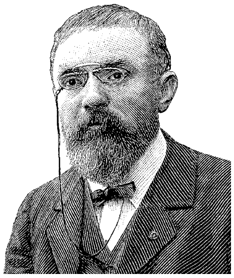

Volume 53, Number 2, 2017
ISSN 0246-0203

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

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Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques (ISSN 0246-0203), Volume 53, Number 2, May 2017. Published quarterly by Association des Publications de l'Institut Henri Poincaré.

POSTMASTER: Send address changes to Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques, Dues and Subscriptions Office, 9650 Rockville Pike, Suite L 2310, Bethesda, Maryland 20814-3998 USA.

Président et directeur de la publication : Cédric Villani
Périodicité : 4 n^{os} / an

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 503–538
DOI: 10.1214/15-AIHP724

Rate of convergence to equilibrium of fractional driven stochastic differential equations with some multiplicative noise

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Abstract. We investigate the problem of the rate of convergence to equilibrium for ergodic stochastic differential equations driven by fractional Brownian motion with Hurst parameter $H > 1/2$ and multiplicative noise component σ. When σ is constant and for every $H \in (0, 1)$, it was proved by Hairer that, under some mean-reverting assumptions, such a process converges to its equilibrium at a rate of order $t^{-\alpha}$ where $\alpha \in (0, 1)$ (depending on H). The aim of this paper is to extend such types of results to some multiplicative noise setting. More precisely, we show that we can recover such convergence rates when $H > 1/2$ and the inverse of the diffusion coefficient σ is a Jacobian matrix. The main novelty of this work is a type of extension of Foster–Lyapunov like techniques to this non-Markovian setting, which allows us to put in place an asymptotic coupling scheme without resorting to deterministic contracting properties.

Résumé. Cet article est consacré à la vitesse de convergence à l'équilibre pour des équations différentielles stochastiques multiplicatives dirigées par un mouvement brownien fractionnaire (fBm). Dans le cas additif, i.e. lorsque le coefficient « diffusif » σ est constant et non dégénéré, cette question a été étudiée par Hairer qui, sous des hypothèses de contraction du coefficient de dérive en dehors d'un compact, a établi par des méthodes de couplage qu'un tel processus converge à l'équilibre à une vitesse dominée par $Ct^{-\alpha}$, où $\alpha \in (0, 1)$ dépend de l'indice de Hurst H du fBm. L'objectif de notre travail est d'étendre ce type de résultat au cadre multiplicatif. Plus précisément, nous montrons que si $H > 1/2$ et si σ^{-1} est une matrice jacobienne, alors le résultat précédent reste vrai avec des bornes identiques sur la vitesse de convergence. La principale nouveauté de ce travail réside dans le développement de techniques de type Foster–Lyapounov dans ce cadre non markovien, nous permettant de mettre en place un schéma de couplage similaire à [9] sans faire appel à des propriétés de contraction déterministes.

MSC: 60G22; 37A25

Keywords: Stochastic differential equations; Fractional Brownian motion; Multiplicative noise; Ergodicity; Rate of convergence to equilibrium; Lyapunov function; Total variation distance

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 539–593
DOI: 10.1214/15-AIHP725

Decomposition of Lévy trees along their diameter[1]

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Abstract. We study the diameter of Lévy trees that are random compact metric spaces obtained as the scaling limits of Galton–Watson trees. Lévy trees have been introduced by Le Gall & Le Jan (*Ann. Probab.* **26** (1998) 213–252) and they generalise Aldous' Continuum Random Tree (1991) that corresponds to the Brownian case. We first characterize the law of the diameter of Lévy trees and we prove that it is realized by a unique pair of points. We prove that the law of Lévy trees conditioned to have a fixed diameter $r \in (0, \infty)$ is obtained by glueing at their respective roots two independent size-biased Lévy trees conditioned to have height $r/2$ and then by uniformly re-rooting the resulting tree; we also describe by a Poisson point measure the law of the subtrees that are grafted on the diameter. As an application of this decomposition of Lévy trees according to their diameter, we characterize the joint law of the height and the diameter of stable Lévy trees conditioned by their total mass; we also provide asymptotic expansions of the law of the height and of the diameter of such normalised stable trees, which generalises the identity due to Szekeres (In *Combinatorial Mathematics, X (Adelaide, 1982)* (1983) 392–397 Springer) in the Brownian case.

