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CONVERGENCE ANALYSIS OF MACHINE LEARNING ALGORITHMS FOR THE NUMERICAL SOLUTION OF MEAN FIELD CONTROL AND GAMES: II—THE FINITE HORIZON CASE

BY RENÉ CARMONA[a] AND MATHIEU LAURIÈRE[b]

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We propose two numerical methods for the optimal control of McKean–Vlasov dynamics in finite time horizon. Both methods are based on the introduction of a suitable loss function defined over the parameters of a neural network. This allows the use of machine learning tools, and efficient implementations of stochastic gradient descent in order to perform the optimization. In the first method, the loss function stems directly from the optimal control problem. The second method tackles a generic forward-backward stochastic differential equation system (FBSDE) of McKean–Vlasov type, and relies on suitable reformulation as a mean field control problem. To provide a guarantee on how our numerical schemes approximate the solution of the original mean field control problem, we introduce a new optimization problem, directly amenable to numerical computation, and for which we rigorously provide an error rate. Several numerical examples are provided. Both methods can easily be applied to certain problems with common noise, which is not the case with the existing technology. Furthermore, although the first approach is designed for mean field control problems, the second is more general and can also be applied to the FBSDEs arising in the theory of mean field games.

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RISK-SENSITIVE CONTROL FOR A CLASS OF DIFFUSIONS WITH JUMPS

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We consider a class of diffusions controlled through the drift and jump size, and driven by a jump Lévy process and a nondegenerate Wiener process, and we study infinite horizon (ergodic) risk-sensitive control problems for this model. We start with the controlled Dirichlet eigenvalue problem in smooth bounded domains, which also allows us to generalize current results in the literature on exit rate control problems. Then we consider the infinite horizon average risk-sensitive minimization and maximization problems on the whole domain. Under suitable hypotheses, we establish existence and uniqueness of a principal eigenfunction for the Hamilton–Jacobi–Bellman (HJB) operator on the whole space, and fully characterize stationary Markov optimal controls as the measurable selectors of this HJB equation.

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WEAK AND STRONG ERROR ANALYSIS FOR MEAN-FIELD RANK-BASED PARTICLE APPROXIMATIONS OF ONE-DIMENSIONAL VISCOUS SCALAR CONSERVATION LAWS

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In this paper, we analyse the rate of convergence of a system of N interacting particles with mean-field rank-based interaction in the drift coefficient and constant diffusion coefficient. We first adapt arguments by Kolli and Shkolnikov (*Ann. Probab.* **46** (2018) 1042–1069) to check trajectorial propagation of chaos with optimal rate $N^{-1/2}$ to the associated stochastic differential equations nonlinear in the sense of McKean. We next relax the assumptions needed by Bossy (*Math. Comp.* **73** (2004) 777–812) to check the convergence in $L^1(\mathbb{R})$ with rate $\mathcal{O}(\frac{1}{\sqrt{N}}+h)$ of the empirical cumulative distribution function of the Euler discretization with step h of the particle system to the solution of a one-dimensional viscous scalar conservation law. Last, we prove that the bias of this stochastic particle method behaves as $\mathcal{O}(\frac{1}{N}+h)$. We provide numerical results which confirm our theoretical estimates.

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CONTINUUM MODELS OF DIRECTED POLYMERS ON DISORDERED DIAMOND FRACTALS IN THE CRITICAL CASE

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We construct and study a family of random continuum polymer measures $\mathbf{M}_r$ corresponding to limiting partition function laws recently derived in a weak-coupling regime for polymer models on hierarchical graphs with marginally relevant disorder. The continuum polymers, which we refer to as *directed paths*, are identified with isometric embeddings of the unit interval [0, 1] into a compact diamond fractal having Hausdorff dimension two, and there is a natural "uniform" probability measure, μ, over the space of directed paths, Γ. Realizations of the random path measures $\mathbf{M}_r$ exhibit strong localization properties in comparison to their subcritical counterparts in which the diamond fractal has dimension less than two. Whereas two paths $p, q \in \Gamma$ sampled independently using the pure measure μ have only finitely many intersections with probability one, a realization of the disordered product measure $\mathbf{M}_r \times \mathbf{M}_r$ a.s. assigns positive weight to the set of pairs of paths (p, q) whose intersection sets are uncountable but of Hausdorff dimension zero. We give a more refined characterization of the size of these dimension-zero sets using generalized (logarithmic) Hausdorff measures. The law of the random measure $\mathbf{M}_r$ cannot be constructed as a subcritical Gaussian multiplicative chaos because the coupling strength to the Gaussian field would, in a formal sense, have to be infinite.

