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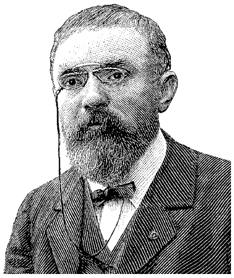

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The infinite Atlas process: Convergence to equilibrium

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Abstract. The semi-infinite Atlas process is a one-dimensional system of Brownian particles, where only the leftmost particle gets a unit drift to the right. Its particle spacing process has infinitely many stationary measures, with one distinguished translation invariant reversible measure. We show that the latter is attractive for a large class of initial configurations of slowly growing (or bounded) particle densities. Key to our proof is a new estimate on the rate of convergence to equilibrium for the particle spacing in a triangular array of finite, large size systems.

Résumé. Let modèle de Atlas demi-infini est un système unidimensional des particules Browniens, où seulement la particule plus à gauche a une vitesse positive. Le processus d'incréments correspondant a une infinité des mesures invariantes, avec une mesure distinguée, reversible et invariante par translations. On montre que cette mesure attire une grande classe des configurations initiales avec densité bornée ou à croissance modérée. Central à notre preuve est une nouvelle estimation de la vitesse de convergence vers l'équilibre du processus d'incréments dans un tableau triangulaire de systèmes finis et de grande taille.

MSC: 60K35; 82C22; 60F17; 60J60

Keywords: Interacting particles; Reflecting Brownian motions; Non-equilibrium hydrodynamics; Infinite Atlas process

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Exponential functionals of spectrally one-sided Lévy processes conditioned to stay positive

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Abstract. We study the properties of the exponential functional $\int_0^{+\infty} e^{-X^{\uparrow}(t)}\,dt$ where $X^{\uparrow}$ is a spectrally one-sided Lévy process conditioned to stay positive. In particular, we study finiteness, self-decomposability, existence of finite exponential moments, asymptotic tail at 0 and smoothness of the density.

Résumé. On étudie les propriétés de la fonctionnelle exponentielle $\int_0^{+\infty} e^{-X^{\uparrow}(t)}\,dt$ où $X^{\uparrow}$ est un processus de Lévy spectralement positifs ou négatifs conditionné à rester positif. On étudie en particulier la finitude, l'auto-décomposabilité, l'existence de moments exponentiels finis, la queue de distribution en 0 et la régularité de la densité.

MSC: 60G51

Keywords: Lévy processes conditioned to stay positive; Exponential functionals; Self-decomposable distributions

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Location of the spectrum of Kronecker random matrices

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Abstract. For a general class of large non-Hermitian random block matrices $\boldsymbol{X}$ we prove that there are no eigenvalues away from a deterministic set with very high probability. This set is obtained from the Dyson equation of the Hermitization of $\boldsymbol{X}$ as the self-consistent approximation of the pseudospectrum. We demonstrate that the analysis of the matrix Dyson equation from (*Probab. Theory Related Fields* (2018)) offers a unified treatment of many structured matrix ensembles.

Résumé. Pour une classe générale de grandes matrices aléatoires par blocs non hermitiennes $\boldsymbol{X}$, nous montrons qu'avec très grande probabilité, il n'y a pas de valeurs propres en dehors d'un ensemble déterministe. Cet ensemble est obtenu à partir de l'équation de Dyson pour l'hermitisation de $\boldsymbol{X}$ comme l'approximation auto-cohérente du pseudo-spectre. Nous démontrons que l'analyse de l'équation de Dyson provenant de (*Probab. Theory Related Fields* (2018)) permet d'étudier de façon unifiée de nombreux ensembles de matrices structurées.

MSC: 60B20; 15B52

Keywords: Outliers; Block matrices; Local law; Non-Hermitian random matrix; Self-consistent pseudospectrum

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Heat kernel estimates for anomalous heavy-tailed random walks

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Abstract. Sub-Gaussian estimates for the natural random walk is typical of many regular fractal graphs. Subordination shows that there exist heavy tailed jump processes whose jump indices are greater than or equal to two. However, the existing machinery used to prove heat kernel bounds for such heavy tailed random walks fail in this case. In this work we extend Davies' perturbation method to obtain transition probability bounds for these anomalous heavy tailed random walks. We prove global upper and lower bounds on the transition probability density that are sharp up to constants. An important feature of our work is that the methods we develop are robust to small perturbations of the symmetric jump kernel.

Résumé. Pour de nombreux graphes réguliers de type fractal, la marche aléatoire simple satisfait des estimations de type sous-Gaussiennes. La technique de la subordination montre alors qu'il existe des processus de saut à queue lourde dont l'indice des sauts est supérieur ou égale a 2. Pour de tels processus, les techniques usuelles pour les estimations loin de la diagonale ne fonctionnent pas. Nous étendons la célèbre méthode de Davies dans le cas de ces processus à sauts « anormaux. » Nous obtenons des bornes supérieures et inférieures précises sur le noyau de transition par des méthodes qui sont stables sous de petites perturbations des sauts.

