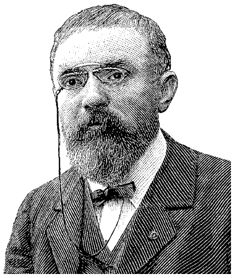

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One-dimensional long-range diffusion-limited aggregation III – The limit aggregate

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Abstract. In this paper we study the structure of the limit aggregate $A_\infty = \bigcup_{n\geq 0} A_n$ of the one-dimensional long range diffusion limited aggregation process defined in (*Ann. Probab.* **44** (2016) 3546–3579). We show (under some regularity conditions) that for walks with finite third moment A_∞ has renewal structure and positive density, while for walks with finite variance the renewal structure no longer exists and A_∞ has 0 density. We define a tree structure on the aggregates and show some results on the degrees and number of ends of these random trees. We introduce a new "harmonic competition" model where different colours compete for harmonic measure, and show how the tree structure is related to coexistence in this model.

Résumé. Nous étudions la structure de l'agrégat limite $A_\infty = \bigcup_{n\geq 0} A_n$ du DLA en dimension 1 avec longue portée, tel qu'introduit dens (*Ann. Probab.* **44** (2016) 3546–3579). Nous montrons (sous des hypothèses de régularité) que pour des marches aléatoires admettant un moment d'ordre 3, A_∞ a une structure de renouvellement et une densité positive, tandis que pour les marches ayant seulement une variance finie, la structure de renouvellement disparaît et la densité est nulle. Nous définissons une structure arborescente sur l'agrégat et montrons quelques résultats sur les degrés et le nombre de bouts de ces arbres aléatoires. Nous introduisons un nouveau modèle de « compétition harmonique » entre des particules de couleurs différentes, et nous montrons que la structure d'arbre est reliée au problème de coexistence dans ce modèle.

MSC: Primary 82C24; secondary 60K35; 97K50; 97K60

Keywords: Diffusion limited aggregation; DLA; Random walk; Harmonic measure; Phase transition; Renewal structure

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BSDEs with diffusion constraint and viscous Hamilton–Jacobi equations with unbounded data

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Abstract. We provide a stochastic representation for a general class of viscous Hamilton–Jacobi (HJ) equations, which has convex and superlinear nonlinearity in its gradient term, via a type of backward stochastic differential equation (BSDE) with constraint in the martingale part. We compare our result with the classical representation in terms of (super)quadratic BSDEs, and show in particular that existence of a viscosity solution to the viscous HJ equation can be obtained under more general growth assumptions on the coefficients, including both unbounded diffusion coefficient and terminal data.

Résumé. Nous donnons une représentation stochastique pour une classe générale d'équations d'Hamilton–Jacobi (HJ) visqueuses, convexes et super-nonlinéaires, au moyen d'équations différentielles stochastiques rétrogrades (EDSR) avec contraintes sur la partie martingale. Nous comparons nos résultats avec la représentation classique en termes d'EDSR (super)quadratiques, et montrons notamment que l'existence d'une solution de viscosité à l'équation visqueuse de HJ peut être obtenue sous des conditions de croissance plus générales, incluant des coefficients et une donnée terminale non bornées.

MSC: 60H30; 35K58

Keywords: Backward stochastic differential equation (BSDE); Randomization; Viscous Hamilton–Jacobi equation; Deterministic KPZ equation; Nonlinear Feynman–Kac formula

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The dichotomy spectrum for random dynamical systems and pitchfork bifurcations with additive noise

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Abstract. We develop the *dichotomy spectrum* for random dynamical systems and demonstrate its use in the characterization of pitchfork bifurcations for random dynamical systems with additive noise.

Crauel and Flandoli (*J. Dynam. Differential Equations* **10** (1998) 259–274) had shown earlier that adding noise to a system with a deterministic pitchfork bifurcation yields a unique attracting random equilibrium with negative Lyapunov exponent throughout, thus "destroying" this bifurcation. Indeed, we show that in this example the dynamics before and after the underlying deterministic bifurcation point are topologically equivalent.

However, in apparent paradox to (*J. Dynam. Differential Equations* **10** (1998) 259–274), we show that there is after all a qualitative change in the random dynamics at the underlying deterministic bifurcation point, characterized by the transition from a hyperbolic to a non-hyperbolic dichotomy spectrum. This breakdown manifests itself also in the loss of uniform attractivity, a loss of experimental observability of the Lyapunov exponent, and a loss of equivalence under uniformly continuous topological conjugacies.

Résumé. Nous développons le *spectre de dichotomie* pour les systèmes dynamiques aléatoires et nous démontrons son utilité pour la caractérisation des bifurcations de fourches dans des systèmes dynamiques aléatoires avec du bruit additif.

Crauel et Flandoli (*J. Dynam. Differential Equations* **10** (1998) 259–274) ont précédemment montré que l'ajout de bruit additif à un système comprenant une bifurcation de fourche déterministe produit un unique équilibre aléatoire attractif avec un exposant de Lyapunov négatif partout, « détruisant » ainsi cette bifurcation. En effet, nous montrons dans cet exemple que la dynamique avant et après le point de bifurcation déterministe sous-jacent sont topologiquement équivalentes.

Cependant, dans un paradoxe apparent avec (*J. Dynam. Differential Equations* **10** (1998) 259–274), nous montrons qu'il y a après tout un changement qualitatif du système aléatoire au point du bifurcation déterministe sous-jacent, caractérisé par la transition du spectre de dichotomie hyperbolique à un spectre non-hyperbolique. Cette rupture se manifeste elle-même aussi dans une perte d'attractivité uniforme, une perte d'observabilité expérimentale de l'exposant de Lyapunov, et une perte d'équivalence sous conjugaisons topologiques uniformes et continues.

MSC: 37H15; 37H20

Keywords: Dichtomy spectrum; Finite-time Lyapunov exponent; Pitchfork bifurcation; Random dynamical system; Topological equivalence

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Noncommutative Burkholder/Rosenthal inequalities associated with convex functions

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Abstract. We prove noncommutative martingale inequalities associated with convex functions. More precisely, we obtain Φ-moment analogues of the noncommutative Burkholder inequalities and the noncommutative Rosenthal inequalities for any convex Orlicz function Φ whose Matuzewska–Orlicz indices p_Φ and q_Φ are such that $1 < p_\Phi \leq q_\Phi < 2$ or $2 < p_\Phi \leq q_\Phi < \infty$. These results generalize the noncommutative Burkholder/Rosenthal inequalities due to Junge and Xu. The key ingredient in our approach is a simultaneous version of the Burkholder inequality recently proved for the case of noncommutative L_p-spaces with $1 < p < 2$.