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SCALING PROPERTIES OF A MOVING POLYMER

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We set up an SPDE model for a moving, weakly self-avoiding polymer with intrinsic length J taking values in $(0, \infty)$. Our main result states that the effective radius of the polymer is approximately $J^{5/3}$; evidently for large J the polymer undergoes stretching. This contrasts with the equilibrium situation without the time variable, where many earlier results show that the effective radius is approximately J.

For such a moving polymer taking values in $\mathbf{R}^2$, we offer a conjecture that the effective radius is approximately $J^{5/4}$.

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CROSSOVER TIMES IN BIPARTITE NETWORKS WITH ACTIVITY CONSTRAINTS AND TIME-VARYING SWITCHING RATES

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In this paper, we study the performance of a bipartite network in which customers arrive at the nodes of the network, but not all nodes are able to serve their customers at all times. Each node can be either active or inactive, and two nodes connected by a bond cannot be active simultaneously. This situation arises in wireless random-access networks where, due to destructive interference, stations that are close to each other cannot use the same frequency band.

We consider a model where the network is bipartite, the active nodes switch themselves off at rate 1 and the inactive nodes switch themselves on at a rate that depends on time and on which half of the bipartite network they are in. An inactive node cannot become active when one of the nodes it is connected to by a bond is active. The switching protocol allows the nodes to share activity among each other. In the limit as the activation rate becomes large, we compute the crossover time between the two states where one-half of the network is active and the other half is inactive. This allows us to assess the overall activity of the network depending on the switching protocol. Our results make use of the metastability analysis for hard-core interacting particle models on finite bipartite graphs derived in an earlier paper. They are valid for a large class of bipartite networks, subject to certain assumptions. Proofs rely on a comparison with switching protocols that are not time varying, through coupling techniques.

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ON CRAMÉR–VON MISES STATISTIC FOR THE SPECTRAL DISTRIBUTION OF RANDOM MATRICES

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Let F_N and F be the empirical and limiting spectral distributions of an $N \times N$ Wigner matrix. The Cramér–von Mises (CvM) statistic is a classical goodness-of-fit statistic that characterizes the distance between F_N and F in L^2-norm. In this paper, we consider a mesoscopic approximation of the CvM statistic for Wigner matrices, and derive its limiting distribution. In the Appendix, we also give the limiting distribution of the CvM statistic (without approximation) for the toy model CUE.

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DISTANCE EVOLUTIONS IN GROWING PREFERENTIAL ATTACHMENT GRAPHS

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We study the evolution of the graph distance and weighted distance between two fixed vertices in dynamically growing random graph models. More precisely, we consider preferential attachment models with power-law exponent $\tau \in (2, 3)$, sample two vertices u_t, v_t uniformly at random when the graph has t vertices and study the evolution of the graph distance between these two fixed vertices as the surrounding graph grows. This yields a discrete-time stochastic process in $t' \geq t$, called the distance evolution. We show that there is a tight strip around the function $4\frac{\log\log(t)-\log(\log(t'/t)\vee 1)}{|\log(\tau-2)|} \vee 2$ that the distance evolution never leaves with high probability as t tends to infinity. We extend our results to weighted distances, where every edge is equipped with an i.i.d. copy of a nonnegative random variable L.