Résumé. Nous étudions le diamètre des arbres de Lévy qui sont des espaces métriques compacts obtenus comme limites d'échelle des arbres de Galton–Watson. Les arbres de Lévy ont été introduits par Le Gall & Le Jan (*Ann. Probab.* **26** (1998) 213–252) et ils généralisent le Continuum Random Tree (1991) d'Aldous qui correspond au cas brownien. Nous caractérisons d'abord la loi du diamètre des arbres de Lévy et nous prouvons qu'une unique paire de points le réalise. Nous prouvons ensuite que la loi des arbres de Lévy conditionnés à avoir leur diamètre égal à $r \in]0, \infty[$ est obtenu en collant à leurs racines respectives deux arbres de Lévy indépendants conditionnés chacuns à avoir une hauteur égale à $r/2$, et à réenraciner uniformément au hasard l'arbre obtenu par ce collage ; nous décrivons également en termes d'une mesure ponctuelle de Poisson, la loi des sous-arbres qui sont attachés le long du diamètre. En application de cette décomposition des arbres de Lévy le long de leur diamètre, nous caractérisons la loi jointe de la hauteur et du diamètre des arbres de Lévy stables conditionnés à avoir une masse totale unité. Nous donnons aussi des développements asymptotiques des lois de la hauteur et du diamètre de ces arbres stables normalisés, ce qui généralise une identité due à Szekeres (In *Combinatorial Mathematics, X (Adelaide, 1982)* (1983) 392–397 Springer) dans le cas brownien.

MSC: Primary 60J80; 60E07; secondary 60E10; 60G52; 60G55

Keywords: Lévy trees; Height process; Diameter; Decomposition; Asymptotic expansion; Stable law

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 594–615
DOI: 10.1214/15-AIHP726

Simple CLE in doubly connected domains

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Abstract. Loop Ensemble (CLE_κ) in doubly connected domains: annuli, the punctured disc, and the punctured plane. We restrict attention to CLE_κ for which the loops are simple, i.e. $\kappa \in (8/3, 4]$. In (*Ann. of Math. (2)* **176** (2012) 1827–1917), simple CLE in the unit disc is introduced and constructed as the collection of outer boundaries of outermost clusters of the Brownian loop soup. For simple CLE in the unit disc, any fixed interior point is almost surely surrounded by some loop of CLE. The gasket of the collection of loops in CLE, i.e. the set of points that are not surrounded by any loop, almost surely has Lebesgue measure zero. In the current paper, simple CLE in an annulus is constructed similarly: it is the collection of outer boundaries of outermost clusters of the Brownian loop soup conditioned on the event that there is no cluster disconnecting the two components of the boundary of the annulus. Simple CLE in the punctured disc can be viewed as simple CLE in the unit disc conditioned on the event that the origin is in the gasket. Simple CLE in the punctured plane can be viewed as simple CLE in the whole plane conditioned on the event that both the origin and infinity are in the gasket. We construct and study these three kinds of CLE's, along with the corresponding exploration processes.

Résumé. Nous étudions l'ensemble des boucles conformes (CLE_κ) dans des domaines connexes du type : anneau, disque percé et plan percé. Nous considérons les cas CLE_κ pour lesquels les boucles sont simples, i.e. $\kappa \in (8/3, 4]$. Dans (*Ann. of Math. (2)* **176** (2012) 1827–1917), l'ensemble CLE dans le disque unité est introduit et construit comme la collection de frontières extérieures des amas les plus excentrés de la soupe de boucles Browniennes. Dans le cas du disque unité, n'importe quel point intérieur est presque sûrement entouré par une boucle du CLE. L'ensemble des points qui ne sont entourés par aucune boucle a une mesure de Lebesgue nulle presque sûrement. Dans notre article, le CLE dans un anneau est construit de façon similaire : il s'agit de la collection de frontières extérieures des amas de la soupe de boucles Browniennes conditionnés sur l'évènement qu'il n'existe pas d'amas séparant les deux composantes de la frontière de l'anneau. Dans le cas du disque percé, le CLE correspond au conditionnement par le fait que l'origine n'est pas entourée par une boucle. Dans le cas du plan percé, le conditionnement est tel que l'origine et l'infini ne sont pas entourés. Nous construisons et étudions ces trois types de CLE et les processus d'exploration correspondants.

MSC: 60J67

Keywords: Schramm Loewner Evolution; Conformal Loop Ensemble; Doubly connected domains; Exploration process

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 616–640
DOI: 10.1214/15-AIHP727

Scaling limits of coalescent processes near time zero

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Abstract. In this paper we obtain scaling limits of Λ-coalescents near time zero under a regularly varying assumption. In particular this covers the case of Kingman's coalescent and beta coalescents. The limiting processes are coalescents with infinite mass, obtained geometrically as tangent cones of Evans metric space associated with the coalescent. In the case of Kingman's coalescent we are able to obtain a simple construction of the limiting space using a two-sided Brownian motion.