MSC: 60J10; 60J75

Keywords: Heavy-tailed random walks; Jump process; Fractals; Heat kernel

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On the exit time and stochastic homogenization of isotropic diffusions in large domains

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Abstract. Stochastic homogenization is achieved for a class of elliptic and parabolic equations describing the lifetime, in large domains, of stationary diffusion processes in random environment which are small, statistically isotropic perturbations of Brownian motion in dimension at least three. Furthermore, the homogenization is shown to occur with an algebraic rate. Such processes were first considered in the continuous setting by Sznitman and Zeitouni (*Invent. Math.* **164** (2006) 455–567), upon whose results the present work relies strongly.

Résumé. On effectue l'homogénéisation stochastique d'une certaine classe d'équations elliptiques et paraboliques. Ces équations décrivent la durée de vie, dans des domaines grands, de processus de diffusion stationnaire en environnement aléatoire qui sont des petites perturbations statistiquement isotropes du mouvement brownien, en dimension au moins trois. On démontre que l'homogénéisation a lieu à vitesse algébrique. De tels processus ont été étudiés dans un cadre continu en premier lieu par Snitzman et Zeitouni (*Invent. Math.* **164** (2006) 455–567), sur les résultats desquels le présent travail s'appuie fortement.

MSC: 35B27; 35J25; 35K20; 60H25; 60J60; 60K37

Keywords: Diffusion processes in random environment; Stochastic homogenization; Elliptic boundary-value problem; Parabolic boundary-value problem

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On the spectral gap of spherical spin glass dynamics

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Abstract. We consider the time to equilibrium for the Langevin dynamics of the spherical p-spin glass model of system size N. We show that the log-Sobolev constant and spectral gap are order 1 at sufficiently high temperatures whereas the spectral gap decays exponentially in N at sufficiently low temperatures. These verify the existence of a dynamical high temperature phase and a dynamical glass phase at the level of the spectral gap. Key to these results are the understanding of the extremal process and restricted free energy of Subag–Zeitouni and Subag.

Résumé. Nous considérons le temps d'atteinte de l'équilibre pour la dynamique de Langevin du modèle de verre de p-spin sphérique de taille N. Nous montrons que la constante de log-Sobolev et le trou spectral sont d'ordre 1 à température suffisamment grande, alors que le trou spectral décroit exponentiellement en N à température suffisamment basse. Ceci confirme l'existence d'une phase dynamique de haute température et d'une phase dynamique verre concernant le trou spectral. Les arguments clés de ces résultats sont la compréhension du processus extrémal et de l'énergie libre restreinte de Subag–Zeitouni et Subag.

MSC: Primary 82C44; secondary 82D30; 82C26; 82B44

Keywords: Spin glass; Langevin dynamics; Glassy dynamics; Spectral gap; Log-Sobolev

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Existence of Stein kernels under a spectral gap, and discrepancy bounds

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Abstract. We establish existence of Stein kernels for probability measures on $\mathbb{R}^d$ satisfying a Poincaré inequality, and obtain bounds on the Stein discrepancy of such measures. Applications to quantitative central limit theorems are discussed, including a new central limit theorem in the Kantorovich–Wasserstein distance W_2 with optimal rate and dependence on the dimension. As a byproduct, we obtain a stable version of an estimate of the Poincaré constant of probability measures under a second moment constraint. The results extend more generally to the setting of converse weighted Poincaré inequalities. The proof is based on simple arguments of functional analysis.

Further, we establish two general properties enjoyed by the Stein discrepancy, holding whenever a Stein kernel exists: Stein discrepancy is strictly decreasing along the CLT, and it controls the third moments of a random vector.

Résumé. Nous prouvons l'existence de noyaux de Stein pour les mesures de probabilités sur $\mathbb{R}^d$ satisfaisant une inégalité de Poincaré, et obtenons des bornes sur la discrépance de Stein de telles mesures. Des applications au théorème central limite sont données, dont une nouvelle borne sur la vitesse de convergence en distance de Kantorovitch–Wasserstein W_2 avec un taux et une dépendance en la dimension optimales. Comme corollaire, nous obtenons une version quantitative d'une borne sur la constante de Poincaré de mesures de probabilités satisfaisant une contrainte sur le moment d'ordre 2. Les résultats sont plus généralement valides dans le cadre de mesures vérifiant une inégalité de Poincaré à poids inversée. La preuve est basée sur des arguments simples d'analyse fonctionnelle.

De plus, nous démontrons deux propriétés générales sur la discrépance de Stein, valide dès lors qu'un noyau de Stein existe : la discrépance de Stein est strictement décroissante le long du TCL, et elle contrôle le moment d'ordre 3 d'un vecteur aléatoire.

