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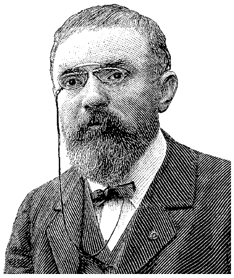

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Geodesic PCA in the Wasserstein space by convex PCA

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Abstract. We introduce the method of Geodesic Principal Component Analysis (GPCA) on the space of probability measures on the line, with finite second moment, endowed with the Wasserstein metric. We discuss the advantages of this approach, over a standard functional PCA of probability densities in the Hilbert space of square-integrable functions. We establish the consistency of the method by showing that the empirical GPCA converges to its population counterpart, as the sample size tends to infinity. A key property in the study of GPCA is the isometry between the Wasserstein space and a closed convex subset of the space of square-integrable functions, with respect to an appropriate measure. Therefore, we consider the general problem of PCA in a closed convex subset of a separable Hilbert space, which serves as basis for the analysis of GPCA and also has interest in its own right. We provide illustrative examples on simple statistical models, to show the benefits of this approach for data analysis. The method is also applied to a real dataset of population pyramids.

Résumé. Nous introduisons la méthode d'Analyse en Composantes Principales Géodésiques (GPCA) dans l'espace des mesures de probabilités à support sur la droite réelle, admettant un moment d'ordre deux, et muni de la métrique de Wasserstein. Nous discutons des avantages de cette approche par rapport à une ACP fonctionnelle standard de densités de probabilités dans l'espace de Hilbert des fonctions de carrés intégrable. Nous établissons la consistence de cette méthode en montrant que la GPCA empirique converge vers sa version population lorsque la taille de l'échantillon tend vers l'infini. Une propriété clé dans l'étude de la GPCA est l'isométrie entre l'espace de Wasserstein et un sous-espace convexe fermé de l'ensemble des fonctions de carrés intégrable, par rapport à une mesure de référence appropriée. De ce fait, nous considérons le problème général de l'ACP dans un sous-ensemble convexe fermé d'un espace de Hilbert séparable, qui sert de base à l'analyse de la GPCA. Nous proposons différents exemples illustratifs à partir de modèles statistiques simples pour montrer les bénéfices de cette approche pour l'analyse de données. La méthode est également appliquée à un exemple réel sur les pyramides des âges.

MSC: Primary 62G05; secondary 62G20

Keywords: Wasserstein space; Geodesic and Convex Principal Component Analysis; Fréchet mean; Functional data analysis; Geodesic space; Inference for family of densities

References

[1] M. Agueh and G. Carlier. Barycenters in the Wasserstein space. *SIAM J. Math. Anal.* **43** (2) (2011) 904–924. MR2801182
[2] L. Ambrosio, N. Gigli and G. Savaré. Gradient flows with metric and differentiable structures, and applications to the Wasserstein space. *Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl.* **15** (3–4) (2004) 327–343. MR2148889
[3] Z. Artstein and R. J. B. Wets. Consistency of minimizers and the SLLN for stochastic programs. *J. Convex Anal.* **2** (1–2) (1995) 1–17. MR1363357
[4] H. Attouch. *Variational Convergence for Functions and Operators. Applicable Mathematic Series*. Pitman, London, 1984. MR0773850
[5] G. Beer. On convergence of closed sets in a metric space and distance functions. *Bull. Aust. Math. Soc.* **31** (1985) 421–432. MR0801600
[6] R. Bhattacharya and V. Patrangenaru. Large sample theory of intrinsic and extrinsic sample means on manifolds. *Ann. Statist.* **31** (1) (2003) 1–29. MR1962498

[7] J. Bigot and T. Klein. Consistent estimation of a population barycenter in the Wasserstein space. Preprint, 2012. Available at arXiv:1212.2562.
[8] Y. Brenier. Polar factorization and monotone rearrangement of vector-valued functions. *Comm. Pure Appl. Math.* **44** (4) (1991) 375–417. MR1100809
[9] O. Chodosh. Optimal transport and Ricci curvature: Wasserstein space over the interval. Preprint, 2011. Available at arXiv:1105.2883.
[10] G. Dal Maso. *An Introduction to Γ-Convergence*. *Progress in Nonlinear Differential Equations and Their Applications* **8**. Birkhäuser, Boston, MA, 1993. MR1201152
[11] J. Dauxois, A. Pousse and Y. Romain. Asymptotic theory for the principal component analysis of a vector random function: Some applications to statistical inference. *J. Multivariate Anal.* **12** (1) (1982) 136–154. MR0650934
[12] P. Delicado. Dimensionality reduction when data are density functions. *Comput. Statist. Data Anal.* **55** (1) (2011) 401–420. MR2736564
[13] P. Embrechts and M. Hofert. A note on generalized inverses. *Math. Methods Oper. Res.* **77** (2013) 423–432. MR3072795
[14] P. T. Fletcher, C. Lu, S. M. Pizer and S. Joshi. Principal geodesic analysis for the study of nonlinear statistics of shape. *IEEE Trans. Med. Imaging* **23** (8) (2004) 995–1005.
[15] S. Gallón, J.-M. Loubes and E. Maza. Statistical properties of the quantile normalization method for density curve alignment. *Math. Biosci.* **242** (2) (2013) 129–142. MR3068678
[16] N. N. Hai and P. T. An. A generalization of Blaschke's convergence theorem in metric spaces. *J. Convex Anal.* **4** (2013) 1013–1024.
[17] S. Huckemann, T. Hotz and A. Munk. Intrinsic shape analysis: Geodesic PCA for Riemannian manifolds modulo isometric lie group actions. *Statist. Sinica* **20** (2010) 1–100. MR2640651
[18] B. E. Johnson. Separate continuity and measurability. *Proc. Amer. Math. Soc.* **20** (2) (1969) 420–422. MR0236345
[19] A. Kneip and K. J. Utikal. Inference for density families using functional principal component analysis. *J. Amer. Statist. Assoc.* **96** (454) (2001) 519–542. With comments and a rejoinder by the authors. MR1946423
[20] M. Ledoux and M. Talagrand. *Probability in Banach Spaces: Isoperimetry and Processes*. *Classics in Mathematics*. Springer-Verlag, Berlin, 2011. Reprint of the 1991 edition. MR2814399
[21] W. K. Newey and D. McFadden. Large sample estimation and hypothesis testing. In *Handbook of Econometrics, Vol. IV* 2111–2245. *Handbooks in Econom*. **2**. North-Holland, Amsterdam, 1994. MR1315971
[22] B. Price. On the completeness of a certain metric space with an application to Blaschke's selection theorem. *Bull. Amer. Math. Soc. (N.S.)* **46** (1940) 278–280. MR0002010
[23] J. O. Ramsay and B. W. Silverman. *Functional Data Analysis*, 2nd edition. *Springer Series in Statistics*. Springer, New York, 2005. MR2168993
[24] R. T. Rockafellar and J. O. Royset. Random variables, monotone relations, and convex analysis. *Math. Program.* **148** (2014) 297–331. MR3274854
[25] W. Rudin. Lebesgue's first theorem. In *Mathematical Analysis and Applications, Part B* 741–747. *Advances in Math. Suppl. Stud* **7b**. Academic Press, New York, 1981. MR0634266
[26] B. W. Silverman. Smoothed functional principal components analysis by choice of norm. *Ann. Statist.* **24** (1) (1996) 1–24. MR1389877
[27] S. Sommer, F. Lauze, S. Hauberg and M. Nielsen. Manifold valued statistics, exact principal geodesic analysis and the effect of linear approximations. In *Computer Vision – ECCV 2010* 43–56. K. Daniilidis, P. Maragos and N. Paragios (Eds). *Lecture Notes in Computer Science* **6316**. Springer, Berlin, 2010.
[28] C. Villani. *Topics in Optimal Transportation*. *Graduate Studies in Mathematics* **58**. Amer. Math. Soc., Providence, RI, 2003. MR1964483
[29] Z. Zhang and H.-G. Müller. Functional density synchronization. *Comput. Statist. Data Anal.* **55** (7) (2011) 2234–2249. MR2786984
[30] H. Ziezold. On expected figures and a strong law of large numbers for random elements in quasi-metric spaces. In *Trans. 7th Prague Conf. Inf. Theory, Stat. Dec. Func., Random Processes, volume A* 591–602. Reidel, Dordrecht, 1977. MR0501230

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Pointwise upper estimates for transition probabilities of continuous time random walks on graphs

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Abstract. Let X be a continuous time random walk on a weighted graph. Given the on-diagonal upper bounds of transition probabilities at two vertices x_1 and x_2, we obtain Gaussian upper estimates for the off-diagonal transition probability $\mathbb{P}_{x_1}(X_t = x_2)$ in terms of an adapted metric introduced by Davies.

Résumé. Soit X une marche aléatoire à temps continu sur un graphe pondéré. Etant données des bornes supérieures sur la transition de probabilité diagonale en deux sommets x_1 et x_2, nous obtenons des estimées supérieures gaussiennes sur la transition de probabilité $\mathbb{P}_{x_1}(X_t = x_2)$ (qui est en dehors de la diagonale) en termes d'une métrique adaptée introduite par Davies.

MSC: 60G50; 30K08

Keywords: Random walk; Transition probability; Heat kernel; Gaussian upper bound

References

[1] S. Andres, M. T. Barlow, J. D. Deuschel and B. M. Hambly. Invariance principle for the random conductance model. *Probab. Theory Related Fields* **156** (2013) 535–580. MR3078279
[2] M. T. Barlow. Random walk on supercritical percolation clusters. *Ann. Probab.* **32** (2004) 3024–3084. MR2094438
[3] M. Barlow and X. Chen. Gaussian bounds and parabolic Harnack inequality on locally irregular graphs. Preprint, 2013. Available at http://www.math.ubc.ca/~barlow/preprints/107_gb_irreg.pdf.
[4] N. Berger, M. Biskup, C. E. Hoffman and G. Kozma. Anomalous heat-kernel decay for random walk among bounded random conductances. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** (2008) 374–392. MR2446329
[5] M. Biskup and T. M. Prescott. Functional CLT for random walk among bounded random conductances. *Electron. J. Probab.* **12** (2007) 1323–1348. MR2354160
[6] O. Boukhadra, T. Kumagai and P. Mathieu. Harnack inequalities and local central limit theorem for the polynomial lower tail random conductance model. *J. Math. Soc. Japan* **67** (2015) 1413–1448. MR3417502
[7] X. Chen. Gaussian bounds and collisions of variable speed random walks on lattices with power law conductances. Available at arXiv:1507.05297v2.
[8] T. Coulhon, A. Grigor'yan and F. Zucca. The discrete integral maximum principle and its applications. *Tohoku Math. J.* **57** (2005) 559–587. MR2203547
[9] E. B. Davies. Analysis on graphs and noncommutative geometry. *J. Funct. Anal.* **111** (1993) 398–430. MR1203460
[10] E. B. Davies. Large deviations for heat kernels on graphs. *J. London Math. Soc.* **47** (1993) 65–72. MR1200978
[11] M. Folz. Gaussian upper bounds for heat kernels of continuous time simple random walks. *Electron. J. Probab.* **16** (2011) 1693–1722. MR2835251
[12] M. Folz. Volume growth and stochastic completeness of graphs. *Trans. Amer. Math. Soc.* **366** (2014) 2089–2119. MR3152724
[13] R. L. Frank, D. Lenz and D. Wingert. Intrinsic metrics for non-local symmetric Dirichlet forms and applications to spectral theory. *J. Funct. Anal.* **266** (2014) 4765–4808. MR3177322
[14] A. Grigor'yan. Gaussian upper bounds for heat kernel on arbitrary manifolds. *J. Differential Geom.* **45** (1997) 33–52. MR1443330
[15] W. Hebisch and L. Saloff-Coste. Gaussian estimates for Markov chains and random walks on groups. *Ann. Probab.* **21** (1993) 673–709. MR1217561
[16] P. Mathieu and E. Remy. Isoperimetry and heat kernel decay on percolation clusters. *Ann. Probab.* **32** (2004) 100–128. MR2040777

[17] K. T. Sturm. Analysis on local Dirichlet spaces I. Recurrence, conservativeness and L^p-Liouville properties. *J. Reine Angew. Math.* **456** (1994) 173–196. MR1301456

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Random walks on quasi one dimensional lattices: Large deviations and fluctuation theorems

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Abstract. Several stochastic processes modeling molecular motors on a linear track are given by random walks (not necessarily Markovian) on quasi 1d lattices and share a common regenerative structure. Analyzing this abstract common structure, we derive information on the large fluctuations of the stochastic process by proving large deviation principles for the first-passage times and for the position. We focus our attention on the Gallavotti–Cohen-type symmetry of the position rate function (fluctuation theorem), showing its equivalence with the independence of suitable random variables. In the special case of Markov random walks, we show that this symmetry is universal only inside a suitable class of quasi 1d lattices.

Résumé. Nous considérons différents processus stochastiques modélisant des moteurs moléculaires : il s'agit de marches aléatoires, non nécessairement markoviennes, le long d'un rail linéaire, presque un réseau unidimensionnel, qui partagent une même structure de régénération. En analysant cette structure abstraite commune nous contrôlons les grandes déviations du processus stochastique, nous établissons des principes de grandes déviations pour les temps de premier passage et pour la variable de position. Nous nous concentrons sur les symétries de type Gallavotti–Cohen de la fonction de taux positionnelle (théorème de fluctuations), en montrant son équivalence avec l'indépendance de certaines variables aléatoires. Dans le cas particulier des marches aléatoires markoviennes, nous montrons que cette symétrie n'est universelle qu'au sein d'une classe particulière de réseaux presque unidimensionnels.

MSC: 60J27; 60F10; 82C05

Keywords: Markov chain; Random time change; Large deviation principle; Gallavotti–Cohen-type symmetry; Fluctuation theorem; Molecular motor

References

[1] D. Andrieux. *Nonequilibrium Statistical Thermodynamics at the Nanoscale: From Maxwell Demon to Biological Information Processing*. VDM Verlag, Saarbrücken, 2009.
[2] H. Brezis. *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. Springer, Heidelberg, 2011. MR2759829
[3] H. Cartan. *Elementary Theory of Analytic Functions of One or Several Complex Variables*. Dover Publications Inc., New York, 1995. MR1348633
[4] R. K. Das and A. B. Kolomeisky. Dynamic properties of molecular motors in the divided-pathway model. *Phys. Chem. Chem. Phys.* **11** (2009) 4815–4820.
[5] A. Dembo, N. Gantert and O. Zeitouni. Large deviations for random walk in random environment with holding times. *Ann. Probab.* **32** (2004) 996–1029. MR2044672
[6] A. Dembo and O. Zeitouni. *Large Deviation Techniques and Applications*, 2nd edition. Springer, New York, 1998. MR1619036
[7] K. Duffy and M. Rodgers-Lee. Some useful functions for functional large deviations. *Stoch. Stoch. Reports* **76** (2004) 267–279. MR2072383
[8] A. Faggionato and V. Silvestri. Discrete kinetic models for molecular motors: Asymptotic velocity and Gaussian fluctuations. *J. Stat. Phys.* **157** (2014) 1062–1096. MR3277760
[9] A. Faggionato and V. Silvestri. Fluctuation theorems for discrete kinetic models of molecular motors. Preprint, 2017. Available at arXiv:1701.01721.

