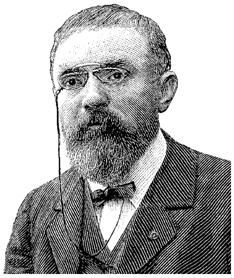

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Pathwise stochastic calculus with local times[1]

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Abstract. We study a notion of local time for a continuous path, defined as a limit of suitable discrete quantities along a general sequence of partitions of the time interval. Our approach subsumes other existing definitions and agrees with the usual (stochastic) local times a.s. for paths of a continuous semimartingale. We establish pathwise version of the Tanaka–Meyer, change of variables and change of time formulae. We provide equivalent conditions for existence of pathwise local time. Finally, we study in detail how the limiting objects, the quadratic variation and the local time, depend on the choice of partitions. In particular, we show that an arbitrary given non-decreasing process can be achieved a.s. by the pathwise quadratic variation of a standard Brownian motion for a suitable sequence of (random) partitions; however, such degenerate behaviour is excluded when the partitions are constructed from stopping times.

Résumé. Nous étudions la notion de temps local pour un processus continu, défini comme la limite de fonctions discrètes le long d'une suite de partitions de l'intervalle de temps. Notre approche englobe les définitions déjà existantes et coincide p.s. avec la définition (stochastique) usuelle des temps locaux pour les trajectoires de semimartingales continues. Nous établissons une version trajectorielle des formules de changement de variables, de temps et de Tanaka–Meyer ansi que plusieures conditions équivalentes pour l'existence du temps local trajectoire par trajectoire. Finalement, nous proposons une étude détaillée de la façon dont le processus limite, son temps local et sa variation quadratique, dépendent du choix de la suite de partitions. Nous montrons en particulier qu'un processus non-décroissant donné peut toujours être obtenu p.s. comme la variation quadratique d'un mouvement Brownien standard le long d'une suite appropriée de partitions (aléatoires). De tels comportements pathologiques sont cependant exclus quand les partitions sont construites à l'aide de temps d'arrêt.

MSC: 60G17; 60H05

Keywords: Pathwise local-time; Itô–Tanaka formula; Random partitions; Brownian variation

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Quantitative homogenization of degenerate random environments

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Abstract. We study discrete linear divergence-form operators with random coefficients, also known as random conductance models. We assume that the conductances are bounded, independent and stationary; the law of a conductance may depend on the orientation of the associated edge. We give a simple necessary and sufficient condition for the relaxation of the environment seen by the particle to be diffusive, in the sense of every polynomial moment. As a consequence, we derive polynomial moment estimates on the corrector.

Résumé. Nous étudions des opérateurs linéaires discrets sous forme divergence à coefficients aléatoires, aussi appelés modèles de conductances aléatoires. Nous supposons que les conductances sont bornées, indépendantes et stationnaires ; la loi d'une conductance peut dépendre de l'orientation de l'arête associée. Nous donnons une condition nécessaire et suffisante simple pour que la relaxation de l'environnement vu par la particule soit diffusive, au sens de tous les moments polynomiaux. Comme conséquence, nous estimons les moments polynomiaux du correcteur.

MSC: 35B27; 35K65; 60K37

Keywords: Quantitative homogenization; Environment viewed by the particle; Mixing of Markov chains; Corrector estimate

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Hydrostatics and dynamical large deviations for a reaction-diffusion model

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Abstract. We consider the superposition of a symmetric simple exclusion dynamics, speeded-up in time, with a spin-flip dynamics in a one-dimensional interval with periodic boundary conditions. We prove the hydrostatics and the dynamical large deviation principle.

Résumé. On considère la superposition de l'exclusion simple symétrique accélérée en temps avec une dynamique non-conservative sur un intervalle uni-dimensionnel avec des conditions périodiques. On démontre le comportement hydrostatique et un principe de grandes déviations dynamique.

MSC: Primary 82C22; secondary 60F10; 82C35

Keywords: Reaction-diffusion equations; Hydrostatics; Dynamical large deviations

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The excursion measure away from zero for spectrally negative Lévy processes

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Abstract. We provide a description of the excursion measure from a point for a spectrally negative Lévy process. The description is based in two main ingredients. The first is building a spectrally negative Lévy process conditioned to avoid zero and the study of its entrance law at zero. The latter is connected with both the excursion measure from zero of the process reflected in its infimum and reflected in its supremum. This leads us to establish a connection between the excursion measure from the state zero and the excursion measure from zero for the process reflected at the infimum and reflected at the supremum, respectively, which is the second main ingredient of our description.