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Keywords: Stein kernels; Quantitative central limit theorems; Poincaré inequalities

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Hausdorff dimension of the scaling limit of loop-erased random walk in three dimensions

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Abstract. Let M_n be the length (number of steps) of the loop-erasure of a simple random walk up to the first exit from a ball of radius n centered at its starting point. It is shown in (*Ann. Probab.* **46** (2) (2018) 687–774) that there exists $\beta \in (1, \frac{5}{3}]$ such that $E(M_n)$ is of order n^{β} in 3 dimensions. In the present article, we show that the Hausdorff dimension of the scaling limit of the loop-erased random walk in 3 dimensions is equal to β almost surely.

Résumé. Soit M_n la longueur (nombre de pas) d'une marche aléatoire simple à boucles effacées considérée jusqu'à la première sortie d'une boule de rayon n centrée en son point de départ. Il est démontré dans (*Ann. Probab.* **46** (2) (2018) 687–774) qu'il existe $\beta \in (1, \frac{5}{3}]$ tel que $E(M_n)$ est d'ordre n^{β} en dimension 3. Dans le présent article, nous montrons que la dimension de Hausdorff de la limite d'échelle de la marche aléatoire effacée en dimension 3 est égale à β presque sûrement.

MSC: 82B41; 60G50

Keywords: Loop-erased random walk; Scaling limit; Hausdorff dimension

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Condensation of a self-attracting random walk

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Abstract. We introduce a Gibbs measure on nearest-neighbour paths of length t in the Euclidean d-dimensional lattice, where each path is penalised by a factor proportional to the size of its boundary and an inverse temperature β. We prove that, for all $\beta > 0$, the random walk condensates to a set of diameter $(t/\beta)^{1/3}$ in dimension $d = 2$, up to a multiplicative constant. In all dimensions $d \geq 3$, we also prove that the volume is bounded above by $(t/\beta)^{d/(d+1)}$ and the diameter is bounded below by $(t/\beta)^{1/(d+1)}$. Similar results hold for a random walk conditioned to have local time greater than β everywhere in its range when β is larger than some explicit constant, which in dimension two is the logarithm of the connective constant.

Résumé. Nous introduisons une mesure de Gibbs sur les chemins de longueur t dans le réseau Euclidien de dimension d, telle qu'un chemin donné est penalisé par un facteur proportionnel à la taille de sa frontière et l'inverse d'une température $\beta > 0$. Nous montrons qu'en dimension $d = 2$, la marche aléatoire se condense dans un ensemble de diamètre $(t/\beta)^{1/3}$ à une constante multiplicative près. En dimensions $d \geq 3$, nous montrons que la marche occupe un volume inférieur à $(t/\beta)^{d/(d+1)}$ et son diamètre est au moins $(t/\beta)^{1/(d+1)}$. Des résultats similaires sont obtenus pour une marche aléatoire conditionnée à avoir un temps local supérieur à β en chaque point visité, pourvu que β soit supérieur à une constante explicite qui en deux dimensions est égale au logarithme de la constante de connectivité.

MSC: 60K35; 60J27; 60F10

Keywords: Gibbs measure; Condensation; Self-attractive random walk; Wulff crystal; Large deviations; Donsker–Varadhan principle

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The speed of biased random walk among random conductances

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Abstract. We consider biased random walk among iid, uniformly elliptic conductances on $\mathbb{Z}^d$, and investigate the monotonicity of the velocity as a function of the bias. It is not hard to see that if the bias is large enough, the velocity is increasing as a function of the bias. Our main result is that if the disorder is small, i.e. all the conductances are close enough to each other, the velocity is always strictly increasing as a function of the bias, see Theorem 1.1. A crucial ingredient of the proof is a formula for the derivative of the velocity, which can be written as a covariance, see Theorem 1.3: it follows along the lines of the proof of the Einstein relation in (*Ann. Probab.* **45** (4) (2017) 2533–2567). On the other hand, we give a counterexample showing that for iid, uniformly elliptic conductances, the velocity is not always increasing as a function of the bias. More precisely, if $d = 2$ and if the conductances take the values 1 (with probability p) and κ (with probability $1 - p$) and p is close enough to 1 and κ small enough, the velocity is *not* increasing as a function of the bias, see Theorem 1.2.

Résumé. Nous étudions des marches aléatoires biaisées dans un milieu aléatoire donné par des poids iid sur les arêtes de $\mathbb{Z}^d$. Les poids sont bornés au-dessus et ils ont une borne inférieure qui est strictement positive. Nous nous intéressons pour la vitesse de la marche en fonction du bias. Un argument connu donne que, pour des biais suffisamment grands, la vitesse est une fonction croissante du biais. Notre résultat principal dit que si le désordre est petit, ce qui veut dire que les poids sont proches les uns aux autres, la vitesse est une fonction croissante du bias, voir Théorème 1.1. Un ingrédient crucial de la preuve est une formule pour la dérivée de la vitesse : cette dérivée peut etre écrit comme une covariance, voir Théorème 1.3. La preuve de Théorème 1.3 suis les arguments de la preuve de la relation d'Einstein dans (*Ann. Probab.* **45** (4) (2017) 2533–2567). Par contre, nous donnons un exemple montrant que pour des poids iid prenant les valeurs 1 (avec probabilité p) et κ (avec probabilité $1 - p$), si p est suffisamment proche de 1 et κ est suffisamment petit, la vitesse n'est *pas* une fonction croissante du bias, voir Théorème 1.2.