[10] A. Faggionato and D. Di Pietro. Gallavotti–Cohen-type symmetry related to cycle decompositions for Markov chains and biochemical applications. *J. Stat. Phys.* **143** (2011) 11–32. MR2787971

[11] M. E. Fisher and A. B. Kolomeisky. The force exerted by a molecular motor. *Proc. Natl. Acad. Sci. USA* **96** (1999) 6597–6602.

[12] M. E. Fisher and A. B. Kolomeisky. Molecular motors and the force they exert. *Phys. A* **274** (1999) 241–266.

[13] N. Gantert. Private communication.

[14] R. C. Gunning and H. Rossi. *Analytic Functions of Several Complex Variables*. Prentice-Hall Inc., Englewood Cliffs, NJ, 1965. MR0180696

[15] F. den Hollander. *Large Deviations*. *Fields Institute Monographs* **14**. Amer. Math. Soc., Providence, RI, 2000. MR1739680

[16] F. Jülicher, A. Ajdari and J. Prost. Modeling molecular motors. *Rev. Modern Phys.* **69** (1997) 1269–1281.

[17] A. B. Kolomeisky. Exact results for parallel-chain kinetic models of biological transport. *J. Chem. Phys.* **115** (2001) 7523.

[18] A. B. Kolomeisky and M. E. Fisher. Extended kinetic models with waiting-time distributions: Exact results. *J. Chem. Phys.* **113** (2000) 10867–10877.

[19] A. B. Kolomeisky and M. E. Fisher. Periodic sequential kinetic models with jumping, branching and deaths. *Phys. A* **279** (2000) 1–20. MR1797136

[20] A. B. Kolomeisky and M. E. Fisher. Molecular motors: A theorist's perspective. *Annu. Rev. Phys. Chem.* **58** (2007) 675–695.

[21] D. Lacoste, A. W. C. Lau and K. Mallick. Fluctuation theorem and large deviation function for a solvable model of a molecular motor. *Phys. Rev. E* **78** (2008) 011915.

[22] D. Lacoste and K. Mallick. *Fluctuation Relations for Molecular Motors*. B. Duplantier and V. Rivasseau (Eds). *Biological Physics. Poincaré Seminar 2009*. *Progress in Mathematical Physics* **60** 61–88. Birkhäuser Verlag, Basel, 2011.

[23] J. L. Lebowitz and H. Spohn. A Gallavotti–Cohen-type symmetry in the large deviation functional for stochastic dynamics. *J. Stat. Phys.* **95** (1999) 333–365. MR1705590

[24] R. Lefevere, M. Mariani and L. Zambotti. Large deviations for renewal processes. *Stochastic Process. Appl.* **121** (2011) 2243–2271. MR2822776

[25] T. Kato. *A Short Introduction to Perturbation Theory for Linear Operators*. Springer, New York, 1982. MR0678094

[26] M. Mariani and L. Zambotti. A renewal version of the Sanov theorem. *Electron. Commun. Probab.* **19** (2014) 1–13. MR3269169

[27] J. R. Norris. *Markov Chains*. Cambridge Univ. Press, Cambridge, 1997. MR1600720

[28] R. Russell. The large deviations of random time changes. Ph.D. thesis, Trinity College, Dublin, 1997.

[29] U. Seifert. Stochastic thermodynamics, fluctuation theorems, and molecular machines. *Rep. Progr. Phys.* **75** (2012) 126001.

[30] D. Tsygankov and M. E. Fisher. Kinetic models for mechanoenzymes: Structural aspects under large loads. *J. Chem. Phys.* **128** (2008) 015102.

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Thick points for Gaussian free fields with different cut-offs

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Abstract. Massive and massless Gaussian free fields can be described as generalized Gaussian processes indexed by an appropriate space of functions. In this article we study various approaches to approximate these fields and look at the fractal properties of the thick points of their cut-offs. Under some sufficient conditions for a centered Gaussian process with logarithmic variance we study the set of thick points and derive their Hausdorff dimension. We prove that various cut-offs for Gaussian free fields satisfy these assumptions. We also give sufficient conditions for comparing thick points of different cut-offs.

Résumé. Les champs libres gaussiens massifs et sans masse peuvent être décrits comme des processus gaussiens généralisées indexés par un espace fonctionnel approprié. Dans cet article nous abordons différentes approches pour approximer ces champs et nous considérons les propriétés fractales des points épais de leur cut-off. Sous certaines conditions suffisantes, pour un processus gaussien avec variance logarithmique nous étudions l'ensemble des points épais et obtenons leur dimension de Hausdorff. Nous prouvons que différents cut-off des champs libres gaussiens satisfont ces hypothèses. Nous donnons aussi des conditions suffisantes pour comparer les points épais des différents cut-off.

MSC: Primary 60G60; secondary 60G15

Keywords: Gaussian multiplicative chaos; Cut-offs; Liouville quantum gravity; Thick points; Hausdorff dimension

References

[1] R. Adler. *An Introduction to Continuity, Extrema, and Related Topics for General Gaussian Processes*. *IMS Lecture Notes – Monograph Series*. Institute of Mathematical Statistics, Hayward, CA, 1990. Available at http://books.google.de/books?id=gHO4KBl2FF0C. MR1088478
[2] P. Balança and E. Herbin. A set-indexed Ornstein–Uhlenbeck process. *Electron. Commun. Probab.* **17** (2012) Article ID 39. MR2970703
[3] S. Chatterjee. Chaos, concentration, and multiple valleys. Preprint, 2008. Available at arXiv:0810.4221.
[4] J. Dubédat. SLE and the free field: Partition functions and couplings. *J. Amer. Math. Soc.* **22** (4) (2009) 995–1054. MR2525778
[5] B. Duplantier, R. Rhodes, S. Sheffield and V. Vargas. Renormalization of critical Gaussian multiplicative chaos and KPZ relation. *Comm. Math. Phys.* **330** (1) (2014) 283–330. MR3215583
[6] B. Duplantier and S. Sheffield. Liouville quantum gravity and KPZ. *Invent. Math.* **185** (2) (2011) 333–393. MR2819163
[7] C. Garban, R. Rhodes and V. Vargas. Liouville Brownian motion. *Ann. Probab.* **44** (4) (2016) 3076–3110. MR3531686
[8] I. M. Gel'fand and N. Y. Vilenkin. *Generalized Functions, Vol. 4: Applications of Harmonic Analysis*. Academic Press, New York, 1964. Translated by Amiel Feinstein. MR0173945
[9] T. Hida. *White Noise: An Infinite Dimensional Calculus*. *Mathematics and Its Applications*. Springer, Berlin, 1993. Available at http://books.google.de/books?id=oWbo7PCguDMC. MR1244577
[10] J. Hu and M. Zähle. Generalized Bessel and Riesz potentials on metric measure spaces. *Potential Anal.* **30** (4) (2009) 315–340. MR2491456
[11] X. Hu, J. Miller and Y. Peres. Thick points of the Gaussian free field. *Ann. Probab.* **38** (2) (2010) 896–926. MR2642894
[12] J.-P. Kahane. Sur le chaos multiplicatif. *Ann. Sci. Math. Québec* **9** (2) (1985) 105–150. MR0829798
[13] G. Lawler. *Conformally Invariant Processes in the Plane*. *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2005. Available at http://books.google.de/books?id=JHMzab3u6U8C. MR2129588
[14] M. Lifshits. *Lectures on Gaussian Processes*. *Springer Briefs in Mathematics*. Springer, Heidelberg, 2012. MR3024389
[15] P. Mörters, Y. Peres, O. Schramm and W. Werner. *Brownian Motion*. *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge University Press, Cambridge, 2010. Available at http://books.google.ch/books?id=e-TbA-dSrzYC. MR2604525

[16] R. Rhodes and V. Vargas. Gaussian multiplicative chaos and applications: A review. *Probab. Surv.* **11** (2014) 315–392. MR3274356
[17] R. Rhodes and V. Vargas. Lectures on Gaussian multiplicative chaos. Random geometry program. University of Cambridge, Cambridge, 2015. Available at https://www.newton.ac.uk/files/seminar/20150119100011001-297514.pdf.
[18] R. Robert and V. Vargas. Gaussian multiplicative chaos revisited. *Ann. Probab.* **38** (2) (2010) 605–631. MR2642887
[19] L. Saloff-Coste. The heat kernel and its estimates. In *Probabilistic Approach to Geometry. Proceedings of the 1st International Conference* 405–436. M. Kotani et al. (Eds). *Advanced Studies in Pure Mathematics* **57**. Mathematical Society of Japan, Tokyo, 2010. MR2648271
[20] A. Shamov. On Gaussian multiplicative chaos. Preprint, 2014. Available at arXiv:1407.4418.
[21] S. Sheffield. Gaussian free fields for mathematicians. *Probab. Theory Related Fields* **139** (3–4) (2007) 521–541. MR2322706
[22] E. M. Stein. *Singular Integrals and Differentiability Properties of Functions*. *Princeton Mathematical Series* **30**. Princeton University Press, Princeton, NJ, 1970. MR0290095
[23] A. Yaglom. Some classes of random fields in n-dimensional space, related to stationary random processes. *Theory Probab. Appl.* **2** (3) (1957) 273–320. MR0094844

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Height fluctuations in interacting dimers

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Abstract. We consider a non-integrable model for interacting dimers on the two-dimensional square lattice. Configurations are perfect matchings of $\mathbb{Z}^2$, i.e. subsets of edges such that each vertex is covered exactly once ("close-packing" condition). Dimer configurations are in bijection with discrete height functions, defined on faces $\boldsymbol{\xi}$ of $\mathbb{Z}^2$. The non-interacting model is "integrable" and solvable via Kasteleyn theory; it is known that all the moments of the height difference $h_{\boldsymbol{\xi}} - h_{\boldsymbol{\eta}}$ converge to those of the massless Gaussian Free Field (GFF), asymptotically as $|\boldsymbol{\xi} - \boldsymbol{\eta}| \to \infty$. We prove that the same holds for small non-zero interactions, as was conjectured in the theoretical physics literature. Remarkably, dimer-dimer correlation functions are instead not universal and decay with a critical exponent that depends on the interaction strength. Our proof is based on an exact representation of the model in terms of lattice interacting fermions, which are studied by constructive field theory methods. In the fermionic language, the height difference $h_{\boldsymbol{\xi}} - h_{\boldsymbol{\eta}}$ takes the form of a non-local operator, consisting of a sum of monomials along an *arbitrary* path connecting $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$. As in the non-interacting case, this path-independence plays a crucial role in the proof.

Résumé. Nous étudions un modèle non integrable de dimères en interaction sur le réseau carré bidimensionnel. Les configurations sont des appariements parfaits de $\mathbb{Z}^2$, i.e. des sous-ensembles d'arêtes tels que tout sommet est contenu dans une et une seule arête (condition de "close-packing"). Les configurations de dimères sont en bijection avec des fonctions de hauteur discrète, définie sur les faces $\boldsymbol{\xi}$ de $\mathbb{Z}^2$. Le modèle sans interaction est "integrable" et resoluble par la théorie de Kasteleyn; il est connu que tous les moments de la différence de hauteur $h_{\boldsymbol{\xi}} - h_{\boldsymbol{\eta}}$ convergent vers ceux du champ Gaussien libre (GFF), dans la limite où $|\boldsymbol{\xi} - \boldsymbol{\eta}| \to \infty$. Nous démontrons que le même résultat est valable quand le paramètre d'interaction est non nul et petit, comme il avait été conjecturé dans la littérature physique. Il est remarquable que, d'autre côté, les fonctions de correlation dimère-dimère ne sont pas universelles et décroissent avec un exposant critique qui dépend de la force de l'interaction. Notre preuve se base sur une representation exacte du modèle en termes de fermions en interaction su réseau, que nous étudions par des outils de la théorie constructive des champs. Dans le language fermionique, la différence de hauteur $h_{\boldsymbol{\xi}} - h_{\boldsymbol{\eta}}$ est un opérateur non local, qui s'écrit comme une somme de monômes le long d'un chemin *arbitraire* qui relie $\boldsymbol{\xi}$ et $\boldsymbol{\eta}$. Tout comme dans le cas sans interaction, l'indépendance par rapport au choix du chemin joue un rôle crucial dans la preuve.