Résumé. Nous obtenons des limites d'échelle de Λ-coalescents en temps zéro sous une hypothèse de variation régulière. Cette hypothèse inclut notamment le coalescent de Kingman ainsi que la famille des Beta-coalescents. Les processus limites sont des processus de coalescence avec masse infinie, construits de manière géométrique comme cônes tangents de l'espace métrique de Evans. Dans le cas particulier du coalescent de Kingman une construction simple du processus limite est donnée à partir d'un mouvement brownien bidirectionnel.

MSC: 60J80; 60J99; 60F99

Keywords: Regularly varying coalescents; Small time asymptotics; Scaling limits; Random metric space; Tangent cones; Gromov–Hausdorff convergence

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 641–657
DOI: 10.1214/15-AIHP728

Path-dependent infinite-dimensional SDE with non-regular drift: An existence result

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Abstract. We establish in this paper the existence of weak solutions of infinite-dimensional shift invariant stochastic differential equations driven by a Brownian term. The drift function is very general, in the sense that it is supposed to be neither bounded or continuous, nor Markov. On the initial law we only assume that it admits a finite specific entropy and a finite second moment.

The originality of our method leads in the use of the specific entropy as a tightness tool and in the description of such infinite-dimensional stochastic process as solution of a variational problem on the path space. Our result clearly improves previous ones obtained for free dynamics with bounded drift.

Résumé. Nous établissons, dans cet article, l'existence de solutions faibles pour un système infini-dimensionnel de diffusions browniennes. Le terme de dérive est véritablement général, au sens où il est supposé n'être ni borné, ni continu, ni Markovien. Nous supposons cependant que la loi initiale admet une entropie spécifique finie.

L'originalité de notre méthode consiste en l'utilisation de la bornitude de l'entropie spécifique comme critère de tension et en l'identification des solutions du système comme solutions d'un problème variationnel sur l'espace des trajectoires. Notre résultat améliore clairement ceux préexistants concernant des dynamiques libres perturbées par des dérives bornées.

MSC: 60H10; 60K35

Keywords: Infinite-dimensional SDE; Non-Markov drift; Non-regular drift; Variational principle; Specific entropy

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 658–678
DOI: 10.1214/15-AIHP729

A dynamical Curie–Weiss model of SOC: The Gaussian case

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Abstract. In this paper, we introduce a Markov process whose unique invariant distribution is the Curie–Weiss model of self-organized criticality (SOC) we designed and studied in (*Ann. Probab.* 44(1):444-478, 2016). In the Gaussian case, we prove rigorously that it is a dynamical model of SOC: the fluctuations of the sum $S_n(\cdot)$ of the process evolve in a time scale of order $\sqrt{n}$ and in a space scale of order $n^{3/4}$ and the limiting process is the solution of a "critical" stochastic differential equation.

Résumé. Dans cet article, nous introduisons un processus de Markov dont l'unique distribution invariante est le modèle d'Ising Curie–Weiss de criticalité auto-organisée que nous avons construit et étudié dans (*Ann. Probab.* 44(1):444-478, 2016). Dans le cas Gaussien, nous montrons rigoureusement qu'il s'agit d'un modèle dynamique de criticalité auto-organisée : les fluctuations de la somme $S_n(\cdot)$ du processus évoluent à une vitesse de temps d'ordre $\sqrt{n}$ et à une échelle spatiale d'ordre $n^{3/4}$ et le processus limite est la solution d'une équation différentielle stochastique « critique ».

MSC: 60J60; 60K35

Keywords: Ising Curie–Weiss; Self-organized criticality; Critical fluctuations; Langevin diffusion; Collapsing processes

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 679–700
DOI: 10.1214/15-AIHP730

Poisson approximation of point processes with stochastic intensity, and application to nonlinear Hawkes processes

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Abstract. We give a general inequality for the total variation distance between a Poisson distributed random variable and a first order stochastic integral with respect to a point process with stochastic intensity, constructed by embedding in a bivariate Poisson process. We apply this general inequality to first order stochastic integrals with respect to a class of nonlinear Hawkes processes, which is of interest in queueing theory, providing explicit bounds for the Poisson approximation, a quantitative Poisson limit theorem, confidence intervals and asymptotic estimates of the moments.

Résumé. Nous donnons une inégalité générale pour la distance en variation totale entre une variable de Poisson aléatoire et une intégrale stochastique par rapport à un processus ponctuel avec une intensité stochastique, construite par plongement dans un processus de Poisson bivarié. Nous appliquons cette inégalité générale aux intégrales stochastiques par rapport à une classe de processus de Hawkes non linéaires, ce qui a un intérêt en théorie des files d'attente, en fournissant des bornes explicites pour l'approximation Possonienne, ainsi qu'un théorème limite Poissonien quantitatif et des intervalles de confiance et estimatées asymptotiques des moments.