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QUENCHED LAW OF LARGE NUMBERS AND QUENCHED CENTRAL LIMIT THEOREM FOR MULTIPLAYER LEAGUES WITH ERGODIC STRENGTHS

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We propose and study a new model for competitions, specifically sports multi-player leagues where the initial strengths of the teams are independent i.i.d. random variables that evolve during different days of the league according to independent ergodic processes. The result of each match is random: the probability that a team wins against another team is determined by a function of the strengths of the two teams in the day the match is played.

Our model generalizes some previous models studied in the physical and mathematical literature and is defined in terms of different parameters that can be statistically calibrated. We prove a quenched—conditioning on the initial strengths of the teams—law of large numbers and a quenched central limit theorem for the number of victories of a team according to its initial strength.

To obtain our results, we prove a theorem of independent interest. For a stationary process $\boldsymbol{\xi} = (\boldsymbol{\xi}_i)_{i\in\mathbb{Z}_{>0}}$ satisfying a mixing condition and an independent sequence of i.i.d. random variables $(\boldsymbol{s}_i)_{i\in\mathbb{Z}_{>0}}$, we prove a quenched—conditioning on $(\boldsymbol{s}_i)_{i\in\mathbb{Z}_{>0}}$—central limit theorem for sums of the form $\sum_{i=1}^n g(\boldsymbol{\xi}_i, \boldsymbol{s}_i)$, where g is a bounded measurable function. We highlight that the random variables $g(\boldsymbol{\xi}_i, \boldsymbol{s}_i)$ are not stationary conditioning on $(\boldsymbol{s}_i)_{i\in\mathbb{Z}_{>0}}$.

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THE DISTRIBUTIONS UNDER TWO SPECIES-TREE MODELS OF THE NUMBER OF ROOT ANCESTRAL CONFIGURATIONS FOR MATCHING GENE TREES AND SPECIES TREES

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For a pair consisting of a gene tree and a species tree, the *ancestral configurations* at a species-tree internal node are the distinct sets of gene lineages that can be present at that node. The enumeration of *root* ancestral configurations—ancestral configurations at the species-tree root—assists in describing the complexity of gene-tree probability calculations in evolutionary biology. Assuming that the gene tree and species tree match in topology, we study the distribution of the number of root ancestral configurations of a random labeled tree topology under the *uniform* and *Yule–Harding* models. We employ analytic combinatorics, considering ancestral configurations in the context of additive tree parameters and using singularity analysis to evaluate asymptotic growth of the coefficients of generating functions. For both models, we obtain asymptotic lognormal distributions for the number of root ancestral configurations. For Yule–Harding random trees, we also obtain the asymptotic mean ($\sim 1.425^n$) and variance ($\sim 2.045^n$) of the number of root ancestral configurations, paralleling previous results for the uniform model (mean $(4/3)^n$, variance $\sim 1.822^n$). A methodological innovation is that to obtain the Yule–Harding asymptotic variance, singularity analysis is conducted from the Riccati differential equation that the generating function satisfies—without possessing the generating function itself.

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COEXISTENCE IN COMPETING FIRST PASSAGE PERCOLATION WITH CONVERSION

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We introduce a two-type first passage percolation competition model on infinite connected graphs as follows. Type 1 spreads through the edges of the graph at rate 1 from a single distinguished site, while all other sites are initially vacant. Once a site is occupied by type 1, it converts to type 2 at rate $\rho > 0$. Sites occupied by type 2 then spread at rate $\lambda > 0$ through vacant sites *and* sites occupied by type 1, whereas type 1 can only spread through vacant sites. If the set of sites occupied by type 1 is nonempty at all times, we say type 1 *survives*. In the case of a regular d-ary tree for $d \geq 3$, we show type 1 can survive when it is slower than type 2, provided ρ is small enough. This is in contrast to when the underlying graph is $\mathbb{Z}^d$, where for any $\rho > 0$, type 1 dies out almost surely if $\lambda > \lambda'$ for some $\lambda' < 1$.