MSC: 82B28; 82B20

Keywords: Dimer model; Constructive renormalization group; Gaussian free field; Universality

References

[1] A. Abdesselam and V. Rivasseau. Explicit fermionic tree expansions. *Lett. Math. Phys.* **44** (1998) 77–88. MR1623695
[2] F. Alet, J. Jacobsen, G. Misguich, V. Pasquier, F. Mila and M. Troyer. Interacting classical dimers on the square lattice. *Phys. Rev. Lett.* **94**, 235702 (2005).
[3] M. Bander and C. Itzykson. Quantum-field-theory calculation of the two-dimensional Ising model correlation function. *Phys. Rev. D* **15** (1977) 463–469.
[4] G. Battle and P. Federbush. A note on cluster expansions, tree graph identities, extra $1/n!$ factors!!! *Lett. Math. Phys.* **8** (1984) 55–57. MR0731201

[5] R. J. Baxter. *Exactly Solved Models in Statistical Mechanics*. Academic Press, London, 1982. MR0690578
[6] G. Benfatto, P. Falco and V. Mastropietro. Functional integral construction of the massive Thirring model: Verification of axioms and massless limit. *Comm. Math. Phys.* **273** (2007) 67–118. MR2308750
[7] G. Benfatto, P. Falco and V. Mastropietro. Massless sine-Gordon and massive Thirring models: Proof of Coleman's equivalence. *Comm. Math. Phys.* **285** (2009) 713–762. MR2461991
[8] G. Benfatto, P. Falco and V. Mastropietro. Universality of one-dimensional Fermi systems, I and II. *Comm. Math. Phys.* **330** (2014) 153–215, 217–282.
[9] G. Benfatto and G. Gallavotti. Perturbation theory of the Fermi surface in a quantum liquid. A general quasiparticle formalism and one-dimensional systems. *J. Stat. Phys.* **59** (1990) 541–664. MR1063175
[10] G. Benfatto and G. Gallavotti. *Renormalization Group*. Princeton Univ. Press, Princeton, NJ, 1995. MR1380265
[11] G. Benfatto, G. Gallavotti, A. Procacci and B. Scoppola. Beta function and Schwinger functions for a many fermions system in one dimension. Anomaly of the Fermi surface. *Comm. Math. Phys.* **160** (1994) 93–171. MR1262193
[12] G. Benfatto and V. Mastropietro. Renormalization group, hidden symmetries and approximate Ward identities in the XYZ model. *Rev. Math. Phys.* **13** (2001) 1323–1435. MR1866190
[13] G. Benfatto and V. Mastropietro. On the density-density critical indices in interacting Fermi systems. *Comm. Math. Phys.* **231** (2002) 97–134. MR1947693
[14] G. Benfatto and V. Mastropietro. Ward identities and chiral anomaly in the Luttinger liquid. *Comm. Math. Phys.* **258** (2005) 609–655. MR2172012
[15] G. Benfatto and V. Mastropietro. Universality relations in non-solvable quantum spin chains. *J. Stat. Phys.* **138** (2010) 1084–1108. MR2601424
[16] P. Billingsley. *Probability and Measure*, 3rd edition. Wiley, New York, 1995. MR1324786
[17] D. Brydges. A short course on cluster expansions. In *Phénomènes Critiques, Systèmes Aléatoires, Théories de Jauge, les Houches Summer School Session* **43** 129–183. North-Holland, Amsterdam, 1986. MR0880525
[18] D. Brydges and P. Federbush. A new form of the Mayer expansion in classical statistical mechanics. *J. Math. Phys.* **19** (1978) 2064–2067. MR0507502
[19] D. Brydges and T. Kennedy. Mayer expansions and the Hamilton–Jacobi equation. *J. Stat. Phys.* **48** (1987) 19–49. MR0914427
[20] J. G. Conlon and T. Spencer. A strong central limit theorem for a class of random surfaces. *Comm. Math. Phys.* **325** (2014) 1–15. MR3182484
[21] V. S. Dotsenko and V. S. Dotsenko. Critical behaviour of the phase transition in the 2D Ising model with impurities. *Adv. in Phys.* **32** (1983) 129–172. MR0710708
[22] J. Dubedat. Dimers and families of Cauchy–Riemann operators I. *J. Amer. Math. Soc.* **28** (2015) 1063–1167. MR3369909
[23] J. Dubedat. Exact bosonization of the Ising model. Available at arXiv:1112.4399.
[24] P. Falco. Interacting fermions picture for dimer models. *Phys. Rev. E* **87**, 060101(R) (2013).
[25] P. Falco. Arrow-arrow correlations for the six-vertex model. *Phys. Rev. E* **88**, 030103(R) (2013).
[26] P. Fendley, R. Moessner and S. L. Sondhi. Classical dimers on the triangular lattice. *Phys. Rev. B* **66**, 214513 (2002).
[27] M. E. Fisher. Statistical mechanics of dimers on a plane lattice. *Phys. Rev. (2)* **124** (1961) 1664–1672. MR0136399
[28] M. E. Fisher and J. Stephenson. Statistical mechanics of dimers on a plane lattice. II. dimer correlations and monomers. *Phys. Rev. (2)* **132** (1963) 1411–1431. MR0158705
[29] J. Fröhlich, R. Götschmann and P. A. Marchetti. The effective gauge field action of a system of non-relativistic electrons. *Comm. Math. Phys.* **173** (1995) 417–452. MR1355632
[30] J. Fröhlich and T. Spencer. The Kosterlitz–Thouless transition in two-dimensional Abelian spin systems and the Coulomb gas. *Comm. Math. Phys.* **81** (1981) 527–602. MR0634447
[31] C. Itzykson and J. Drouffe. *Statistical Field Theory: From Brownian Motion to Renormalization and Lattice Gauge Theory*. Cambridge Univ. Press, Cambridge, 1991.
[32] G. Gallavotti. Renormalization theory and ultraviolet stability for scalar fields via renormalization group methods. *Rev. Modern Phys.* **57** (1985) 471–562. MR0789582
[33] G. Gallavotti and F. Nicolò. Renormalization theory in four-dimensional scalar fields. Part I. *Comm. Math. Phys.* **100** (1985) 545–590. MR0806252
[34] G. Gentile and V. Mastropietro. Renormalization group for one-dimensional fermions. A review on mathematical results. *Phys. Rep.* **352** (2001) 273–438. MR1862624
[35] G. Giacomin, S. Olla and H. Spohn. Equilibrium fluctuations for the $\nabla\phi$ interface model. *Ann. Probab.* **29** (2001) 1138–1172. MR1872740
[36] A. Giuliani. The ground state construction of the two-dimensional Hubbard model on the honeycomb lattice. In *Quantum Theory from Small to Large Scales. Lecture Notes of the les Houches Summer School* **95**. Oxford Univ. Press, Oxford, 2012. DOI: 10.1093/acprof:oso/9780199652495.003.0010.
[37] A. Giuliani and V. Mastropietro. Anomalous universality in the anisotropic Ashkin–Teller model. *Comm. Math. Phys.* **256** (2005) 681–735. *Phys. Rev. Lett.* **93** (2004) 190603. MR2161276
[38] A. Giuliani, R. L. Greenblatt and V. Mastropietro. The scaling limit of the energy correlations in non integrable Ising models. *J. Math. Phys.* **53**, 095214 (2012). MR2905796
[39] L. P. Kadanoff. Connections between the critical behavior of the planar model and that of the eight-vertex model. *Phys. Rev. Lett.* **39** (1977) 903–905. MR0446269
[40] P. W. Kasteleyn. The statistics of dimers on a lattice: I. The number of dimer arrangements on a quadratic lattice. *Physica* **27** (1961) 1209–1225. *J. Math. Phys.* **4** (1963) 287–293. MR0153427
[41] R. Kenyon. Local statistics of lattice dimers. *Ann. Inst. Henri Poincaré Probab. Stat.* **33** (1997) 591–618. MR1473567

[42] R. Kenyon. Conformal invariance of domino tiling. *Ann. Probab.* **28** (2000) 759–795. MR1782431
[43] R. Kenyon. Dominos and the Gaussian free field. *Ann. Probab.* **29** (2001) 1128–1137. MR1872739
[44] R. Kenyon. Lectures on dimers. In *Statistical Mechanics* 191–230. *IAS/Park City Math. Ser.* **16**. Amer. Math. Soc., Providence, RI, 2009. MR2523460
[45] R. Kenyon, A. Okounkov and S. Sheffield. Dimers and amoebae. *Ann. of Math. (2)* **163** (2006) 1019–1056. MR2215138
[46] K. Johnson. Solution of the equations for the Green's functions of a two dimensional relativistic field theory. *Nuovo Cimento (10)* **20** (1961) 773–790. MR0137503
[47] B. Laslier and F. L. Toninelli. How quickly can we sample a uniform domino tiling of the $2L \times 2L$ square via Glauber dynamics? *Probab. Theory Related Fields* **161** (2015) 509–559. MR3334275
[48] V. Mastropietro. Ising models with four spin interaction at criticality. *Comm. Math. Phys.* **244** (2004) 595–642. MR2034489
[49] V. Mastropietro. Nonperturbative Adler–Bardeen theorem. *J. Math. Phys.* **48** (2007) 22302–22334. MR2297954
[50] V. Mastropietro. *Non-perturbative Renormalization*. World Scientific, Hackensack, NJ, 2008. MR2414874
[51] D. C. Mattis and E. H. Lieb. Exact solution of a many-fermion system and its associated boson field. *J. Math. Phys.* **6** (1965) 304–312. MR0172638
[52] J. Miller. Fluctuations for the Ginzburg–Landau $\nabla\phi$ interface model on a bounded domain. *Comm. Math. Phys.* **308** (2011) 591–639. MR2855536
[53] A. Naddaf and T. Spencer. On homogenization and scaling limit of some gradient perturbations of a massless free field. *Comm. Math. Phys.* **183** (1997) 55–84. MR1461951
[54] J. W. Negele and H. Orland. *Quantum Many-Particle Systems*. Westview Press, Boulder, CO, 2008.
[55] S. Papanikolaou, E. Luijten and E. Fradkin. Quantum criticality, lines of fixed points, and phase separation in doped two-dimensional quantum dimer models. *Phys. Rev. B* **76**, 134514 (2007).
[56] H. Pinson. Rotational invariance and discrete analyticity in the 2d dimer model. *Comm. Math. Phys.* **245** (2004) 355–382. MR2039700
[57] S. Samuel. The use of anticommuting variable integrals in statistical mechanics. I. The computation of partition functions. *J. Math. Phys.* **21** (1980) 2806–2814. II. The computation of correlation functions. *J. Math. Phys.* **21** (1980) 2815–2819. MR0597600
[58] S. Sheffield. Gaussian free fields for mathematicians. *Probab. Theory Related Fields* **139** (2007) 521–541. MR2322706
[59] S. Smirnov. Critical percolation in the plane: Conformal invariance, Cardy's formula, scaling limits percolation. *C. R. Acad. Sci. Paris Ser. I Math.* **333** (2001) 239–244. MR1851632
[60] S. Smirnov. Conformal invariance in random cluster models. I. Holomorphic fermions in the Ising model. *Ann. of Math. (2)* **172** (2010) 1435–1467. MR2680496
[61] N. V. Temperley and M. E. Fisher. Dimer problem in statistical mechanics-an exact result. *Philos. Mag. (8)* **6** (1961) 1061–1063. MR0136398
[62] W. Thirring. A soluble relativistic field theory. *Ann. Phys.* **3** (1958) 91–112. MR0091788
[63] C. N. Yang and C. P. Yang. One-dimensional chain of anisotropic spin-spin interactions. I, II and III. *Phys. Rev.* **150** 321–327, 327-339, and **151** 258–264.
[64] J. B. Zuber and C. Itzykson. Quantum field theory and the two-dimensional Ising model. *Phys. Rev. D* **15** (1977) 2875–2884.

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A simpler proof for the dimension of the graph of the classical Weierstrass function[1]

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Abstract. Let $W_{\lambda,b}(x)=\sum_{n=0}^{\infty}\lambda^n g(b^n x)$ where $b\geq 2$ is an integer and $g(u)=\cos(2\pi u)$ (classical Weierstrass function). Building on work by Ledrappier (In *Symbolic Dynamics and Its Applications* (1992) 285–293), Barański, Bárány and Romanowska (*Adv. Math.* **265** (2014) 32–59) and Tsujii (*Nonlinearity* **14** (2001) 1011–1027), we provide an elementary proof that the Hausdorff dimension of $W_{\lambda,b}$ equals $2+\frac{\log\lambda}{\log b}$ for all $\lambda\in(\lambda_b,1)$ with a suitable $\lambda_b<1$. This reproduces results by Barański, Bárány and Romanowska (*Adv. Math.* **265** (2014) 32–59) without using the dimension theory for hyperbolic measures of Ledrappier and Young (*Ann. of Math. (2)* **122** (1985) 540–574; *Comm. Math. Phys.* **117** (1988) 529–548), which is replaced by a simple telescoping argument together with a recursive multi-scale estimate.

Résumé. Soit $W_{\lambda,b}(x)=\sum_{n=0}^{\infty}\lambda^n g(b^n x)$, où $b\geq 2$ est un nombre entier et $g(u)=\cos(2\pi u)$ (fonction de Weierstrass classique). En utilisant des idées et résultats de Ledrappier (In *Symbolic Dynamics and Its Applications* (1992) 285–293), de Barański, Bárány et Romanowska (*Adv. Math.* **265** (2014) 32–59) et de Tsujii (*Nonlinearity* **14** (2001) 1011–1027), nous présentons une démonstration élémentaire du fait que la dimension de Hausdorff de $W_{\lambda,b}$ est égale à $2+\frac{\log\lambda}{\log b}$ pour tout $\lambda\in(\lambda_b,1)$ avec $\lambda_b<1$ approprié. Cela reproduit des résultats de Barański, Bárány et Romanowska (*Adv. Math.* **265** (2014) 32–59) sans utiliser la théorie de dimension des mesures hyperboliques de Ledrappier et Young (*Ann. of Math. (2)* **122** (1985) 540–574 ; *Comm. Math. Phys.* **117** (1988) 529–548), laquelle est remplacée par un argument téléscopique élémentaire conjointement avec une estimation récursive multi-échelle.

MSC: 37D20; 37D45; 37G35; 37H20

Keywords: Weierstrass function; Hausdorff dimension

References

[1] K. Barański, B. Bárány and J. Romanowska. On the dimension of the graph of the classical Weierstrass function. *Adv. Math.* **265** (2014) 32–59. MR3255455
[2] L. Biacino. On the Hausdorff dimension of the graph of a Weierstrass type function. *Stud. Univ. Babeş-Bolyai Math.* **56** (4) (2011) 7–17. MR2896736
[3] S. Fu. On the Hausdorff dimension of the graph of the Weierstrass function. *Far East J. Dyn. Syst.* **17** (2011) 85–137. MR2952868
[4] B. Hunt. The Hausdorff dimension of graphs of Weierstrass functions. *Proc. Amer. Math. Soc.* **126** (3) (1998) 791–800. MR1452806
[5] J. Kaplan, J. Mallet-Paret and J. Yorke. The Lyapunov dimension of a nowhere differentiable attracting torus. *Ergodic Theory Dynam. Systems* **4** (1984) 261–281. MR0766105
[6] G. Keller. Stability index for chaotically driven concave maps. *J. Lond. Math. Soc. (2)* **89** (2014) 603–622. MR3188635
[7] F. Ledrappier. On the dimension of some graphs. In *Symbolic Dynamics and Its Applications (New Haven, CT, 1991)* 285–293. *Contemp. Math.* **135**, 1992. MR1185095
[8] F. Ledrappier and L.-S. Young. The metric entropy of diffeomorphisms. II. Relations between entropy, exponents and dimension. *Ann. of Math. (2)* **122** (1985) 540–574. MR0819557
[9] F. Ledrappier and L.-S. Young. Dimension formula for random transformations. *Comm. Math. Phys.* **117** (1988) 529–548. MR0953818
[10] B. Mandelbrot. *Fractals: Form, Chance, and Dimension*. Freeman, San Francisco, 1977. MR0471493

[11] Y. Peres and W. Schlag. Smoothness of projections, Bernoulli convolutions, and the dimension of exceptions. *Duke Math. J.* **102** (2) (2000) 193–251. MR1749437

[12] Y. Peres, W. Schlag and B. Solomyak. Sixty years of Bernoulli convolutions. In *Fractal Geometry and Stochastics II* 1–28. Birkhäuser, Basel, 2000. MR1785620

[13] Y. Peres and B. Solomyak. Absolute continuity of Bernoulli convolutions, a simple proof. *Math. Res. Lett.* **3** (1996) 231–239. MR1386842

[14] W. Shen. Hausdorff dimension of the graphs of the classical Weierstrass functions. Available at arXiv:1505.03986.