MSC: 60K37; 60G42; 60J10

Keywords: Random walk in random environment; Random conductances; Effective velocity; Regeneration times

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How round are the complementary components of planar Brownian motion?

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Abstract. Consider a Brownian motion W in $\mathbf{C}$ started from 0 and run for time 1. Let $A(1), A(2), \dots$ denote the bounded connected components of $\mathbf{C} - W([0,1])$. Let $R(i)$ (resp. $r(i)$) denote the out-radius (resp. in-radius) of $A(i)$ for $i \in \mathbf{N}$. Our main result is that $\mathbf{E}[\sum_i R(i)^2 |\log R(i)|^\theta] < \infty$ for any $\theta < 1$. We also prove that $\sum_i r(i)^2 |\log r(i)| = \infty$ almost surely. These results have the interpretation that most of the components $A(i)$ have a rather regular or round shape.

Résumé. Soit W un mouvement brownien dans $\mathbf{C}$ issu de 0. Soit $A(1), A(2), \dots$ les composantes connexes bornées de $\mathbf{C} \setminus W([0,1])$. Soit $R(i)$ (resp. $r(i)$) le rayon extérieur (resp. le rayon intérieur) de $A(i)$, pour $i \in \mathbf{N}$. Notre résultat principal est que $\mathbf{E}[\sum_i R(i)^2 |\log R(i)|^\theta] < \infty$ pour tout $\theta < 1$. Nous montrons aussi que $\sum_i r(i)^2 |\log r(i)|] = \infty$ presque surement. Ces résultats peuvent s'interpréter comme le fait que la plupart des composantes $A(i)$ ont une forme assez régulière, ou ronde.

MSC: 60G17

Keywords: Planar Brownian motion; Complementary components of planar Brownian motion

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Matrix Dirichlet processes

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Abstract. Matrix Dirichlet processes, in reference to their reversible measure, appear in a natural way in many different models in probability. Applying the language of diffusion operators and the theory of boundary equations, we describe Dirichlet processes on the matrix simplex and provide two models of matrix Dirichlet processes, which can be realized by various projections, through the Brownian motion on the special unitary group and also through Wishart processes.

Résumé. Les processus de Dirichlet matriciels, en référence à leur mesure réversible, apparaissent de manière naturelle dans de nombreux modèles différents en probabilité. En utilisant la langage des opérateurs de diffusion et la théorie des équations de bord, nous décrivont les processus de Dirichlet sur le simplexe matriciel et proposont deux modèles pour les processus de Dirichlet matriciels, qui peuvent être réalisés par les projections diverses, par le movement brownien sur le groupe unitaire spécial et par les processus de Wishart.

MSC: 47B25; 60B20

Keywords: Matrix Dirichlet processes; Diffusion operators; Wishart processes

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Parabolic Anderson model with rough or critical Gaussian noise

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Abstract. This paper considers the parabolic Anderson equation

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\frac{\partial^{d+1} W^{\mathbf{H}}}{\partial t \partial x_1 \cdots \partial x_d}$$

generated by a $(d+1)$-dimensional fractional noise with the Hurst parameter $\mathbf{H} = (H_0, H_1, \dots, H_d)$. The existence/uniqueness, Feynman–Kac's moment formula and the precise intermittency exponents are formulated in the case when some of $H_1, \dots, H_d$ are less than one half, and in the case when the Dalang's condition

$$d - \sum_{k=1}^n H_j < 1 \quad \text{is replaced by } d - \sum_{k=1}^n H_j = 1.$$

Some partial result is also achieved for the case when $H_0 < 1/2$ which brings insight on what to expect as the Gaussian noise is rough in time.

Résumé. Cet article s'intéresse à l'équation d'Anderson parabolique

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\frac{\partial^{d+1} W^{\mathbf{H}}}{\partial t \partial x_1 \cdots \partial x_d}$$

engendrée par un bruit fractionnaire de dimension $(d+1)$ et de paramètre de Hurst $\mathbf{H} = (H_0, H_1, \dots, H_d)$. L'existence et l'unicité, la formule des moments de Feynman–Kac et les exposants précis d'intermittence sont formulés dans le cas où l'un des paramètres $H_1, \dots, H_d$ est inférieur à un demi, et dans le cas où la condition de Dalang

$$d - \sum_{k=1}^n H_j < 1 \quad \text{est remplacée par } d - \sum_{k=1}^n H_j = 1.$$

Des résultats partiels sont aussi obtenus dans la cas $H_0 < 1/2$, ce qui donne une intuition de ce qui doit être attendu dans le cas où le bruit Gaussien est rugueux en temps.