Résumé. Nous prouvons des inégalités de martingales non commutatives associées à des fonctions convexes. Plus précisément, nous obtenons des analogues des inégalités de Burkholder non commutatives et des inégalités de Rosenthal non commutatives pour des Φ-moments associés à toute fonction convexe Φ dont les indices de Matuzewska–Orlicz p_Φ et q_Φ vérifient $1 < p_\Phi \leq q_\Phi < 2$ ou $2 < p_\Phi \leq q_\Phi < \infty$. Ces résultats généralisent les inégalités de Burkholder/Rosenthal non commutatives obtenues par Junge et Xu. L'ingrédient clé de notre approche est une version simultanée de l'inégalité de Burkholder récemment demontrée dans le cas des espaces L_p non commutatifs pour $1 < p < 2$.

MSC: Primary 46L53; 46L52; secondary 47L05; 60G42

Keywords: Noncommutative Burkholder inequalities; Noncommutative Rosenthal inequalities; Orlicz functions; Moment inequalities; Interpolations

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Universality for random matrix flows with time-dependent density

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Abstract. We show that the Dyson Brownian Motion exhibits local universality after a very short time assuming that local rigidity and level repulsion of the eigenvalues hold. These conditions are verified, hence bulk spectral universality is proven, for a large class of Wigner-like matrices, including deformed Wigner ensembles and ensembles with non-stochastic variance matrices whose limiting densities differ from Wigner's semicircle law.

Résumé. Nous démontrons que le mouvement Brownien de Dyson établit l'universalité des statistiques spectrales locales après un temps très court, en supposant la rigidité locale et la répulsion de valeurs propres. Ces conditions sont satisfaites, et donc l'universalité spectrale est démontrée au centre du spectre, pour une large classe des matrices aléatoires du type Wigner, y compris les ensembles de Wigner deformés et des ensembles dont la matrice des variances est non-stochastique, dont les densités asymptotiques diffèrent de la loi du demi-cercle de Wigner.

MSC: 60B20; 60H30

Keywords: Random matrix; Local eigenvalue statistics; Universality; Dyson Brownian motion

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General random walk in a random environment defined on Galton–Watson trees

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Abstract. We consider a particle performing a random walk on a Galton–Watson tree, when the probabilities of jumping from a vertex to any one of its neighbours are determined by a random process. We introduce a method for deriving conditions under which the walk is either transient or recurrent. We first suppose that the weights are i.i.d., and re-prove a result of Lyons and Pemantle (*Ann. Probab.* **20** (1992) 125–136). We then assume a Markovian environment along each line of descent, and finally consider a random walk in a Markovian environment that itself changes the environment. Our approach involves studying the typical behaviour of the walk on fixed lines of descent, which we then show determines the behaviour of the process on the whole tree.

Résumé. Nous considérons le mouvement d'une particule sur un arbre de Galton–Watson, lorsque les probabilités de saut d'un nœud à l'un de ses voisins sont déterminées par un processus aléatoire. Conditionnellement à l'arbre, des poids positifs sont affectés aux arcs de telle sorte que, vu le long d'une ligne de descente, ils évoluent comme un processus aléatoire. Afin de présenter notre méthode pour prouver la récurrence ou la transience du processus, nous supposons d'abord que les poids sont i.i.d., redémontrant ainsi un résultat de Lyons et Pemantle. Nous étendons ensuite l'argument pour permettre un environnement Markovien, et enfin à une marche aléatoire sur un environnement Markovien qui modifie l'environnement. Notre approche consiste à étudier le comportement typique des processus sur les lignes de descente fixes, dont nous montrons ensuite qu'il détermine le comportement du processus sur l'ensemble de l'arbre.

MSC: Primary 60K37; secondary 60J20; 60K35

Keywords: Random walk in random environment; Galton–Watson; Reinforcement

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Minimax goodness-of-fit testing in ill-posed inverse problems with partially unknown operators

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Abstract. We consider a Gaussian sequence model that contains ill-posed inverse problems as special cases. We assume that the associated operator is partially unknown in the sense that its singular functions are known and the corresponding singular values are unknown but observed with Gaussian noise. For the considered model, we study the minimax goodness-of-fit testing problem. Working with certain ellipsoids in the space of square-summable sequences of real numbers, with a ball of positive radius removed, we obtain lower and upper bounds for the minimax separation radius in the non-asymptotic framework, i.e., for fixed values of the involved noise levels. Examples of mildly and severely ill-posed inverse problems with ellipsoids of ordinary-smooth and super-smooth sequences are examined in detail and minimax rates of goodness-of-fit testing are obtained for illustrative purposes.

Résumé. Nous considérons un modèle séquentiel gaussien incluant des modèles de type « problème inverse » comme cas particulier. Nous supposons que l'opérateur associé est partiellement connu au sens où les fonctions propres associées à la décomposition en valeurs singulières sont connues, mais pas les valeurs propres. Ces dernières sont malgré tout observables en présence d'un bruit gaussien. Dans ce cadre, nous nous concentrons sur l'étude d'un problème de test d'adéquation. En utilisant à la fois une condition d'énergie sur le signal considéré ainsi que des propriétés de régularité, nous établissons des bornes supérieures et inférieures pour la vitesse minimax de séparation dans un cadre asymptotique, i.e. pour des valeurs fixées des niveaux de bruit impliqués dans les observations. Pour finir, des exemples particuliers de problèmes inverses et de régularités sont considérés et les vitesses minimax de séparation correspondantes sont discutées en détail afin d'illustrer les résultats obtenus.

MSC: 62G05; 62K20

Keywords: Ellipsoids; Compact operators; Gaussian sequence model; Gaussian white noise model; Ill-posed inverse problems; Minimax goodness-of-fit testing; Minimax signal detection; Singular value decomposition

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A variational approach to some transport inequalities

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Abstract. In this paper, we relate transport-entropy inequalities to the minimization of certain functionals defined on the space of probability measures. This approach leads in particular to a new proof of a result by Otto and Villani (*J. Funct. Anal.* **173** (2000) 361–400) showing that the logarithmic Sobolev inequality implies Talagrand's transport inequality.

Résumé. Dans cet article, nous proposons une approche des inégalités de transport-entropie fondée sur la minimisation de certaines fonctionnelles définies sur l'espace des mesures de probabilité. Cette approche nous permet en particulier de donner une nouvelle preuve d'un résultat d'Otto et Villani (*J. Funct. Anal.* **173** (2000) 361–400) montrant que l'inégalité de Sobolev logarithmique entraîne l'inégalité de transport de Talagrand.