Résumé. Dans ce travail on décrit la mesure d'excursions en dehors de zéro pour les processus de Lévy spectralement négatifs. Cette description utilise deux ingrédients principaux. Le premier consiste à construire un processus de Lévy spectralement négatif conditionné à éviter zéro et étudier sa loi d'entrée. Celle ci est intimement liée aux mesures d'excursions en dehors de zéro du processus réfléchi dans l'infimum passé, et, respectivement, dans le supremum passé. Ceci établit un lien naturel entre la mesure d'excursions en dehors de zéro et ces deux dernières mesures d'excursions. Ceci est le deuxième ingrédient de notre description.

MSC: 60G51; 60G17

Keywords: Lévy processes; Excursion theory from a point; Local times; Fluctuation theory

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Parametrix construction of the transition probability density of the solution to an SDE driven by α-stable noise[1]

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Abstract. Let $L := -a(x)(-\Delta)^{\alpha/2} + (b(x), \nabla)$, where $\alpha \in (0, 2)$, and $a : \mathbb{R}^d \to (0, \infty)$, $b : \mathbb{R}^d \to \mathbb{R}^d$. Under certain regularity assumptions on the coefficients a and b, we associate with the $C_\infty(\mathbb{R}^d)$-closure of $(L, C^2_\infty(\mathbb{R}^d))$ a Feller Markov process X, which possesses a transition probability density $p_t(x, y)$. To construct this transition probability density and to obtain the two-sided estimates on it, we develop a new version of the parametrix method, which even allows us to handle the case $0 < \alpha \le 1$ and $b \neq 0$, i.e. when the gradient part of the generator is not dominated by the jump part.

Résumé. Soit $L := -a(x)(-\Delta)^{\alpha/2} + (b(x), \nabla)$, avec $\alpha \in (0, 2)$, et $a : \mathbb{R}^d \to (0, \infty)$, $b : \mathbb{R}^d \to \mathbb{R}^d$. Sous certaines hypothèses de régularité des coefficients a et b, nous associons à la $C_\infty(\mathbb{R}^d)$-fermeture de $(L, C^2_\infty(\mathbb{R}^d))$ un processus de Markov fellerien X, possédant une densité de probabilité de transition $p_t(x, y)$. Afin de construire cette densité, et d'en obtenir des bornes supérieures et inférieures, nous développons une nouvelle version de la méthode parametrix, qui permet même de traiter le cas où $0 < \alpha \le 1$ et $b \neq 0$, c'est-à-dire quand la partie de gradient du générateur n'est pas dominée par la partie de saut.

MSC: Primary 60J35; secondary 60J75; 35S05; 35S10; 47G30

Keywords: Pseudo-differential operator; Generator of a Markov process; Transition probability density; Martingale problem; SDE; Levi's parametrix method

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Lace expansion for dummies

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Abstract. We show Green's function asymptotic upper bound for the two-point function of weakly self-avoiding walk in $d > 4$, revisiting a classic problem. Our proof relies on Banach algebras to analyse the lace-expansion fixed point equation and is simpler than previous approaches in that it avoids Fourier transforms.

Résumé. Nous montrons une domination asymptotique de la fonction à deux points de la marche faiblement auto-évitante en dimension $d > 4$ par la fonction de Green, revisitant ainsi un problème classique. Notre preuve s'appuie sur des techniques d'algèbres de Banach pour analyser le point fixe de l'équation de développement en lacets. Elle est plus simple que les approches précédentes car elle ne passe pas par la transformée de Fourier.

MSC: 84B41; 60K35

Keywords: Self-avoiding walk; Lace expansion; Banach algebra; Deconvolution; Edgeworth expansion

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A unified approach to *a priori* estimates for supersolutions of BSDEs in general filtrations

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Abstract. We provide a unified approach to *a priori* estimates for supersolutions of BSDEs in general filtrations, which may not be quasi left-continuous. Unlike the previous related approaches in simpler settings, our results do not only rely on a simple application of Itô's formula and classical estimates, but use crucially appropriate generalizations of deep estimates for supermartingales obtained by Meyer. As an example of application, we prove that reflected BSDEs are well-posed in a general framework which has not been covered so far in the existing literature.