MSC: 60F10; 60H15; 60H40; 60J65; 81U10

Keywords: Parabolic Anderson equation; Dalang's condition; Fractional, rough and critical Gaussian noises; Feynman–Kac's representation; Brownian motion; Moment asymptotics

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A new approach to the existence of invariant measures for Markovian semigroups

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Abstract. We give a new, two-step approach to prove existence of finite invariant measures for a given Markovian semigroup. First, we fix a convenient auxiliary measure and then we prove conditions equivalent to the existence of an invariant finite measure which is absolutely continuous with respect to it. As applications, we obtain a unifying generalization of different versions for Harris' ergodic theorem which provides an answer to an open question of Tweedie. Also, we show that for a nonlinear SPDE on a Gelfand triple, the strict coercivity condition is sufficient to guarantee the existence of a unique invariant probability measure for the associated semigroup, once it satisfies a Harnack type inequality with power. A corollary of the main result shows that any uniformly bounded semigroup on L^p possesses an invariant measure and we give some applications to sectorial perturbations of Dirichlet forms.

Résumé. On établit une approche en deux étapes pour démontrer l'existence des mesure invariantes finies pour un semigroupe de Markov donné. En fixant d'abord une mesure auxiliaire convenable, on démontre ensuite des conditions équivalentes à l'existence d'une mesure invariante finie qui est absolument continue par rapport à elle. Comme applications, on obtient une généralisation unificatrice des diverses versions du théorème ergodique de Harris et on fournit une réponse à une question ouverte de Tweedie. On montre aussi que pour une EDP stochastique sur un triplet de Gelfand, la condition de coercivité stricte est suffisante pour garantir l'existence d'une seule mesure de probabilité pour le semigroupe associé, si une inégalité de type Harnack avec puissance est satisfaite. Un corollaire du résultat central montre que tout semigroupe uniformément borné sur L^p possède une mesure invariante ; on donne des applications aux perturbations sectorielles des formes de Dirichlet.

MSC: Primary 37C40; secondary 37A30; 37L40; 60J35; 60J25; 31C25; 82B10

Keywords: Invariant measure; Markovian semigroup; Transition function; Lyapunov function; Krylov–Bogoliubov theorem; Harris' ergodic theorem; Uniformly bounded C_0-semigroup on L^p; Komlós lemma; Sobolev inequality

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Statistical physics on a product of trees

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Abstract. Let G be the product of finitely many trees $T_1 \times T_2 \times \cdots \times T_N$, each of which is regular with degree at least three. We consider Bernoulli bond percolation and the Ising model on this graph, giving a short proof that the model undergoes a second order phase transition with mean-field critical exponents in each case. The result concerning percolation recovers a result of Kozma (2013), while the result concerning the Ising model is new.

We also present a new proof, using similar techniques, of a lemma of Schramm concerning the decay of the critical two-point function along a random walk, as well as some generalizations of this lemma.

Résumé. Soit G le produit d'un nombre fini d'arbres $T_1 \times T_2 \times \cdots \times T_N$ ayant chacun un degré constant supérieur à trois, nous étudions la percolation de Bernoulli et le modèle d'Ising sur ce graphe et présentons une preuve simple de l'existence d'une transition de phase de second ordre ayant les mêmes exposants critiques que le modèle en champs moyen. Le résultat pour la percolation est une preuve alternative d'un résultat de Kozma (2013), tandis que le résultat pour le modèle d'Ising est nouveau.

Nous présentons également une nouvelle preuve, reposant sur des techniques similaires, du lemme de Schramm concernant la vitesse de décroissance de la fonction à deux points le long de la marche aléatoire, ainsi que des généralisations de ce lemme.

MSC: Primary 60K35; secondary 60B99

Keywords: Percolation; Ising model; Triangle condition; Bubble diagram; Nonuniqueness; Nonamenable groups; Mean-field; Nonunimodular

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Finite-time singularity of the stochastic harmonic map flow

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Abstract. We investigate the influence of an infinite dimensional Gaussian noise on the bubbling phenomenon for the stochastic harmonic map flow $u(t,\cdot):\mathbb{D}^2\to\mathbb{S}^2$, from the two-dimensional unit disc onto the sphere. The diffusion term is assumed to have range one pointwisely in the tangent space $T_{u(t,x)}\mathbb{S}^2$, so that the noise preserves the 1-corotational symmetry of solutions. Under the assumption that its space-correlation is of trace class (in some appropriate Hilbert space), we prove that the noise generates blow-up with positive probability. This scenario happens no matter how we choose the initial data, provided it fulfills the latter symmetry assumption.