MSC: 60E15; 26D10; 58E99

Keywords: Optimal transport; Transport-entropy inequalities

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Hydrodynamic limit for a system of independent, sub-ballistic random walks in a common random environment

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Abstract. We consider a system of independent random walks in a common random environment. Previously, a hydrodynamic limit for the system of RWRE was proved under the assumption that the random walks were transient with positive speed (*Electron. J. Probab.* **15** (2010) 1024–1040). In this paper we instead consider the case where the random walks are transient but with a sublinear speed of the order n^κ for some $\kappa \in (0,1)$ and prove a quenched hydrodynamic limit for the system of random walks with time scaled by $n^{1/\kappa}$ and space scaled by n. The most interesting feature of the hydrodynamic limit is that the influence of the environment does not average out under the hydrodynamic scaling; that is, the asymptotic particle density depends on the specific environment chosen. The hydrodynamic limit for the system of RWRE is obtained by first proving a hydrodynamic limit for a system of independent particles in a directed trap environment.

Résumé. Nous considérons un système de particules indépendantes évoluant dans un milieu aléatoire commun. Auparavant, la limite hydrodynamique de ce système de particules en milieu aléatoire a été obtenue quand les particules sont transientes avec une vitesse positive (*Electron. J. Probab.* **15** (2010) 1024–1040). Dans cet article nous considérons le cas où les particules sont transientes mais ont une vitesse sous-linéaire d'ordre n^κ pour $\kappa \in (0,1)$ et nous montrons l'existence d'une limite hydrodynamique du système de particules avec une échelle du temps $n^{1/\kappa}$ et une échelle spatiale n. La propriété la plus intéressante de cette limite hydrodynamique est que le milieu n'est pas moyenné par la limite ; c'est-à-dire, la densité asymptotique des particules dépend de la réalisation du milieu choisi. La limite hydrodynamique du système de particules est déduite à partir de la limite hydrodynamique d'un système de particules indépendantes dans un milieu aléatoire dirigé.

MSC: Primary 60K35; secondary 60K37

Keywords: Hydrodynamic limit; Random walk in random environment; Directed traps

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Gradient estimates for porous medium and fast diffusion equations by martingale method

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Abstract. In this paper, we establish several local and global gradient estimates for positive solutions of Porous Medium Equations (PMEs) and Fast Diffusion Equations (FDEs). Our proof is probabilistic and uses martingale techniques.

Résumé. Dans cet article, nous établissons plusieurs estimées (locales et globales) des gradients des solutions positives des équations aux milieux poreux et des équations de la diffusion rapide. Notre preuve est probabiliste et utilise des techniques de martingales.

MSC: 60H10; 60H30

Keywords: Gradient estimate; Porous medium equation; Fast diffusion equation; Martingale technique

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A rice method proof of the null-space property over the Grassmannian

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Abstract. The Null-Space Property (NSP) is a necessary and sufficient condition for the recovery of the largest coefficients of solutions to an under-determined system of linear equations. Interestingly, this property governs also the success and the failure of recent developments in high-dimensional statistics, signal processing, error-correcting codes and the theory of polytopes.

Although this property is the keystone of ℓ_1-minimization techniques, it is an open problem to derive a closed form for the phase transition on NSP. In this article, we provide the first proof of NSP using random processes theory and the Rice method. As a matter of fact, our analysis gives non-asymptotic bounds for NSP with respect to unitarily invariant distributions. Furthermore, we derive a simple sufficient condition for NSP.

Résumé. La propriété du noyau (NSP en anglais) est une condition nécessaire et suffisante pour estimer les plus grands coefficients d'un système linéaire sous-déterminé d'équations. De manière intéressante, cette propriété gouverne aussi le succès ou l'échec de récentes approches en statistique en grandes dimensions, traitement du signal, codes correcteurs d'erreurs et la théorie des polytopes.

Bien que cette propriété soit au centre des techniques de minimisation L1, un problème ouvert reste l'obtention d'une forme explicite de la transition de phase de la propriété NSP. Dans cet article, nous donnons la première preuve de la propriété NSP du point de vue de la théorie des processus aléatoires et de la méthode de Rice. Ainsi, notre analyse conduit à de nouvelles bornes non asymptotiques pour la propriété NSP pour toute distribution invariante par rotation. De plus, nous déduisons une condition suffisante simple pour établir la propriété NSP.

MSC: 62J05; 62H12; 62H20

Keywords: Rice Method; High-dimensional statistics; ℓ_1-minimization; Null-Space Property; Random processes theory

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Limit theorems for the left random walk on $\mathrm{GL}_d(\mathbb{R})$

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Abstract. Motivated by a recent work of Benoist and Quint and extending results from the PhD thesis of the third author, we obtain limit theorems for products of independent and identically distributed elements of $\mathrm{GL}_d(\mathbb{R})$, such as the Marcinkiewicz–Zygmund strong law of large numbers, the CLT (with rates in Wasserstein's distances) and almost sure invariance principles with rates.

Résumé. Motivés par un travail récent de Benoist et Quint, nous étendons certains résultats issus de la thèse de doctorat du troisième auteur puis établissons des théorèmes limite pour les produits de matrices indépendantes et identiquement distribuées de $\mathrm{GL}_d(\mathbb{R})$. Nous nous intéressons notamment aux lois fortes de Marcinkiewicz–Zygmund, au TLC (avec vitesses en distance de Wasserstein) et au principe d'invariance presque-sûr avec vitesse.

MSC: 60B15; 60F05; 60F17

Keywords: Central Limit theorem; Random walks on $\mathrm{GL}_d(\mathbb{R})$; Strong invariance principles

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Brownian motion correlation in the peanosphere for $\kappa > 8$

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Abstract. The peanosphere (or "mating of trees") construction of Duplantier, Miller, and Sheffield encodes certain types of γ-Liouville quantum gravity (LQG) surfaces ($\gamma \in (0,2)$) decorated with an independent SLE_κ ($\kappa = 16/\gamma^2 > 4$) in terms of a correlated two-dimensional Brownian motion and provides a framework for showing that random planar maps decorated with statistical physics models converge to LQG decorated with an SLE. Previously, the correlation for the Brownian motion was only explicitly identified as $-\cos(4\pi/\kappa)$ for $\kappa \in (4,8]$ and unknown for $\kappa > 8$. The main result of this work is that this formula holds for all $\kappa > 4$. This supplies the missing ingredient for proving convergence results of the aforementioned type for $\kappa > 8$. Our proof is based on the calculation of a certain tail exponent for SLE_κ on a quantum wedge and then matching it with an exponent which is well-known for Brownian motion.

Résumé. La sphère de Peano (ou «Accouplement d'arbres») construite par Duplantier, Miller, et Sheffield encode certains types de surfaces de γ-gravité quantique de Liouville (LQG) décorées par un SLE_κ (pour $\gamma \in (0,2)$ et $\kappa = 16/\gamma^2 > 4$), en termes d'un mouvement Brownien 2-dimensionnel corrélé et fournit un cadre pour montrer que les cartes planaires décorées par un modèle de physique statistique convergent vers un LQG décoré par un SLE. Précédemment, la corrélation du mouvement Brownien était seulement explicitement identifiée à $-\cos(4\pi/\kappa)$ pour $\kappa \in (4,8]$, mais inconnue pour $\kappa > 8$. Le résultat principal de ce travail est que cette formule reste vraie pour $\kappa > 8$. Cela donne l'ingrédient manquant pour prouver les résultats de convergence mentionnés précédemment pour $\kappa > 8$. Notre preuve est basée sur le calcul d'un exposant de queue pour le SLE_κ sur un coin quantique et sur son identification avec un exposant bien connu pour le mouvement Brownien.