[15] P. Shmerkin. On the exceptional set for absolute continuity of Bernoulli convolutions. *Geom. Funct. Anal.* **24** (3) (2014) 946–958. MR3213835

[16] M. Tsujii. Fat solenoidal attractors. *Nonlinearity* **14** (2001) 1011–1027. MR1862809

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Higher moments of the natural parameterization for SLE curves

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Abstract. In this paper, we will show that the higher moments of the natural parametrization of *SLE* curves in any bounded domain in the upper half plane is finite. We prove this by estimating the probability that an *SLE* curve gets near n given points.

Résumé. Dans cet article, nous montrons que les grands moments de la paramétrisation naturelle d'une courbe *SLE* dans n'importe quel domaine borné du demi-plan supérieur sont finis. Nous prouvons ceci en estimant la probabilité qu'une courbe *SLE* soit proche d'un nombre n de points fixés.

MSC: 60J67

Keywords: SLE curve; Natural parametrization; Green's function

References

[1] T. Alberts and M. Kozdron. Intersection probabilities for a chordal SLE path and a semicircle. *Electron. Commun. Probab.* **13** (2008) 448–460. MR2430712
[2] V. Beffara. The dimension of SLE curves. *Ann. Probab.* **36** (2008) 1421–1452. MR2435854
[3] G. Lawler. *Conformally Invariant Processes in the Plane*. Amer. Math. Soc., Providence, RI, 2005. MR2129588
[4] G. Lawler. Schramm–Loewner evolution. In *Statistical Mechanics* 231–295. S. Sheffield and T. Spencer (Eds). *IAS/Park City Mathematical Series*. AMS, Providence, RI, 2009. MR2523461
[5] G. Lalwer. Minkowski content of the intersection of a Schramm–Loewner evolution (SLE) curve with the real line. Preprint, 2014.
[6] G. Lawler and M. Rezaei. Minkowski content and natural parameterization for the Schramm–Loewner evolution. *Ann. Probab.* **43** (2015) 1082–1120. MR3342659
[7] G. Lawler and M. Rezaei. Up-to-constants bounds on the two-point Green's function for SLE curves. *Electron. Commun. Probab.* **20** (2015) 1–13. MR3358967
[8] G. Lawler and S. Sheffield. A natural parametrization for the Schramm–Loewner evolution. *Ann. Probab.* **39** (2011) 1896–1937. MR2884877
[9] G. Lawler, O. Schramm and W. Werner. Conformal invariance of planar loop-erased random walks and uniform spanning trees. *Ann. Probab.* **32** (2004) 939–995. MR2044671
[10] G. Lawler and B. Werness. Multi-point Green's function for SLE and an estimate of Beffara. *Ann. Probab.* **41** (2013) 1513–1555. MR3098683
[11] C. Pommerenke. *Boundary Behaviour of Conformal Maps*. Springer, Berlin, 1991. MR1217706
[12] S. Rohde and O. Schramm. Basic properties of SLE. *Ann. of Math. (2)* **161** (2005) 879–920. MR2153402
[13] O. Schramm. Scaling limits of loop-erased random walks and uniform spanning trees. *Israel J. Math.* **118** (2000) 221–288. MR1776084
[14] O. Schramm and S. Sheffield. Harmonic explorer and its convergence to SLE(4). *Ann. Probab.* **33** (2005) 2127–2148. MR2184093
[15] S. Smirnov. Critical percolation in the plane: Conformal invariance, Cardy's formula, scaling limits. *C. R. Acad. Sci. Paris Sér. I Math.* **333** (2001) 239–244. MR1851632
[16] S. Smirnov. Conformal invariance in random cluster models. I. Holomorphic fermions in the Ising model. *Ann. of Math. (2)* **172** (2009) 1435–1467. MR2680496

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Scaling limit of multitype Galton–Watson trees with infinitely many types

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Abstract. We introduce a certain class of 2-type Galton–Watson trees with edge lengths. We prove that, after an adequate rescaling, the weighted height function of a forest of such trees converges in law to the reflected Brownian motion. We then use this to deduce under mild conditions an invariance principle for multitype Galton–Watson trees with a countable number of types, thus extending a result of G. Miermont on multitype Galton–Watson trees with finitely many types.

Résumé. Nous introduisons une certaine classe d'arbres de Galton–Watson à deux types avec longueurs d'arêtes. Nous prouvons qu'après une renormalisation adéquate, la fonction de hauteur pondérée d'une forêt de tels arbres converge en loi vers le mouvement brownien réfléchi. Nous déduisons ensuite de ceci, sous des hypothèses raisonnables, un principe d'invariance sur les arbres de Galton–Watson multitypes à ensemble de type dénombrable, étendant ainsi un résultat de G. Miermont sur les arbres de Galton–Watson multitypes à ensemble de types fini.

MSC: 60J80; 60F17

Keywords: Galton–Watson tree; Multitype Galton–Watson tree; Infinitely many types; Edge lengths; Scaling limit

References

[1] E. Aïdékon and L. de Raphélis. Scaling limit of the recurrent biased random walk on a Galton–Watson tree. Preprint, 2015. Available at http://arxiv.org/abs/1509.07383.
[2] D. J. Aldous. The continuum random tree I. *Ann. Probab.* **19** (1991) 1–28. MR1085326
[3] D. J. Aldous. The continuum random tree II: An overview. In *Proc. Durham Symp. Stochastic Analysis 1990* 23–70. M. T. Barlow and N. H. Bingham (Eds). Cambridge University Press, Cambridge, 1991. MR1166406
[4] D. J. Aldous. The continuum random tree III. *Ann. Probab.* **21** (1993) 248–289. MR1207226
[5] J. D. Biggins. Random walk conditioned to stay positive. *J. Lond. Math. Soc. (2)* **67** (1) (2003) 259–272. MR1942425
[6] N. Curien and Y. Peres. Random laminations and multitype branching processes. *Electron. Commun. Probab.* **16** (2011) 435–446. MR2831082
[7] T. Duquesne and J.-F. Le Gall. Random trees, Lévy processes and spatial branching processes. *Astérisque* **281** (2002). MR1954248
[8] R. Durrett, H. Kesten and E. Waymire. On weighted heights of random trees. *J. Theoret. Probab.* **4** (1) (1991) 223–237. MR1088403
[9] W. Feller. *An Introduction to Probability Theory and Its Applications II*, 2nd edition. Wiley, New York, 1971. MR0270403
[10] T. E. Harris. *The Theory of Branching Processes*. *Die Grundlehren der Mathematischen Wissenschaften* **119**. Springer, Berlin, 1963. MR0163361
[11] P. Jagers. General branching processes as Markov fields. *Stochastic Process. Appl.* **32** (1989) 183–212. MR1014449
[12] T. G. Kurtz, R. Lyons, R. Permantle and Y. Peres. A conceptual proof of the Kesten–Stigum theorem for multi-type branching processes. In *Classical and Modern Branching Processes* 181–185. *IMA Vol. Math. Appl.* **84**, 1997. MR1601737
[13] J.-F. Le Gall. Random trees and applications. *Probab. Surv.* **2** (2005) 245–311. MR2203728
[14] J.-F. Le Gall and Y. Le Jan. Branching processes in Lévy processes: The exploration process. *Ann. Probab.* **26** (1) (1998) 213–252. MR1617047
[15] R. Lyons, R. Permantle and Y. Peres. Conceptual proofs of $L \log L$ criteria for mean behavior of branching processes. *Ann. Probab.* **23** (3) (1995) 1125–1138. MR1349164

[16] S. P. Meyn and R. L. Tweedie. *Markov Chains and Stochastic Stability*. Springer, New York, 1993. MR1287609

[17] G. Miermont. Invariance principles for spatial multitype Galton–Watson trees. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** (2008) 1128–1161. MR2469338

[18] J. Neveu. Arbres et processus de Galton–Watson. *Ann. Inst. Henri Poincaré Probab. Stat.* **22** (1986) 199–207. MR0850756

[19] M. Ossiander, E. Waymire and Q. Zhang. Some width function asymptotics for weighted trees. *Ann. Appl. Probab.* **7** (4) (1997) 972–995. MR1484794

[20] E. Seneta. *Non-negative Matrices and Markov Chains*, 2nd edition. Springer, New York, 1981. MR2209438

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The many-to-few lemma and multiple spines

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Abstract. We develop a simple and intuitive identity for calculating expectations of weighted k-fold sums over particles in branching processes, generalising the well-known many-to-one lemma.

Résumé. On développe une identité simple et intuitive pour calculer l'espérance des sommations k-plier sur particules dans les processus de branchement, la généralisation du lemme bien connu 'many-to-one'.

MSC: 60J80

Keywords: Branching processes; Many-to-one; Many-to-few; Spine

References

[1] E. Aïdékon and S. C. Harris. Near-critical survival probability of branching Brownian motion with an absorbing barrier. Preprint.
[2] E. Aidekon. Convergence in law of the minimum of a branching random walk. *Ann. Probab.* **41** (3A) (2013) 1362–1426. MR3098680
[3] S. Albeverio, L. V. Bogachev, S. A. Molchanov and E. B. Yarovaya. Annealed moment Lyapunov exponents for a branching random walk in a homogeneous random branching environment. *Markov Process. Related Fields* **6** (4) (2000) 473–516. MR1805091
[4] V. Bansaye, J.-F. Delmas, L. Marsalle and V.C. Tran. Limit theorems for Markov processes indexed by continuous time Galton–Watson trees. *Ann. Appl. Probab.* **21** (6) (2011) 2263–2314. MR2895416
[5] P. Carmona and Y. Hu. The spread of a catalytic branching random walk. *Ann. Inst. Henri Poincaré Probab. Stat.* **50** (2) (2014) 327–351. MR3189074
[6] B. Chauvin and A. Rouault. KPP equation and supercritical branching Brownian motion in the subcritical speed area. Application to spatial trees. *Probab. Theory Related Fields* **80** (2) (1988) 299–314. MR0968823
[7] X. Chen. Waiting times for particles in a branching Brownian motion to reach the rightmost position. *Stochastic Process. Appl.* **123** (8) (2013) 3153–3182. MR3062441
[8] D. A. Dawson and E. A. Perkins. *Historical Processes*. Amer. Math. Soc., Providence, RI, 1991. MR1079034
[9] L. Döring and M. I. Roberts. Catalytic branching processes via spine techniques and renewal theory. In *Séminaire de Probabilités, XLV* 305–322. *Lecture Notes in Math.* **2078**. Springer, Berlin, 2013. MR3185920
[10] L. G. Gorostiza and A. Wakolbinger. Persistence criteria for a class of critical branching particle systems in continuous time. *Ann. Probab.* **19** (1) (1991) 266–288. MR1085336
[11] O. Gün, W. König and O. Sekulovic. Moment asymptotics for branching random walks in random environment. *Electron. J. Probab.* **18** (63) (2013) 1–18. MR3078022
[12] O. Kallenberg. Stability of critical cluster fields. *Math. Nachr.* **77** (1) (1977) 7–43. MR0443078
[13] R. Lyons, R. Pemantle and Y. Peres. Conceptual proofs of $L \log L$ criteria for mean behavior of branching processes. *Ann. Probab.* **23** (3) (1995) 1125–1138. MR1349164
[14] P. Maillard. Branching Brownian motion with selection of the N right-most particles: An approximate model. Ph.D. thesis, 2012.
[15] P. Maillard. Speed and fluctuations of N-particle branching Brownian motion with spatial selection. *Probab. Theory Related Fields* **166** (3) (2016) 1061–1173. MR3568046
[16] M. I. Roberts. Spine changes of measure and branching diffusions. Ph.D. thesis, University of Bath, 2010. Available at http://people.bath.ac.uk/mir20/thesis.pdf.
[17] M. I. Roberts. Fine asymptotics for the consistent maximal displacement of branching Brownian motion. *Electron. J. Probab.* **20** (2015) article 28. MR3325098
[18] S. Sawyer. Branching diffusion processes in population genetics. *Adv. in Appl. Probab.* **8** (4) (1976) 659–689. MR0432250

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Some examples of quenched self-averaging in models with Gaussian disorder

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Abstract. In this paper we give an elementary approach to several results of Chatterjee in (Disorder chaos and multiple valleys in spin glasses (2013) arXiv:0907.3381, *Comm. Math. Phys.* **337** (2015) 93–102), as well as some generalizations. First, we prove quenched disorder chaos for the bond overlap in the Edwards–Anderson type models with Gaussian disorder. The proof extends to systems at different temperatures and covers a number of other models, such as the mixed p-spin model, Sherrington–Kirkpatrick model with multi-dimensional spins and diluted p-spin model. Next, we adapt the same idea to prove quenched self-averaging of the bond magnetization for one system and use it to show quenched self-averaging of the site overlap for random field models with positively correlated spins. Finally, we show self-averaging for certain modifications of the random field itself.

Résumé. Dans cet article, nous présentons une approche élémentaire de plusieurs résultats de Chatterjee (Disorder chaos and multiple valleys in spin glasses (2013) arXiv:0907.3381, *Comm. Math. Phys.* **337** (2015) 93–102), et quelques généralisations. D'abord, nous prouvons, dans le cas d'un désordre quenched, un résultat de chaos pour le recouvrement par arêtes dans les modèles de type Edwards–Anderson avec désordre gaussien. La preuve s'étend à des systèmes à différentes températures et couvre d'autres modèles comme le modèle p-spins mixte, le modèle de Sherrington–Kirkpatrick avec des spins multi-dimensionnels et le modèle p-spin dilué. Ensuite, nous adaptons la même idée pour prouver la propriété d'auto-moyennisation du recouvrement par site et nous l'utilisons pour montrer le même résultat pour des modèles avec un champs aléatoire et des spins positivement corrélés. Enfin, nous montrons la propriété d'auto-moyennisation pour certaines modifications du champs aléatoire.