MSC: Primary 60F05; 60G55

Keywords: Chen–Stein's method; Clark–Ocone formula; Confidence interval; Erlang loss system; Hawkes process; Malliavin's calculus; Poisson approximation; Stochastic intensity

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 701–717
DOI: 10.1214/15-AIHP732

Any orthonormal basis in high dimension is uniformly distributed over the sphere

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Abstract. Let $\mathbb{X}^d$ be a real or complex Hilbert space of finite but large dimension d, let $\mathbb{S}(\mathbb{X}^d)$ denote the unit sphere of $\mathbb{X}^d$, and let u denote the normalized uniform measure on $\mathbb{S}(\mathbb{X}^d)$. For a finite subset B of $\mathbb{S}(\mathbb{X}^d)$, we may test whether it is approximately uniformly distributed over the sphere by choosing a partition $A_1, \dots, A_m$ of $\mathbb{S}(\mathbb{X}^d)$ and checking whether the fraction of points in B that lie in A_k is close to $u(A_k)$ for each $k = 1, \dots, m$. We show that if B is any orthonormal basis of $\mathbb{X}^d$ and m is not too large, then, if we randomize the test by applying a random rotation to the sets $A_1, \dots, A_m$, B will pass the random test with probability close to 1. This statement is related to, but not entailed by, the law of large numbers. An application of this fact in quantum statistical mechanics is briefly described.

Résumé. Soit $\mathbb{X}^d$ un espace de Hilbert réel ou complexe de dimension finie mais grande d et soit $\mathbb{S}(\mathbb{X}^d)$ la sphère unité de $\mathbb{X}^d$, on note u la mesure uniforme normalisée sur $\mathbb{S}(\mathbb{X}^d)$. Pour un sous ensemble fini B de $\mathbb{S}(\mathbb{X}^d)$, nous pouvons tester s'il est approximativement uniformément distribué sur la sphère en choisissant une partition $A_1, \dots, A_m$ de $\mathbb{S}(\mathbb{X}^d)$ et en vérifiant si la fraction des points dans B qui se trouvent dans A_k est proche de $u(A_k)$ pour tout $k = 1, \dots, m$. Nous montrons que si B est n'importe quelle base orthonormée de $\mathbb{X}^d$ et que si m n'est pas trop grand, alors si on randomise le test en appliquant une rotation aléatoire aux ensembles $A_1, \dots, A_m$, l'ensemble B va passer le test avec une probabilité proche de 1. Ce résultat est relié à la loi des grands nombres. Une application de ce résultat en mécanique statistique quantique est décrite brièvement.

MSC: 60F05; 82B10; 28C10

Keywords: Law of large numbers; Haar measure on the orthogonal or unitary groups; Asymptotics in high dimension; Irreducible representations of the orthogonal or unitary groups; random orthonormal basis

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 718–752
DOI: 10.1214/15-AIHP734

Typical behavior of the harmonic measure in critical Galton–Watson trees

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Abstract. We study the typical behavior of the harmonic measure of balls in large critical Galton–Watson trees whose offspring distribution has finite variance. The harmonic measure considered here refers to the hitting distribution of height n by simple random walk on a critical Galton–Watson tree conditioned to have height greater than n. We prove that, with high probability, the mass of the harmonic measure carried by a random vertex uniformly chosen from height n is approximately equal to $n^{-\lambda}$, where the constant $\lambda > 1$ does not depend on the offspring distribution. This universal constant λ is equal to the first moment of the asymptotic distribution of the conductance of size-biased Galton–Watson trees minus 1.

Résumé. Nous étudions le comportement typique de la mesure harmonique au bord des boules dans les grands arbres de Galton–Watson critiques, dont la loi de reproduction est de variance finie. On comprend par mesure harmonique la loi du premier point d'atteinte de la hauteur n par une marche aléatoire simple sur un arbre de Galton–Watson critique conditionné à avoir une hauteur supérieure à n. Nous prouvons que, avec une grande probabilité, la masse de la mesure harmonique portée par un sommet choisi uniformément au hasard de la hauteur n est approximativement égale à $n^{-\lambda}$, où la constante $\lambda > 1$ ne dépend pas de la loi de reproduction. Cette constante universelle λ est égale au moment d'ordre 1 de la distribution asymptotique de la conductance de l'arbre de Galton–Watson biaisé par la taille moins 1.

MSC: 60J80; 60G50; 60K37

Keywords: Size-biased Galton–Watson tree; Harmonic measure; Uniform measure; Simple random walk and Brownian motion on trees

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 753–765
DOI: 10.1214/15-AIHP735

One-dependent coloring by finitary factors

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Abstract. Holroyd and Liggett recently proved the existence of a stationary 1-dependent 4-coloring of the integers, the first stationary k-dependent q-coloring for any k and q, and arguably the first natural finitely dependent process that is not a block factor of an i.i.d. process. That proof specifies a consistent family of finite-dimensional distributions, but does not yield a probabilistic construction on the whole integer line. Here we prove that the process can be expressed as a finitary factor of an i.i.d. process. The factor is described explicitly, and its coding radius obeys power-law tail bounds.