Résumé. Nous analysons ici l'influence d'un bruit gaussien infini-dimensionnel sur le phénomène de bubbling relatif au flot stochastique des applications harmoniques $u(t,\cdot):\mathbb{D}^2\to\mathbb{S}^2$, du disque unité vers la sphère. On suppose que le terme de diffusion est ponctuellement de rang un dans le plan tangent $T_{u(t,x)}\mathbb{S}^2$, de sorte que le bruit préserve la symétrie 1-corotationnelle des solutions. Sous l'hypothèse que sa corrélation spatiale est de trace finie (dans un espace de Hilbert *ah hoc*), nous montrons que le bruit engendre une singularité avec probabilité non nulle. Ce scénario se produit indépendamment du choix de la condition initiale, pourvu qu'elle soit 1-corotationnelle.

MSC: 60H15 (35R60); 58E20; 35K55; 35B44

Keywords: Stochastic partial differential equation; Harmonic Maps; Blow-up

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Transversal fluctuations for a first passage percolation model

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Abstract. In 1996, Licea, Newman, and Piza proved that for a rather convoluted definition of the transversal fluctuation exponent in first passage percolation, that exponent is bounded below by $3/5$. In this paper we introduce a new first passage percolation model in a Poissonian environment on $\mathbb{R}^2$, and prove the same estimate for a natural clean notion of the exponent.

Résumé. En 1996, Licea, Newman et Piza ont démontré que, pour une définition plutôt compliquée de l'exposant de la fluctuation transversale en percolation de premier passage, cet exposant est borné inférieurement par $3/5$. Dans cet article, nous introduisons un nouveau modèle de percolation de premier passage dans un environnement poissonien sur $\mathbb{R}^2$ et démontrons la même estimée pour une notion naturelle de l'exposant.

MSC: 60K35; 82B43

Keywords: First passage percolation; Transversal fluctuation; Poissonian potential; Superdiffusivity

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Continuous-state branching processes, extremal processes and super-individuals

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Abstract. The long-term behavior of flows of continuous-state branching processes are characterized through subordinators and extremal processes. The extremal processes arise in the case of supercritical processes with infinite mean and of subcritical processes with infinite variation. The jumps of these extremal processes are interpreted as specific initial individuals whose progenies overwhelm the population. These individuals, which correspond to the records of a certain Poisson point process embedded in the flow, are called super-individuals. They radically increase the growth rate to $+\infty$ in the supercritical case, and slow down the rate of extinction in the subcritical one.

Résumé. Les comportements en temps long des flots de processus de branchement en temps et espace continus sont caractérisés par des subordinateurs et des processus extrémaux. Les processus extrémaux apparaissent dans le cas des processus sur-critiques de moyenne infinie et des processus sous-critiques à variation infinie. Les sauts de ces processus extrémaux sont interprétés comme des individus initiaux spécifiques dont les descendances envahissent la population. Ces individus, qui correspondent aux instants de records d'un certain processus ponctuel de Poisson, sont appelés super-individus. Ils augmentent de façon radicale la vitesse de divergence dans le cas sur-critique et diminuent celle d'extinction dans le cas sous-critique.

MSC: Primary 60J80; secondary 60G70; 60G55

Keywords: Continuous-state branching process; Subordinator; Extremal process; Infinite mean; Infinite variation; Super-exponential growth; Grey martingale; Non-linear renormalisation

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Bayesian nonparametric analysis of Kingman's coalescent

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Dedicated to the memory of Paul Joyce

Abstract. Kingman's coalescent is one of the most popular models in population genetics. It describes the genealogy of a population whose genetic composition evolves in time according to the Wright–Fisher model, or suitable approximations of it belonging to the broad class of Fleming–Viot processes. Ancestral inference under Kingman's coalescent has had much attention in the literature, both in practical data analysis, and from a theoretical and methodological point of view. Given a sample of individuals taken from the population at time $t > 0$, most contributions have aimed at making frequentist or Bayesian parametric inference on quantities related to the genealogy of the sample. In this paper we propose a Bayesian nonparametric predictive approach to ancestral inference. That is, under the prior assumption that the composition of the population evolves in time according to a neutral Fleming–Viot process, and given the information contained in an initial sample of m individuals taken from the population at time $t > 0$, we estimate quantities related to the genealogy of an additional unobservable sample of size $m' \geq 1$. As a by-product of our analysis we introduce a class of Bayesian nonparametric estimators (predictors) which can be thought of as Good–Turing type estimators for ancestral inference. The proposed approach is illustrated through an application to genetic data.

