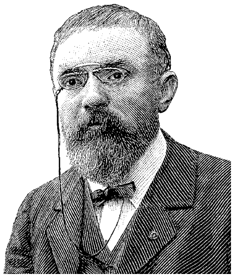

Volume 55, Number 3, 2019
ISSN 0246-0203

ANNALES DE L'INSTITUT HENRI POINCARÉ
PROBABILITÉS ET STATISTIQUES

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ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Vol. 55, No. 3 (August, 2019) 1203–1813

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques (ISSN 0246-0203), Volume 55, Number 3, August 2019. Published quarterly by Association des Publications de l'Institut Henri Poincaré.

POSTMASTER: Send address changes to Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques, Dues and Subscriptions Office, 9650 Rockville Pike, Suite L 2310, Bethesda, Maryland 20814-3998 USA.

Président et directeur de la publication : Cédric Villani
Périodicité : 4 n^{os} / an

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1203–1225
https://doi.org/10.1214/18-AIHP916

Limit law of a second class particle in TASEP with non-random initial condition

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Abstract. We consider the totally asymmetric simple exclusion process (TASEP) with non-random initial condition and density λ on $\mathbb{Z}_-$ and ρ on $\mathbb{Z}_+$, and a second class particle initially at the origin. For $\lambda < \rho$, there is a shock and the second class particle moves with speed $1-\lambda-\rho$. For large time t, we show that the position of the second class particle fluctuates on the $t^{1/3}$ scale and determine its limiting law. We also obtain the limiting distribution of the number of steps made by the second class particle until time t.

Résumé. On considère le processus d'exclusion simple totalement asymétrique avec des condition initiales déterministes, densité λ sur $\mathbb{Z}_-$ et ρ sur $\mathbb{Z}_+$. Initialement on place une particule de deuxième classe à l'origine. Si $\lambda < \rho$, un choc est créé et la particule de deuxième classe le suit avec vitesse $1-\lambda-\rho$. Dans la limite $t \to \infty$, on démontre que les fluctuations de la position de la particule de deuxième classe sont de l'ordre $t^{1/3}$ et on obtient sa loi limite. On détermine aussi la loi limite du nombre de sauts faits par la particule de deuxième classe jusqu'à l'instant t.

MSC: 82C22; 60K35; 60B20

Keywords: Exclusion process; Second class particle; Shock; Scaling limit; KPZ universality class

References

[1] J. Baik, G. Ben Arous and S. Péché. Phase transition of the largest eigenvalue for non-null complex sample covariance matrices. *Ann. Probab.* **33** (2005) 1643–1697. MR2165575
[2] J. Baik, P. L. Ferrari and S. Péché. Convergence of the two-point function of the stationary TASEP. In *Singular Phenomena and Scaling in Mathematical Models* 91–110. Springer, Berlin, 2014. MR3205038
[3] M. Balázs, E. Cator and T. Seppäläinen. Cube root fluctuations for the corner growth model associated to the exclusion process. *Electron. J. Probab.* **11** (2006) 1094–1132. MR2268539
[4] A. Borodin and V. Gorin. Lectures on integrable probability, 2012. Available at arXiv:1212.3351. MR3526828
[5] A. Borodin and S. Shlosman. Gibbs ensembles of nonintersecting paths. *Comm. Math. Phys.* **293** (2010) 145–170. MR2563801
[6] E. Cator and L. Pimentel. On the local fluctuations of last-passage percolation models. *Stochastic Process. Appl.* **125** (2015) 879–903. MR3293293
[7] I. Corwin. The Kardar–Parisi–Zhang equation and universality class. *Random Matrices Theory Appl.* **1** (2012) 1130001. MR2930377
[8] I. Corwin, P. L. Ferrari and S. Péché. Universality of slow decorrelation in KPZ models. *Ann. Inst. Henri Poincaré Probab. Stat.* **48** (2012) 134–150. MR2919201
[9] P. A. Ferrari. Shock fluctuations in asymmetric simple exclusion. *Probab. Theory Related Fields* **91** (1992) 81–101. MR1142763
[10] P. A. Ferrari and L. Fontes. Shock fluctuations in the asymmetric simple exclusion process. *Probab. Theory Related Fields* **99** (1994) 305–319. MR1278887
[11] P. A. Ferrari and C. Kipnis. Second class particles in the rarefaction fan. *Ann. Inst. Henri Poincaré* **31** (1995) 143–154. MR1340034
[12] P. A. Ferrari, J. B. Martin and L. P. R. Pimentel. A phase transition for competition interfaces. *Ann. Appl. Probab.* **19** (2009) 281–317. MR2498679
[13] P. A. Ferrari and L. P. R. Pimentel. Competition interfaces and second class particles. *Ann. Probab.* **33** (2005) 1235–1254. MR2150188

[14] P. L. Ferrari. Polynuclear growth on a flat substrate and edge scaling of GOE eigenvalues. *Comm. Math. Phys.* **252** (2004) 77–109. MR2103905
[15] P. L. Ferrari. Slow decorrelations in KPZ growth. *J. Stat. Mech.* (2008) P07022.
[16] P. L. Ferrari. From interacting particle systems to random matrices. *J. Stat. Mech.* (2010) P10016. MR2800495
[17] P. L. Ferrari and P. Nejjar. Anomalous shock fluctuations in TASEP and last passage percolation models. *Probab. Theory Related Fields* **161** (2015) 61–109. MR3304747
[18] P. L. Ferrari and P. Nejjar. Fluctuations of the competition interface in presence of shocks. *ALEA Lat. Am. J. Probab. Math. Stat.* **14** (2017) 299–325. MR3637438
[19] P. L. Ferrari and A. Occelli. Universality of the GOE Tracy–Widom distribution for TASEP with arbitrary particle density. Preprint, 2017. Available at arXiv:1704.01291.
[20] P. L. Ferrari and H. Spohn. Scaling limit for the space–time covariance of the stationary totally asymmetric simple exclusion process. *Comm. Math. Phys.* **265** (2006) 1–44. MR2217295
[21] P. L. Ferrari and H. Spohn. Random growth models. In *The Oxford Handbook of Random Matrix Theory* 782–801. G. Akemann, J. Baik and P. Di Francesco (Eds). Oxford Univ. Press, Oxford, 2011. MR2932658
[22] J. Gärtner and E. Presutti. Shock fluctuations in a particle system. *Ann. Inst. Henri Poincaré A, Phys. Théor.* **53** (1990) 1–14. MR1077463
[23] K. Johansson. Shape fluctuations and random matrices. *Comm. Math. Phys.* **209** (2000) 437–476. MR1737991
[24] M. Kardar, G. Parisi and Y. Z. Zhang. Dynamic scaling of growing interfaces. *Phys. Rev. Lett.* **56** (1986) 889–892.
[25] T. M. Liggett. *Interacting Particle Systems*. Springer Verlag, Berlin, 1985. MR0776231
[26] T. M. Liggett. *Stochastic Interacting Systems: Contact, Voter and Exclusion Processes*. Springer Verlag, Berlin, 1999. MR1717346
[27] T. Mountford and H. Guiol. The motion of a second class particle for the TASEP starting from a decreasing shock profile. *Ann. Appl. Probab.* **15** (2005) 1227–1259. MR2134103
[28] P. Nejjar. Transition to shocks in TASEP and decoupling of last passage times. Preprint, 2017. Available at arXiv:1705.08836.
[29] L. P. R. Pimentel. Local behavior of airy processes. Preprint, 2017. Available at arXiv:1704.01903.
[30] M. Prähofer and H. Spohn. Universal distributions for growth processes in $1+1$ dimensions and random matrices. *Phys. Rev. Lett.* **84** (2000) 4882–4885.
[31] M. Prähofer and H. Spohn. Current fluctuations for the totally asymmetric simple exclusion process. In *In and Out of Equilibrium*, V. Sidoravicius (Ed.). *Progress in Probability*. Birkhäuser, Basel, 2002. MR1901953
[32] J. Quastel. Introduction to KPZ. In *Current Developments in Mathematics* 125–194, 2012. MR3098078
[33] J. Quastel and H. Spohn. The one-dimensional KPZ equation and its universality class. *J. Stat. Phys.* **160** (2015) 965–984. MR3373647
[34] C. A. Tracy and H. Widom. On orthogonal and symplectic matrix ensembles. *Comm. Math. Phys.* **177** (1996) 727–754. MR1385083

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1226–1257
https://doi.org/10.1214/18-AIHP917

Homogenization theory for the random conductance model with degenerate ergodic weights and unbounded-range jumps

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Abstract. We study homogenization properties of the discrete Laplace operator with random conductances on a large domain in $\mathbb{Z}^d$. More precisely, we prove almost-sure homogenization of the discrete Poisson equation and of the top of the Dirichlet spectrum.