Résumé. Nous proposons dans cet article une approche unifiée permettant l'obtention d'estimées a priori pour des sur-solutions d'EDSR adaptées à des filtrations générales, en particulier non nécessairement quasi-continues à gauche. Contrairement aux approches antérieures de ce problème dans des cadres plus simples, nos résultats ne sont pas la conséquence directe de la formule d'Itô et d'estimées classiques, mais dépendent de manière cruciale de versions appropriées à notre contexte d'estimées obtenues par Meyer pour des sur-martingales. Nous proposons entre autres une application de nos résultats à l'étude de l'existence et de l'unicité de solutions d'EDSR réfléchies dans un cadre général non-couvert par les résultats précédents dans la littérature.

MSC: 60H99

Keywords: Backward SDE; Supersolution; Doob–Meyer decomposition; Reflected backward SDE

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Level-set percolation for the Gaussian free field on a transient tree

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Abstract. We investigate level-set percolation of the Gaussian free field on transient trees, for instance on super-critical Galton–Watson trees conditioned on non-extinction. Recently developed Dynkin-type isomorphism theorems provide a comparison with percolation of the vacant set of random interlacements, which is more tractable in the case of trees. If h_* and u_* denote the respective (non-negative) critical values of level-set percolation of the Gaussian free field and of the vacant set of random interlacements, we show here that $h_* < \sqrt{2u_*}$ in a broad enough set-up, but provide an example where $0 = h_* = u_*$ occurs. We also obtain some sufficient conditions ensuring that $h_* > 0$.

Résumé. Nous étudions la percolation de niveau pour le champ libre gaussien sur des arbres transients, par exemple sur des arbres de Galton–Watson surcritiques conditionnés à survivre. Des théorèmes de type isomorphisme de Dynkin récemment obtenus offrent un outil de comparaison avec la percolation de l'ensemble vacant pour les entrelacs aléatoires, qui se trouve être plus simple à étudier dans le cas des arbres. Si h_* et u_* désignent les valeurs critiques respectives de la percolation de niveau du champ libre gaussien, et de l'ensemble vacant des entrelacs aléatoires, nous montrons dans un cadre assez général que $h_* < \sqrt{2u_*}$, mais présentons un exemple pour lequel on a les égalités $0 = h_* = u_*$. Nous obtenons aussi des conditions suffisantes qui impliquent que $h_* > 0$.

MSC: 60K35; 60G15; 60J10; 60J80; 82B43

Keywords: Level-set percolation; Gaussian free field; Transient trees; Random interlacements

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Support theorem for a singular SPDE: The case of gPAM

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Abstract. We consider the generalized parabolic Anderson equation (gPAM) in 2 dimensions with periodic boundary. This is an example of a singular semilinear stochastic partial differential equation in the subcritical regime, with (renormalized) solutions only recently understood via Hairer's regularity structures and, in some cases equivalently, paracontrollled distributions by Gubinelli, Imkeller and Perkowski. In the present paper we utilise the paracontrolled machinery and obtain a (Stroock–Varadhan) type support description for the law of gPAM. In the spirit of rough paths, the crucial step is to identify the support of the enhanced noise in a sufficiently fine topology. The renormalization is seen to affect the support description.

Résumé. On considère l'équation d'Anderson parabolique généralisée (gPAM) en dimension 2 avec condition au bord périodique. Cette équaton est une équation aux dérivées partielles stochastique singulière qui a été étudiée en utilisant les structures de régularité introduites par M. Hairer ou de manière équivalente par les distributions paracontrolées introduites M. Gubinelli, P. Imkeller et N. Perkpowski. Dans ce travail on se propose d'utiliser la notion de distribution paracontrolée afin d'obtenir le support (de type Stroock–Varadhan) de la loi de gPAM. Dans le même esprit que les chemins rugueux, le point crucial est d'identifier le support du bruit augmenté dans une topologie assez fine. On voit dans ce modèle que la renormalisation affecte le support de la solution.

MSC: 60H15

Keywords: Support theorem; Parabolic Anderson model; Paracontrolled distribution; SPDE

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The Bismut–Elworthy–Li formula for mean-field stochastic differential equations

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Abstract. We generalise the so-called Bismut–Elworthy–Li formula to a class of stochastic differential equations whose coefficients might depend on the law of the solution. We give some examples of where this formula can be applied to in the context of finance and the computation of Greeks and provide a simple but rather illustrative simulation experiment showing that the use of the Bismut–Elworthy–Li formula, also known as Malliavin method, is more efficient compared to the finite difference method.