Résumé. Holroyd et Liggett ont récemment prouvé l'existence d'un 4-coloriage stationnaire et 1-dépendant des entiers, le premier q-coloriage stationnaire et k-dépendant pour tous k et q, et probablement le premier processus naturel à dépendance finie qui ne corresponde pas à un facteur par blocs d'un processus i.i.d. Cette preuve définit une famille consistante de distributions finidimensionnelles, mais n'offre pas de construction probabiliste sur tous les entiers. On montre ici que l'on peut exprimer le processus comme un facteur par blocs d'un processus i.i.d. Le facteur a une description explicite, et son rayon de codage satisfait des bornes de type loi de puissance.

MSC: 60G10; 05C15; 60C05

Keywords: Proper coloring; One-dependence; Stationary process; Finitary factor

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 766–801
DOI: 10.1214/15-AIHP736

Supercritical behavior of asymmetric zero-range process with sitewise disorder

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Abstract. We establish necessary and sufficient conditions for weak convergence to the upper invariant measure for one-dimensional asymmetric nearest-neighbour zero-range processes with non-homogeneous jump rates. The class of "environments" considered is close to that considered by (*Stochastic Process. Appl.* **90** (2000) 67–81), while our class of processes is broader. We also give in arbitrary dimension a simpler proof of the result of (In *Asymptotics: Particles, Processes and Inverse Problems* (2007) 108–120 Inst. Math. Statist.) with weaker assumptions.

Résumé. Nous établissons des conditions nécessaires et suffisantes de convergence faible vers la mesure invariante maximale pour le processus de zero-range asymétrique à plus proche voisin en dimension un, avec des taux de sauts inhomogènes. La classe d'« environnements » considérée est proche de celle considérée dans (*Stochastic Process. Appl.* **90** (2000) 67–81), mais la classe de processus concernée est plus large. Nous donnons également, en dimension quelconque, une preuve plus simple du résultat de (In *Asymptotics: Particles, Processes and Inverse Problems* (2007) 108–120 Inst. Math. Statist.) sous des hypothèses plus faibles.

MSC: Primary 60K35; 60K37; secondary 82C22

Keywords: Zero-range process; Site disorder; Supercritical initial condition; Large-time convergence; Critical invariant measure; Escape of mass; Hydrodynamic limit

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 802–818
DOI: 10.1214/15-AIHP737

Point process convergence for branching random walks with regularly varying steps

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Abstract. We consider the limiting behaviour of the point processes associated with a branching random walk with supercritical branching mechanism and balanced regularly varying step size. Assuming that the underlying branching process satisfies Kesten–Stigum condition, it is shown that the point process sequence of properly scaled displacements coming from the nth generation converges weakly to a Cox cluster process. In particular, we establish that a conjecture of (*J. Stat. Phys.* **143** (3) (2011) 420–446) remains valid in this setup, investigate various other issues mentioned in their paper and recover the main result of (*Z. Wahrsch. Verw. Gebiete* **62** (2) (1983) 165–170) in our framework.

Résumé. Nous étudions le comportement limite de processus ponctuels associés à la marche aléatoire branchante avec branchement surcritique et une loi de déplacement à variation régulière. Si le processus de branchement sous-jacent satisfait une condition de Kesten–Stigum, nous montrons que le processus ponctuel de la suite des déplacements changés d'échelle provenant de la n-ième génération converge faiblement vers un processus de Cox. En particulier, nous prouvons qu'une conjecture de (*J. Stat. Phys.* **143** (3) (2011) 420–446) reste valable dans ce contexte, nous étudions plusieurs questions soulevées dans leur article et retrouvons le résultat principal de (*Z. Wahrsch. Verw. Gebiete* **62** (2) (1983) 165–170) dans notre cadre.

MSC: Primary 60J70; 60G55; secondary 60J80

Keywords: Branching random walk; Maxima; Galton–Watson process; Extreme value theory; Point process; Cox process

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 819–841
DOI: 10.1214/15-AIHP738

Moment asymptotics for parabolic Anderson equation with fractional time-space noise: In Skorokhod regime

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Abstract. In this paper, we consider the parabolic Anderson equation that is driven by a Gaussian noise fractional in time and white or fractional in space, and is solved in a mild sense defined by Skorokhod integral. Our objective is the precise moment Lyapunov exponent and high moment asymptotics. As far as the long term asymptotics are concerned, some feature given in our theorems is different from what have been observed in the Stratonovich-regime and in the setting of the white time noise. While the difference disappears when it comes to the high moment asymptotics. To achieve our goal, we introduce a variational inequality and use some newly developed tools such as time-space LDP of Feynman–Kac type, linearization by tangent approximation, together with some techniques developed along the line of probability in Banach spaces.