We assume that the conductances are stationary, ergodic and nearest-neighbor conductances are positive. In contrast to earlier results, we do not require uniform ellipticity but certain integrability conditions on the lower and upper tails of the conductances. We further allow jumps of arbitrary length.

Without the long-range connections, the integrability condition on the lower tail is optimal for spectral homogenization. It coincides with a necessary condition for the validity of a local central limit theorem for the random walk among random conductances. As an application of spectral homogenization, we prove a quenched large deviation principle for the normalized and rescaled local times of the random walk in a growing box.

Our proofs are based on a compactness result for the Laplacian's Dirichlet energy, Poincaré inequalities, Moser iteration and two-scale convergence.

Résumé. Nous considérons les propriétés d'homogénéisation de l'opérateur de Laplace discret avec des conductances aléatoires. Nous démontrons l'homogénéisation de l'équation de Poisson discrète et des plus hauts éléments du spectre de l'opérateur de Dirichlet dans un domaine limité.

Nous supposons les conductances stationnaires, ergodiques et strictement positives à plus proches voisins. Comparé aux résultats précédents, nous remplaçons l'ellipticité uniforme par des conditions d'intégrabilité des moments des conductances. De plus, nous autorisons des sauts de tailles arbitraires.

En l'absence de sauts longs, les conditions sur les moments sont optimales pour l'homogénéisation spectrale. Elles correspondent à la condition nécessaire du théorème central limite pour les marches aléatoires en conductances aléatoires. Nous utilisons l'homogénéisation spectrale pour démontrer un principe de grandes déviations gelé pour le temps local normalisé de la marche aléatoire dans une suite croissante de boites.

Nos démonstrations sont basées sur un résultat de compacité pour l'énergie de Dirichlet, les inégalités de Poincaré, l'itération de Moser et la convergence à deux échelles.

MSC: 60H25; 60K37; 35B27; 35R60; 47B80; 47A75

Keywords: Random conductance model; Homogenization; Dirichlet eigenvalues; Local times; Percolation

References

[1] S. Andres, J.-D. Deuschel and M. Slowik. Invariance principle for the random conductance model in a degenerate ergodic environment. *Ann. Probab.* **43** (4) (2015) 1866–1891. MR3353817
[2] S. Andres, J.-D. Deuschel and M. Slowik. Harnack inequalities on weighted graphs and some applications to the random conductance model. *Probab. Theory Related Fields* **164** (3–4) (2016) 931–977. MR3477784

[3] G. Allaire. Homogenization and two-scale convergence. *SIAM J. Math. Anal.* **23** (6) (1992) 1482–1518. MR1185639
[4] D. Boivin and J. Depauw. Spectral homogenization of reversible random walks on $\mathbb{Z}^d$ in a random environment. *Stochastic Process. Appl.* **104** (1) (2003) 29–56. MR1956471
[5] M. Biskup, R. Fukushima and W. König. Eigenvalue fluctuations for lattice Anderson Hamiltonians. *SIAM J. Math. Anal.* **48** (4) (2016) 2674–2700. MR3537879
[6] M. Biskup, R. Fukushima and W. König. Eigenvalue fluctuations for lattice Anderson Hamiltonians: Unbounded potentials. *Interdiscip. Inform. Sci.* **25** (1) (2018) 59–76.
[7] J.-P. Bouchaud and A. Georges. Anomalous diffusion in disordered media: Statistical mechanisms, models and physical applications. *Phys. Rep.* **195** (4–5) (1990) 127–293. MR1081295
[8] M. Biskup. Recent progress on the random conductance model. *Probab. Surv.* **8** (2011) 294–373. MR2861133
[9] H. Brezis. *Functional Analysis, Sobolev Spaces and Partial Differential Equations*. *Universitext*. Springer, New York, 2011. MR2759829
[10] T. Coulhon. Espaces de Lipschitz et inégalités de Poincaré. *J. Funct. Anal.* **136** (1) (1996) 81–113. MR1375154
[11] J.-D. Deuschel, T. A. Nguyen and M. Slowik. Quenched invariance principles for the random conductance model on a random graph with degenerate ergodic weights. *Probab. Theory Related Fields* **170** (1–2) (2018) 363–386. MR3748327
[12] M. D. Donsker and S. R. S. Varadhan. Asymptotic evaluation of certain Markov process expectations for large time, I. *Comm. Pure Appl. Math.* **28** (1975) 1–47. MR0386024
[13] L. C. Evans. *Partial Differential Equations*, 2nd edition. *Graduate Studies in Mathematics* **19**. American Mathematical Society, Providence, RI, 2010. MR2597943
[14] A. Faggionato. Random walks and exclusion processes among random conductances on random infinite clusters: Homogenization and hydrodynamic limit. *Electron. J. Probab.* **13** (73) (2008) 2217–2247. MR2469609
[15] A. Faggionato. Spectral analysis of 1D nearest-neighbor random walks and applications to subdiffusive trap and barrier models. *Electron. J. Probab.* **17** (15) (2012) 36pp. MR2892328
[16] F. Flegel. Localization of the principal Dirichlet eigenvector in the heavy-tailed random conductance model. *Electron. J. Probab.* **23** (68) (2018) 43pp.
[17] N. Gantert, W. König and Z. Shi. Annealed deviations of random walk in random scenery. *Ann. Inst. Henri Poincaré Probab. Stat.* **43** (1) (2007) 47–76. MR2288269
[18] A. Giunti and J.-C. Mourrat. Quantitative homogenization of degenerate random environments. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (1) (2018) 22–50. MR3765879
[19] V. V. Jikov, S. M. Kozlov and O. A. Oleĭnik. *Homogenization of Differential Operators and Integral Functionals*. Springer-Verlag, Berlin, 1994. MR1329546
[20] H. Kesten. Aspects of first passage percolation. In *École D'été de Probabilités de Saint-Flour, XIV – 1984* 125–264. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876084
[21] W. König and T. Wolff. Large deviations for the local times of a random walk among random conductances in a growing box. *Markov Process. Related Fields* **21** (3) (2015) 591–638. MR3469270
[22] T.-W. Ma. *Banach–Hilbert Spaces, Vector Measures and Group Representations*. World Scientific Publishing Co., Inc., River Edge, NJ, 2002. MR1908733
[23] G. Dal Maso. *An Introduction to* Γ*-Convergence*. *Progress in Nonlinear Differential Equations and Their Applications* **8**. Birkhäuser Boston, Inc., Boston, MA, 1993. MR1201152
[24] P. Mathieu and A. Piatnitski. Quenched invariance principles for random walks on percolation clusters. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **463** (2085) (2007) 2287–2307. MR2345229
[25] S. Neukamm, M. Schäffner and A. Schlömerkemper. Stochastic homogenization of nonconvex discrete energies with degenerate growth. *SIAM J. Math. Anal.* **49** (3) (2017) 1761–1809. MR3650427
[26] G. Nguetseng. A general convergence result for a functional related to the theory of homogenization. *SIAM J. Math. Anal.* **20** (3) (1989) 608–623. MR0990867
[27] A. Piatnitski and E. Zhizhina. Periodic homogenization of nonlocal operators with a convolution-type kernel. *SIAM J. Math. Anal.* **49** (1) (2017) 64–81. MR3595876
[28] L. Saloff-Coste. Lectures on finite Markov chains. In *Lectures on Probability Theory and Statistics* 301–413. *Saint-Flour*, *1996*. *Lecture Notes in Math.* **1665**. Springer, Berlin, 1997. MR1490046
[29] E. Seneta. *Non-negative Matrices and Markov Chains*. *Springer Series in Statistics*. Springer, New York, 2006. Revised reprint of the second (1981) edition. MR2209438
[30] V. V. Zhikov and A. L. Pyatnitskiĭ. Homogenization of random singular structures and random measures. *Izv. Ross. Akad. Nauk Ser. Mat.* **70** (1) (2006) 23–74. MR2212433

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1258–1277
https://doi.org/10.1214/18-AIHP918

Activated random walk on a cycle

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Abstract. We consider Activated Random Walk (ARW), a particle system with mass conservation, on the cycle $\mathbb{Z}/n\mathbb{Z}$. One starts with a mass density $\mu > 0$ of initially active particles, each of which performs a simple symmetric random walk at rate one and falls asleep at rate $\lambda > 0$. Sleepy particles become active on coming in contact with other active particles. There have been several recent results concerning fixation/non-fixation of the ARW dynamics on infinite systems depending on the parameters μ and λ. On the finite graph $\mathbb{Z}/n\mathbb{Z}$, unless there are more than n particles, the process fixates (reaches an absorbing state) almost surely in finite time. In a first rigorous result for a finite system, establishing well known beliefs in the statistical physics literature, we show that the number of steps the process takes to fixate is linear in n (up to poly-logarithmic terms), when the density is sufficiently low compared to the sleep rate, and exponential in n when the sleep rate is sufficiently small compared to the density, reflecting the fixation/non-fixation phase transition in the corresponding infinite system as established in (*Invent. Math.* **188** (2012) 127–150).