Résumé. Nous généralisons la formule dite Bismut–Elworthy–Li à une classe d'équations différentielles stochastiques dont les coefficients pourrait dépendre de la loi de la solution. Nous donnons quelques exemples où cette formule peut être appliquée dans le contexte de la finance et le calcul des Grecs et de fournir une expérience de simulation simple mais significative montrant que l'utilisation de la formule Bismut–Elworthy–Li, également connu comme méthode de Malliavin, est plus efficace que la méthode des différences finies.

MSC: 60H07; 60H10; 60J60; 65C05

Keywords: Bismut–Elworthy–Li formula; Malliavin calculus; Monte Carlo methods; Stochastic differential equations; Integration by parts formulas

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On sensitivity of uniform mixing times[1]

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Abstract. We show that the order of the L_∞-mixing time of simple random walks on a sequence of uniformly bounded degree graphs of size n may increase by an optimal factor of $\Theta(\log\log n)$ as a result of a bounded perturbation of the edge weights. This answers a question and a conjecture of Kozma.

Résumé. Nous montrons que le temps de mélange pour la distance L_∞ d'une marche aléatoire sur une suite de graphe de taille n et de degré uniformément borné peut être multiplié par un facteur d'ordre $\log\log n$ (optimal) en perturbant le poids des arrêtes du graphe de manière uniformément bornée. Ceci résout une question et une conjecture de Kozma.

MSC: 60J10

Keywords: Sensitivity; Mxing-time; Sensitivity of mixing times; Hitting times

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Liouville Brownian motion and thick points of the Gaussian free field

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Abstract. We find a lower bound for the Hausdorff dimension of times that a Liouville Brownian motion spends in thick points of the Gaussian Free Field, as a function of the thickness parameter. This completes a conjecture in Berestycki (*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 947–964), where the corresponding upper bound was shown, thereby charactarising the multifractal spectrum of LBM.

In the course of the proof, we obtain estimates on the (Euclidean) local diffusivity exponent, which depends strongly on the thickness of the starting point. For a Liouville typical point, it is $1/(2-\frac{\gamma^2}{2})$. In particular, for $\gamma > \sqrt{2}$, the path is Lebesgue – almost everywhere differentiable, almost surely. However, depending on the thickness of the point it can be both locally sub- and super-diffusive.

Résumé. Nous trouvons une limite inférieure pour la dimension Hausdorff de l'ensemble des temps qu'un mouvement brownien de Liouville (LBM) passe dans les points épais du GFF, en fonction du paramètre d'épaisseur. Ceci démontre une conjecture de Berestycki (*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 947–964), où la limite supé-rieure correspondante était obtenue, caractérisant le spectre multifractal du LBM.

Au cours de la preuve, nous obtenons des estimations sur l'exposant local de diffusivité (euclidien), qui dépend fortement de l'épaisseur du point de départ. Pour un point Liouville typique, nous trouvons $1/(2-\frac{\gamma^2}{2})$. Notamment, pour $\gamma > \sqrt{2}$, la trajectoire est Lebesgue – presque partout dérivable, presque sûrement.

MSC: 60J60; 60D05; 28A80; 81T40

Keywords: Liouville quantum gravity; Liouville Brownian motion; Gaussian multiplicative chaos

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The sharp interface limit for the stochastic Cahn–Hilliard equation

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Abstract. We study the ε-dependent two and three dimensional stochastic Cahn–Hilliard equation in the sharp interface limit $\varepsilon \to 0$. The parameter ε is positive and measures the width of transition layers generated during phase separation. We also couple the noise strength to this parameter. Using formal asymptotic expansions, we identify the limit. In the right scaling, our results indicate that the stochastic Cahn–Hilliard equation converge to a Hele-Shaw problem with stochastic forcing on the curvature equation. In the case when the noise is sufficiently small, we rigorously prove that the limit is a deterministic Hele-Shaw problem. Finally, we discuss which estimates are necessary in order to extend the rigorous result to larger noise strength.

Résumé. Nous étudions l'équation de Cahn–Hilliard stochastique dépendante en ε, posée en dimensions deux et trois, dans la limite de l'interface nette $\varepsilon \to 0$. Le paramètre ε est positif et mesure la largeur de couches de transition générées pendant la séparation de phase. Nous couplons aussi la puissance de bruit à ce paramètre. Nous déterminons la limite à l'aide de séries asymptotiques formelles. Dans l'échelle appropriée, nos résultats indiquent que l' équation de Cahn–Hilliard stochastique converge vers un problème de Hele-Shaw avec un forçage stochastique dans l'équation de la courbure. Dans le cas d'un bruit suffisamment petit, nous prouvons rigoureusement que la limite est un problème Hele-Shaw déterministe. Finalement, nous discutons des estimations nécessaires afin d'étendre le résultat rigoureux en présence de bruit d'une intensité plus grande.