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THE STOCHASTIC AIRY OPERATOR AT LARGE TEMPERATURE

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It was shown in (*J. Amer. Math. Soc.* **24** (2011) 919–944) that the edge of the spectrum of β ensembles converges in the large N limit to the bottom of the spectrum of the stochastic Airy operator. In the present paper, we obtain a complete description of the bottom of this spectrum when the temperature $1/\beta$ goes to ∞: we show that the point process of appropriately rescaled eigenvalues converges to a Poisson point process on $\mathbf{R}$ of intensity $e^x\,dx$ and that the eigenfunctions converge to Dirac masses centered at i.i.d. points with exponential laws. Furthermore, we obtain a precise description of the microscopic behavior of the eigenfunctions near their localization centers.

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FIRST-ORDER BEHAVIOR OF THE TIME CONSTANT IN BERNOULLI FIRST-PASSAGE PERCOLATION

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We consider the standard model of first-passage percolation on $\mathbb{Z}^d$ ($d \geq 2$), with i.i.d. passage times associated with either the edges or the vertices of the graph. We focus on the particular case where the distribution of the passage times is the Bernoulli distribution with parameter $1 - \varepsilon$. These passage times induce a random pseudo-metric T_ε on $\mathbb{R}^d$. By subadditive arguments, it is well known that for any $z \in \mathbb{R}^d \setminus \{0\}$, the sequence $T_\varepsilon(0, nz)/n$ converges a.s. toward a constant $\mu_\varepsilon(z)$ called the time constant. We investigate the behavior of $\varepsilon \mapsto \mu_\varepsilon(z)$ near 0, and prove that $\mu_\varepsilon(z) = \|z\|_1 - C(z)\varepsilon^{1/d_1(z)} + o(\varepsilon^{1/d_1(z)})$, where $d_1(z)$ is the number of nonnull coordinates of z, and $C(z)$ is a constant whose dependence on z is partially explicit.

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THE MICROSTRUCTURE OF STOCHASTIC VOLATILITY MODELS WITH SELF-EXCITING JUMP DYNAMICS

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We provide a general probabilistic framework within which we establish scaling limits for a class of continuous-time stochastic volatility models with self-exciting jump dynamics. In the scaling limit, the joint dynamics of asset returns and volatility is driven by independent Gaussian white noises and two independent Poisson random measures that capture the arrival of exogenous shocks and the arrival of self-excited shocks, respectively. Various well-studied stochastic volatility models with and without self-exciting price/volatility co-jumps are obtained as special cases under different scaling regimes. We analyze the impact of external shocks on the market dynamics, especially their impact on jump cascades and show in a mathematically rigorous manner that many small external shocks may trigger endogenous jump cascades in asset returns and stock price volatility.

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LARGE DEVIATIONS FOR MARKOV JUMP PROCESSES IN PERIODIC AND LOCALLY PERIODIC ENVIRONMENTS

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The paper deals with a family of jump Markov process defined in a medium with a periodic or locally periodic microstructure. We assume that the generator of the process is a zero order convolution type operator with rapidly oscillating locally periodic coefficient and, under natural ellipticity and localization conditions, show that the family satisfies the large deviation principle in the path space equipped with Skorokhod topology. The corresponding rate function is defined in terms of a family of auxiliary periodic spectral problems. It is shown that the corresponding Lagrangian is a convex function of velocity that has a superlinear growth at infinity. However, neither the Lagrangian nor the corresponding Hamiltonian need not be strictly convex, we only claim their strict convexity in some neighbourhood of infinity. It then depends on the profile of the generator kernel whether the Lagrangian is strictly convex everywhere or not.

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DIMENSION RESULTS FOR THE SPECTRAL MEASURE OF THE CIRCULAR β ENSEMBLES

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We study the dimension properties of the spectral measure of the circular β-ensembles. For $\beta \geq 2$ it was previously shown by Simon that the spectral measure is almost surely singular continuous with respect to Lebesgue measure on $\partial\mathbb{D}$ and the dimension of its support is $1 - 2/\beta$. We reprove this result with a combination of probabilistic techniques and the so-called Jitomirskaya–Last inequalities. Our method is simpler in nature and mostly self-contained, with an emphasis on the probabilistic aspects rather than the analytic. We also extend the method to prove a large deviations principle for norms involved in the Jitomirskaya–Last analysis.