Résumé. Nous considérons la marche aléatoire activée (*Activated Random Walk*, ARW), un système de particules avec conservation de masse sur le cycle $\mathbb{Z}/n\mathbb{Z}$. Partant d'un état initial avec une densité $\mu > 0$ de particules actives, chacune d'entre elles évolue selon une marche simple symétrique à taux 1, et s'endort à taux $\lambda > 0$. Les particules endormies deviennent actives lorsqu'elles entrent en contact avec d'autres particules actives. Plusieurs résultats récents se sont penchés sur la fixation ou la non-fixation de la dynamique en volume infini, en fonction des paramètres μ et λ. Sur le graphe fini $\mathbb{Z}/n\mathbb{Z}$, à moins qu'il y ait plus de n particules, le processus se fixe (en atteignant un état absorbant) presque sûrement en temps fini. Nous établissons un premier résultat rigoureux sur ces systèmes finis, confirmant des prédictions bien connues de la littérature de physique statistique, en montrant que le nombre d'étapes avant fixation est linéaire en n (à des termes poly-logarithmiques près) lorsque la densité est suffisamment petite par rapport au taux d'endormissement, et exponentielle en n lorsque le taux d'endormissement est suffisamment petit par rapport à la densité, ce qui reflète la transition de phase entre fixation et non-fixation établie dans (*Invent. Math.* **188** (2012) 127–150) pour le système infini.

MSC: 60K35; 82C20; 82C22

Keywords: Activated random walk; Diaconis–Fulton representation; Abelian property; Self-ogranized criticality

References

[1] G. Amir and O. Gurel-Gurevich. On fixation of activated random walks. *Electron. Commun. Probab.* **12** (2010) 119–123. MR2643591
[2] R. Basu, S. Ganguly and C. Hoffman. Non-fixation of symmetric activated random walk on the line for small sleep rate. *Comm. Math. Phys.* To appear, 2018. Preprint. Available at arXiv:1508.05677.
[3] P. Diaconis and W. Fulton. A growth model, a game, an algebra, Lagrange inversion, and characteristic classes. *Rend. Semin. Mat. Univ. Politec. Torino* **49** (1) (1991) 95–119. MR1218674
[4] R. Dickman. Nonequilibrium phase transitions in epidemics and sandpiles. *Phys. A* **306** (2002) 90–97. MR1933220
[5] R. Dickman, M. Alava, M. A. Muñoz, J. Peltola, A. Vespignani and S. Zapperi. Critical behaviour of a one-dimensional fixed-energy stochastic sandpile. *Phys. Rev. E* **64** (2001), 56104.
[6] R. Dickman, M. A. Muñoz, A. Vespignani and S. Zapperi. Paths to self-organized criticality. *Braz. J. Phys.* **30** (2000) 27–41.

[7] R. Dickman, A. Vespignani and S. Zapperi. Self-organized criticality as an absorbing-state phase transition. *Phys. Rev. E* **57** (1998) 5095–5105. MR1628210
[8] R. Dickman, L. T. Rolla and V. Sidoravicius. Activated random walkers: Facts, conjectures and challenges. *J. Stat. Phys.* **138** (1–3) (2010) 126–142. MR2594894
[9] K. Eriksson. Chip-firing games on mutating graphs. *SIAM J. Discrete Math.* **9** (1) (1996) 118–128. MR1375419
[10] A. Fey, L. Levine and D. B. Wilson. Driving sandpiles to criticality and beyond. *Phys. Rev. Lett.* **104** (14) (2010) Article ID 145703.
[11] B. Hough, D. Jerison and L. Levine. Sandpiles on the square lattice. Preprint, 2017. Available at arXiv:1703.00827.
[12] S. Janson. Tail bounds for sums of geometric and exponential random variables. *Statist. Probab. Lett.* **135** (2018) 1–6. MR3758253
[13] H. Kesten and V. Sidoravicius. Branching random walk with catalysts. *Electron. J. Probab.* **8** (2003) 1–51. MR1961167
[14] H. Kesten and V. Sidoravicius. The spread of a rumor or infection in a moving population. *Ann. Probab.* **33** (2005) 2402–2462. MR2184100
[15] H. Kesten and V. Sidoravicius. A phase transition in a model for the spread of an infection. *Illinois J. Math.* **50** (2006) 547–634. MR2247840
[16] H. Kesten and V. Sidoravicius. A shape theorem for the spread of an infection. *Ann. of Math. (2)* **167** (2008) 701–766. MR2415386
[17] L. Levine. Threshold state and a conjecture of Poghosyan, Poghosyan, Priezzhev and Ruelle. *Comm. Math. Phys.* **335** (2) (2015) 1003–1017. MR3316648
[18] L. Levine and Y. Peres. Internal erosion and the exponent $3/4$. Available at http://www.math.cornell.edu/~levine/erosion.pdf.
[19] S. Lübeck. Universal scaling behavior of non-equilibrium phase transitions. *Internat. J. Modern Phys. B* **18** (31–32) (2004) 3977–4118.
[20] S. S. Manna. Large-scale simulation of avalanche cluster distribution in sand pile model. *J. Stat. Phys.* **59** (1990) 509–521.
[21] S. S. Manna. Two-state model of self-organized criticality. *J. Phys. A: Math. Gen.* **24** (1991) L363–L369.
[22] L. T. Rolla and V. Sidoravicius. Absorbing-state phase transition for driven-dissipative stochastic dynamics on $\mathbb{Z}$. *Invent. Math.* **188** (1) (2012) 127–150. MR2897694
[23] E. Shellef. Nonfixation for activated random walks. *ALEA Lat. Am. J. Probab. Math. Stat.* **7** (2010) 137–149. MR2651824
[24] V. Sidoravicius and A. Teixeira. Absorbing-state transition for stochastic sandpiles and activated random walks. *Electron. J. Probab.* **22** (33) (2017) Article ID 22. MR3646059
[25] A. Stauffer and L. Taggi. Critical density of activated random walks on transitive graphs. *Ann. Probab.* **46** (2018) 2190–2220. MR3813989
[26] L. T. Rolla and L. Tournier. Non-fixation for biased activated random walks. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2) (2018) 938–951. MR3795072
[27] L. Taggi. Absorbing-state phase transition in biased activated random walk. *Electron. J. Probab.* **21** (2016) Article ID 13. MR3485355
[28] A. Vespignani, R. Dickman, M. A. Muñoz and S. Zapperi. Driving, conservation, and absorbing states in sandpiles. *Phys. Rev. Lett.* **81** (1998) 5676–5679.
[29] A. Vespignani, R. Dickman, M. A. Muñoz and S. Zapperi. Absorbing-state phase transitions in fixed-energy sandpiles. *Phys. Rev. E* **62** (2000) 4564–4582.
[30] R. Vidigal and R. Dickman. Asymptotic behavior of the order parameter in a stochastic sandpile. *J. Stat. Phys.* **118** (1) (2005) 1–25. MR2122547

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1278–1315
https://doi.org/10.1214/18-AIHP919

Non-Gaussian limit theorem for non-linear Langevin equations driven by Lévy noise

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Abstract. In this paper, we study the small noise behaviour of solutions of a non-linear second order Langevin equation $\ddot{x}^\varepsilon_t + |\dot{x}^\varepsilon_t|^\beta = \dot{Z}^\varepsilon_{\varepsilon t}$, $\beta \in \mathbb{R}$, driven by symmetric non-Gaussian Lévy processes Z^ε. This equation describes the dynamics of a one-degree-of-freedom mechanical system subject to non-linear friction and noisy vibrations. For a compound Poisson noise, the process x^ε on the macroscopic time scale t/ε has a natural interpretation as a non-linear filter which responds to each single jump of the driving process. We prove that a system driven by a general symmetric Lévy noise exhibits essentially the same asymptotic behaviour under the principal condition $\alpha + 2\beta < 4$, where $\alpha \in [0,2]$ is the "uniform" Blumenthal–Getoor index of the family $\{Z^\varepsilon\}_{\varepsilon>0}$.