MSC: 35K55; 35K40; 60H30; 60H15

Keywords: Multi-dimensional stochastic Cahn–Hilliard equation; Additive Noise; stochastic sharp interface limit; Hele Shaw problem; Interface motion

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Tube estimates for diffusion processes under a weak Hörmander condition

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Abstract. We consider a diffusion process under a local weak Hörmander condition on the coefficients. We find Gaussian estimates for the density in short time and exponential lower and upper bounds for the probability that the diffusion remains in a small tube around a deterministic trajectory (skeleton path). These bounds depend explicitly on the radius of the tube and on the energy of the skeleton path. We use a norm which reflects the non-isotropic structure of the problem, meaning that the diffusion propagates in $\mathbb{R}^2$ with different speeds in the directions σ and $[\sigma, b]$. We establish a connection between this norm and the standard control distance.

Résumé. On considère une diffusion dont les coefficients satisfont une condition d'Hörmander faible locale. On obtient des estimées gaussiennes de la densité en temps court et des bornes inférieures et supérieures exponentielles pour la probabilité que la diffusion reste dans un petit tube autour d'une trajectoire déterministe (« squelette »). Ces bornes dépendent explicitement du rayon du tube et de l'énergie du squelette. On utilise une norme qui prend en compte la structure non isotrope du problème, dans le sens où la diffusion se propage dans $\mathbb{R}^2$ avec des vitesses différentes dans la direction de σ et $[\sigma, b]$. On établit un lien entre cette norme et la distance de contrôle standard.

MSC: Primary 60H30; secondary 60H07

Keywords: Density estimates; Tube estimates; Hypoellipticity; Hörmander condition; Malliavin Calculus

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Deep factorisation of the stable process II: Potentials and applications

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Abstract. Here, we propose a different perspective of the *deep factorisation* in (*Electron. J. Probab.* **21** (2016) Paper No. 23, 28) based on determining potentials. Indeed, we factorise the inverse of the MAP-exponent associated to a stable process via the Lamperti–Kiu transform. Here our factorisation is completely independent from the derivation in (*Electron. J. Probab.* **21** (2016) Paper No. 23, 28), moreover there is no clear way to invert the factors in (*Electron. J. Probab.* **21** (2016) Paper No. 23, 28) to derive our results. Our method gives direct access to the potential densities of the ascending and descending ladder MAP of the Lamperti-stable MAP in closed form.

In the spirit of the interplay between the classical Wiener–Hopf factorisation and the fluctuation theory of the underlying Lévy process, our analysis will produce a collection of new results for stable processes. We give an identity for the law of the point of closest reach to the origin for a stable process with index $\alpha \in (0,1)$ as well as an identity for the the law of the point of furthest reach before absorption at the origin for a stable process with index $\alpha \in (1,2)$. Moreover, we show how the deep factorisation allows us to compute explicitly the limiting distribution of stable processes multiplicatively reflected in such a way that it remains in the strip $[-1,1]$.

Résumé. On propose une perspective différente à la factorisation du type Wiener–Hopf, dite *deep factorisation* en anglais, obtenue dans (*Electron. J. Probab.* **21** (2016) Paper No. 23, 28), qui consiste en une factorisation de la matrice exposant caractéristique du processus de Markov additif (MAP) associé à un processus stable via la transformation de Lamperti–Kiu. Ici on décrit les mesures potentiel, au lieu de la mesure de Lévy, la dérive et le terme de mort. Les méthodes utilisés ici sont complètement différentes de celles de (*Electron. J. Probab.* **21** (2016) Paper No. 23, 28), ceci est dû, d'un part, au fait qu'il n'y a pas de méthode claire pour inverser les facteurs apparaissant dans cette référence, et, d'autre part, nos méthodes nous permettent d'obtenir explicitement les mesures potentiel des processus d'échelle croissant et décroissant.

D'une manière analogue à la conjonction entre la factorisation de Wiener–Hopf et la théorie des fluctuations des processus de Lévy, notre analyse nous permet de produire une collection de résultats nouveaux pour les processus stables. On donne une identité pour la loi du point le plus proche de l'origine pour un processus stable d'indice $\alpha \in (0,1)$, ainsi qu'une identité pour la loi du point le plus lointain avant le premier temps d'atteinte de zéro pour un processus stable d'indice $\alpha \in (1,2)$. De plus, nos résultats nous permettent de calculer explicitement la limite en loi du processus stable réfléchi multiplicativement de telle sorte à rester dans l'intervalle $[-1,1]$.