Résumé. Nous considérons l'équation d'Anderson parabolique conduite par un bruit gaussien, fractionnaire en temps, et blanc ou fractionnaire en espace, qu'on résout dans un sens faible défini par une intégrale de Skorokhod. Notre objectif est de donner l'exposant de Lyapounov pour les moments, et les asymptotiques des grands moments. Pour les asymptotiques en temps long, nos résultats mettent en évidence des phénomènes différents de ceux observés pour le régime Stratonovich, et dans le cas d'un bruit blanc en temps. Ces différences s'effacent néanmoins lorsque l'on considère les asymptotiques des grands moments. Nos résultats sont obtenus en introduisant une nouvelle inégalité variationnelle, et à l'aide d'outils nouveaux tels qu'un principe de grandes déviations de type Feynman–Kac, la linéarisation par des approximations tangentes, et des techniques inspirées des probabilités dans les espaces de Banach.

MSC: 60F10; 60H15; 60H40; 60J65; 81U10

Keywords: Lyapunov exponent; High moment asymptotics; White and fractional noise; Brownian motion; Parabolic Anderson equation; Feynman–Kac's representation

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 842–864
DOI: 10.1214/16-AIHP739

Conditional speed of branching Brownian motion, skeleton decomposition and application to random obstacles

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Abstract. We study a branching Brownian motion Z in $\mathbb{R}^d$, among obstacles scattered according to a Poisson random measure with a radially decaying intensity. Obstacles are balls with constant radius and each one works as a trap for the whole motion when hit by a particle. Considering a general offspring distribution, we derive the decay rate of the annealed probability that none of the particles of Z hits a trap, asymptotically in time t. This proves to be a rich problem motivating the proof of a more general result about the speed of branching Brownian motion conditioned on non-extinction. We provide an appropriate "skeleton" decomposition for the underlying Galton–Watson process when supercritical and show that the "doomed" particles do not contribute to the asymptotic decay rate.

Résumé. On étudie un mouvement brownien branchant Z dans $\mathbb{R}^d$ qui se déplace parmi des obstacles qui sont dispersés par rapport à une mesure aléatoire de Poisson d'une intensité déclinant radialement. Les obstacles sont des boules de rayon constant, et lorsqu'une particule rencontre un tel obstacle, tout le mouvement s'arrête. En considérant une distribution générale pour le nombre de descendants, on calcule le taux de décroissance de la probabilité «annealed» que toutes les particules de Z évitent les pièges, asymptotiquement en temps. Cela se révèle être un problème riche qui motive la preuve d'un résultat plus général concernant la vitesse du mouvement brownien branchant conditionné à survivre. Dans le cas sur-critique on fournit une décomposition «en squelette» appropriée pour le processus de Galton–Watson sous-jacent, et on montre que les particules «condamnées» ne contribuent pas au taux de décroissance asymptotique.

MSC: 60J80; 60K37; 60F10

Keywords: Branching Brownian motion; Poissonian traps; Random environment; Hard obstacles; Rightmost particle

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 865–899
DOI: 10.1214/16-AIHP740

SPDEs on narrow domains and on graphs: An asymptotic approach

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Abstract. We introduce here a class of stochastic partial differential equations defined on a graph and we show how they are obtained as the limit of suitable stochastic partial equations defined in a narrow channel, as the width of the channel goes to zero.

Résumé. Nous introduisons ici une classe d'équations aux dérivées partielles stochastiques définies sur un graphe et nous montrons comme ils sont obtenus sous la limite d'équations aux dérivées partielles stochastiques appropriées définies dans un canal étroit, lorsque la largeur du canal tend vers zéro.

MSC: 60H15; 70K65; 35R02

Keywords: Averaging principle; Stochastic Skorohod problem; Marlov processes on graphs; Stochastic partial differential equations

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 900–915
DOI: 10.1214/16-AIHP741

Scaling limits of random outerplanar maps with independent link-weights[1]

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Abstract. The scaling limit of large simple outerplanar maps was established by Caraceni using a bijection due to Bonichon, Gavoille and Hanusse. The present paper introduces a new bijection between outerplanar maps and trees decorated with ordered sequences of edge-rooted dissections of polygons. We apply this decomposition in order to provide a new, short proof of the scaling limit that also applies to the general setting of first-passage percolation. We obtain sharp tail-bounds for the diameter and recover the asymptotic enumeration formula for outerplanar maps. Our methods also enable us to treat subclasses such as bipartite outerplanar maps.