Résumé. Dans cet article, nous étudions le comportement du bruit des solutions d'une équation de Langevin non linéaire du second ordre $\ddot{x}^\varepsilon_t + |\dot{x}^\varepsilon_t|^\beta = \dot{Z}^\varepsilon_{\varepsilon t}$, $\beta \in \mathbb{R}$, dirigée par un processus de Lévy Z^ε symétrique et non Gaussien. Cette équation décrit la dynamique d'un système mécanique à un degré de liberté soumis à un frottement non linéaire et à des vibrations aléatoires. Pour un bruit de Grenaille (quantique, shot noise), le processus x^ε sur l'échelle de temps macroscopique t/ε s'interprète naturellement comme un filtre non linéaire qui réagit à chaque saut du processus Z. Nous montrons qu'un système conduit par un bruit de Lévy symétrique présente le même comportement asymptotique sous la condition principale $\alpha + 2\beta < 4$, où $\alpha \in [0,2]$ est l'indice Blumenthal–Getoor « uniforme » de la famille $\{Z^\varepsilon\}_{\varepsilon>0}$.

MSC: Primary 60F05; secondary 60G51; 60H10; 60J25; 70F40; 70L05

Keywords: Lévy process; Langevin equation; Non-linear friction; Hölder-continuous drift; Singular drift; Stable Lévy process; Blumenthal–Getoor index; Ergodic Markov process; Lyapunov function

References

[1] H. Al-Talibi, A. Hilbert and V. Kolokoltsov. Nelson-type limit for a particular class of Lévy processes. *AIP Conference Proceedings* **1232** (2010).
[2] O. E. Barndorff-Nielsen and N. Shephard. Non-Gaussian Ornstein–Uhlenbeck-based models and some of their uses in Financial Economics. *J. Roy. Statist. Soc. Ser. B* **63** (2001) 167–241. MR1841412
[3] O. E. Barndorff-Nielsen and N. Shephard. Integrated OU processes and non-Gaussian OU-based stochastic volatility models. *Scand. J. Stat.* **30** (2003) 277–296. MR1983126
[4] A. Baule and P. Sollich. Singular features in noise-induced transport with dry friction. *Europhys. Lett.* **97** (2012) 20001.
[5] P. J. Blau. *Friction Science and Technology. From Concepts to Applications*, 2nd edition. CRC Press, Boca Raton, FL, 2009.
[6] A. Chechkin, V. Gonchar, J. Klafter, R. Metzler and L. Tanatarov. Stationary states of non-linear oscillators driven by Lévy noise. *Chem. Phys.* **284** (2002) 233–251.
[7] A. V. Chechkin, V. Yu. Gonchar, J. Klafter and R. Metzler. Natural cutoff in Lévy flights caused by dissipative nonlinearity. *Phys. Rev. E* **72** (2005) 010101.
[8] A. V. Chechkin, V. Yu. Gonchar, J. Klafter, R. Metzler and L. V. Tanatarov. Lévy flights in a steep potential well. *J. Stat. Phys.* **115** (2004) 1505–1535.

[9] A. V. Chechkin, V. Yu. Gonchar and M. Szydłowski. Fractional kinetics for relaxation and superdiffusion in a magnetic field. *Phys. Plasmas* **9** (2002) 78–88.

[10] A. S. Cherny and H. J. Engelbert. *Singular Stochastic Differential Equations*. *Lecture Notes in Mathematics* **1858**. Springer, Berlin, 2005. MR2112227

[11] P. G. de Gennes. Brownian motion with dry friction. *J. Stat. Phys.* **119** (2005) 953–962. MR2157853

[12] A. Dubkov and B. Spagnolo. Langevin approach to Lévy flights in fixed potentials: Exact results for stationary probability distributions. *Acta Phys. Polon. B* **38** (2007) 1745–1758. MR2308844

[13] B. Dybiec, E. Gudowska-Nowak and I. M. Sokolov. Underdamped stochastic harmonic oscillator driven by Lévy noise. *Phys. Rev. E* **96** (2017) 042118.

[14] B. Dybiec, I. M. Sokolov and A. V. Chechkin. Stationary states in single-well potentials under symmetric Lévy noises. *Journal of Statistical Mechanics: Theory and Experiment* (2010). Available at P07008.

[15] A. Eberle, A. Guillin and R. Zimmer. Quantitative Harris type theorems for diffusions and McKean–Vlasov processes, 2016. Preprint. Available at arXiv:1606.06012.

[16] R. Eon and M. Gradinaru. Gaussian asymptotics for a non-linear Langevin type equation driven by a symmetric α-stable Lévy noise. *Electron. J. Probab.* **20** (2015) 1–19. MR3407217

[17] W. Feller. *An Introduction to Probability Theory and Its Applications 2*, 3rd edition. John Wiley & Sons, New York, NY, 1971. MR0270403

[18] P. S. Goohpattader and M. K. Chaudhury. Diffusive motion with nonlinear friction: Apparently Brownian. *J. Chem. Phys.* **133** (2010) 024702.

[19] M. I. Gordin. The central limit theorem for stationary processes. *Sov. Math., Dokl.* **10** (1969) 1174–1176. MR0251785

[20] M. I. Gordin and B. A. Lifshits. The central limit theorem for stationary Markov processes. *Sov. Math., Dokl.* **19** (1978) 392–394.

[21] M. Hairer. *Convergence of Markov Processes*. *Lecture Notes*, 2016. Available at www.hairer.org/notes/Convergence.pdf.

[22] H. Hayakawa. Langevin equation with Coulomb friction. *Phys. D* **205** (2005) 48–56.

[23] R. Hintze and I. Pavlyukevich. Small noise asymptotics and first passage times of integrated Ornstein–Uhlenbeck processes driven by α-stable Lévy processes. *Bernoulli* **20** (2014) 265–281. MR3160582

[24] O. Kallenberg. *Foundations of Modern Probability*, 2nd edition. *Probability and Its Applications* Springer, New York, 2002. MR1876169

[25] A. Kawarada and H. Hayakawa. Non-Gaussian velocity distribution function in a vibrating granular bed. *J. Phys. Soc. Jpn.* **73** (2004) 2037–2040.

[26] A. Kulik. *Ergodic Behavior of Markov Processes. With Applications to Limit Theorems*. De Gryuter, Berlin, 2017.

[27] V. Lakshmikantham and S. Leela. *Differential and Integral Inequalities: Theory and Applications. Volume I: Ordinary Differential Equations*. *Mathematics in Science and Engineering* **55**. Academic Press, New York, 1969. MR0379934

[28] B. Lindner. The diffusion coefficient of nonlinear Brownian motion. *New J. Phys.* **9** (2007) 136.

[29] B. Lindner. Diffusion coefficient of a Brownian particle with a friction function given by a power law. *J. Stat. Phys.* **130** (2008) 523–533. MR2378163

[30] B. Lindner. Diffusion of particles subject to nonlinear friction and a colored noise. *New J. Phys.* **12** (2010) 063026.

[31] V. Lisý, J. Tóthová and L. Glod. Diffusion in a medium with nonlinear friction. *Int. J. Thermophys.* **35** (2014) 2001–2010.

[32] Y. Lü and J. D. Bao. Inertial Lévy flight with nonlinear friction. *Chin. Phys. Lett.* **28** (2011) 110201.

[33] A. Mauger. Anomalous motion generated by the Coulomb friction in the Langevin equation. *Phys. A* **367** (2006) 129–135.

[34] B. J. N. Persson. *Sliding Friction: Physical Principles and Applications*, 2nd edition. Springer, Berlin, 2000. MR1488960

[35] V. L. Popov. *Contact Mechanics and Friction: Physical Principles and Applications*. Springer, Heidelberg, 2010.

[36] N. I. Portenko. Some perturbations of drift-type for symmetric stable processes. *Random Oper. Stoch. Equ.* **2** (1994) 211–224. MR1310558

[37] G. Samorodnitsky and M. Grigoriu. Tails of solutions of certain nonlinear stochastic differential equations driven by heavy tailed Lévy motions. *Stochastic Process. Appl.* **105** (2003) 69–97. MR1972289

[38] V. P. Sergienko and S. N. Bukharov. *Noise and Vibration in Friction Systems*. *Springer Series in Materials Science* **212**. Springer, Cham, 2015. MR3497806

[39] R. Situ. *Theory of Stochastic Differential Equations with Jumps and Applications*. Springer, Berlin, 2005. MR2160585

[40] I. M. Sokolov, W. Ebeling and B. Dybiec. Harmonic oscillator under Lévy noise: Unexpected properties in the phase space. *Phys. Rev. E* **83** (2011) 041118.