MSC: 60G18; 60G52; 60G51

Keywords: Stable processes; Self-similar Markov processes; Wiener–Hopf factorisation; Radial reflection

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Quenched invariance principle for random walk in time-dependent balanced random environment

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Abstract. We prove a quenched central limit theorem for balanced random walks in time-dependent ergodic random environments which is not necessarily nearest-neighbor. We assume that the environment satisfies appropriate ergodicity and ellipticity conditions. The proof is based on the use of a maximum principle for parabolic difference operators.

Résumé. Nous démontrons un théorème de loi limite centrale presque sûr pour des marches aléatoires équilibrées non nécessairement aux plus proches voisins, dans un milieu aléatoire ergodique. Nous supposons que l'environnement satisfait des conditions d'ergodicité et d'ellipicité appropriées. Notre preuve est basée sur un principe du maximum pour des opérateurs aux différences paraboliques.

MSC: 35K10; 60K37; 82D30

Keywords: Random walk in random environment; Maximum principle; Quenched central limit theorem

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Stein's method for positively associated random variables with applications to the Ising and voter models, bond percolation, and contact process

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Abstract. We provide nonasymptotic L^1 bounds to the normal for four well-known models in statistical physics and particle systems in $\mathbb{Z}^d$; the ferromagnetic nearest-neighbor Ising model, the supercritical bond percolation model, the voter model and the contact process. In the Ising model, we obtain an L^1 distance bound between the total magnetization and the normal distribution at any temperature when the magnetic moment parameter is nonzero, and when the inverse temperature is below critical and the magnetic moment parameter is zero. In the percolation model we obtain such a bound for the total number of points in a finite region belonging to an infinite cluster in dimensions $d \ge 2$, in the voter model for the occupation time of the origin in dimensions $d \ge 7$, and for finite time integrals of nonconstant increasing cylindrical functions evaluated on the one dimensional supercritical contact process started in its unique invariant distribution.

The tool developed for these purposes is a version of Stein's method adapted to positively associated random variables. In one dimension, letting $\boldsymbol{\xi} = (\xi_1, \dots, \xi_m)$ be a positively associated mean zero random vector with components that obey the bound $|\xi_i| \le B, i = 1, \dots, m$, and whose sum $W = \sum_{i=1}^m \xi_i$ has variance 1, it holds that

$$d_1\big(\mathcal{L}(W), \mathcal{L}(Z)\big) \le 5B + \sqrt{\frac{8}{\pi}} \sum_{i \ne j} \mathbb{E}[\xi_i \xi_j],$$

where Z has the standard normal distribution and $d_1(\cdot, \cdot)$ is the L^1 metric. Our methods apply in the multidimensional case with the L^1 metric replaced by a smooth function metric.

Résumé. Nous obtenons des bornes L^1 non asymptotiques montrant l'approximation gaussienne pour quatre modèles classiques de physique statistique et de systèmes de particules dans $\mathbb{Z}^d$: le modèle d'Ising ferromagnétique au plus proche voisin, la percolation par arêtes sur-critique, le modèle du votant et le processus de contact. Pour le modèle d'Ising, nous obtenons une borne en distance L^1 entre la magnétisation totale et la loi normale, soit à toute température lorsque le paramètre de moment magnétique est non nul, soit, dans la phase sous-critique, si ce paramètre est nul. Pour le modèle de percolation, nous obtenons une telle borne pour le nombre total de points dans une région finie qui appartiennent à un cluster infini, en dimensions $d \ge 2$. Pour le modèle du votant nous considérons le temps d'occupation à l'origine en dimensions $d \ge 7$, et pour le processus de contact, nous nous intéressons à des intégrales temporelles à horizon fini de fonctions cylindriques croissantes, évaluées en le processus de contact unidimensionnel sous l'unique mesure invariante.