Résumé. La limite d'échelle des cartes planaires extérieures simples a été établie par Caraceni via une bijection de Bonichon, Gavoille et Hanusse. Dans ce papier, nous construisons une nouvelle bijection entre les cartes planaires extérieures, et les arbres décorés par des suites ordonnées de dissections de polygones enracinées. Nous utilisons cette décomposition pour obtenir une nouvelle preuve, plus courte, du résultat de Caraceni, qui s'étend en outre au cadre général de la percolation de premier passage. Nous obtenons des bornes précises sur la queue de distribution du diamètre des cartes planaires extérieures, et retrouvons les formules d'énumération asymptotiques de ces cartes. Nos méthodes nous permettent également de traiter le cas de sous-classes comme les cartes planaires extérieures biparties.

MSC: Primary 60F17; 60C05; secondary 05C80

Keywords: Continuum random tree; Scaling limits; Random graphs; Outerplanar maps

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 916–936
DOI: 10.1214/16-AIHP742

Unimodality for free Lévy processes

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Abstract. We will prove that: (1) A symmetric free Lévy process is unimodal if and only if its free Lévy measure is unimodal; (2) Every free Lévy process with boundedly supported Lévy measure is unimodal in sufficiently large time. (2) is completely different property from classical Lévy processes. On the other hand, we find a free Lévy process such that its marginal distribution is not unimodal for any time $s > 0$ and its free Lévy measure does not have a bounded support. Therefore, we conclude that the boundedness of the support of free Lévy measure in (2) cannot be dropped. For the proof we will (almost) characterize the existence of atoms and the existence of continuous probability densities of marginal distributions of a free Lévy process in terms of Lévy–Khintchine representation.

Résumé. Nous montrons que: (1) Un processus de Lévy libre symétrique est unimodal si et seulement si sa mesure de Lévy libre est unimodale; (2) Chaque processus de Lévy libre avec mesure de Lévy à support borné est unimodal en temps suffisamment grand. (2) est une propriété tout à fait différente des processus de Lévy classiques. D'autre part, nous trouvons un processus de Lévy libre tel que la distribution marginale n'est pas unimodale pour tout temps $s > 0$ et dont la mesure de Lévy libre n'est pas de support borné. Par conséquent, nous concluons que l'hypothèse sur le support de la mesure de Lévy libre dans (2) ne peut pas être supprimée. Pour la preuve, nous caractérisons (presque) l'existence d'atomes et l'existence de densités de probabilité continues pour les distributions marginales d'un processus libre Lévy en termes de sa représentation de Lévy–Khintchine.

MSC: 46L54; 60G51

Keywords: Free probability; Free convolution; Free Lévy process; Unimodality

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 937–956
DOI: 10.1214/16-AIHP743

Maximal inequalities for stochastic convolutions driven by compensated Poisson random measures in Banach spaces[1]

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Abstract. We consider a Banach space $(E, \|\cdot\|)$ such that, for some $q \geq 2$, the function $x \mapsto \|x\|^q$ is of C^2 class and its kth, $k = 1, 2$, Fréchet derivatives are bounded by some constant multiples of the $(q-k)$th power of the norm. We also consider a C_0-semigroup S of contraction type on $(E, \|\cdot\|)$. Finally we consider a compensated Poisson random measure $\tilde{N}$ on a measurable space $(Z, \mathcal{Z})$.

We study the following stochastic convolution process

$$u(t) = \int_0^t \int_Z S(t-s)\xi(s,z)\tilde{N}(\mathrm{d}s, \mathrm{d}z), \quad t \geq 0,$$

where $\xi : [0, \infty) \times \Omega \times Z \to E$ is an $\mathbb{F} \otimes \mathcal{Z}$-predictable function.

We prove that there exists a càdlàg modification $\tilde{u}$ of the process u which satisfies the following maximal type inequality

$$\mathbb{E} \sup_{0 \leq s \leq t} \|\tilde{u}(s)\|^{q'} \leq C\mathbb{E}\left(\int_0^t \int_Z \|\xi(s,z)\|^p N(\mathrm{d}s, \mathrm{d}z)\right)^{\frac{q'}{p}},$$

for all $q' \geq q$ and $1 < p \leq 2$ with $C = C(q, p)$.

Résumé. On considère un espace de Banach $(E, \|\cdot\|)$ tel que, pour $q \geq 2$, la fonction $x \mapsto \|x\|^q$ est de classe C^2 avec des dérivées $k^{\text{ième}}$, $k = 1, 2$, au sens de Fréchet bornées par des constantes multiples de la puissance $(q-k)$ de la norme. On considère également un C_0-semigroupe de contraction S sur $(E, \|\cdot\|)$. Finalement, on considère une mesure de Poisson compensée $\tilde{N}$ sur un espace mesurable $(Z, \mathcal{Z})$.