[41] H. Tanaka, M. Tsuchiya and S. Watanabe. Perturbation of drift-type for Lévy processes. *J. Math. Kyoto Univ.* **14** (1974) 73–92. MR0368146

[42] H. Touchette, E. Van der Straeten and W. Just. Brownian motion with dry friction: Fokker–Planck approach. *J. Phys. A* **43** (2010) 445002. MR2733815

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1316–1348
https://doi.org/10.1214/18-AIHP920

Estimating functions for SDE driven by stable Lévy processes

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Abstract. This paper is concerned with parametric inference for a stochastic differential equation driven by a pure-jump Lévy process, based on high frequency observations on a fixed time period. Assuming that the Lévy measure of the driving process behaves like that of an α-stable process around zero, we propose an estimating functions based method which leads to asymptotically efficient estimators for any value of $\alpha \in (0,2)$ and does not require any integrability assumptions on the process. The main limit theorems are derived thanks to a control in total variation distance between the law of the normalized process, in small time, and the α-stable distribution. This method is an alternative to the non Gaussian quasi-likelihood estimation method proposed by Masuda (*Stochastic Process. Appl.* (2018) To appear) where the Blumenthal–Getoor index α is restricted to belong to the interval $[1,2)$.

Résumé. Dans cet article, nous étudions l'estimation des paramètres d'une équation différentielle stochastique dirigée par un processus de saut pur, à partir d'observations hautes fréquences du processus sur un intervalle de temps fixe. En supposant que la mesure de Lévy du processus de saut qui dirige l'équation se comporte autour de zéro comme la mesure de Lévy d'un processus α-stable, nous proposons une méthode d'estimation basée sur des fonctions estimantes qui conduit à des estimateurs asymptotiquement efficaces des paramètres de tendance et d'échelle, pour toute valeur de $\alpha \in (0,2)$, et qui ne nécessite pas de conditions d'intégrabilité du processus. Les principaux résultats asymptotiques sont obtenus grâce à un contrôle en variation totale entre la loi du processus renormalisé, en temps petit, et la loi α-stable. Cette méthode est une alternative à la méthode de quasi-vraisemblance non gaussienne proposée par Masuda (*Stochastic Process. Appl.* (2018) To appear), où l'indice de Blumenthal–Getoor α appartient à l'interval $[1,2)$.

MSC: Primary 60G51; 60G52; 60J75; 62F12; secondary 60H07; 60F05

Keywords: Lévy process; Stable process; Stochastic differential equation; Parametric inference; Estimating functions; Malliavin calculus

References

[1] Y. Aït-Sahalia and J. Jacod. Volatility estimators for discretely sampled Lévy processes. *Ann. Statist.* **35** (2007) 355–392. MR2332279
[2] Y. Aït-Sahalia and J. Jacod. Fisher's information for discretely sampled Lévy processes. *Econometrica* **76** (2008) 727–761. MR2433480
[3] Y. Aït-Sahalia and J. Jacod. Estimating the degree of activity of jumps in high frequency data. *Ann. Statist.* **37** (2009) 2202–2244. MR2543690
[4] O. Barndorff-Nielsen and M. Sørensen. A review of some aspects of asymptotic likelihood theory for stochastic processes. *Int. Stat. Rev.* **62** (1994) 133–165.
[5] B. M. Bibby and M. Sørensen. Martingale estimation functions for discretely observed diffusion processes. *Bernoulli* **1** (1995) 17–39. MR1354454
[6] K. Bichteler, J.-B. Gravereaux and J. Jacod. *Malliavin Calculus for Processes with Jumps*. *Stochastics Monographs* **2**. Gordon and Breach Science Publishers, New York, 1987. MR1008471
[7] E. Clément and A. Gloter. Local asymptotic mixed normality property for discretely observed stochastic differential equations driven by stable Lévy processes. *Stochastic Process. Appl.* **125** (2015) 2316–2352. MR3322866
[8] E. Clément, A. Gloter and H. Nguyen. LAMN property for the drift and volatility parameters of a SDE driven driven by a stable Lévy process. *ESAIM Probab. Stat.* To appear, 2018.
[9] E. Clément, A. Gloter and H. Nguyen. Asymptotics in small time for the density of a stochastic differential equation driven driven by a stable Lévy process. *ESAIM Probab. Stat.* To appear, 2018.

[10] I. A. Ibragimov and R. Has'minskiĭ. *Statistical Estimation: Asymptotic Theory*. *Applications of Mathematics* **16**. Springer-Verlag, New York, 1981. Translated from the Russian by Samuel Kotz. MR0620321

[11] D. Ivanenko, A. M. Kulik and H. Masuda. Uniform LAN property of locally stable Lévy process observed at high frequency. *ALEA Lat. Am. J. Probab. Math. Stat.* **12** (2015) 835–862. MR3453298

[12] J. Jacod. The Euler scheme for Lévy driven stochastic differential equations: Limit theorems. *Ann. Probab.* **32** (2004) 1830–1872. MR2073179

[13] J. Jacod and P. Protter. *Discretization of Processes*. *Stochastic Modelling and Applied Probability* **67**. Springer, Heidelberg, 2012. MR2859096

[14] J. Jacod and M. Sørensen. A review of asymptotic theory of estimating functions. *Stat. Inference Stoch. Process.* **21** (2018) 415–434. MR3824976

[15] R. Kawai and H. Masuda. On the local asymptotic behavior of the likelihood function for Meixner Lévy processes under high-frequency sampling. *Statist. Probab. Lett.* **81** (2011) 460–469. MR2765166

[16] R. Kawai and H. Masuda. Local asymptotic normality for normal inverse Gaussian Lévy processes with high-frequency sampling. *ESAIM Probab. Stat.* **17** (2013) 13–32. MR3002994

[17] M. Kessler and M. Sørensen. Estimating equations based on eigenfunctions for a discretely observed diffusion process. *Bernoulli* **5** (1999) 299–314. MR1681700

[18] A. Kulik. On weak uniqueness and distributional properties of a solution to an SDE with α-stable noise, 2015. Available at arXiv:1511.00106v1.

[19] H. Luschgy and G. Pagès. Moment estimates for Lévy processes. *Electron. Commun. Probab.* **13** (2008) 422–434. MR2430710

[20] H. Masuda. Joint estimation of discretely observed stable Lévy processes with symmetric Lévy density. *J. Japan Statist. Soc.* **39** (2009) 49–75. MR2571802

[21] H. Masuda. Non-Gaussian quasi-likelihood estimation of SDE driven by locally stable Lévy process. *Stochastic Process. Appl.* To appear, 2018.

[22] P. A. Meyer. Les résultats récents de Burkholder, Davis et Gundy. In *Séminaire de Probabilités, VI (Univ. Strasboug, année universitaire 1970–1971)* 151–158. *Lecture Notes in Math.* **258**. Springer, Berlin, 1972. MR0383520

[23] M. Sørensen. On asymptotics of estimating functions. *Braz. J. Probab. Stat.* **13** (1999) 111–136. MR1803041

[24] V. Todorov. Jump activity estimation for pure-jump semimartingales via self-normalized statistics. *Ann. Statist.* **43** (2015) 1831–1864. MR3357880

[25] V. Todorov and G. Tauchen. Limit theorems for power variations of pure-jump processes with application to activity estimation. *Ann. Appl. Probab.* **21** (2011) 546–588. MR2807966

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1349–1376
https://doi.org/10.1214/18-AIHP921

Inverting the cut-tree transform

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Abstract. We consider fragmentations of an $\mathbb{R}$-tree T driven by cuts arriving according to a Poisson process on $T \times [0, \infty)$, where the first co-ordinate specifies the location of the cut and the second the time at which it occurs. The genealogy of such a fragmentation is encoded by the so-called *cut-tree*, which was introduced by Bertoin and Miermont (*Ann. Appl. Probab.* **23** (4) (2013) 1469–1493) for a fragmentation of the Brownian continuum random tree. The cut-tree was generalised by Dieuleveut (*Ann. Appl. Probab.* **25** (4) (2015) 2215–2262) to a fragmentation of the α-stable trees, $\alpha \in (1, 2)$, and by Broutin and Wang (*Bernoulli* **23** (4A) (2017) 2380–2433) to the inhomogeneous continuum random trees of Aldous and Pitman (*Probab. Theory Related Fields* **118** (4) (2000) 455–482). In the first two cases, the projections of the forest-valued fragmentation processes onto the sequence of masses of their constituent subtrees yield an important family of examples of Bertoin's self-similar fragmentations (*Ann. Inst. Henri Poincaré Probab. Stat.* **38** (3) (2002) 319–340); in the first case the time-reversal of the fragmentation gives an additive coalescent. Remarkably, in all of these cases, the law of the cut-tree is the same as that of the original $\mathbb{R}$-tree.