Résumé. La coalescence de Kingman est l'un des modèles les plus populaires en génétique des populations. Il décrit la généalogie d'une population dont la composition génétique évolue dans le temps selon le modèle de Wright–Fisher, ou des approximations appropriées de celle-ci appartenant à la grande classe des processus de Fleming–Viot. L'inférence ancestrale sous la coalescence de Kingman a reçu beaucoup d'attention dans la littérature, à la fois dans l'analyse des données, et d'un point de vue théorique et méthodologique. Étant donné un échantillon d'individus échantillonnés dans la population au temps $t > 0$, la plupart des contributions existantes visaient l'inférence paramétrique, fréquentiste ou bayésienne, sur des quantités liées à la généalogie de l'échantillon. Dans cet article, nous proposons une approche prédictive bayésienne non paramétrique de l'inférence ancestrale. C'est-à-dire, sous l'hypothèse préalable que la composition de la population évolue dans le temps selon un processus de Fleming–Viot neutre, et compte tenu de l'information contenue dans un échantillon initial de m individus dans la population au temps $t > 0$, nous estimons des quantités liées à la généalogie d'un échantillon additionnel non observable de taille $m' \geq 1$. En corollaire de notre analyse, nous introduisons une classe d'estimateurs bayésiens non paramétriques (prédicteurs) qui peuvent être considérés comme des estimateurs de type Good–Turing pour l'inférence ancestrale. L'approche proposée est illustrée par une application sur données génétiques.

MSC: Primary 62C10; secondary 62M05

Keywords: Ancestral inference; Bayesian nonparametrics; Dirichlet process; Kingman's coalescent; Lineages distributions; Predictive probability

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Comparing mixing times on sparse random graphs

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Abstract. It is natural to expect that nonbacktracking random walk will mix faster than simple random walks, but so far this has only been proved in regular graphs. To analyze typical irregular graphs, let G be a random graph on n vertices with minimum degree 3 and a degree distribution that has exponential tails. We determine the precise worst-case mixing time for simple random walk on G, and show that, with high probability, it exhibits cutoff at time $\mathbf{h}^{-1}\log n$, where $\mathbf{h}$ is the asymptotic entropy for simple random walk on a Galton–Watson tree that approximates G locally. (Previously this was only known for typical starting points.) Furthermore, we show this asymptotic mixing time is strictly larger than the mixing time of nonbacktracking walk, via a delicate comparison of entropies on the Galton–Watson tree.

Résumé. Il est naturel de s'attendre à ce que la marche aléatoire sans rebroussement mélange plus vite que la marche aléatoire simple, mais jusqu'ici, cela n'était prouvé que dans le cas des graphes réguliers. Pour analyser le cas de graphes irréguliers typiques, soit G un graphe aléatoire à n sommets de degrés au moins 3 et distribués selon une loi à queue exponentielle. On détermine le temps de mélange partant du pire point de départ pour la marche aléatoire simple sur G, et l'on montre qu'avec grande probabilité, cette marche présente le phénomène de cutoff au temps $\mathbf{h}^{-1}\log n$, où $\mathbf{h}$ est l'entropie asymptotique de la marche aléatoire simple sur un arbre de Galton–Watson qui est une approximation locale de G. (Précédemment, cela n'était connu que pour des points de départ typiques.) De plus, on montre que ce temps de mélange est strictement plus grand que celui de la marche aléatoire sans rebroussement, via une comparison délicate des entropies sur l'arbre de Galton–Watson.

MSC: Primary 60J10; secondary 05C80

Keywords: Random graphs; Random walks; Nonbacktracking vs. Simple random walk; Mixing times of Markov chains

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An isomorphism between branched and geometric rough paths

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Abstract. We exhibit an explicit natural isomorphism between spaces of branched and geometric rough paths. This provides a multi-level generalisation of the isomorphism of Lejay–Victoir [*J. Differential Equations* **225** (2006) 103–133] as well as a canonical version of the Itô–Stratonovich correction formula of Hairer–Kelly [*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 207–251]. Our construction is elementary and uses the property that the Grossman–Larson algebra is isomorphic to a tensor algebra.

We apply this isomorphism to study signatures of branched rough paths. Namely, we show that the signature of a branched rough path is trivial if and only if the path is tree-like, and construct a non-commutative Fourier transform for probability measures on signatures of branched rough paths. We use the latter to provide sufficient conditions for a random signature to be determined by its expected value, thus giving an answer to the uniqueness moment problem for branched rough paths.

Résumé. Nous explicitons un isomorphisme naturel entre les espaces de chemins rugueux branchants et géométriques. Ceci fournit une généralisation multi-échelle de l'isomorphisme de Lejay–Victoir [*J. Differential Equations* **225** (2006) 103–133], ainsi qu'une version canonique de la formule pour le terme correctif d'Itô–Stratonovich d'Hairer et Kelly [*Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 207–251]. Notre construction est élémentaire et utilise la propriété que l'algèbre de Grossman–Larson est isomorphe à une algèbre tensorielle.

Nous appliquons cet isomorphisme pour étudier la signature des chemins rugueux branchants. Plus précisément, nous montrons que la signature d'un chemin rugueux branchant est triviale si et seulement si le chemin a une structure arborescente, et nous construisons une transformée de Fourier non commutative pour les mesures de probabilités sur les signatures de chemins rugueux branchants. Nous utilisons cette dernière pour donner des conditions suffisantes pour qu'une signature aléatoire soit déterminée par sa valeur moyenne, fournissant ainsi une réponse au problème d'unicité des moments pour les chemins rugueux branchants.