MSC: 60J67; 60D05

Keywords: Schramm–Loewner evolution; Liouville quantum gravity; Peanosphere

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Joint convergence of random quadrangulations and their cores

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Abstract. We show that a uniform quadrangulation, its largest 2-connected block, and its largest simple block jointly converge to the same Brownian map in distribution for the Gromov–Hausdorff–Prokhorov topology. We start by deriving a local limit theorem for the asymptotics of maximal block sizes, extending the result in (*Random Structures Algorithms* **19** (2001) 194–246). The resulting diameter bounds for pendant submaps of random quadrangulations straightforwardly lead to Gromov–Hausdorff convergence. To extend the convergence to the Gromov–Hausdorff–Prokhorov topology, we show that exchangeable "uniformly asymptotically negligible" attachments of mass simply yield, in the limit, a deterministic scaling of the mass measure.

Résumé. Nous montrons qu'une quadrangulation uniformément aléatoire, sa plus grande composante 2-connexe, et sa plus grande composante simple convergent conjointement en loi vers la même carte brownienne dans le sens Gromov–Hausdorff–Prokhorov. En premier, nous étendons l'analyse de (*Random Structures Algorithms* **19** (2001) 194–246) afin de démontrer un théorème limite local pour les tailles des plus grandes composantes. Les bornes sur les diamètres ainsi obtenues impliquent directement la convergence dans le sens Gromov–Hausdorff. Pour obtenir la convergence pour la topologie Gromov–Hausdorff–Prokhorov, nous prouvons que l'effet de l'attachement des masses sur l'objet limite est déterministe, si les masses sont attachées de manière échangeable et les masses sont uniformément asymptotiquement négligeables.

MSC: Primary 60C05; secondary 68P10; 68W40

Keywords: Brownian map; Gromov–Hausdorff–Prokhorov convergence; Singularity analysis; Connectivity; Random quadrangulations

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Crossing probabilities for critical Bernoulli percolation on slabs

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Abstract. We prove that in the critical Bernoulli percolation on graphs $\mathbb{Z}^2 \times \{0, \dots, k-1\}^{d-2}$, for each $\rho > 0$, the probability of open left-right crossing of rectangle $[0, \rho N] \times [0, N] \times [0, k-1]^{d-2}$ is uniformly positive.

Résumé. On démontre que dans la percolation de Bernoulli critique sur le graphe $\mathbb{Z}^2 \times \{0, \dots, k-1\}^{d-2}$, pour chaque $\rho > 0$, la probabilité d'avoir un passage de gauche à droite ouvert dans $[0, \rho N] \times [0, N] \times [0, k-1]^{d-2}$ est uniformément positive.

MSC: 60K35; 82B43

Keywords: Critical Bernoulli percolation; Slab; Russo–Seymour–Welsh theorem

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Limit theorems for longest monotone subsequences in random Mallows permutations

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Abstract. We study the lengths of monotone subsequences for permutations drawn from the Mallows measure. The Mallows measure was introduced by Mallows in connection with ranking problems in statistics. Under this measure, the probability of a permutation π is proportional to $q^{\mathrm{inv}(\pi)}$ where q is a positive parameter and $\mathrm{inv}(\pi)$ is the number of inversions in π.

In our main result we show that when $0 < q < 1$, then the limiting distribution of the longest increasing subsequence (LIS) is Gaussian, answering an open question in (*Probab. Theory Related Fields* **161** (2015) 719–780). This is in contrast to the case when $q = 1$ where the limiting distribution of the LIS when scaled appropriately is the GUE Tracy–Widom distribution. We also obtain a law of large numbers for the length of the longest decreasing subsequence (LDS) and identify the limiting constant, answering a further open question in (*Probab. Theory Related Fields* **161** (2015) 719–780).

Résumé. Nous étudions les longueurs des sous-suites monotones de permutations aléatoires tirées sous la mesure de Mallows. La mesure de Mallows a été introduite par Mallows dans le contexte des problèmes de classement en statistique. Sous cette mesure la probabilité d'une permutation π est proportionnelle à $q^{\mathrm{inv}(\pi)}$ où q est un paramètre positif et $\mathrm{inv}(\pi)$ est le nombre d'inversions de π.

Notre résultat principal montre que lorsque $0 < q < 1$, la loi de la plus longue sous-suite croissante est Gaussienne, répondant ainsi à une question posée dans (*Probab. Theory Related Fields* **161** (2015) 719–780). Notons le contraste avec le cas $q = 1$, où la loi limite de la plus longue sous-suite croissante proprement normalisée est la distribution du GUE Tracy–Widom. Nous obtenons aussi une loi des grands nombres pour la longueur de la plus longue sous-suite décroissante et identifions la limite, répondant ainsi à une autre question posée dans (*Probab. Theory Related Fields* **161** (2015) 719–780).

MSC: 60C05; 60F05

Keywords: Mallows permutations; Longest increasing subsequence; Central limit theorem

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A discrete log-Sobolev inequality under a Bakry–Émery type condition

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Abstract. We consider probability mass functions V supported on the positive integers using arguments introduced by Caputo, Dai Pra and Posta, based on a Bakry–Émery condition for a Markov birth and death operator with invariant measure V. Under this condition, we prove a new modified logarithmic Sobolev inequality, generalizing and strengthening results of Wu, Bobkov and Ledoux, and Caputo, Dai Pra and Posta. We show how this inequality implies results including concentration of measure and hypercontractivity, and discuss how it may extend to higher dimensions.

Résumé. Nous considérons des distributions de probabilité V à support dans l'ensemble des entiers positifs, en utilisant des arguments introduits par Caputo, Dai Pra et Posta, basés sur une condition de Bakry–Émery pour une chaîne de naissance et mort avec mesure invariante V. Sous cette condition, nous prouvons une nouvelle inégalité de Sobolev logarithmique modifiée, en généralisant et améliorant des résultats de Wu, de Bobkov et Ledoux, et de Caputo, Dai Pra et Posta. Nous montrons comment cette inégalité implique des résultats tels que la concentration de la mesure et l'hypercontractivité, et nous discutons d'une extension possible aux dimensions supérieures.