In this paper, we develop a clean general framework for the study of cut-trees of $\mathbb{R}$-trees. We then focus particularly on the problem of *reconstruction*: how to recover the original $\mathbb{R}$-tree from its cut-tree. This has been studied in the setting of the Brownian CRT by Broutin and Wang (*Electron. J. Probab.* **22** (2017) 80), who prove that it is possible to reconstruct the original tree in distribution. We describe an enrichment of the cut-tree transformation, which endows the cut-tree with information we call a *consistent collection of routings*. We show this procedure is well-defined under minimal conditions on the $\mathbb{R}$-trees. We then show that, for the case of the Brownian CRT and the α-stable trees with $\alpha \in (1, 2)$, the original tree and the Poisson process of cuts thereon can both be almost surely reconstructed from the enriched cut-trees. For the latter results, our methods make essential use of the self-similarity and re-rooting invariance of these trees.

Résumé. Nous considérons des fragmentations d'un $\mathbb{R}$-arbre T dirigées par des coupures qui arrivent selon un processus de Poisson sur $T \times [0, \infty)$, où la première composante désigne le point auquel se produit la coupure et le deuxième, l'instant auquel elle a lieu. La généalogie d'une telle fragmentation est codée par *l'arbre des coupes*, qui a été introduit par Bertoin et Miermont (*Ann. Appl. Probab.* **23** (4) (2013) 1469–1493) pour une fragmentation de l'arbre brownien. L'arbre des coupes a ensuite été généralisé par Dieuleveut (*Ann. Appl. Probab.* **25** (4) (2015) 2215–2262) à une fragmentation des arbres α-stables, pour $\alpha \in (1, 2)$, et par Broutin et Wang (*Bernoulli* **23** (4A) (2017) 2380–2433) aux arbres continus inhomogènes d'Aldous et Pitman (*Probab. Theory Related Fields* **118** (4) (2000) 455–482). Dans les deux premiers cas, l'évolution de la suite des masses des sous-arbres apparaissant dans le processus de fragmentation constitue une famille importante de processus de fragmentation auto-similaires de Bertoin (*Ann. Inst. Henri Poincaré Probab. Stat.* **38** (3) (2002) 319–340); dans le premier cas, une fois inversée dans le temps, la fragmentation devient un coalescent additif. Remarquablement, dans tous ces cas, la loi de l'arbre des coupes est la même que celle du $\mathbb{R}$-arbre initial.

Dans cet article, nous développons un cadre général pour l'étude de l'arbre des coupes d'un $\mathbb{R}$-arbre. Par la suite, nous nous concentrons particulièrement sur le problème de la *reconstruction* : comment retrouver le $\mathbb{R}$-arbre initial à partir de son arbre des coupes. Ce problème a été étudié pour l'arbre brownien par Broutin et Wang (*Electron. J. Probab.* **22** (2017) 80), qui démontrent qu'il est possible de reconstruire l'arbre initial en loi. Nous décrivons un enrichissement de la construction de l'arbre des coupes, qui dote l'arbre des coupes d'une structure supplémentaire que nous appelons une *collection cohérente de routages*. Nous démontrons que ce procédé est bien défini sous des conditions minimales sur le $\mathbb{R}$-arbre. Ensuite, nous démontrons que pour l'arbre brownien ainsi que pour l'arbre α-stable avec $\alpha \in (1, 2)$, l'arbre initial muni de son processus de Poisson de coupures peut être reconstruit presque sûrement à partir de l'arbre des coupes enrichi. Pour ces derniers résultats, nous utilisons de façon essentielle l'auto-similarité et l'invariance par réenracinement de ces $\mathbb{R}$-arbres aléatoires.

MSC: Primary 60C05; secondary 05C05; 60G52; 60J80

Keywords: $\mathbb{R}$-tree; Cut-tree; Fragmentation

References

[1] R. Abraham and J.-F. Delmas. The forest associated with the record process on a Lévy tree. *Stochastic Process. Appl.* **123** (9) (2013) 3497–3517. MR3071387

[2] R. Abraham, J.-F. Delmas and P. Hoscheit. A note on the Gromov–Hausdorff–Prokhorov distance between (locally) compact metric measure spaces. *Electron. J. Probab.* **18** (2013) 14, 21 pp. (electronic). MR3035742

[3] L. Addario-Berry, N. Broutin and C. Holmgren. Cutting down trees with a Markov chainsaw. *Ann. Appl. Probab.* **24** (6) (2014) 2297–2339. MR3262504

[4] M. Albenque and C. Goldschmidt. The Brownian continuum random tree as the unique solution to a fixed point equation. *Electron. Commun. Probab.* **20** (2015) 61, 14 pp. (electronic). MR3399812

[5] D. Aldous. The continuum random tree. I. *Ann. Probab.* **19** (1) (1991) 1–28. MR1085326

[6] D. Aldous. The continuum random tree II: An overview. In *Stochastic Analysis*. Cambridge University Press, 1991. MR1166406

[7] D. Aldous. The continuum random tree III. *Ann. Probab.* **21** (1) (1993) 248–289. MR1207226

[8] D. Aldous. Recursive self-similarity for random trees, random triangulations and Brownian excursion. *Ann. Probab.* **22** (2) (1994) 527–545. MR1288122

[9] D. Aldous and J. Pitman. The standard additive coalescent. *Ann. Probab.* **26** (4) (1998) 1703–1726. MR1675063

[10] D. Aldous and J. Pitman. A family of random trees with random edge lengths. *Random Structures Algorithms* **15** (2) (1999) 176–195. MR1704343

[11] D. Aldous and J. Pitman. Inhomogeneous continuum random trees and the entrance boundary of the additive coalescent. *Probab. Theory Related Fields* **118** (4) (2000) 455–482. MR1808372

[12] D. J. Aldous and A. Bandyopadhyay. A survey of max-type recursive distributional equations. *Ann. Appl. Probab.* **15** (2) (2005) 1047–1110. MR2134098

[13] E. Baur and J. Bertoin. Cutting edges at random in large recursive trees. In *Stochastic Analysis and Applications 2014* 51–76. *Springer Proc. Math. Stat.* **100**. Springer, Cham, 2014. MR3332709

[14] J. Bertoin. Self-similar fragmentations. *Ann. Inst. Henri Poincaré Probab. Stat.* **38** (3) (2002) 319–340. MR1899456

[15] J. Bertoin. Fires on trees. *Ann. Inst. Henri Poincaré Probab. Stat.* **48** (4) (2012) 909–921. MR3052398

[16] J. Bertoin. The cut-tree of large recursive trees. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2) (2015) 478–488. MR3335011

[17] J. Bertoin and G. Miermont. The cut-tree of large Galton–Watson trees and the Brownian CRT. *Ann. Appl. Probab.* **23** (4) (2013) 1469–1493. MR3098439

[18] N. H. Bingham, C. M. Goldie and J. L. Teugels. *Regular Variation*. *Encyclopedia of Mathematics and Its Applications* **27**. Cambridge University Press, Cambridge, 1989. MR0898871

[19] N. Broutin and M. Wang. Cutting down **p**-trees and inhomogeneous continuum random trees. *Bernoulli* **23** (4A) (2017) 2380–2433. MR3648034

[20] N. Broutin and M. Wang. Reversing the cut tree of the Brownian continuum random tree. *Electron. J. Probab.* **22** (2017) 80, 23 pp. (electronic). MR3710800

[21] D. Burago, Y. Burago and S. Ivanov. *A Course in Metric Geometry*. *Graduate Studies in Mathematics* **33**. American Mathematical Society, Providence, RI, 2001. MR1835418

[22] M. Camarri and J. Pitman. Limit distributions and random trees derived from the birthday problem with unequal probabilities. *Electron. J. Probab.* **5** (2000) 2, 18 pp. (electronic). MR1741774

[23] E. Crane. Steady state clusters and the Ráth–Tóth mean field forest fire model. Available at arXiv:1809.03462.