L'outil que nous développons à cette fin est une version de la méthode de Stein adaptée à des variables aléatoires associées positivement. En dimension 1, si $\boldsymbol{\xi} = (\xi_1, \dots, \xi_m)$ désigne un vecteur aléatoire centré et positivement associé dont les composantes satisfont la borne $|\xi_i| \le B, i = 1, \dots, m$, et dont la somme $W = \sum_{i=1}^m \xi_i$ est de variance 1, alors

$$d_1\big(\mathcal{L}(W), \mathcal{L}(Z)\big) \le 5B + \sqrt{\frac{8}{\pi}} \sum_{i \ne j} \mathbb{E}[\xi_i \xi_j]$$

où Z est une variable aléatoire gaussienne centrée réduite et $d_1(\cdot,\cdot)$ est la distance L^1. Nos méthodes s'appliquent au cas multidimensionnel si la distance L^1 est remplacée par une distance plus faible.

MSC: 60F05; 82B30; 60G60

Keywords: Random fields; Block dependence; Correlation inequality; Positive dependence

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A new computation of the critical point for the planar random-cluster model with $q \geq 1$

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Abstract. We present a new computation of the critical value of the random-cluster model with cluster weight $q \geq 1$ on $\mathbb{Z}^2$. This provides an alternative approach to the result in (*Probab. Theory Related Fields* **153** (2012) 511–542). We believe that this approach has several advantages. First, most of the proof can easily be extended to other planar graphs with sufficient symmetries. Furthermore, it invokes RSW-type arguments which are not based on self-duality. And finally, it contains a new way of applying sharp threshold results which avoid the use of symmetric events and periodic boundary conditions. Some of the new methods presented in this paper have a larger scope than the planar random-cluster model, and may be useful to investigate sharp threshold phenomena for more general dependent percolation processes in arbitrary dimensions.

Résumé. Nous proposons une nouvelle preuve du fait que le point critique du modèle de percolation de Fortuin–Kasteleyn sur le réseau carré vaut $\sqrt{q}/(1+\sqrt{q})$ lorsque $q \geq 1$. Cette preuve est une alternative à la stratégie implémentée dans (*Probab. Theory Related Fields* **153** (2012) 511–542). Cette approche a plusieurs avantages. Tout d'abord, la grande majorité des arguments peuvent être généralisés aux autres graphes planaires (ayant suffisamment de symétries). De plus, elle n'invoque pas d'argument de type RSW basés sur l'auto-dualité du modèle. Enfin, elle repose sur une nouvelle façon d'appliquer les théorèmes de seuil qui n'utilise pas la symmétrie des évènements de croisement et les conditions de bord périodiques. Certaines de ces nouvelles méthodes ont un champs d'application qui dépasse largement le cas du modèle de percolation de Fortuin–Kasteleyn et pourrait être utile pour prouver la décroissance exponentielles d'autres modèles de percolation dépendante.

MSC: 82B20; 60K35; 82B26; 82B43

Keywords: Phase transition; Random-cluster model; Potts model; Critical point; Sharp phase transition

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Joint exceedances of random products[1]

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Abstract. We analyze the joint extremal behavior of n random products of the form $\prod_{j=1}^m X_j^{a_{ij}}$, $1 \le i \le n$, for non-negative, independent regularly varying random variables $X_1, \dots, X_m$ and general coefficients $a_{ij} \in \mathbb{R}$. Products of this form appear for example if one observes a linear time series with gamma type innovations at n points in time. We combine arguments of linear optimization and a generalized concept of regular variation on cones to show that the asymptotic behavior of joint exceedance probabilities of these products is determined by the solution of a linear program related to the matrix $\mathbf{A} = (a_{ij})$.

Résumé. Nous étudions le comportement extrémal multivarié de n produits aléatoires de la forme $\prod_{j=1}^m X_j^{a_{ij}}$, $1 \le i \le n$, pour des variables aléatoires positives, indépendantes et identiquement distribuées $X_1, \dots, X_m$ et des coefficients $a_{ij} \in \mathbb{R}$ quelconques. De tels produits apparaissent notamment lorsqu'on observe un échantillon de taille n issu d'une série temporelle linéaire dont les innovations sont de type gamma. En combinant des arguments d'optimisation linéaire et le concept de variations régulières étendu à des cônes, nous montrons que le comportement asymptotique des probabilités de dépassement de seuil multiples pour de tels produits est déterminé par la solution d'un problème de programmation linéaire associé à la matrice $\mathbf{A} = (a_{ij})$.

MSC: Primary 60G70; secondary 60B10; 28A33

Keywords: Extreme value theory; Linear programming; M-convergence; Random products; Regular variation

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Range and critical generations of a random walk on Galton–Watson trees

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Abstract. In this paper we consider a random walk in random environment on a tree and focus on the boundary case for the underlying branching potential. We study the range R_n of this walk up to time n and obtain its correct asymptotic in probability which is of order $n/\log n$. This result is a consequence of the asymptotical behavior of the number of visited sites at generations of order $(\log n)^2$, which turn out to be the most visited generations. Our proof which involves a quenched analysis gives a description of the typical environments responsible for the behavior of R_n.