On étudie le processus stochastique suivant :

$$u(t) = \int_0^t \int_Z S(t-s)\xi(s,z)\tilde{N}(\mathrm{d}s, \mathrm{d}z), \quad t \geq 0,$$

où $\xi : [0, \infty[\times \Omega \times Z \to E$ est une fonction $\mathbb{F} \otimes \mathcal{Z}$-prévisible.

On prouve qu'il existe une modification càdlàg $\tilde{u}$ du processus u qui vérifie l'inégalité de type maximale suivante :

$$\mathbb{E} \sup_{0 \leq s \leq t} \|\tilde{u}(s)\|^{q'} \leq C\mathbb{E}\left(\int_0^t \int_Z \|\xi(s,z)\|^p N(\mathrm{d}s, \mathrm{d}z)\right)^{\frac{q'}{p}},$$

pour tout $q' \geq q$ et $1 < p \leq 2$ avec $C = C(q, p)$.

MSC: 60H15; 60F10; 60H05; 60G57; 60J75

Keywords: Stochastic convolution; Martingale type p Banach space; Poisson random measure

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 957–996
DOI: 10.1214/16-AIHP745

Strong stationary times for one-dimensional diffusions

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Abstract. A necessary and sufficient condition is obtained for the existence of strong stationary times for ergodic one-dimensional diffusions, whatever the initial distribution. The strong stationary times are constructed through intertwinings with dual processes, in the Diaconis–Fill sense, taking values in the set of segments of the extended line $\mathbb{R} \sqcup \{-\infty, +\infty\}$. They can be seen as natural Doob transforms of the extensions to the diffusion framework of the evolving sets of Morris–Peres. Starting from a singleton set, the dual process begins by evolving into true segments in the same way a Bessel process of dimension 3 escapes from 0. The strong stationary time corresponds to the first time the full segment $[-\infty, +\infty]$ is reached. The benchmark Ornstein–Uhlenbeck process cannot be treated in this way; it will nevertheless be seen how to use other strong times to recover its optimal exponential rate of convergence to equilibrium in the total variation sense.

Résumé. Une condition nécessaire et suffisante est obtenue pour l'existence de temps fort de stationnarité, quelque soit la condition initiale. Les temps forts de stationnarité sont construits par le biais d'entrelacements avec des processus duaux, au sens de Diaconis–Fill, prenant leurs valeurs dans l'ensemble des segments de la droite étendue $\mathbb{R} \sqcup \{-\infty, +\infty\}$. Ils peuvent être vus comme des transformées de Doob d'extensions au cadre diffusif des ensembles évoluants de Morris–Peres. Partant d'un singleton, le processus dual commence par évoluer en segments de la même manière qu'un processus de Bessel de dimension 3 s'échappe de 0. Le temps fort de stationnarité correspond au premier temps d'atteinte de $[-\infty, +\infty]$. Le processus d'Ornstein–Uhlenbeck ne peut pas être traiter de la sorte, il est toutefois possible d'utiliser d'autres temps forts pour retrouver son taux exponentiel optimal de convergence à l'équilibre en variation totale.

MSC: Primary 60J60; secondary 37A30; 47A10; 60J35; 60E15

Keywords: Strong (stationary) time; Ergodic one-dimensional diffusion; Intertwining; Dual process; Evolving segments; Explosion time; Bessel process; Ornstein–Uhlenbeck process; Spectral decomposition and quasi-stationary measure

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Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2017, Vol. 53, No. 2, 997–1005
DOI: 10.1214/16-AIHP746

Homogenization via sprinkling

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Abstract. We show that a superposition of an ε-Bernoulli bond percolation and any everywhere percolating subgraph of $\mathbb{Z}^d$, $d \geq 2$, results in a connected subgraph, which after a renormalization dominates supercritical Bernoulli percolation. This result, which confirms a conjecture from (*J. Math. Phys.* **41** (2000) 1294–1297), is mainly motivated by obtaining finite volume characterizations of uniqueness for general percolation processes.

Résumé. On considère un sous-graphe de $\mathbb{Z}^d$, $d \geq 2$, dont toutes les composantes connexes sont infinies. On montre que la superposition d'un tel sous-graphe avec une ε-percolation forme un graphe connexe qui, convenablement renormalisé, domine une percolation de Bernoulli surcritique. Ce résultat confirme une conjecture énoncée dans (*J. Math. Phys.* **41** (2000) 1294–1297), et sa motivation principale est d'obtenir des caractérisations en volume fini de l'unicité de l'amas infini pour des processus de percolation généraux.

MSC: 60K35; 05C80

Keywords: Percolation; Sprinkling; Finite-size criterion; Random geometry

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