[24] E. Crane, N. Freeman and B. Tóth. Cluster growth in the dynamical Erdős–Rényi process with forest fires. *Electron. J. Probab.* **20** (2015) 101, 33 pp. (electronic). MR3407218

[25] D. Dieuleveut. The vertex-cut-tree of Galton–Watson trees converging to a stable tree. *Ann. Appl. Probab.* **25** (4) (2015) 2215–2262. MR3349006

[26] M. Drmota, A. Iksanov, M. Moehle and U. Roesler. A limiting distribution for the number of cuts needed to isolate the root of a random recursive tree. *Random Structures Algorithms* **34** (3) (2009) 319–336. MR2504401

[27] T. Duquesne and J.-F. Le Gall. *Random Trees, Lévy Processes and Spatial Branching Processes*. *Asterisque* **281**. SMF, 2002. MR1954248

[28] T. Duquesne and J.-F. Le Gall. Probabilistic and fractal aspects of Lévy trees. *Probab. Theory Related Fields* **131** (4) (2005) 553–603. MR2147221

[29] R. Durrett. *Probability: Theory and Examples*, 4th edition. *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge University Press, Cambridge, 2010. MR2722836

[30] R. Durrett and T. M. Liggett. Fixed points of the smoothing transformation. *Z. Wahrsch. Verw. Gebiete* **64** (3) (1983) 275–301. MR0716487

[31] S. N. Evans, J. Pitman and A. Winter. Rayleigh processes, real trees, and root growth with re-grafting. *Probab. Theory Related Fields* **134** (1) (2006) 81–126. MR2221786

[32] S. N. Evans and A. Winter. Subtree prune and regraft: A reversible real tree-valued Markov process. *Ann. Probab.* **34** (3) (2006) 918–961. MR2243874

[33] J. A. Fill, N. Kapur and A. Panholzer. Destruction of very simple trees. *Algorithmica* **46** (3–4) (2006) 345–366. MR2291960

[34] K. Fukaya. Collapsing of Riemannian manifolds and eigenvalues of Laplace operator. *Invent. Math.* **87** (3) (1987) 517–547. MR0874035

[35] C. Goldschmidt and B. Haas. A line-breaking construction of the stable trees. *Electron. J. Probab.* **20** (2015) 16, 24 pp. (electronic). MR3317158

[36] A. Greven, P. Pfaffelhuber and A. Winter. Convergence in distribution of random metric measure spaces (Λ-coalescent measure trees). *Probab. Theory Related Fields* **145** (1) (2009) 285–322. MR2520129

[37] M. Gromov. *Metric Structures for Riemannian and Non-Riemannian Spaces*, English edition. *Modern Birkhäuser Classics*. Birkhäuser Boston, Inc., Boston, MA, 2007. Based on the 1981 French original, With appendices by M. Katz, P. Pansu and S. Semmes, Translated from the French by Sean Michael Bates. MR2307192

[38] B. Haas and G. Miermont. Scaling limits of Markov branching trees with applications to Galton–Watson and random unordered trees. *Ann. Probab.* **40** (6) (2012) 2589–2666. MR3050512

[39] B. Haas, J. Pitman and M. Winkel. Spinal partitions and invariance under re-rooting of continuum random trees. *Ann. Probab.* **37** (4) (2009) 1381–1411. MR2546748

[40] C. Holmgren. Random records and cuttings in binary search trees. *Combin. Probab. Comput.* **19** (3) (2010) 391–424. MR2607374

[41] A. Iksanov and M. Möhle. A probabilistic proof of a weak limit law for the number of cuts needed to isolate the root of a random recursive tree. *Electron. Commun. Probab.* **12** (2007) 28–35. MR2407414

[42] S. Janson. Random cutting and records in deterministic and random trees. *Random Structures Algorithms* **29** (2) (2006) 139–179. MR2245498

[43] I. Kortchemski. Invariance principles for Galton–Watson trees conditioned on the number of leaves. *Stochastic Process. Appl.* **122** (9) (2012) 3126–3172. MR2946438

[44] A. Meir and J. W. Moon. Cutting down random trees. *J. Aust. Math. Soc.* **11** (1970) 313–324. MR0284370

[45] A. Meir and J. W. Moon. Cutting down recursive trees. *Math. Biosci.* **21** (1974) 173–181.

[46] G. Miermont. Self-similar fragmentations derived from the stable tree. I. Splitting at heights. *Probab. Theory Related Fields* **127** (3) (2003) 423–454. MR2018924

[47] G. Miermont. Self-similar fragmentations derived from the stable tree. II. Splitting at nodes. *Probab. Theory Related Fields* **131** (3) (2005) 341–375. MR2123249

[48] G. Miermont. Tessellations of random maps of arbitrary genus. *Ann. Sci. Éc. Norm. Supér. (4)* **42** (5) (2009) 725–781. MR2571957

[49] A. Panholzer. Cutting down very simple trees. *Quaest. Math.* **29** (2) (2006) 211–227. MR2233368

[50] J. Pitman. Combinatorial stochastic processes. In *Lectures from the 32nd Summer School on Probability Theory Held in Saint-Flour, July 7–24, 2002, with a Foreword by Jean Picard*. *Lecture Notes in Mathematics* **1875**. Springer-Verlag, Berlin, 2006. MR2245368

[51] B. Ráth and B. Tóth. Erdős–Rényi random graphs + forest fires = self-organized criticality. *Electron. J. Probab.* **14** (2009) 45, 1290–1327. MR2511285

[52] D. Rizzolo. Scaling limits of Markov branching trees and Galton–Watson trees conditioned on the number of vertices with out-degree in a given set. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2) (2015) 512–532. MR3335013

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1377–1401
https://doi.org/10.1214/18-AIHP922

A large deviation principle for empirical measures on Polish spaces: Application to singular Gibbs measures on manifolds

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Abstract. We prove a large deviation principle for a sequence of point processes defined by Gibbs probability measures on a Polish space. This is obtained as a consequence of a more general Laplace principle for the non-normalized Gibbs measures. We consider four main applications: Conditional Gibbs measures on compact spaces, Coulomb gases on compact Riemannian manifolds, the usual Gibbs measures in the Euclidean space and the zeros of Gaussian random polynomials. Finally, we study the generalization of Fekete points and prove a deterministic version of the Laplace principle known as Γ-convergence. The approach is partly inspired by the works of Dupuis and co-authors. It is remarkably natural and general compared to the usual strategies for singular Gibbs measures.

Résumé. On montre un principe de grandes déviations pour une suite de processus ponctuels définis par des mesures de probabilités de Gibbs dans un espace polonais. Il est obtenu comme conséquence d'un principe de Laplace pour des mesures de Gibbs non normalisées. On considère quatre applications: Des mesures de Gibbs conditionnées dans des espaces compacts, des gaz de Coulomb sur des variétés riemanniennes compactes, les mesures de Gibbs habituelles sur l'espace euclidien et les zéros des polynômes aléatoires gaussiens. Finalement, on étudie la généralisation des points Fekete et on prouve une version déterministe du principe de Laplace appelée Γ-convergence. Notre approche est partiellement inspirée par les travaux de Dupuis et ses coauteurs. C'est notablement naturelle et générale en comparaison avec les stratégies habituelles pour les mesures de Gibbs singulières.

MSC: 60F10; 60K35; 82C22; 30C15

Keywords: Gibbs measure; Coulomb gas; Empirical measure; Large deviation principle; Interacting particle system; Singular potential; Constant curvature; Relative entropy; Random polynomials; Fekete points

References

[1] G. W. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, 2010. MR2760897
[2] T. Aubin. *Some Nonlinear Problems in Riemannian Geometry*. *Springer Monographs in Mathematics*. Springer-Verlag, Berlin, 1998. MR1636569
[3] C. Beltrán, N. Corral and J. G. Criado del Rey. Discrete and continuous Green energy on compact manifolds. Preprint. Available at https://arxiv.org/abs/1702.00864.
[4] G. Ben Arous and A. Guionnet. Large deviations for Wigner's law and Voiculescu's non-commutative entropy. *Probab. Theory Related Fields* **108** (4) (1997) 517–542. MR1465640
[5] G. Ben Arous and O. Zeitouni. Large deviations from the circular law. *ESAIM Probab. Stat.* **2** (1998) 123–134. MR1660943
[6] R. J. Berman. Kähler–Einstein metrics emerging from free fermions and statistical mechanics. *J. High Energy Phys.* **2011** (10) (2011) 106, 31 pp. MR2909692
[7] R. J. Berman. A thermodynamical formalism for Monge–Ampère equations, Moser–Trudinger inequalities and Kähler–Einstein metrics. *Adv. Math.* **248** (2013) 1254–1297. MR3107540
[8] R. J. Berman. On large deviations for Gibbs measures, mean energy and gamma-convergence. *Constr. Approx.* **48** (1) (2018) 3–30. MR3825945
[9] T. Bodineau and A. Guionnet. About the stationary states of vortex systems. *Ann. Inst. Henri Poincaré Probab. Stat.* **35** (2) (1999) 205–237. MR1678526