MSC: 39B62; 60J10

Keywords: Bakry–Émery condition; Birth and death chain; Concentration of measure; Discrete probability measure; Log-concavity; Log-Sobolev inequality

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Integration formulas for Brownian motion on classical compact Lie groups

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Abstract. Combinatorial formulas for the moments of the Brownian motion on classical compact Lie groups are obtained. These expressions are deformations of formulas of B. Collins and P. Śniady for moments of the Haar measure and yield a proof of the First Fundamental Theorem of invariant theory and of classical Schur–Weyl dualities based on stochastic calculus.

Résumé. On obtient ici des formules combinatoires pour les moments du mouvement Brownien sur les groupes de Lie classiques compacts. Elles sont des déformations de celles obtenues par B. Collins et P. Śniady pour les moments de la mesure de Haar et permettent de donner une preuve fondée sur le calcul stochastique du premier théorème fondamental de la théorie des invariants et de la dualité de Schur–Weyl.

MSC: 46L53; 60J65; 60H30; 60G15; 14L35; 13A50; 15A72; 16W22; 05E10

Keywords: Brownian motion on Compact Lie groups; Weingarten calculus; Schur–Weyl duality; First Fundamental Theorem of invariant theory

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A boundedness trichotomy for the stochastic heat equation[1]

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Abstract. We consider the stochastic heat equation with a multiplicative white noise forcing term under standard "intermitency conditions." The main finding of this paper is that, under mild regularity hypotheses, the a.s.-boundedness of the solution $x \mapsto u(t,x)$ can be characterized generically by the decay rate, at $\pm\infty$, of the initial function u_0. More specifically, we prove that there are 3 generic boundedness regimes, depending on the numerical value of $\mathbf{\Lambda} := \lim_{|x|\to\infty} |\log u_0(x)|/(\log|x|)^{2/3}$.

Résumé. Nous nous intéressons à l'équation de la chaleur stochastique avec un bruit blanc multiplicatif sous des «conditions d'intermittence» standard. Le résultat principal de cet article est que, sous des hypothèses de régularité raisonnables, le caractère presque sûrement borné de la solution $x \mapsto u(t,x)$ est entièrement déterminé par la vitesse de décroissance en $\pm\infty$ de la condition initiale u_0. Plus précisément, nous démontrons qu'il existe trois régimes distincts selon la valeur de $\mathbf{\Lambda} := \lim_{|x|\to\infty} |\log u_0(x)|/(\log|x|)^{2/3}$.

MSC: Primary 60H15; secondary 35R60

Keywords: The stochastic heat equation

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Non-intersecting Brownian bridges and the Laguerre Orthogonal Ensemble

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Abstract. We show that the squared maximal height of the top path among N non-intersecting Brownian bridges starting and ending at the origin is distributed as the top eigenvalue of a random matrix drawn from the Laguerre Orthogonal Ensemble. This result can be thought of as a pre-asymptotic version of K. Johansson's result (*Comm. Math. Phys.* **242** (2003) 277–329) that the supremum of the Airy_2 process minus a parabola has the Tracy–Widom GOE distribution, and as such it provides an explanation for how this distribution arises in models belonging to the KPZ universality class with flat initial data. The result can be recast in terms of the probability that the top curve of the stationary Dyson Brownian motion hits an hyperbolic cosine barrier. Our proof is based on a formula, derived in (*Ann. Inst. Henri Poincaré B, Calc. Probab. Stat.* **51** (2015) 28–58), for the probability that Dyson Brownian motion stays below a curve on a finite interval, which is given in terms of the Fredholm determinant of a certain "path-integral" kernel.

Résumé. On montre que le carré de la hauteur maximale de la trajectoire supérieure parmi N ponts browniens non-intersectants issus et terminés en 0 a la même loi que la plus grande valeur propre d'une matrice aléatoire tirée de l'Ensemble Orthogonal de Laguerre. Ce résultat peut être vu comme une version pré-asymptotique du résultat de K. Johansson (*Comm. Math. Phys.* **242** (2003) 277–329) qui établit que le supremum du processus d'Airy_2 moins une parabole est distribué selon la loi de Tracy–Widom GOE, et fournit ainsi une explication sur la façon dont cette distribution apparaît dans des modèles appartenant à la classe d'universalité de KPZ avec donnée initiale plate. Le résultat peut être reformulé en termes de la probabilité que la plus haute courbe du mouvement brownien de Dyson stationnaire atteigne une barrière de cosinus hyperbolique. Notre preuve repose sur une formule, obtenue dans (*Ann. Inst. Henri Poincaré B, Calc. Probab. Stat.* **51** (2015) 28–58), pour la probabilité que le mouvement Brownien de Dyson reste sous une courbe dans un intervalle fini, qui est donnée en termes du déterminant de Fredholm d'un certain « noyau d'intégrale de chemins ».

MSC: 60K35; 60B20; 60J65

Keywords: Non-intersecting Brownian motions; KPZ universality class; Random matrices; Airy_2 process

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The power of averaging at two consecutive time steps: Proof of a mixing conjecture by Aldous and Fill

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Abstract. Let $(X_t)_{t=0}^{\infty}$ be an irreducible reversible discrete-time Markov chain on a finite state space Ω. Denote its transition matrix by P. To avoid periodicity issues (and thus ensuring convergence to equilibrium) one often considers the continuous-time version of the chain $(X_t^{\mathrm{c}})_{t\ge 0}$ whose kernel is given by $H_t := e^{-t}\sum_k (tP)^k/k!$. Another possibility is to consider the associated averaged chain $(X_t^{\mathrm{ave}})_{t=0}^{\infty}$, whose distribution at time t is obtained by replacing P^t by $A_t := (P^t + P^{t+1})/2$.

A sequence of Markov chains is said to exhibit (total-variation) cutoff if the convergence to stationarity in total-variation distance is abrupt. Let $(X_t^{(n)})_{t=0}^{\infty}$ be a sequence of irreducible reversible discrete-time Markov chains. In this work we prove that the sequence of associated continuous-time chains exhibits total-variation cutoff around time t_n iff the sequence of the associated averaged chains exhibits total-variation cutoff around time t_n. Moreover, we show that the width of the cutoff window for the sequence of associated averaged chains is at most that of the sequence of associated continuous-time chains. In fact, we establish more precise quantitative relations between the mixing-times of the continuous-time and the averaged versions of a reversible Markov chain, which provide an affirmative answer to a problem raised by Aldous and Fill [2002, Open Problem 4.17].

Résumé. Soit $(X_t)_{t=0}^{\infty}$ une chaîne de Markov en temps discret, irréductible et réversible, à valeurs dans un espace d'états fini Ω. Soit P sa matrice de transition. Pour éviter les problèmes de périodicité (et ainsi garantir la convergence vers l'équilibre), on considère souvent la version à temps continu $(X_t^{\mathrm{c}})_{t\ge 0}$, dont le noyau est donné par $H_t := e^{-t}\sum_k (tP)^k/k!$. Une alternative consiste à considérer la chaîne moyennée $(X_t^{\mathrm{ave}})_{t=0}^{\infty}$, dont la loi au temps t est obtenue en remplaçant P^t par $A_t := (P^t + P^{t+1})/2$.