MSC: 60K35; 82B44

Keywords: Self-averaging; Gaussian disorder; Spin glasses

References

[1] A. J. Bray and M. A. Moore. Chaotic nature of the spin-glass phase. *Phys. Rev. Lett.* **58** (1) (1987) 57–60.
[2] S. Chatterjee. Disorder chaos and multiple valleys in spin glasses, 2013. Available at arXiv:0907.3381.
[3] S. Chatterjee. *Superconcentration and Related Topics*. *Springer Monographs in Mathematics*. Springer, Berlin–Heidelberg, 2014. MR3157205
[4] S. Chatterjee. Absence of replica symmetry breaking in the random field Ising model. *Comm. Math. Phys.* **337** (2015) 93–102. MR3324156
[5] W.-K. Chen. Disorder chaos in the Sherrington–Kirkpatrick model with external field. *Ann. Probab.* **41** (5) (2013) 3345–3391. MR3127885
[6] W.-K. Chen and D. Panchenko. An approach to chaos in some mixed p-spin models. *Probab. Theory Related Fields* **157** (1) (2013) 389–404. MR3101851
[7] W.-K. Chen. Chaos problem in the mixed even-spin model. *Comm. Math. Phys.* **328** (3) (2014) 867–901. MR3201215
[8] D. S. Fisher and D. A. Huse. Ordered phase of short-range Ising spin glasses. *Phys. Rev. Lett.* **56** (15) (1986) 1601–1604.
[9] C. M. Fortuin, P. W. Kasteleyn and J. Ginibre. Correlation inequalities on some partially ordered sets. *Comm. Math. Phys.* **22** (1971) 89–103. MR0309498
[10] M. Talagrand. *Mean-Field Models for Spin Glasses*. *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge A Series of Modern Surveys in Mathematics* **54**. Springer-Verlag, Berlin, 2011. MR2731561

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Strong existence and uniqueness for degenerate SDE with Hölder drift

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Abstract. In this paper, we prove pathwise uniqueness for stochastic degenerate systems with a Hölder drift, for a Hölder exponent larger than the critical value $2/3$. This work extends to the degenerate setting the earlier results obtained by Zvonkin (*Mat. Sb. (N.S.)* **93(135)** (1974) 129–149, 152), Veretennikov (*Mat. Sb. (N.S.)* **111(153)** (1980) 434–452, 480), Krylov and Röckner (*Probab. Theory Related Fields* **131**(2) (2005) 154–196) from non-degenerate to degenerate cases. The existence of a threshold for the Hölder exponent in the degenerate case may be understood as the price to pay to balance the degeneracy of the noise. Our proof relies on regularization properties of the associated PDE, which is degenerate in the current framework and is based on a parametrix method.

Résumé. Dans ce travail, on montre qu'un système hypoelliptique, composé d'une composante diffusive et d'une composante totalement dégénérée, est fortement résoluble lorsque l'exposant de la régularité Hölder de la dérive par rapport à la composante dégénérée est strictement supérieur à $2/3$. Ce travail étend au cadre dégénéré les travaux antérieurs de Zvonkin (*Mat. Sb. (N.S.)* **93(135)** (1974) 129–149, 152), Veretennikov (*Mat. Sb. (N.S.)* **111(153)** (1980) 434–452, 480), Krylov et Röckner (*Probab. Theory Related Fields* **131**(2) (2005) 154–196). L'apparition d'un seuil critique pour l'exposant peut-être vue comme le prix à payer pour la dégénérescence. La preuve repose sur des résultats de régularité de la solution de l'EDP associée, qui est dégénérée, et est basée sur une méthode parametrix.

MSC: 60H10; 60H30; 34F05

Keywords: Strong uniqueness; Degeneracy; Hölder drift; Parametrix; Stochastic differential equation

References

[1] R. F. Bass. *Diffusions and Elliptic Operators*. *Probability and Its Applications (New York)*. Springer-Verlag, New York, 1998. MR1483890
[2] A. M. Davie. Uniqueness of solutions of stochastic differential equations. *Int. Math. Res. Not. IMRN* **24** (2007) Article ID rnm124. MR2377011
[3] F. Delarue and S. Menozzi. Density estimates for a random noise propagating through a chain of differential equations. *J. Funct. Anal.* **259** (6) (2010) 1577–1630. MR2659772
[4] M. Di Francesco and S. Polidoro. Schauder estimates, Harnack inequality and Gaussian lower bound for Kolmogorov-type operators in non-divergence form. *Adv. Differential Equations* **11** (11) (2006) 1261–1320. MR2277064
[5] E. Fedrizzi and F. Flandoli. Pathwise uniqueness and continuous dependence of SDEs with non-regular drift. *Stochastics* **83** (3) (2011) 241–257. MR2810591
[6] F. Flandoli. *Random Perturbation of PDEs and Fluid Dynamic Models*. *Lecture Notes in Mathematics* **2015**. Springer, Heidelberg, 2011. MR2796837
[7] M. I. Freidlin. *Functional Integration and Partial Differential Equations*. Princeton University Press, Princeton, NJ, 1985. MR0833742
[8] A. Friedman. *Partial Differential Equations of Parabolic Type*. Prentice-Hall, Englewood Cliffs, NJ, 1964. MR0181836
[9] A. Friedman. *Stochastic Differential Equations and Applications*. Dover, Mineola, NY, 2006. MR2295424
[10] L. Hörmander. Hypoelliptic second order differential equations. *Acta Math.* **119** (1967) 147–171. MR0222474
[11] A. Kolmogorov. Zufällige bewegungen (zur theorie der brownschen bewegung). *Ann. of Math. (2)* **35** (1934) 116–117. MR1503147
[12] N. V. Krylov and M. Röckner. Strong solutions of stochastic equations with singular time dependent drift. *Probab. Theory Related Fields* **131** (2) (2005) 154–196. MR2117951

[13] H. Kunita. Stochastic differential equation and stochastic flows of diffeomorphisms. In *École d'été de probabilités de Saint-Flour, XII – 1982* 143–303. Springer-Verlag, Berlin, 1982. MR0876080
[14] S. Menozzi. Parametrix techniques and martingale problems for some degenerate Kolmogorov equations. *Electron. Commun. Probab.* **16** (2011) 234–250. MR2802040
[15] D. W. Stroock and S. R. S. Varadhan. *Multidimensional Diffusion Processes*. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **233**. Springer-Verlag, Berlin, 1979. MR0532498
[16] A. J. Veretennikov. Strong solutions and explicit formulas for solutions of stochastic integral equations. *Mat. Sb. (N.S.)* **111(153)** (1980) 434–452, 480. MR0568986
[17] A. Y. Veretennikov. Stochastic equations with diffusion that degenerates with respect to part of the variables. *Izv. Ross. Akad. Nauk Ser. Mat.* **47** (1) (1983) 189–196. MR0688921
[18] X. Zhang. Well-posedness and large deviation for degenerate SDEs with Sobolev coefficients. *Rev. Mat. Iberoamericana* **29** (1) (2010) 25–52. MR3010120
[19] X. Zhang. Stochastic homeomorphism flows of SDEs with singular drifts and Sobolev diffusion coefficients. *Electron. J. Probab.* **16** (2011) 1096–1116. MR2820071
[20] A. K. Zvonkin. A transformation of the phase space of a diffusion process that will remove the drift. *Mat. Sb. (N.S.)* **93(135)** (1974) 129–149. 152. MR0336813

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Hammersley's harness process: Invariant distributions and height fluctuations

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Abstract. We study the invariant distributions of Hammersley's serial harness process in all dimensions and height fluctuations in one dimension. Subject to mild moment assumptions there is essentially one unique invariant distribution, and all other invariant distributions are obtained by adding harmonic functions of the averaging kernel. We identify one Gaussian case where the invariant distribution is i.i.d. Height fluctuations in one dimension obey the stochastic heat equation with additive noise (Edwards–Wilkinson universality). We prove this for correlated initial data subject to fast enough polynomial decay of strong mixing coefficients, including process-level tightness in the Skorohod space of space–time trajectories.

Résumé. Nous étudions la mesure invariante du processus de harnais de Hammersley en dimension arbitraire et les fluctuations de la hauteur en dimension un. Sous des hypothèses douces sur les moments, il y a essentiellement une mesure invariante unique, et toutes les autres mesures invariantes sont obtenues par l'addition de fonctions harmoniques du noyau. Nous identifions un cas Gaussien ou la mesure invariante est i.i.d. Les fluctuations de la hauteur en dimension un obéissent à l'équation stochastique de chaleur à bruit additif (universalité d'Edwards–Wilkinson). Nous démontrons ce résultat dans le cas de données initiales corrélées dont les coefficients de mélange fort décroissent assez rapidement, y compris l'étroitesse au niveau du processus dans l'espace de Skorohod de trajectoires spatio-temporelles.

MSC: 60K35; 60F05

Keywords: Harness; Gaussian process; Edwards–Wilkinson universality class; Random walk; Fractional Brownian motion; Fluctuations; Interface; Process tightness; Strong mixing coefficients; Linear process; Harmonic crystal; Stochastic heat equation

References

[1] R. Arratia. The motion of a tagged particle in the simple symmetric exclusion system on **Z**. *Ann. Probab.* **11** (2) (1983) 362–373. MR0690134
[2] M. Balázs, F. Rassoul-Agha and T. Seppäläinen. The random average process and random walk in a space–time random environment in one dimension. *Comm. Math. Phys.* **266** (2) (2006) 499–545. MR2238887
[3] P. J. Bickel and M. J. Wichura. Convergence criteria for multiparameter stochastic processes and some applications. *Ann. Math. Stat.* **42** (1971) 1656–1670. MR0383482
[4] A. Borodin and V. Gorin. Lectures on integrable probability. Available at arXiv:1212.3351.
[5] R. C. Bradley. Basic properties of strong mixing conditions. A survey and some open questions. *Probab. Surv.* **2** (2005) 107–144. Update of, and a supplement to, the 1986 original. MR2178042
[6] P. Caputo and J.-D. Deuschel. Large deviations and variational principle for harmonic crystals. *Comm. Math. Phys.* **209** (3) (2000) 595–632. MR1743610
[7] I. Corwin. The Kardar–Parisi–Zhang equation and universality class. *Random Matrices Theory Appl.* **1** (1) (2012) 1130001. MR2930377
[8] A. De Masi and P. A. Ferrari. Flux fluctuations in the one dimensional nearest neighbors symmetric simple exclusion process. *J. Stat. Phys.* **107** (3–4) (2002) 677–683. MR1898853
[9] D. Dürr, S. Goldstein and J. Lebowitz. Asymptotics of particle trajectories in infinite one-dimensional systems with collisions. *Comm. Pure Appl. Math.* **38** (5) (1985) 573–597. MR0803248

[10] R. Durrett. *Probability: Theory and Examples*, 4th edition. *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge University Press, Cambridge, 2010. MR2722836
[11] W. Feller. *An Introduction to Probability Theory and Its Applications, Vol. II*, 2nd edition. Wiley, New York, 1971. MR0270403
[12] P. A. Ferrari and L. R. G. Fontes. Fluctuations of a surface submitted to a random average process. *Electron. J. Probab.* **3** (1998) 6 (electronic). MR1624854
[13] P. A. Ferrari and B. M. Niederhauser. Harness processes and harmonic crystals. *Stochastic Process. Appl.* **116** (6) (2006) 939–956. MR2254666
[14] N. G. Gamkrelidze. A measure of "smoothness" of multidimensional distributions of integer-valued random vectors. *Theory Probab. Appl.* **30** (2) (1985) 427–431. MR0792636
[15] J. M. Hammersley. Harnesses. In *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability (Berkeley, CA, 1965/66), Vol. III: Physical Sciences* 89–117. University of California Press, Berkeley, CA, 1967. MR0224144
[16] T. E. Harris. Diffusion with "collisions" between particles. *J. Appl. Probab.* **2** (1965) 323–338. MR0184277
[17] C. T. Hsiao. Stochastic processes with Gaussian interaction of components. *Z. Wahrsch. Verw. Gebiete* **59** (1) (1982) 39–53. MR0643787
[18] C. T. Hsiao. Infinite systems with locally additive interaction of components. *Chinese J. Math.* **13** (2) (1985) 83–95. MR0805447
[19] M. D. Jara and C. Landim. Nonequilibrium central limit theorem for a tagged particle in symmetric simple exclusion. *Ann. Inst. Henri Poincaré Probab. Stat.* **42** (5) (2006) 567–577. MR2259975
[20] J. Mathew, F. Rassoul-Agha and T. Seppäläinen. Independent particles in a dynamical random environment. Available at arXiv:1110.1889.
[21] R. Kumar. Space–time current process for independent random walks in one dimension. *ALEA Lat. Am. J. Probab. Math. Stat.* **4** (2008) 307–336. MR2456971
[22] G. F. Lawler and V. Limic. *Random Walk: A Modern Introduction*. *Cambridge Studies in Advanced Mathematics* **123**. Cambridge University Press, Cambridge, 2010. MR2677157
[23] T. M. Liggett. *Interacting Particle Systems*. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **276**. Springer-Verlag, New York, 1985. MR0776231
[24] T. M. Liggett. Interacting particle systems – An introduction. In *School and Conference on Probability Theory* 1–56. *ICTP Lecture Notes* **XVII**. Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2004 (electronic). MR2198846
[25] M. Peligrad and S. Sethuraman. On fractional Brownian motion limits in one dimensional nearest-neighbor symmetric simple exclusion. *ALEA Lat. Am. J. Probab. Math. Stat.* **4** (2008) 245–255. MR2448774
[26] M. Peligrad and S. Utev. Central limit theorem for linear processes. *Ann. Probab.* **25** (1) (1997) 443–456. MR1428516
[27] J. Peterson and T. Seppäläinen. Current fluctuations of a system of one-dimensional random walks in random environment. *Ann. Probab.* **38** (6) (2010) 2258–2294. MR2683630
[28] M. A. Pinsky. *Introduction to Fourier Analysis and Wavelets*. *Brooks/Cole Series in Advanced Mathematics*. Brooks/Cole, Pacific Grove, CA, 2002. MR2100936
[29] E. Rio. Inequalities and limit theorems for weakly dependent sequences. Lecture notes for 3ème cycle. Available at https://cel.archives-ouvertes.fr/cel-00867106, 2013.
[30] H. Rost and M. E. Vares. Hydrodynamics of a one-dimensional nearest neighbor model. In *Particle Systems, Random Media and Large Deviations* 329–342. *Contemp. Math.* **41**. Amer. Math. Soc., Providence, RI, 1985. MR0814722
[31] T. Seppäläinen. Translation invariant exclusion processes. Lecture notes. Available at http://www.math.wisc.edu/~seppalai/excl-book/etusivu.html.
[32] T. Seppäläinen. Second-order fluctuations and current across characteristic for a one-dimensional growth model of independent random walks. *Ann. Probab.* **33** (2) (2005) 759–797. MR2123209
[33] T. Seppäläinen. *Current Fluctuations for Stochastic Particle Systems with Drift in One Spatial Dimension*. *Ensaios Matemáticos [Mathematical Surveys]* **18**. Sociedade Brasileira de Matemática, Rio de Janeiro, 2010. MR2724800
[34] F. Spitzer. *Principles of Random Walks*, 2nd edition. *Graduate Texts in Mathematics* **34**. Springer-Verlag, New York, 1976. MR0388547
[35] A. Toom. Tails in harnesses. *J. Stat. Phys.* **88** (1–2) (1997) 347–364. MR1468388
[36] J. B. Walsh. An introduction to stochastic partial differential equations. In *École d'été de probabilités de Saint-Flour, XIV – 1984* 265–439. *Lecture Notes in Mathematics* **1180**. Springer, Berlin, 1986. MR0876085
[37] J. Yu. Edwards–Wilkinson fluctuations in the Howitt–Warren flows. Available at arXiv:1412.3911.
[38] Y. Zhai. Discrete time harness processes. Ph.D. thesis, University of Wisconsin, 2015. Available at arXiv:1506.02116. MR3407309

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Scaling limits for the peeling process on random maps

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Abstract. We study the scaling limit of the volume and perimeter of the discovered regions in the Markovian explorations known as peeling processes for infinite random planar maps such as the uniform infinite planar triangulation (UIPT) or quadrangulation (UIPQ). In particular, our results apply to the metric exploration or peeling by layers algorithm, where the discovered regions are (almost) completed balls, or hulls, centered at the root vertex. The scaling limits of the perimeter and volume of hulls can be expressed in terms of the hull process of the Brownian plane studied in our previous work. Other applications include the metric exploration of the dual graph of our infinite random lattices, and first-passage percolation with exponential edge weights on the dual graph, also known as the Eden model or uniform peeling.