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SPECIES TREE ESTIMATION UNDER JOINT MODELING OF COALESCENCE AND DUPLICATION: SAMPLE COMPLEXITY OF QUARTET METHODS

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We consider species tree estimation under a standard stochastic model of gene tree evolution that incorporates incomplete lineage sorting (as modeled by a coalescent process) and gene duplication and loss (as modeled by a branching process). Through a probabilistic analysis of the model, we derive sample complexity bounds for widely used quartet-based inference methods that highlight the effect of the duplication and loss rates in both subcritical and supercritical regimes.

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CONVERGENCE OF PERSISTENCE DIAGRAM IN THE SPARSE REGIME

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The objective of this paper is to examine the asymptotic behavior of persistence diagrams associated with Čech filtration. A persistence diagram is a graphical descriptor of a topological and algebraic structure of geometric objects. We consider Čech filtration over a scaled random sample $r_n^{-1}\mathcal{X}_n = \{r_n^{-1}X_1, \dots, r_n^{-1}X_n\}$, such that $r_n \to 0$ as $n \to \infty$. We treat persistence diagrams as a point process and establish their limit theorems in the sparse regime: $nr_n^d \to 0$, $n \to \infty$. In this setting, we show that the asymptotics of the kth persistence diagram depends on the limit value of the sequence $n^{k+2}r_n^{d(k+1)}$. If $n^{k+2}r_n^{d(k+1)} \to \infty$, the scaled persistence diagram converges to a deterministic Radon measure almost surely in the vague metric. If r_n decays faster so that $n^{k+2}r_n^{d(k+1)} \to c \in (0,\infty)$, the persistence diagram weakly converges to a limiting point process without normalization. Finally, if $n^{k+2}r_n^{d(k+1)} \to 0$, the sequence of probability distributions of a persistence diagram should be normalized, and the resulting convergence will be treated in terms of the $\mathcal{M}_0$-topology.

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A CONCENTRATION OF MEASURE AND RANDOM MATRIX APPROACH TO LARGE-DIMENSIONAL ROBUST STATISTICS

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This article studies the *robust covariance matrix estimation* of a data collection $X = (x_1, \dots, x_n)$ with $x_i = \sqrt{\tau_i} z_i + m$, where $z_i \in \mathbb{R}^p$ is a *concentrated vector* (e.g., an elliptical random vector), $m \in \mathbb{R}^p$ a deterministic signal and $\tau_i \in \mathbb{R}$ a scalar perturbation of possibly large amplitude, under the assumption where both n and p are large. This estimator is defined as the fixed point of a function which we show is contracting for a so-called *stable semi-metric*. We exploit this semi-metric along with concentration of measure arguments to prove the existence and uniqueness of the robust estimator as well as evaluate its limiting spectral distribution.

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PROPERTIES OF EIGENVALUES AND EIGENVECTORS OF LARGE-DIMENSIONAL SAMPLE CORRELATION MATRICES

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This paper is to study the properties of eigenvalues and eigenvectors of high-dimensional sample correlation matrices. We first improve the result of Jiang (*Sankhyā* **66** (2004) 35–48), Xiao and Zhou (*J. Theoret. Probab.* **23** (2010) 1–20) and the Theorem 1 of El Karoui (*Ann. Appl. Probab.* **19** (2009) 2362–2405), both concerning the limiting spectral distribution and the extreme eigenvalues of sample correlation matrices, by allowing a more general fourth moment condition. Then, we establish a central limit theorem (CLT) for the linear statistics of the eigenvectors of large sample correlation matrices. We discover that the difference between the functional CLT of the sample covariance matrix and the sample correlation matrix is fundamentally influenced by the direction of a nonrandom projection vector. In the special case where the square root of the correlation matrix is identity, the difference will be determined by the sum of the fourth powers of the entries of the projection vector. These results also indicate that the eigenmatrix of sample correlation matrices is *not* asymptotically Haar if the underlying distribution is Gaussian. In other words, the normalization based on the sample variances affects the asymptotic properties of the eigenmatrix of the Wishart matrix. Furthermore, we establish a theorem concerning CLT for the linear statistics of the eigenvectors of large sample covariance matrices. This theorem improves the main results in Bai, Miao and Pan (*Ann. Probab.* **35** (2007) 1532–1572), which requires the assumption that the fourth moment of the underlying variable matches the one of Gaussian distribution, as well as Theorem 1.3 in Pan and Zhou (*Ann. Appl. Probab.* **18** (2008) 1232–1270), which relaxed the Gaussian like fourth moment requirement but assumes the maximum entries of the projection vector converge to 0 uniformly. We illustrate the usefulness of the theoretical results through an application in communications.