Pour une suite de chaînes de Markov, on parle de cutoff (en variation totale) lorsque la convergence à l'équilibre (mesurée par la distance en variation totale) est abrupte. Soit $(X_t^{(n)})_{t=0}^{\infty}$ une suite de chaînes irréductibles et réversibles à temps discret. Dans ce travail, nous montrons que la suite de chaînes à temps continu associées satisfait un cutoff en variation totale au temps t_n si et seulement si la suite de chaînes moyennées associées satisfait un cutoff en variation totale au temps t_n. De plus, nous montrons que la largeur de la fenêtre de cutoff pour la suite de chaînes moyennées est majorée par celle de la suite de chaînes à temps continu. Nous établissons en fait des relations quantitatives plus précises entre les temps de mélange de la version moyennée et de la version à temps continu d'une chaîne de Markov réversible quelconque. Cela répond de manière affirmative à une question soulevée par Aldous et Fill [2002, Open Problem 4.17].

MSC: 60J10

Keywords: Mixing-time; Finite reversible Markov chains; Averaged chain; Maximal inequalities; Cutoff

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Doubly probabilistic representation for the stochastic porous media type equation

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Abstract. The purpose of the present paper consists in proposing and discussing a doubly probabilistic representation for a stochastic porous media equation in the whole space $\mathbb{R}^1$ perturbed by a multiplicative colored noise. For almost all random realizations ω, one associates a stochastic differential equation in law with random coefficients, driven by an independent Brownian motion.

Résumé. Cet article propose et discute une représentation doublement probabiliste pour une équation des milieux poreux stochatique dans l'espace tout entier $\mathbb{R}^1$, perturbée par un bruit multiplicatif coloré. Pour presque toute réalisation ω de l'aléa, on associe une équation différentielle stochastique en loi avec coefficients aléatoires, dirigée par un mouvement brownien indépendant.

MSC: 35R60; 60H15; 60H30; 60H10; 60G46; 35C99; 58J65; 82C31

Keywords: Stochastic partial differential equations; Infinite volume; Singular porous media type equation; Doubly probabilistic representation; Multiplicative noise; Singular random Fokker–Planck type equation; Filtering

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Pólya tree posterior distributions on densities

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Abstract. Pólya trees form a popular class of prior distributions used in Bayesian nonparametrics. For some choice of parameters, Pólya trees are prior distributions on density functions. In this paper we carry out a frequentist analysis of the induced posterior distributions in the density estimation model. We investigate the contraction rate of Pólya tree posterior densities in terms of the supremum loss and study the limiting shape distribution. A nonparametric Bernstein–von Mises theorem is established, as well as a Bayesian Donsker theorem for the posterior cumulative distribution function.

Résumé. Les arbres de Pólya constituent une classe de lois a priori très utilisée en bayésien non-paramétrique. Pour certains choix de paramètres, les arbres de Pólya induisent des lois à densité. Nous menons une analyse fréquentiste des lois a posteriori bayésiennes correspondantes dans le modèle d'estimation de densité. La concentration a posteriori des densités–arbre de Pólya est étudiée en terme de la norme–sup et nous déterminons la loi a posteriori limite après renormalisation. Un théorème de Bernstein–von Mises non-paramétrique est établi, ainsi qu'un théorème de Donsker bayésien pour la fonction de répartition a posteriori.

MSC: Primary 62G20; secondary 62G07; 62G15

Keywords: Bayesian nonparametrics; Pólya tree distribution; Supremum norm convergence; Minimax rate; Bernstein–von Mises theorem; Bayesian Donsker theorem

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Edgeworth expansions for profiles of lattice branching random walks

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Abstract. Consider a branching random walk on $\mathbb{Z}$ in discrete time. Denote by $L_n(k)$ the number of particles at site $k \in \mathbb{Z}$ at time $n \in \mathbb{N}_0$. By the *profile* of the branching random walk (at time n) we mean the function $k \mapsto L_n(k)$. We establish the following asymptotic expansion of $L_n(k)$, as $n \to \infty$:

$$\mathrm{e}^{-\varphi(0)n} L_n(k) = \frac{\mathrm{e}^{-\frac{1}{2}x_n^2(k)}}{\sqrt{2\pi\varphi''(0)n}} \sum_{j=0}^{r} \frac{F_j(x_n(k))}{n^{j/2}} + o\bigl(n^{-\frac{r+1}{2}}\bigr) \quad \text{a.s.},$$

where $r \in \mathbb{N}_0$ is arbitrary, $\varphi(\beta) = \log \sum_{k\in\mathbb{Z}} \mathrm{e}^{\beta k} \mathbb{E} L_1(k)$ is the cumulant generating function of the intensity of the branching random walk and

$$x_n(k) = \frac{k - \varphi'(0)n}{\sqrt{\varphi''(0)n}}.$$

The expansion is valid uniformly in $k \in \mathbb{Z}$ with probability 1 and the F_j's are polynomials whose random coefficients can be expressed through the derivatives of φ and the derivatives of the limit of the Biggins martingale at 0. Using exponential tilting, we also establish more general expansions covering the whole range of the branching random walk except its extreme values. As an application of this expansion for $r = 0, 1, 2$ we recover in a unified way a number of known results and establish several new limit theorems. In particular, we study the a.s. behavior of the individual occupation numbers $L_n(k_n)$, where $k_n \in \mathbb{Z}$ depends on n in some regular way. We also prove a.s. limit theorems for the mode $\arg\max_{k\in\mathbb{Z}} L_n(k)$ and the height $\max_{k\in\mathbb{Z}} L_n(k)$ of the profile. The asymptotic behavior of these quantities depends on whether the drift parameter $\varphi'(0)$ is integer, non-integer rational, or irrational. Applications of our results to profiles of random trees including binary search trees and random recursive trees will be given in a separate paper.