Résumé. Nous étudions la limite d'échelle du processus des volumes et des périmètres des régions explorées par un algorithme « d'épluchage » sur les cartes infinies aléatoires telles que l'UIPT (la triangulation infinie uniforme du plan) ou son analogue quadrangulaire l'UIPQ. Nos résultats s'appliquent en particulier à l'exploration des boules (pour la distance de graphe) complétées et centrées à la racine de la carte. Dans ce cas, la limite d'échelle coïncide avec le processus du périmètre et du volume des boules complétées dans le plan brownien. Parmi les autres applications, mentionnons l'exploration des boules complétées sur la carte duale et la percolation de premier passage avec poids exponentiels sur la carte duale. Ce dernier modèle, équivalent au modèle d'Eden sur la carte initiale, correspond à l'algorithme d'épluchage uniforme.

MSC: Primary 05C80; 60F17; secondary 60J10

Keywords: Random planar maps; Peeling process; Scaling limits; Lévy process

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References

[1] J. Ambjørn and T. Budd. Multi-point functions of weighted cubic maps. *Ann. Inst. Henri Poincaré D* **3** (1) (2016) 1–44. MR3462628
[2] O. Angel. Scaling of percolation on infinite planar maps, I. Available at arXiv:math/0501006.
[3] O. Angel. Growth and percolation on the uniform infinite planar triangulation. *Geom. Funct. Anal.* **13** (2003) 935–974. MR2024412
[4] O. Angel and N. Curien. Percolations on infinite random maps, half-plane models. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 405–431. MR3335009
[5] O. Angel and G. Ray. Classification of half-planar maps. *Ann. Probab.* **43** (2015) 1315–1349. MR3342664
[6] O. Angel and O. Schramm. Uniform infinite planar triangulation. *Comm. Math. Phys.* **241** (2003) 191–213. MR2013797
[7] I. Benjamini and N. Curien. Simple random walk on the uniform infinite planar quadrangulation: Subdiffusivity via pioneer points. *Geom. Funct. Anal.* **23** (2013) 501–531. MR3053754
[8] J. Bertoin. *Lévy Processes*. *Cambridge Tracts in Mathematics* **121**. Cambridge University Press, Cambridge, 1996. MR1406564
[9] J. Bertoin and R. A. Doney. On conditioning a random walk to stay nonnegative. *Ann. Probab.* **22** (1994) 2152–2167. MR1331218
[10] J. Bouttier and E. Guitter. Distance statistics in quadrangulations with a boundary, or with a self-avoiding loop. *J. Phys. A* **42** (2009) 465208. MR2552016
[11] T. Budd. The peeling process of infinite Boltzmann planar maps. *Electron. J. Combin.* **23** (1) (**2016**) Paper 1.28. MR3484733
[12] F. Caravenna and L. Chaumont. Invariance principles for random walks conditioned to stay positive. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** (2008) 170–190. MR2451576
[13] P. Chassaing and B. Durhuus. Local limit of labeled trees and expected volume growth in a random quadrangulation. *Ann. Probab.* **34** (2006) 879–917. MR2243873
[14] N. Curien. Planar stochastic hyperbolic triangulations. *Probab. Theory Related Fields* **165** (3–4) (2016) 509–540. MR3520011

[15] N. Curien. A glimpse of the conformal structure of random planar maps. *Comm. Math. Phys.* **333** (2015) 1417–1463. MR3302638

[16] N. Curien and J.-F. Le Gall. First-passage percolation and local modifications of distances in random planar maps. Available at arxiv:1511.04264.

[17] N. Curien and J.-F. Le Gall. The hull process of the Brownian plane. *Probab. Theory Related Fields* **166** (1–2) (2016) 187–231. MR3547738

[18] N. Curien and J.-F. Le Gall. The Brownian plane. *J. Theoret. Probab.* **27** (2014) 1249–1291. MR3278940

[19] I. A. Ibragimov and Y. V. Linnik. *Independent and Stationary Sequences of Random Variables*. Wolters-Noordhoff Publishing, Groningen, 1971. MR0322926

[20] T. Jeulin. Sur la convergence absolue de certaines intégrales. In *Séminaire de Probabilités XVI* 248–256. *Lecture Notes in Mathematics* **920**. Springer, Berlin, 1982. MR0658688

[21] M. Krikun. Local structure of random quadrangulations. Available at arXiv:0512304.

[22] M. Krikun. A uniformly distributed infinite planar triangulation and a related branching process. *J. Math. Sci. (N. Y.)* **131** (2005) 5520–5537. MR2050691

[23] M. Krikun. Explicit enumeration of triangulations with multiple boundaries. *Electron. J. Combin.* **14** (2007) P61. MR2336338

[24] R. Lyons and Y. Peres. Probability on trees and networks. Available at http://mypage.iu.edu/~rdlyons/.

[25] L. Ménard. The two uniform infinite quadrangulations of the plane have the same law. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** (2010) 190–208. MR2641776

[26] L. Ménard and P. Nolin. Percolation on uniform infinite planar maps. *Electron. J. Probab.* **19** (2014) 79. MR3256879

[27] J. Miller and S. Sheffield. Quantum Loewner evolution. *Duke Math. J.* **165** (17) (2016) 3241–3378. MR3572845

[28] J. Pitman. *Combinatorial Stochastic Processes*. *Lecture Notes in Mathematics* **1875**. Springer, Berlin, 2006. MR2245368

[29] G. Ray. Geometry and percolation on half planar triangulations. *Electron. J. Probab.* **19** (2014) 47. MR3217335

[30] R. Stephenson. Local convergence of large critical multi-type Galton–Watson trees and applications to random maps. Available at arXiv:1412.6911 and http://link.springer.com/article/10.1007/s10959-016-0707-3.

[31] Y. Watabiki. Construction of non-critical string field theory by transfer matrix formalism in dynamical triangulation. *Nuclear Phys. B* **441** (1995) 119–163. MR1329946

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On comparison principle and strict positivity of solutions to the nonlinear stochastic fractional heat equations

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Abstract. In this paper, we prove a sample-path comparison principle for the nonlinear stochastic fractional heat equation on $\mathbb{R}$ with measure-valued initial data. We give quantitative estimates about how close to zero the solution can be. These results extend Mueller's comparison principle on the stochastic heat equation to allow more general initial data such as the (Dirac) delta measure and measures with heavier tails than linear exponential growth at $\pm\infty$. These results generalize a recent work by Moreno Flores (*Ann. Probab.* **42** (2014) 1635–1643), who proves the strict positivity of the solution to the stochastic heat equation with the delta initial data. As one application, we establish the *full intermittency* for the equation. As an intermediate step, we prove the Hölder regularity of the solution starting from measure-valued initial data, which generalizes, in some sense, a recent work by Chen and Dalang (*Stoch. Partial Differ. Equ. Anal. Comput.* **2** (2014) 316–352).

Résumé. Dans ce papier, nous montrons un principe de comparaison trajectoriel pour l'équation de la chaleur stochastique, fractionnaire, nonlinéaire sur $\mathbb{R}$ avec une donnée initiale à valeur mesure. Nous donnons des estimations quantitatives de la proximité à zéro d'une solution. Ces résultats étendent le principe de comparaison de Mueller pour l'équation de la chaleur stochastique et permettent de considérer des données initiales plus générales telles que des mesures de Dirac et des mesures à queue plus lourde qu'une croissance exponentielle linéaire en $\pm\infty$. Ces résultats généralisent un travail récent par Moreno Flores (*Ann. Probab.* **42** (2014) 1635–1643), qui a prouvé la stricte positivité de l'équation de la chaleur stochastique partant d'un Dirac. Comme application, nous établissons la *complète intermittence* pour l'équation. Dans une étape intermédiaire, nous prouvons la régularité Hölder de solutions partant d'une donnée initiale à valeur mesure ce qui généralise, dans un certain sens, un travail récent de Chen and Dalang (*Stoch. Partial Differ. Equ. Anal. Comput.* **2** (2014) 316–352).

MSC: Primary 60H15; secondary 60G60; 35R60

Keywords: Nonlinear stochastic fractional heat equation; Parabolic Anderson model; Comparison principle; Measure-valued initial data; Stable processes

References

[1] T. Alberts, K. Khanin and J. Quastel. The intermediate disorder regime for directed polymers in dimension $1+1$. *Ann. Probab.* **42** (3) (2014) 1212–1256. MR3189070
[2] S. Assing. Comparison of systems of stochastic partial differential equations. *Stochastic Process. Appl.* **82** (2) (1999) 259–282. MR1700009
[3] D. Baĭnov and P. Simeonov. *Integral Inequalities and Applications*. *Mathematics and Its Applications (East European Series)* **57**. Kluwer Academic Publishers Group, Dordrecht, 1992. MR1171448
[4] L. Bertini and N. Cancrini. The stochastic heat equation: Feynman–Kac formula and intermittence. *J. Stat. Phys.* **78** (5–6) (1995) 1377–1401. MR1316109
[5] R. A. Carmona and S. A. Molchanov. Parabolic Anderson problem and intermittency. *Mem. Amer. Math. Soc.* **108** (518) (1994) 1–125. MR1185878
[6] L. Chen and R. C. Dalang. Hölder-continuity for the nonlinear stochastic heat equation with rough initial conditions. *Stoch. Partial Differ. Equ. Anal. Comput.* **2** (3) (2014) 316–352. MR3255231

[7] L. Chen and R. C. Dalang Moments and growth indices for nonlinear stochastic heat equation with rough initial conditions. *Ann. Probab.* **43** (6) (2015) 3006–3051. MR3433576

[8] L. Chen and R. C. Dalang. Moment bounds and asymptotics for the stochastic wave equation. *Stochastic Process. Appl.* **125** (4) (2015) 1605–1628. MR3310358

[9] L. Chen and R. C. Dalang. Moments, intermittency, and growth indices for the nonlinear fractional stochastic heat equation. *Stoch. Partial Differ. Equ. Anal. Comput.* **3** (3) (2015) 360–397. MR3383450

[10] D. Conus, M. Joseph and D. Khoshnevisan. Correlation-length bounds, and estimates for intermittent islands in parabolic SPDEs. *Electron. J. Probab.* **17** (2012) 102. MR3005720

[11] D. Conus, M. Joseph, D. Khoshnevisan and S.-Y. Shiu. Initial measures for the stochastic heat equation. *Ann. Inst. Henri Poicaré Probab. Stat.* **50** (1) (2014) 136–153. MR3161526

[12] J. T. Cox, K. Fleischmann and A. Greven. Comparison of interacting diffusions and an application to their ergodic theory. *Probab. Theory Related Fields* **105** (4) (1996) 513–528. MR1402655

[13] L. Debbi. Explicit solutions of some fractional partial differential equations via stable subordinators. *J. Appl. Math. Stoch. Anal.* **2006** (2006) Article ID 93502. MR2221004

[14] L. Debbi and M. Dozzi. On the solutions of nonlinear stochastic fractional partial differential equations in one spatial dimension. *Stochastic Process. Appl.* **115** (11) (2005) 1764–1781. MR2172885

[15] M. Foondun and D. Khoshnevisan. Intermittence and nonlinear parabolic stochastic partial differential equations. *Electron. J. Probab.* **14** (21) (2009) 548–568. MR2480553

[16] B. Hajek. Mean stochastic comparison of diffusions. *Z. Wahrsch. Verw. Gebiete* **68** (3) (1985) 315–329. MR0771469

[17] N. Ikeda and S. Watanabe. *Stochastic Differential Equations and Diffusion Processes*, 2nd edition. *North-Holland Mathematical Library* **24**. North-Holland Publishing Co., Amsterdam, 1989. MR1011252

[18] S. Jacka and R. Tribe. Comparisons for measure valued processes with interactions. *Ann. Probab.* **31** (3) (2003) 1679–1712. MR1989447

[19] M. Joseph, D. Khoshnevisan and C. Mueller. Strong invariance and noise-comparison principles for some parabolic stochastic PDEs. Preprint, 2014. Available at arXiv:1404.6911.

[20] M. Kardar, G. Parisi and Y.-C. Zhang. Dynamic scaling of growing interfaces. *Phys. Rev. Lett.* **56** (9) (1986) 889–892.

[21] P. Kotelenez. Comparison methods for a class of function valued stochastic partial differential equations. *Probab. Theory Related Fields* **93** (1) (1992) 1–19. MR1172936

[22] H. Kunita. *Stochastic Flows and Stochastic Differential Equations*. *Cambridge Studies in Advanced Mathematics* **24**. Cambridge University Press, Cambridge, 1990. MR1070361

[23] F. Mainardi, Y. Luchko and G. Pagnini. The fundamental solution of the space–time fractional diffusion equation. *Fract. Calc. Appl. Anal.* **4** (2) (2001) 153–192. MR1829592

[24] A. Milian. Comparison theorems for stochastic evolution equations. *Stoch. Stoch. Rep.* **72** (1–2) (2002) 79–108. MR1896438

[25] G. R. Moreno Flores. On the (strict) positivity of solutions of the stochastic heat equation. *Ann. Probab.* **42** (4) (2014) 1635–1643. MR3262487

[26] C. Mueller. On the support of solutions to the heat equation with noise. *Stoch. Stoch. Rep.* **37** (4) (1991) 225–245. MR1149348

[27] C. Mueller. Some tools and results for parabolic stochastic partial differential equations. In *A Minicourse on Stochastic Partial Differential Equations* 111–144. *Lecture Notes in Math.* **1962**. Springer, Berlin, 2009. MR2508775

[28] C. Mueller and D. Nualart. Regularity of the density for the stochastic heat equation. *Electron. J. Probab.* **13** (74) (2008) 2248–2258. MR2469610

[29] F. W. J. Olver, D. W. Lozier, R. F. Boisvert and C. W. Clark (Eds). *NIST Handbook of Mathematical Functions*. U.S. Department of Commerce National Institute of Standards and Technology, Washington, DC, 2010. MR2723248

[30] L. C. G. Rogers and D. Williams. *Diffusions, Markov Processes, and Martingales. Vol. 2*. *Cambridge Mathematical Library*. Cambridge University Press, Cambridge, 2000. MR1780932

[31] T. Shiga. Two contrasting properties of solutions for one-dimensional stochastic partial differential equations. *Canad. J. Math.* **46** (2) (1994) 415–437. MR1271224

[32] V. V. Uchaikin and V. M. Zolotarev. *Chance and Stability. Modern Probability and Statistics*. VSP, Utrecht, 1999. MR1745764

[33] J. B. Walsh. An introduction to stochastic partial differential equations. In *École d'Été de Probabilités de Saint-Flour XIV—1984* 265–439. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876085

[34] V. M. Zolotarev. *One-Dimensional Stable Distributions*. *Translations of Mathematical Monographs* **65**. American Mathematical Society, Providence, RI, 1986. MR0854867

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Penalized maximum likelihood estimation and effective dimension

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Abstract. This paper extends some prominent statistical results including *Fisher Theorem and Wilks phenomenon* to the penalized maximum likelihood estimation with a quadratic penalization. It appears that sharp expansions for the penalized MLE $\widetilde{\boldsymbol{\theta}}_G$ and for the penalized maximum likelihood can be obtained without involving any asymptotic arguments, the results only rely on smoothness and regularity properties of the of the considered log-likelihood function. The error of estimation is specified in terms of the effective dimension $\mathtt{p}_G$ of the parameter set which can be much smaller than the true parameter dimension and even allows an infinite dimensional functional parameter. In the i.i.d. case, the Fisher expansion for the penalized MLE can be established under the constraint "$\mathtt{p}_G^2/n$ is small" while the remainder in the Wilks result is of order $\sqrt{\mathtt{p}_G^3/n}$.