Résumé. Dans cet article nous considérons une marche aléatoire en milieu aléatoire sur un arbre, en nous concentrant sur le cas frontière du potentiel branchant sous-jacent. Nous étudions le nombre de points visités par cette marche avant l'instant n, R_n, et obtenons son comportement asymptotique en probabilité qui est de l'ordre de $n/\log n$. Ce résultat est une conséquence du comportement asymptotique du nombre de points visités par la marche au niveau des générations critiques, c'est à dire en $(\log n)^2$. La preuve permet une description des environnements typiques qui conduisent au comportement de R_n.

MSC: 60K37; 60J80; 60G50

Keywords: Random walks; Range; Random environment; Branching random walk

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Product blocking measures and a particle system proof of the Jacobi triple product

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Abstract. We review product form blocking measures in the general framework of nearest neighbor asymmetric one dimensional misanthrope processes. This class includes exclusion, zero range, bricklayers, and many other models. We characterize the cases when such measures exist in infinite volume, and when finite boundaries need to be added. By looking at inter-particle distances, we extend the construction to some 0-1 valued particle systems e.g., q-ASEP and the Katz-Lebowitz-Spohn process, even outside the misanthrope class. Along the way we provide a full ergodic decomposition of the product blocking measure into components that are characterized by a non-trivial conserved quantity. Substituting in simple exclusion and zero range has an interesting consequence: a purely probabilistic proof of the Jacobi triple product, a famous identity that mostly occurs in number theory and the combinatorics of partitions. Surprisingly, here it follows very naturally from the exclusion – zero range correspondence.

Résumé. Nous passons en revue l'existence de mesures bloquantes de forme produit dans le contexte général des processus misanthropes asymétriques, au plus proche voisin, en dimension 1. Cela recouvre les modèles dits d'exclusion, de « zero range », de « bricklayers », et bien d'autres. Nous caractérisons les cas où de telles mesures existent en volume infini, et les cas où des frontières doivent être ajoutées. En nous intéressant aux distances entre particules, nous étendons la construction à certains systèmes de particules à valeurs dans {0, 1} qui ne sont pas misanthropes, tels que le q-ASEP et le processus de Katz-Lebowitz-Spohn. Au passage, nous obtenons une décomposition ergodique des mesures bloquantes de forme produit en des composantes caractérisées par une quantité conservée non triviale. Une conséquence intéressante, dans le cas de l'exclusion simple et du processus « zero range », est que cela donne une preuve purement probabiliste du triple produit de Jacobi, une identité célèbre qui intervient en théorie des nombres et dans la combinatoire des partitions. De façon surprenante, dans notre contexte, cette formule découle très naturellement de la correspondance entre l'exclusion et le processus « zero range ».

MSC: 60K35; 82C41

Keywords: Blocking measure; Interacting particle systems; Reversible stationary distribution; Jacobi triple product

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Limit theorems for affine Markov walks conditioned to stay positive

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Abstract. Consider the real Markov walk $S_n = X_1 + \cdots + X_n$ with increments $(X_n)_{n\geq 1}$ defined by a stochastic recursion starting at $X_0 = x$. For a starting point $y > 0$, denote by τ_y the exit time of the process $(y + S_n)_{n\geq 1}$ from the positive part of the real line. We investigate the asymptotic behaviour of the probability of the event $\tau_y \geq n$ and of the conditional law of $y + S_n$ given $\tau_y \geq n$ as $n \to +\infty$.

Résumé. On considère une marche Markovienne réelle $S_n = X_1 + \cdots + X_n$ dont les accroissements $(X_n)_{n\geq 1}$ sont définis par une récursion stochastique partant de $X_0 = x$. Pour un point de départ $y > 0$, on note par τ_y le temps de sortie du processus $(y + S_n)_{n\geq 1}$ de la partie positive de la droite des réels. On s'intéresse au comportement asymptotique de la probabilité de l'évènement $\tau_y \geq n$ ainsi qu'à la loi conditionnelle de $y + S_n$ sachant $\tau_y \geq n$ quand $n \to +\infty$.

MSC: Primary 60J05; 60J50; 60G50; secondary 60J70; 60G42

Keywords: Exit time; Stochastic recursion; Markov chains; Harmonic function

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