Résumé. Nous considérons une marche branchante sur $\mathbb{Z}$ en temps discret. Soit $L_n(k)$ le nombre de particules au site $k \in \mathbb{Z}$ au temps $n \in \mathbb{N}_0$. Nous appelons *profil* de la marche branchante (au temps n) la fonction $k \mapsto L_n(k)$. Nous établissons le développement asymptotique suivant pour $L_n(k)$, lorsque $n \to \infty$:

$$\mathrm{e}^{-\varphi(0)n} L_n(k) = \frac{\mathrm{e}^{-\frac{1}{2}x_n^2(k)}}{\sqrt{2\pi\varphi''(0)n}} \sum_{j=0}^{r} \frac{F_j(x_n(k))}{n^{j/2}} + o\bigl(n^{-\frac{r+1}{2}}\bigr) \quad \text{p.s.},$$

où $r \in \mathbb{N}_0$ est arbitraire, $\varphi(\beta) = \log \sum_{k\in\mathbb{Z}} \mathrm{e}^{\beta k} \mathbb{E} L_1(k)$ est la fonction génératrice des cumulants de l'intensité de la marche branchante, et

$$x_n(k) = \frac{k - \varphi'(0)n}{\sqrt{\varphi''(0)n}}.$$

Le développement est valable uniformément en $k \in \mathbb{Z}$ avec probabilité 1 et les F_j sont des polynômes dont les coefficients aléatoires s'expriment à l'aide des dérivées de φ et des dérivées de la limite de la martingale de Biggins en 0. En utilisant une déformation exponentielle, nous établissons aussi des développements plus généraux qui couvrent tout le spectre de la marche branchante à l'exception des valeurs extrêmes. Comme application de ce développement pour $r = 0, 1, 2$ nous retrouvons de façon unifiée plusieurs résultats connus et montrons de nouveaux théorèmes limite. En particulier, nous étudions le comportement p.s. des nombres d'occupation $L_n(k_n)$, où $k_n \in \mathbb{Z}$ dépend de n de façon régulière. Nous montrons aussi un théorème limite p.s. pour le mode $\arg\max_{k\in\mathbb{Z}} L_n(k)$ et la hauteur $\max_{k\in\mathbb{Z}} L_n(k)$ du profil. Le comportement asymptotique de ces quantités dépend de si le paramètre de la dérive $\varphi'(0)$ est entier, rationnel, ou irrationnel. D'autres applications de nos résultats aux profils d'arbres aléatoires, incluant les arbres de recherche binaires et les arbres aléatoires récursifs, seront donnés dans un autre article.

MSC: Primary 60G50; secondary 60F05; 60J80; 60F10; 60F15

Keywords: Branching random walk; Edgeworth expansion; Central limit theorem; Profile; Biggins martingale; Random analytic function; Mod-φ-convergence; Height; Mode

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Scaling limits for the threshold window: When does a monotone Boolean function flip its outcome?

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With an appendix by Gábor Pete[c]

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Abstract. Consider a monotone Boolean function $f:\{0,1\}^n\to\{0,1\}$ and the canonical monotone coupling $\{\eta_p : p\in[0,1]\}$ of an element in $\{0,1\}^n$ chosen according to product measure with intensity $p\in[0,1]$. The random point $p\in[0,1]$ where $f(\eta_p)$ flips from 0 to 1 is often concentrated near a particular point, thus exhibiting a threshold phenomenon. For a sequence of such Boolean functions, we peer closely into this threshold window and consider, for large n, the limiting distribution (properly normalized to be nondegenerate) of this random point where the Boolean function switches from being 0 to 1. We determine this distribution for a number of the Boolean functions which are typically studied and pay particular attention to the functions corresponding to iterated majority and percolation crossings. It turns out that these limiting distributions have quite varying behavior. In fact, we show that any nondegenerate probability measure on $\mathbb{R}$ arises in this way for some sequence of Boolean functions.

Résumé. Soit $f:\{0,1\}^n\to\{0,1\}$ une fonction booléenne monotone, et $\{\eta_p : p\in[0,1]\}$ le couplage monotone canonique d'éléments de $\{0,1\}^n$ choisis selon la mesure produit d'intensité $p\in[0,1]$. Le point aléatoire $p\in[0,1]$ en lequel $f(\eta_p)$ bascule de 0 à 1 est souvent concentré près d'une valeur particulière, présentant ainsi un effet de seuil. Pour une suite de telles fonctions booléennes, nous étudions de plus près la fenêtre de seuil correspondante en considérant la loi limite lorsque n tend vers l'infini (proprement normalisée pour être non-dégénérée) de ce point aléatoire critique où la fonction booléenne bascule. Nous déterminons cette loi pour de nombreuses fonctions booléennes classiques, en portant une attention particulière aux cas de la majorité itérée et des croisements de percolation. Il se trouve que ces lois limites ont des comportements d'une grande variété : en fait, nous montrons que toute mesure de probabilité non-dégénérée sur $\mathbb{R}$ peut être obtenue de cette façon à partir d'une suite bien choisie de fonctions booléennes.

MSC: 06E30; 60F20; 60F99; 60K35

Keywords: Boolean functions; Sharp thresholds; Influences; Iterated majority function; Near-critical percolation

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Spectral asymptotics for V-variable Sierpinski gaskets

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Abstract. The family of V-variable fractals provides a means of interpolating between two families of random fractals previously considered in the literature; scale irregular fractals ($V = 1$) and random recursive fractals ($V = \infty$). We consider a class of V-variable affine nested fractals based on the Sierpinski gasket with a general class of measures. We calculate the spectral exponent for a general measure and find the spectral dimension for these fractals. We show that the spectral properties and on-diagonal heat kernel estimates for V-variable fractals are closer to those of scale irregular fractals, in that it is the fluctuations in scale that determine their behaviour but that there are also effects of the spatial variability.

Résumé. La famille des fractales V-variables donne un moyen d'interpolation entre deux familles de fractales aléatoires étudiées dans la littérature : les fractales à échelle irrégulière ($V = 1$) et les fractales récursives aléatoires ($V = \infty$). Nous considérons une classe de fractales V-variables affines emboîtées, construites à partir du tamis de Sierpinski muni d'une classe générale de mesures. Nous calculons l'exposant spectral d'une mesure générale, et déterminons la dimension spectrale de ces fractales. Nous montrons que les propriétés spectrales, de même que les estimées de noyau de la chaleur sur la diagonale, sont plus proches de celles des fractales à échelle irrégulière, du fait que ce sont les fluctuations d'échelle qui déterminent leurs comportements. Néanmoins, la variabilité spatiale a aussi une influence.

MSC: Primary 35P20; secondary 28A80; 31C25; 35K08; 60J60

Keywords: Random fractals; Laplace operator; Eigenvalue counting function; Spectral dimension; Heat kernel estimates; Spectral asymptotics; V-variable; Sierpinski gasket

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Mean-field interaction of Brownian occupation measures, I: Uniform tube property of the Coulomb functional

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Abstract. We study the transformed path measure arising from the self-interaction of a three-dimensional rownian motion via an exponential tilt with the Coulomb energy of the occupation measures of the motion by time t. The logarithmic asymptotics of the partition function were identified in the 1980s by Donsker and Varadhan (*Comm. Pure Appl. Math.* **505** (1983) 505–528) in terms of a variational formula. Recently (Brownian occupations measures, compactness and large deviations (2014) Preprint) a new technique for studying the path measure itself was introduced, which allows for proving that the normalized occupation measure asymptotically concentrates around the set of all maximizers of the formula. In the present paper, we show that likewise the Coulomb functional of the occupation measure concentrates around the set of corresponding Coulomb functionals of the maximizers in the uniform topology. This is a decisive step on the way to a rigorous proof of the convergence of the normalized occupation measures towards an explicit mixture of the maximizers, derived in (Mean-field interaction of Brownian occupation measures, II: A rigorous construction of the Pekar process. Preprint). Our methods rely on deriving Hölder-continuity of the Coulomb functional of the occupation measure with exponentially small deviation probabilities and invoking the large deviation theory developed in (Brownian occupations measures, compactness and large deviations (2014) Preprint) to a certain shift-invariant functional of the occupation measures.