Résumé. Cet article généralise certains résultats statistiques importants dont le *Théorème de Fisher* et le *phénomène de Wilks* à l'estimation du maximum de vraisemblance pénalisée de façon quadratique. Il apparaît que des développements précis pour l'EMV pénalisée $\widetilde{\boldsymbol{\theta}}_G$ et le maximum de vraisemblance pénalisé peuvent être obtenus sans arguments asymptotiques, les résultats reposent alors seulement sur la régularité et les propriétés de la fonction de log-vraisemblance. L'erreur d'estimation est spécifiée en fonction de la dimension effective $\mathtt{p}_G$ de l'ensemble des paramètres qui peut être beaucoup plus petite que la véritable dimension et permet ainsi de considérer un cas infini dimensionnel. Dans le cas i.i.d., le développement de Fisher pour l'EMV pénalisée peut être établi sous la contrainte « $\mathtt{p}_G^2/n$ est petit » tandis que le reste dans le résultat de Wilks est d'ordre $\sqrt{\mathtt{p}_G^3/n}$.

MSC: Primary 62F10; secondary 62J12; 62F25; 62H12

Keywords: Penalty; Wilks and Fisher expansions

References

[1] A. Andresen and V. Spokoiny. Critical dimension in profile semiparametric estimation. *Electron. J. Stat.* **8** (2) (2014) 3077–3125. MR3301302
[2] A. Barron, L. Birgé and P. Massart. Risk bounds for model selection via penalization. *Probab. Theory Related Fields* **113** (3) (1999) 301–413. MR1679028
[3] A. Belloni and V. Chernozhukov. On the computational complexity of MCMC-based estimators in large samples. *Ann. Statist.* **37** (4) (2009) 2011–2055. MR2533478
[4] L. Birgé and P. Massart. Minimum contrast estimators on sieves: Exponential bounds and rates of convergence. *Bernoulli* **4** (3) (1998) 329–375. MR1653272
[5] L. Birgé and P. Massart. Gaussian model selection. *J. Eur. Math. Soc. (JEMS)* **3** (3) (2001) 203–268. MR1848946
[6] L. Birgé and P. Massart. Minimal penalties for Gaussian model selection. *Probab. Theory Related Fields* **138** (1–2) (2007) 33–73. MR2288064
[7] S. Boucheron and P. Massart. A high-dimensional Wilks phenomenon. *Probab. Theory Related Fields* **150** (2011) 405–433. MR2824862
[8] J. Fan, C. Zhang and J. Zhang. Generalized likelihood ratio statistics and Wilks phenomenon. *Ann. Statist.* **29** (1) (2001) 153–193. MR1833962
[9] S. Ghosal. Asymptotic normality of posterior distributions in high-dimensional linear models. *Bernoulli* **5** (2) (1999) 315–331. MR1681701
[10] S. Ghosal. Asymptotic normality of posterior distributions for exponential families when the number of parameters tends to infinity. *J. Multivariate Anal.* **74** (1) (2000) 49–68. MR1790613

[11] Y. Golubev and V. Spokoiny. Exponential bounds for minimum contrast estimators. *Electron. J. Stat.* **3** (2009) 712–746. MR2534199

[12] P. J. Green and B. W. Silverman. *Nonparametric Regression and Generalized Linear Models: A Roughness Penalty Approach*. Chapman & Hall, London, 1994. MR1270012

[13] P. J. Huber. The behavior of maximum likelihood estimates under nonstandard conditions. In *Proc. 5th Berkeley Symp. Math. Stat. Probab., Univ. Calif. 1965/66, 1* 221–233. Univ. California Press, Berkeley, CA, 1967. MR0216620

[14] I. A. Ibragimov and R. Z. Khas'minskij. *Statistical Estimation. Asymptotic Theory*. Springer, New York, 1981. Transl. from the Russian by Samuel Kotz. MR0620321

[15] Y. Kim. The Bernstein–von Mises theorem for the proportional hazard model. *Ann. Statist.* **34** (4) (2006) 1678–1700. MR2283713

[16] R. Koenker, P. Ng and S. Portnoy. Quantile smoothing splines. *Biometrika* **81** (4) (1994) 673–680. MR1326417

[17] B. Laurent and P. Massart. Adaptive estimation of a quadratic functional by model selection. *Ann. Statist.* **28** (5) (2000) 1302–1338.

[18] E. Mammen. Empirical process of residuals for high-dimensional linear models. *Ann. Statist.* **24** (1) (1996) 307–335. MR1389892

[19] S. Portnoy. Asymptotic behavior of M-estimators of p regression parameters when p^2/n is large. I. Consistency. *Ann. Statist.* **12** (4) (1984) 1298–1309. MR0760690

[20] S. Portnoy. Asymptotic behavior of M estimators of p regression parameters when p^2/n is large. II. Normal approximation. *Ann. Statist.* **13** (4) (1985) 1403–1417. MR0811499

[21] S. Portnoy. Asymptotic behavior of the empiric distribution of M-estimated residuals from a regression model with many parameters. *Ann. Statist.* **14** (1986) 1152–1170. MR0856812

[22] S. Portnoy. Asymptotic behavior of likelihood methods for exponential families when the number of parameters tends to infinity. *Ann. Statist.* **16** (1) (1988) 356–366. MR0924876

[23] X. Shen. On methods of sieves and penalization. *Ann. Statist.* **25** (6) (1997) 2555–2591. MR1604416

[24] X. Shen and W. H. Wong. Convergence rate of sieve estimates. *Ann. Statist.* **22** (2) (1994) 580–615. MR1292531

[25] V. Spokoiny. Parametric estimation. Finite sample theory. *Ann. Statist.* **40** (6) (2012) 2877–2909. MR3097963

[26] V. Spokoiny, W. Wang and W. Härdle. Local quantile regression (with rejoinder). *J. Statist. Plann. Inference* **143** (7) (2013) 1109–1129. MR3049611

[27] V. Spokoiny and M. Zhilova. Bootstrap confidence sets under model misspecification. *Ann. Statist.* **43** (2015) 2653–2675. MR3405607

[28] S. van de Geer. M-estimation using penalties or sieves. *J. Statist. Plann. Inference* **108** (1–2) (2002) 55–69. MR1947391

[29] S. A. Van de Geer. *Applications of Empirical Process Theory*. *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge University Press, Cambridge, 2000. MR1739079

[30] A. van der Vaart and J. A. Wellner. *Weak Convergence and Empirical Processes. With Applications to Statistics*. *Springer Series in Statistics*. Springer, New York, 1996. MR1385671

[31] A. Zaitsev, E. Burnaev and V. Spokoiny. Properties of the posterior distribution of a regression model based on Gaussian random fields. *Autom. Remote Control* **74** (10) (2013) 1645–1655. MR3219856

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The high-temperature behavior for the directed polymer in dimension $1+2$

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Abstract. We investigate the high-temperature behavior of the directed polymer model in dimension $1+2$. More precisely we study the difference $\Delta\mathtt{F}(\beta)$ between the *quenched* and *annealed* free energies for small values of the inverse temperature β. This quantity is associated to localization properties of the polymer trajectories, and is related to the overlap fraction of two replicas. Adapting recent techniques developed by the authors in the context of the disordered pinning model (Berger and Lacoin, 2015), we identify the sharp asymptotic high temperature behavior

$$\lim_{\beta\to 0}\beta^2\log\Delta\mathtt{F}(\beta)=-\pi.$$

Résumé. Nous analysons le comportement du modèle de polymère dirigé en dimension $1+2$, dans la limite de haute température. Plus précisément, nous étudions la différence $\Delta\mathtt{F}(\beta)$ entre les énergies libres *gelées* et *recuites*, pour les petites valeurs de la température inverse β. Cette quantité est associée à des propriétés de localisation des trajectoires du polymère, et est reliée à la fraction de superposition de deux répliques. En adaptant des techniques récemment développées par les auteurs dans le contexte du modèle d'accrochage désordonné (Berger et Lacoin, 2015), nous identifions le comportement asymptotique précis dans la limite de haute température

$$\lim_{\beta\to 0}\beta^2\log\Delta\mathtt{F}(\beta)=-\pi.$$

MSC: 82D60; 60K37; 82B44

Keywords: Disordered systems; Directed polymer; Free energy; Localization

References

[1] T. Alberts, K. Khanin and J. Quastel. The intermediate disorder regime for directed polymers in dimension $1+1$. *Ann. Probab.* **42** (2014) 1212–1256. MR3189070
[2] K. S. Alexander and G. Yildirim. Directed polymers in a random environment with a defect line. *Electron. J. Probab.* **20** (2015) Article ID 6. MR3311219
[3] K. S. Alexander and N. Zygouras. Subgaussian concentration and rates of convergence in directed polymers. *Electron. J. Probab.* **18** (2013) Article ID 5. MR3024099
[4] Q. Berger and H. Lacoin Pinning on a defect line: Characterization of marginal disorder relevance and sharp asymptotics for the critical point shift. Preprint, 2015. Available at arXiv:1503.07315 [math-ph].
[5] E. Bolthausen. A note on diffusion of directed polymers in a random environment. *Comm. Math. Phys.* **123** (1989) 529–534. MR1006293
[6] F. Caravenna, R. Sun and N. Zygouras. Polynomial chaos and scaling limits of disordered systems. *J. Eur. Math. Soc. (JEMS)*. To appear.
[7] F. Caravenna, R. Sun and N. Zygouras. Universality in marginally relevant disordered systems. Preprint, 2015. Available at arxiv:1510.06287 [math.PR].

[8] F. Caravenna, F. Toninelli and N. Torri Universality for the pinning model in the weak coupling regime. Preprint, 2015. Available at arXiv:1505.04927v1 [math.PR].

[9] P. Carmona and Y. Hu. On the partition function of a directed polymer in a random Gaussian environment. *Probab. Theory Related Fields* **124** (3) (2002) 431–457. MR1939654

[10] P. Carmona and Y. Hu. Strong disorder implies strong localization for directed polymers in a random environment. *ALEA Lat. Am. J. Probab. Math. Stat.* **2** (2006) 217–229. MR2249669

[11] F. Comets. Weak disorder for low dimensional polymers: The model of stable laws. *Markov Process. Related Fields* **13** (4) (2007) 681–696. MR2381596

[12] F. Comets and V. Vargas. Majorizing multiplicative cascades for directed polymers in random media. *ALEA Lat. Am. J. Probab. Math. Stat.* **2** (2006) 267–277. MR2249671

[13] F. Comets, T. Shiga and N. Yoshida. Directed polymers in a random environment: Strong disorder and path localization. *Bernoulli* **9** (4) (2003) 705–723. MR1996276

[14] F. Comets, T. Shiga and N. Yoshida. Probabilistic analysis of directed polymers in a random environment: A review. *Adv. Stud. Pure Math.* **39** (2004) 115–142. MR2073332

[15] F. Comets and N. Yoshida. Directed polymers in a random environment are diffusive at weak disorder. *Ann. Probab.* **34** (5) (2006) 1746–1770. MR2271480

[16] B. Derrida, G. Giacomin, H. Lacoin and F. L. Toninelli. Fractional moment bounds and disorder relevance for pinning models. *Comm. Math. Phys.* **287** (2009) 867–887. MR2486665

[17] G. Giacomin, H. Lacoin and F. L. Toninelli. Disorder relevance at marginality and critical point shift. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** (2011) 148–175. MR2779401

[18] D. A. Huse and C. L. Henley. Pinning and roughening of domain wall in Ising systems due to random impurities. *Phys. Rev. Lett.* **54** (1985) 2708–2711.

[19] H. Lacoin. New bounds for the free energy of directed polymers in dimension $1+1$ and $1+2$. *Comm. Math. Phys.* **294** (2010) 471–503. MR2579463

[20] M. Ledoux. *The Concentration of Measure Phenomenon*. American Mathematical Society, Providence, RI, 2005. MR1849347

[21] M. Nakashima. A remark on the bound for the free energy of directed polymers in random environment in $1+2$ dimension. *J. Math. Phys.* **55** (2014) 093304. MR3390799

[22] M. Miura, Y. Tawara and K. Tsuchida. Strong and weak disorder for Lévy directed polymers in random environment. *Stoch. Anal. Appl.* **26** (5) (2008) 1000–1012. MR2440912

[23] T. Sasamoto and H. Spohn. Exact height distributions for the KPZ equation with narrow wedge initial condition. *Nuclear Phys. B* **834** (2010) 523–542. MR2628936

[24] T. Sasamoto and H. Spohn. The one-dimensional KPZ equation: An exact solution and its universality. *Phys. Rev. Lett.* **104** (2010) 230602.

[25] F. Watbled. Sharp asymptotics for the free energy of $1+1$ dimensional directed polymers in an infinitely divisible environment. *Electron. Commun. Probab.* **17** (2012) 53. MR2999981

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An irreversible local Markov chain that preserves the six vertex model on a torus

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Abstract. We construct an irreversible local Markov dynamics on configurations of up-right paths on a discrete two-dimensional torus, that preserves the Gibbs measures for the six vertex model. An additional feature of the dynamics is a conjecturally nontrivial drift of the height function.