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ASYMPTOTIC OPTIMALITY OF THE BINOMIAL-EXHAUSTIVE POLICY FOR POLLING SYSTEMS WITH LARGE SWITCHOVER TIMES

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We study an optimal-control problem of polling systems with large switchover times, when a holding cost is incurred on the queues. In particular, we consider a stochastic network with a single server that switches between several buffers (queues) according to a pre-specified order, assuming that the switchover times between the queues are large relative to the processing times of individual jobs. Due to its complexity, computing an optimal control for such a system is prohibitive, and so we instead search for an asymptotically optimal control. To this end, we first solve an optimal control problem for a deterministic relaxation (namely, for a fluid model), that is represented as a hybrid dynamical system. We then "translate" the solution to that fluid problem to a binomial-exhaustive policy for the underlying stochastic system, and prove that this policy is asymptotically optimal in a large-switchover-time scaling regime, provided a certain uniform integrability (UI) condition holds. Finally, we demonstrate that the aforementioned UI condition holds in the following cases: (i) the holding cost has (at most) linear growth, and all service times have finite second moments; (ii) the holding cost grows at most at a polynomial rate (of any degree), and the service-time distributions possess finite moment generating functions.

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DEPOSITION, DIFFUSION, AND NUCLEATION ON AN INTERVAL

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Motivated by nanoscale growth of ultra-thin films, we study a model of deposition, on an interval substrate, of particles that perform Brownian motions until any two meet, when they nucleate to form a static island, which acts as an absorbing barrier to subsequent particles. This is a continuum version of a lattice model studied in the applied literature. We show that the associated interval-splitting process converges in the sparse deposition limit to a Markovian process (in the vein of Brennan and Durrett) governed by a splitting density with a compact Fourier series expansion but, apparently, no simple closed form. We show that the same splitting density governs the fixed deposition rate, large time asymptotics of the normalized gap distribution, so these asymptotics are independent of deposition rate. The splitting density is derived by solving an exit problem for planar Brownian motion from a right-angled triangle, extending work of Smith and Watson.

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AN IMPOSSIBILITY RESULT FOR PHYLOGENY RECONSTRUCTION FROM k-MER COUNTS

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We consider phylogeny estimation under a two-state model of sequence evolution by site substitution on a tree. In the asymptotic regime where the sequence lengths tend to infinity, we show that for any fixed k no statistically consistent phylogeny estimation is possible from k-mer counts over the full leaf sequences alone. Formally, we establish that the joint distribution of k-mer counts over the entire leaf sequences on two distinct trees have total variation distance bounded away from 1 as the sequence length tends to infinity. Our impossibility result implies that statistical consistency requires more sophisticated use of k-mer count information, such as block techniques developed in previous theoretical work.

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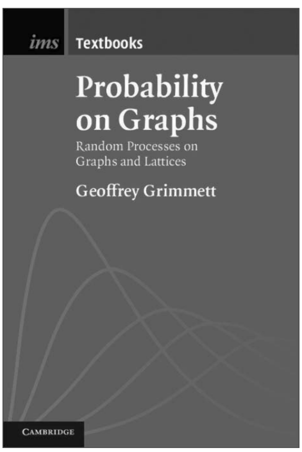

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