berry–Esseen type bound which gives optimal bounds in many situations under suitable moment assumptions. By combining the general bound with Palm theory, we deduce a new error bound for assessing the accuracy of normal approximation to statistics arising from random measures, including stochastic geometry. We illustrate the use of the bound in four examples: completely random measures, excursion random measure of a locally dependent random process, and the total edge length of Ginibre–Voronoi tessellations and of Poisson–Voronoi tessellations. Moreover, we apply the general bound to Stein couplings and discuss the special cases of local dependence and additive functionals in occupancy problems.

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A WEAK SOLUTION THEORY FOR STOCHASTIC VOLTERRA EQUATIONS OF CONVOLUTION TYPE

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We obtain general weak existence and stability results for stochastic convolution equations with jumps under mild regularity assumptions, allowing for non-Lipschitz coefficients and singular kernels. Our approach relies on weak convergence in L^p spaces. The main tools are new a priori estimates on Sobolev–Slobodeckij norms of the solution, as well as a novel martingale problem that is equivalent to the original equation. This leads to generic approximation and stability theorems in the spirit of classical martingale problem theory. We also prove uniqueness and path regularity of solutions under additional hypotheses. To illustrate the applicability of our results, we consider scaling limits of nonlinear Hawkes processes and approximations of stochastic Volterra processes by Markovian semimartingales.

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ASYMPTOTIC ANALYSIS FOR EXTREME EIGENVALUES OF PRINCIPAL MINORS OF RANDOM MATRICES

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Consider a standard white Wishart matrix with parameters n and p. Motivated by applications in high-dimensional statistics and signal processing, we perform asymptotic analysis on the maxima and minima of the eigenvalues of all the $m \times m$ principal minors, under the asymptotic regime that n, p, m go to infinity. Asymptotic results concerning extreme eigenvalues of principal minors of real Wigner matrices are also obtained. In addition, we discuss an application of the theoretical results to the construction of compressed sensing matrices, which provides insights to compressed sensing in signal processing and high-dimensional linear regression in statistics.

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ASYMPTOTIC BEHAVIOR OF THE OCCUPANCY DENSITY FOR OBLIQUELY REFLECTED BROWNIAN MOTION IN A HALF-PLANE AND MARTIN BOUNDARY

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Let $\boldsymbol{\pi}$ be the occupancy density of an obliquely reflected Brownian motion in the half plane and let (ρ, α) be the polar coordinates of a point in the upper half plane. This work determines the exact asymptotic behavior of $\boldsymbol{\pi}(\rho, \alpha)$ as $\rho \to \infty$ with $\alpha \in (0, \pi)$. We find explicit functions a, b, c such that

$$\boldsymbol{\pi}(\rho, \alpha) \underset{\rho \to \infty}{\sim} a(\alpha)\rho^{b(\alpha)} e^{-c(\alpha)\rho}.$$

This closes an open problem first stated by Professor J. Michael Harrison in August 2013. We also compute the exact asymptotics for the tail distribution of the boundary occupancy measure and we obtain an explicit integral expression for $\boldsymbol{\pi}$. We conclude by finding the Martin boundary of the process and giving all of the corresponding harmonic functions satisfying an oblique Neumann boundary problem.

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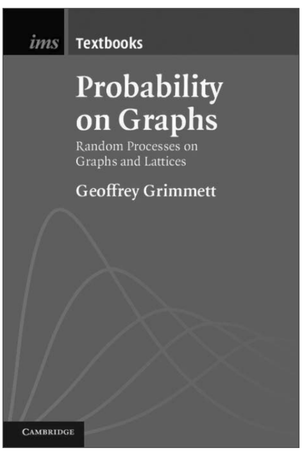

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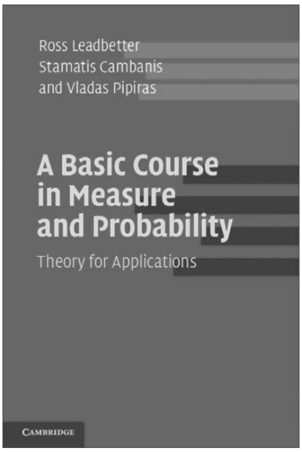

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