Résumé. Nous construisons une dynamique de Markov locale et irréversible sur des configurations de chemins sur un tore bidimensionnel qui préserve la mesure de Gibbs du modèle à six sommets. Une caractéristique de cette dynamique est qu'elle devrait induire une dérive non triviale de la fonction de hauteur.

MSC: Primary 60K35; secondary 60J10

Keywords: Six vertex model; Markov dynamics; Gibbs measure

References

[1] A. Borodin. Schur dynamics of the Schur processes. *Adv. Math.* **228** (2011) 2268–2291. MR2836121
[2] A. Borodin. On a family of symmetric rational functions. Preprint. Available at arXiv:1410.0976.
[3] A. Borodin, A. Bufetov and G. Olshanski. Limit shapes for growing extreme characters of $U(\infty)$. *Ann. Appl. Probab.* **25** (2015) 2339–2381. MR3349009
[4] A. Borodin and I. Corwin. Macdonald processes. *Probab. Theory Related Fields* **158** (2014) 225–400. MR3152785
[5] A. Borodin and P. L. Ferrari. Anisotropic KPZ growth in $2+1$ dimensions: Fluctuations and covariance structure. *J. Stat. Mech.* (2009) P02009. MR2485688
[6] A. Borodin and P. L. Ferrari. Anisotropic growth of random surfaces in $2+1$ dimensions. *Comm. Math. Phys.* **325** (2014) 603–684. MR3148098
[7] A. Borodin and L. Petrov. Integrable probability: From representation theory to Macdonald processes. *Probab. Surv.* **11** (2014) 1–58. MR3178541
[8] I. Corwin. The Kardar–Parisi–Zhang equation and universality class. *Random Matrices Theory Appl.* **1** (2012). Available at arXiv:1106.1596 [math.PR]. MR2930377
[9] I. Corwin and F. L. Toninelli. Stationary measure of the driven two-dimensional q-Whittaker particle system on the torus. Preprint, 2015. Available at arXiv:1509.01605.
[10] P. L. Ferrari and H. Spohn. Random growth models. In *The Oxford Handbook of Random Matrix Theory*, G. Akemann, J. Baik and P. Di Francesco (Eds), 2011. MR2920518
[11] D. J. Gates and M. Westcott. Stationary states of crystal growth in three dimensions. *J. Stat. Phys.* **81** (1995) 681–715.
[12] R. Kenyon, A. Okounkov and S. Sheffield. Dimers and amoebae. *Ann. of Math.* **163** (2006) 1019–1056. MR2215138
[13] C. Kipnis and C. Landim. *Scaling Limits of Interacting Particle Systems*. *Grundlehren der mathematischen Wissenschaften* **320**. Springer, Berlin, 1999. MR1707314
[14] K. Palamarchuk and N. Reshetikhin. The 6-vertex model with fixed boundary conditions. In *Workshop Bethe Ansatz: 75 Years Laters*, *Brussels, Belgium*, 2006. Available at arXiv:1010.5011 [math-ph].

[15] S. Sheffield. *Random Surfaces*. *Astérisque*, 2005. MR2251117
[16] F. L. Toninelli. A $(2+1)$-dimensional growth process with explicit stationary measures. Preprint, 2015. Available at arXiv:1503.05339.

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On the Malliavin differentiability of BSDEs

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Abstract. In this paper we provide new conditions for the Malliavin differentiability of solutions of Lipschitz or quadratic BSDEs. Our results rely on the interpretation of the Malliavin derivative as a Gâteaux derivative in the directions of the Cameron–Martin space. Incidentally, we provide a new formulation for the characterization of the Malliavin–Sobolev type spaces $\mathbb{D}^{1,p}$.

Résumé. Dans cet article, nous donnons de nouvelles conditions nous assurant que les solutions d'EDSR à générateurs lipschitziens ou à croissance quadratique sont différentiables au sens de Malliavin, en utilisant l'interprétation de la dérivée de Malliavin comme dérivée de Gâteaux directionnelle par rapport à l'espace de Cameron–Martin. Ce résultat est en outre basé sur une nouvelle caractérisation des espaces de Malliavin–Sobolev $\mathbb{D}^{1,p}$ que nous fournissons.

MSC: Primary 60H10; secondary 60H07

Keywords: Malliavin's calculus; Abstract Wiener space; BSDEs

References

[1] O. Aboura and S. Bourguin. Density estimates for solutions to one dimensional backward SDE's. *Potential Anal.* **38** (2) (2013) 573–587. MR3015365
[2] S. Ankirchner, P. Imkeller and G. dos Reis. Classical and variational differentiability of bsdes with quadratic growth. *Electron. J. Probab.* **12** (53) (2007) 1418–1453. MR2354164
[3] F. Antonelli and A. Kohatsu-Higa. Densities of one-dimensional backward SDEs. *Potential Anal.* **22** (3) (2005) 263–287. MR2134722
[4] J.-M. Bismut. Contrôle des systèmes linéaires quadratiques: Applications de l'intégrale stochastique. In *Séminaire de Probabilités, XII (Univ. Strasbourg, Strasbourg, 1976/1977)* 180–264. *Lecture Notes in Math.* **649**. Springer, Berlin, 1978. MR0520007
[5] V. I. Bogachev. *Differentiable Measures and the Malliavin Calculus*. *Mathematical Surveys and Monographs* **164**, Am. Math. Soc., Providence, 2010. MR2663405
[6] P. Briand and C. Labart. Simulation of BSDEs by Wiener chaos expansion. *Ann. Appl. Probab.* **24** (3) (2014) 1129–1171. MR3199982
[7] P. Briand, B. Delyon, Y. Hu, E. Pardoux and L. Stoica. L^p solutions of backward stochastic differential equations. *Stochastic Process. Appl.* **108** (1) (2003) 109–129. MR2008603
[8] G. dos Reis. On some properties of solutions of quadratic growth BSDE and applications in finance and insurance. Ph.D. thesis, Humboldt University, 2010.
[9] N. El Karoui, S. Peng and M. C. Quenez. Backward stochastic differential equations in finance. *Math. Finance* **7** (1) (1997) 1–71. MR1434407
[10] Y. Hu, D. Nualart and X. Zhong. Malliavin calculus for backward stochastic differential equations and application to numerical solutions. *Ann. Appl. Probab.* **21** (6) (2011) 2379–2423. MR2895419
[11] P. Imkeller and G. dos Reis. Path regularity and explicit convergence rate for BSDE with truncated quadratic growth. *Stochastic Process. Appl.* **120** (3) (2010) 348–379. MR2584898
[12] P. Imkeller, A. Réveillac and A. Richter. Differentiability of quadratic BSDEs generated by continuous martingales. *Ann. Appl. Probab.* **22** (1) (2012) 285–336. MR2932548
[13] P. Imkeller, T. Mastrolia, D. Possamaï and A. Réveillac. A note on the Malliavin–Sobolev spaces. *Statist. Probab. Lett.* **19** (2016) 45–53. MR3434959
[14] S. Janson. *Gaussian Hilbert Spaces*. Cambridge University Press, Cambridge, 1997. MR1474726

[15] S. Kusuoka. Dirichlet forms and diffusion processes on Banach spaces. *J. Fac. Sci., Univ. Tokyo, Sect. 1A, Math.* **29** (1) (1982) 79–95. MR0657873

[16] P. Malliavin. Stochastic calculus of variation and hypoelliptic operators. In *Proceedings of the International Symposium on Stochastic Differential Equations (Res. Inst. Math. Sci., Kyoto Univ., Kyoto, 1976)* 195–263. Wiley, New York–Chichester–Brisbane, 1978. MR0536013

[17] T. Mastrolia, D. Possamaï and A. Réveillac. Density analysis of BSDEs. *Ann. Probab.* To appear, 2014. Available at arXiv:1402.4416.

[18] D. Nualart. *The Malliavin Calculus and Related Topics*, 2nd edition. *Probability and Its Applications*. Springer, Berlin, 2006. MR2200233

[19] E. Pardoux and S. Peng. Adapted solution of a backward stochastic differential equation. *Systems Control Lett.* **14** (1) (1990) 55–61. MR1037747

[20] E. Pardoux and S. Peng. Backward stochastic differential equations and quasilinear parabolic partial differential equations. In *Stochastic Partial Differential Equations and Their Applications* 200–217. Springer, Berlin, 1992. MR1176785

[21] P. E. Protter. *Stochastic Integration and Differential Equations*. 2nd edition. *Stochastic Modelling and Applied Probability* **21**. Springer, Berlin, 2005. Version 2.1, Corrected third printing. MR2273672

[22] I. Shigekawa. Derivatives of Wiener functionals and absolute continuity of induced measures. *J. Math. Kyoto Univ.* **20** (2) (1980) 263–289. MR0582167

[23] D. W. Stroock. The Malliavin calculus and its application to second order parabolic differential equations. I. *Math. Syst. Theory* **14** (1) (1981) 25–65. MR0603973

[24] H. Sugita. On a characterization of the Sobolev spaces over an abstract Wiener space. *J. Math. Kyoto Univ.* **25** (4) (1985) 717–725. MR0810975

[25] A. S. Üstünel and M. Zakai. *Transformation of Measure on Wiener Space*. Springer, Berlin, 2000. MR1736980

[26] M. Veraar. The stochastic Fubini theorem revisited. *Stochastics* **84** (4) (2012) 543–551. MR2966093

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Nonexistence of Lyapunov exponents for matrix cocycles

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Abstract. It follows from Oseledec Multiplicative Ergodic Theorem (or Kingman's Sub-additive Ergodic Theorem) that the Lyapunov-irregular set of points for which the Oseledec averages of a given continuous cocycle diverge has zero measure with respect to any invariant probability measure. In strong contrast, for any dynamical system $f : X \to X$ with exponential specification property and a Hölder continuous matrix cocycle $A : X \to \mathrm{GL}(m, \mathbb{R})$, we show here that if there exist ergodic measures with different Lyapunov spectrum, then the Lyapunov-irregular set of A is residual (i.e., containing a dense G_δ set). Here we point out that exponential specification is introduced and plays critical role, and it is still unknown whether specification is enough. The above result can be used not only for all mixing hyperbolic systems but also for some non-hyperbolic systems.

Résumé. Le théorème ergodique multiplicatif d'Oseledets (ou le théorème ergodique sous-additif de Kingman) implique que l'ensemble Lyapounov-irrégulier (les points pour lesquels la moyenne d'Oseledets d'un cocycle continu donné diverge) est de mesure nulle pour toute mesure de probabilité invariante. Par contraste avec ce fait, nous montrons que pour tout système dynamique $f : X \to X$ satisfaisant la spécification exponentielle, et pour tout cocycle de matrices $A : X \to \mathrm{GL}(m, \mathbb{R})$ Hölder continu, s'il existe des mesures ergodiques avec des spectres de Lyapounov distincts, alors l'ensemble Lyapounov-irrégulier de A est résiduel (i.e., il contient un G_δ-dense). Nous mettons donc en évidence le rôle critique de la spécification exponentielle. Il n'est pas connu si cette propriété est suffisante. Notre résultat s'applique à tous les systèmes hyperboliques mélangeants et à certains systèmes non-hyperboliques.

MSC: 37H15; 37D20; 37D25; 37C50

Keywords: Oseledec Multiplicative Ergodic Theorem; Kingman's Sub-additive Ergodic Theorem; Lyapunov exponents and cocycles; (Exponential) specification property; Uniformly hyperbolic systems

References

[1] F. Abdenur, C. Bonatti and S. Crovisier. Nonuniform hyperbolicity of C^1-generic diffeomorphisms. *Israel J. Math.* **183** (1) (2011) 1–60. MR2811152
[2] S. Albeverio, M. Pratsiovytyi and G. Torbin. Topological and fractal properties of subsets of real numbers which are not normal. *Bull. Sci. Math.* **129** (2005) 615–630. MR2166730
[3] L. Barreira, J. Li and C. Valls. Irregular sets for ratios of Birkhoff averages are residual. *Publ. Mat.* **58** (2014) 49–62. MR3211826
[4] L. Barreira and Y. B. Pesin. *Nonuniform Hyperbolicity*. Cambridge Univ. Press, Cambridge, 2007. MR2348606
[5] R. Bowen. Periodic points and measures for Axiom A diffeomorphisms. *Trans. Amer. Math. Soc.* **154** (1971) 377–397. MR0282372
[6] R. Bowen. Periodic orbits for hyperbolic flows. *Amer. J. Math.* **94** (1972) 1–30. MR0298700
[7] M. Denker, C. Grillenberger and K. Sigmund. *Ergodic Theory on Compact Space*. *Lecture Notes in Mathematics* **527**, 1976. MR0457675
[8] A. Furman. On the multiplicative ergodic theorem for uniquely ergodic systems. *Ann. Inst. Henri Poincaré Probab. Stat.* **33** (6) (1997) 797–815. MR1484541
[9] A. Gogolev. Diffeomorphisms Hölder conjugate to Anosov diffeomorphisms. *Ergodic Theory Dynam. Systems* **30** (2) (2010) 441–456. MR2599887
[10] B. Kalinin. Livšic theorem for matrix cocycles. *Ann. of Math.* **173** (2) (2011) 1025–1042. MR2776369
[11] A. Katok and B. Hasselblatt. *Introduction to the Modern Theory of Dynamical Systems*. *Encyclopedia of Mathematics and Its Applications* **54**. Cambridge Univ. Press, Cambridge, 1995. MR1326374

[12] D. Lenz. Existence of non-uniform cocycles on uniquely ergodic systems. *Ann. Inst. Henri Poincaré Probab. Stat.* **40** (2) (2004) 197–206. MR2044815

[13] J. Li and M. Wu. The sets of divergence points of self-similar measures are residual. *J. Math. Anal. Appl.* **404** (2013) 429–437. MR3045184

[14] J. Li and M. Wu. Generic property of irregular sets in systems satisfying the specification property. *Discrete Contin. Dyn. Syst.* **34** (2014) 635–645. MR3094597

[15] R. Mañé. *Ergodic Theory and Differentiable Dynamics*. Springer-Verlag, Berlin, London, 1987. MR0889254

[16] L. Olsen. Extremely non-normal numbers. *Math. Proc. Cambridge Philos. Soc.* **137** (2004) 43–53. MR2075041

[17] V. I. Oseledec. Multiplicative ergodic theorem, Liapunov characteristic numbers for dynamical systems. *Trans. Moscow Math. Soc.* **19** (1968) 197–221. Translated from Russian. MR0240280

[18] X. Tian. Lyapunov 'non-typical' points of matrix cocycles and topological entropy. Preprint. Available at arXiv:1505.04345.

[19] P. Walters. Unique ergodicity and random matrix products. In *Lyapunov Exponents* 37–55. Springer, Berlin, Heidelberg, 1986. MR0850069

[20] Z. Wang and W. Sun. Lyapunov exponents of hyperbolic measures and hyperbolic periodic orbits. *Trans. Amer. Math. Soc.* **362** (2010) 4267–4282. MR2608406