Résumé. Nous étudions la mesure de trajectoire transformée engendrée par l'auto-interaction d'un mouvement Brownien tridimensionnel en utilisant un biais exponentiel par l'énergie de Coulomb des mesures d'occupation de ce mouvement au temps t. Les asymptotes logarithmiques de la fonction de partition ont été identifiées dans les années 1980 par Donsker et Varadhan [*Comm. Pure Appl. Math.* **505** (1983) 505–528] au moyen d'une formule variationnelle. Récemment, dans (Brownian occupations measures, compactness and large deviations (2014) Preprint), une nouvelle technique pour étudier la mesure de chemins elle-même a été introduite. Elle permet de prouver que la mesure d'occupation normalisée se concentre asymptotiquement autour de l'ensemble des maximums de la formule.

Dans le présent article, nous prouvons que la fonctionnelle de Coulomb de la mesure d'occupation se concentre elle aussi, dans la topologie uniforme, autour de l'ensemble des fonctionnelles de Coulomb correspondant aux maximums. Ceci représente une étape décisive vers une preuve rigoureuse de la convergence des mesures d'occupation normalisées vers un mélange explicite des maximums, dérivée dans (Mean-field interaction of Brownian occupation measures, II: A rigorous construction of the Pekar process. Preprint). Nos méthodes reposent sur l'obtention de la continuité hölderienne de la fonctionnelle de Coulomb de la mesure d'occupation avec des probabilités de déviations exponentiellement petites, en invoquant la théorie des grandes déviations développée dans (Brownian occupations measures, compactness and large deviations (2014) Preprint) pour une certaine fonctionnelle, invariante par dćalage, des mesures d'occupation.

MSC: 60J65; 60J55; 60F10

Keywords: Gibbs measures; Interacting Brownian motions; Coulomb functional; Polaron problem

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Level lines of the Gaussian free field with general boundary data

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Abstract. We study the level lines of a Gaussian free field in a planar domain with general boundary data F. We show that the level lines exist as continuous curves under the assumption that F is regulated (i.e., admits finite left and right limits at every point), and satisfies certain inequalities. Moreover, these level lines are a.s. determined by the field. This allows us to define and study a generalization of the $\mathrm{SLE}_4(\underline{\rho})$ process, now with a continuum of force points. A crucial ingredient is a monotonicity property in terms of the boundary data which strengthens a result of Miller and Sheffield and is also of independent interest.

Résumé. Nous étudions les lignes de niveau d'un champ libre Gaussien dans un domaine D du plan, avec condition au bord générale donnée par une fonction F. Nous montrons que ces lignes existent comme courbes continues sous l'hypothèse que F est une fonction réglée (i.e., F admet une limite à droite et à gauche en tous points) et satisfait certaines inégalités. De plus, ces lignes de niveau sont presque sûrement determinées par le champ. Cela nous permet de définir et d'étudier une généralisation des courbes $\mathrm{SLE}_4(\underline{\rho})$ avec un continuum de points marqués. Un ingrédient essentiel de la preuve est une propriété de monotonicité en termes de la condition au bord, d'un intérêt indépendant, qui améliore un théorème de Miller et Sheffield.

MSC: 60K35; 60D05

Keywords: Gaussian free field; Level lines; Schramm Loewner evolution

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Spectral measures of factor of i.i.d. processes on vertex-transitive graphs

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Abstract. We prove that a measure on $[-d, d]$ is the spectral measure of a factor of i.i.d. process on a vertex-transitive infinite graph if and only if it is absolutely continuous with respect to the spectral measure of the graph. Moreover, we show that the set of spectral measures of factor of i.i.d. processes and that of $\bar{d}_2$-limits of factor of i.i.d. processes are the same.

Résumé. On prouve qu'une mesure est la mesure spectrale d'un processus facteur de i.i.d. sur un graphe infini nœud-transitive si et seulement si elle est absolument continue par rapport à la mesure spectrale de ce graphe. De plus, on montre que l'ensemble des mesures spectrales des processus facteur de i.i.d. et celui des $\bar{d}_2$-limites des processus facteur de i.i.d. sont les mêmes.

MSC: 60G15

Keywords: Factor of i.i.d.; Gaussian process; Spectral measure

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Infinite systems of competing Brownian particles

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Abstract. Consider a system of infinitely many Brownian particles on the real line. At any moment, these particles can be ranked from the bottom upward. Each particle moves as a Brownian motion with drift and diffusion coefficients depending on its current rank. The gaps between consecutive particles form the (infinite-dimensional) gap process. We find a stationary distribution for the gap process. We also show that if the initial value of the gap process is stochastically larger than this stationary distribution, this process converges back to this distribution as time goes to infinity. This continues the work by Pal and Pitman (*Ann. Appl. Probab.* **18** (2008) 2179–2207). Also, this includes infinite systems with asymmetric collisions, similar to the finite ones from Karatzas, Pal and Shkolnikov (*Ann. Inst. H. Poincare* **52** (2016) 323–354).

Résumé. Nous considérons un système infini de particules browniennes sur la droite réelle. À tout moment ces particules peuvent être ordonnées de façon croissante. Chaque particule se déplace suivant un mouvement brownien dont les coefficients de dérive et de diffusion dépendent du rang de la particule. Les distances entre les particules successives forment le processus (infini dimensionnel) des écarts. Nous trouvons une mesure stationnaire du processus des écarts. Nous montrons aussi que si la distribution initiale du processus des écarts domine stochastiquement la distribution stationnaire, le processus converge vers cette distribution en grand temps. Ce travail poursuit donc l'étude de Pal et Pitman (*Ann. Appl. Probab.* **18** (2008) 2179–2207). Il inclut aussi le cas des systèmes infinis avec collisions asymétriques, similaire au cas fini de Karatzas et Shkolnikov (*Ann. Inst. H. Poincare* **52** (2016) 323–354).

MSC: 60K35; 60J60; 60J65; 60H10; 91B26

Keywords: Reflected Brownian motion; Competing Brownian particles; Asymmetric collisions; Interacting particle systems; Weak convergence; Stochastic comparison; Triple collisions; Stationary distribution

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