MSC: Primary 60H10; secondary 16T05; 60B15

Keywords: Branched rough paths; Butcher group; Signature; Non-commutative Fourier transform

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On random walk on growing graphs[1]

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Abstract. Random walk on changing graphs is considered. For sequences of finite graphs increasing monotonically towards a limiting infinite graph, we establish transition probability upper bounds. It yields sufficient transience criteria for simple random walk on slowly growing graphs, upon knowing the volume and Cheeger constant of each graph. For much more specialized cases, we establish matching lower bounds, and deduce sufficient (weak) recurrence criteria. We also address recurrence directly in relation to a universality conjecture of (*Electron. J. Probab.* **19** (2014) Article ID 106). We answer a related question of (*Ann. Probab.* **39** (2011) 1161–1203, Problem 1.8) about "inhomogeneous merging" in the negative.

Résumé. Nous considérons un modèle de marche aléatoire sur un graphe dynamique. Pour une suite de graphes finis croissant vers un graphe limite infini, nous montrons une borne supérieure pour la probabilité de transition. Cela donne un critère de transience pour la marche simple, pour des graphes à croissante lente, à partir du volume et de la constante de Cheeger de chaque graphe. Pour des cas plus particuliers, nous montrons une borne inférieure du même ordre et déduisons un critère de récurrence (dans un sens faible). Nous répondons aussi à la question de la récurrence directement, en lien avec une conjecture d'universalité de (*Electron. J. Probab.* **19** (2014) Article ID 106). Nous répondons aussi négativement à une question reliée de (*Ann. Probab.* **39** (2011) 1161–1203, Problem 1.8), à propos du « plongement inhomogène ».

MSC: Primary 60J10; secondary 60J35; 60K37

Keywords: Random walk; Time-inhomogeneity; Evolving sets; Recurrence; Transience; Heat kernel bounds; Merging

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On the equivalence between some jumping SDEs with rough coefficients and some non-local PDEs

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Abstract. We study some jumping SDE and the corresponding Fokker–Planck (or Kolmogorov forward) equation, which is a non-local PDE. We assume only some measurability and growth conditions on the coefficients. We prove that for any weak solution $(f_t)_{t\in[0,T]}$ of the PDE, there exists a weak solution to the SDE of which the time marginals are given by $(f_t)_{t\in[0,T]}$. As a corollary, we deduce that for any given initial condition, existence for the PDE is equivalent to weak existence for the SDE and uniqueness in law for the SDE implies uniqueness for the PDE. This extends some ideas of Figalli (*J. Funct. Anal.* **254** (2008) 109–153) concerning continuous SDEs and local PDEs.

Résumé. On étudie certaines EDS à sauts et les équations de Fokker–Planck (ou Kolmogorov progressives) correspondantes, qui sont des EDP non-locales. On suppose seulement que les coefficients sont mesurables et à croissance au plus linéaire. On montre que pour toute solution faible $(f_t)_{t\in[0,T]}$ de l'EDP, il existe une solution faible à l'EDS, dont les lois marginales sont données par $(f_t)_{t\in[0,T]}$. On en déduit que pour toute donnée initiale, l'existence pour l'EDP est équivalente à l'existence faible pour l'EDS, et que l'unicité en loi pour l'EDS implique l'unicité pour l'EDP. Nous étendons ainsi des idées de Figalli (*J. Funct. Anal.* **254** (2008) 109–153) concernant des EDS continues et des EDP locales.

MSC: 60H10; 60J75; 40K05

Keywords: Existence and uniqueness; Weak solution; Jumping SDEs; Non-local PDEs

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Local fluctuations of critical Mandelbrot cascades

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Abstract. We investigate so-called generalized Mandelbrot cascades at the freezing (critical) temperature. It is known that, after a proper rescaling, a sequence of multiplicative cascades converges weakly to some continuous random measure. Our main question is how the limiting measure μ fluctuates. For any given point x, denoting by $B_n(x)$ the ball of radius 2^{-n} centered around x, we present optimal lower and upper estimates of $\mu(B_n(x))$ as $n \to \infty$.

Résumé. Nous étudions les cascades de Mandelbrot généralisées à la température (critique) de *freezing*. Il est connu qu'après une mise à l'échelle appropriée, une telle suite de cascades multiplicatives converge faiblement vers une certaine mesure aléatoire continue. La question est alors de savoir à quel point la mesure limite μ fluctue. Pour tout point x donné, et en notant $B_n(x)$ la boule de rayon 2^{-n} centrée en x, nous présentons des bornes supérieures et inférieures optimales pour $\mu(B_n(x))$ lorsque $n \to \infty$.

MSC: 60J80; 60G57

Keywords: Mandelbrot cascades; Branching random walk; Derivative martingale; Conditioned random walk

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