[10] V. S. Borkar. *Probability Theory. An Advanced Course*. *Universitext*. Springer-Verlag, New York, 1995. MR1367959

[11] R. Butez. Large deviations for the empirical measure of random polynomials: Revisit of the Zeitouni–Zelditch theorem. *Electron. J. Probab.* **21** (2016) 73, 37 pp. MR3592204

[12] J.-B. Castéras. A mean field type flow part I: Compactness of solutions to a perturbed mean field type equation. *Calc. Var. Partial Differ. Equ.* **53** (1–2) (2015) 221–246. MR3336318

[13] D. Chafaï, N. Gozlan and P.-A. Zitt. First-order global asymptotics for confined particles with singular pair repulsion. *Ann. Appl. Probab.* **24** (6) (2014) 2371–2413. MR3262506

[14] P. Dupuis and R. S. Ellis. *A Weak Convergence Approach to the Theory of Large Deviations*. *Wiley Series in Probability and Statistics: Probability and Statistics*. A Wiley-Interscience Publication, New York, 1997. MR1431744

[15] P. Dupuis, V. Laschos and K. Ramanan. Large deviations for empirical measures generated by Gibbs measures with singular energy functionals. Preprint. Available at https://arxiv.org/abs/1511.06928.

[16] D. García-Zelada. A large deviation principle for empirical measures on Polish spaces: Application to singular Gibbs measures on manifolds. Preprint. Available at https://arxiv.org/abs/1703.02680v1.

[17] H.-O. Georgii. Large deviations and maximum entropy principle for interacting random fields on $\mathbb{Z}^d$. *Ann. Probab.* **21** (4) (1993) 1845–1875. MR1245292

[18] D. Gianni and Maso. *An Introduction to* Γ*-Convergence*. *Progress in Nonlinear Differential Equations and Their Applications* **8**. Birkhäuser, Boston, 1993. MR1201152

[19] A. Hardy. A note on large deviations for 2D Coulomb gas with weakly confining potential. *Electron. Commun. Probab.* **17** (2012) 19, 12 pp. MR2926763

[20] F. Hiai and D. Petz. Logarithmic energy as an entropy functional. In *Advances in Differential Equations and Mathematical Physics* 205–221. *Contemporary Mathematics* **217**. American Mathematical Society, Providence, RI, 1998. MR1606719

[21] F. Hiai and D. Petz. A large deviation theorem for the empirical eigenvalue distribution of random unitary matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **36** (1) (2000) 71–85. MR1743092

[22] M. Mariani. A Γ-convergence approach to large deviations. Preprint. Available at https://arxiv.org/abs/1204.0640.

[23] E. Sandier and S. Serfaty. From the Ginzburg–Landau model to vortex lattice problems. *Comm. Math. Phys.* **313** (3) (2012) 635–743. MR2945619

[24] O. Zeitouni and S. Zelditch. Large deviations of empirical measures of zeros of random polynomials. *Int. Math. Res. Not.* **2010** (20) (2010) 3935–3992. MR2738347

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1402–1438
https://doi.org/10.1214/18-AIHP923

Edge of spiked beta ensembles, stochastic Airy semigroups and reflected Brownian motions

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Abstract. We access the edge of Gaussian beta ensembles with one spike by analyzing high powers of the associated tridiagonal matrix models. In the classical cases $\beta = 1, 2, 4$, this corresponds to studying the fluctuations of the largest eigenvalues of additive rank one perturbations of the GOE/GUE/GSE random matrices. In the infinite-dimensional limit, we arrive at a one-parameter family of random Feynman–Kac type semigroups, which features the stochastic Airy semigroup of Gorin and Shkolnikov (*Ann. Probab.* **46** (2018) 2287–2344) as an extreme case. Our analysis also provides Feynman–Kac formulas for the spiked stochastic Airy operators, introduced by Bloemendal and Virág (*Probab. Theory Related Fields* **156** (2013) 795–825). The Feynman–Kac formulas involve functionals of a reflected Brownian motion and its local times, thus, allowing to study the limiting operators by tools of stochastic analysis. We derive a first result in this direction by obtaining a new distributional identity for a reflected Brownian bridge conditioned on its local time at zero. A key feature of our proof consists of a novel strong invariance result for certain non-negative random walks and their occupation times that is based on the Skorokhod reflection map.

Résumé. Nous accédons à l'extrémité du spectre des ensembles bêta gaussiens avec perturbation de rang un par l'entremise de grandes puissances des matrices tridiagonales qui y sont associées. Pour les valeurs traditionnelles $\beta = 1, 2, 4$, ceci correspond à l'étude des fluctuations des valeurs propres maximales des matrices aléatoires GOE/GUE/GSE assujetties à une perturbation additive de rang un. En dimensions infinies, nos résultats nous mènent vers une famille de semi-groupes de type Feynman–Kac qui, dans un cas extrême, correspond au *stochastic Airy semigroup* introduit par Gorin et Shkolnikov (*Ann. Probab.* **46** (2018) 2287–2344). De plus, nos résultats ont pour corollaire des formules de Feynman–Kac pour les *spiked stochastic Airy operators* introduits par Bloemendal et Virág (*Probab. Theory Related Fields* **156** (2013) 795–825). Ces formules sont exprimées à l'aide de certaines fonctionnelles du mouvement brownien réfléchi et de ses temps locaux. Ce faisant, les opérateurs en question peuvent être étudiés à l'aide du calcul stochastique. Nous obtenons un premier résultat dans cette lignée en démontrant une nouvelle identité décrivant la distribution du mouvement brownien réfléchi ayant été conditionné sur son temps local à zéro. La principale innovation de notre démonstration consiste en la preuve d'un nouveau résultat sur l'approximation forte du mouvement brownien réfléchi et de son temps local par une marche aléatoire non négative en utilisant la méthode de réflexion de Skorokhod.

MSC: 60B20; 60H25; 47D08; 60J55

Keywords: Beta ensembles; Feynman–Kac formulas; Local times; Low rank perturbations; Moments method; Operator limits; Path transformations; Random tridiagonal matrices; Reflected Brownian motions; Skorokhod map; Stochastic Airy semigroups; Strong invariance principles

References

[1] G. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, 2010. MR2760897

[2] J. Baik, G. Ben Arous and S. Péché. Phase transition of the largest eigenvalue for nonnull complex sample covariance matrices. *Ann. Probab.* **33** (2005) 1643–1697. MR2165575

[3] R. Bass and D. Khoshnevisan. Strong approximations to Brownian local time. *Progr. Probab.* **33** (1992) 43–65. MR1278076

[4] R. F. Bass and D. Khoshnevisan. Laws of the iterated logarithm for local times of the empirical process. *Ann. Probab.* **23** (1995) 388–399. MR1330775

[5] K. E. Bassler, P. J. Forrester and N. E. Frankel. Edge effects in some perturbations of the Gaussian unitary ensemble. *J. Math. Phys.* **51** (2010), Article ID 123305. MR2779601
[6] R. N. Bhattacharya and R. R. Rao. *Normal Approximation and Asymptotic Expansions*. John Wiley & Sons, New York, 1976. MR0436272
[7] P. Billingsley. *Convergence of Probability Measures*, 2nd edition. John Wiley & Sons, New York, 1999. MR1700749
[8] A. Bloemendal and B. Virág. Limits of spiked random matrices I. *Probab. Theory Related Fields* **156** (2013) 795–825. MR3078286
[9] A. Bloemendal and B. Virág. Limits of spiked random matrices II. *Ann. Probab.* **44** (2016) 2726–2769. MR3531679
[10] X. Chen. *Random Walk Intersections: Large Deviations and Related Topics*. *Mathematical Surveys and Monographs* **157**. American Mathematical Society, Providence, 2010. MR2584458
[11] R. Dudley. *Uniform Central Limit Theorems*. *Cambridge Studies in Advanced Mathematics* **63**. Cambridge University Press, Cambridge, 1999. MR1720712
[12] I. Dumitriu and A. Edelman. Matrix models for beta ensembles. *J. Math. Phys.* **43** (2002) 5830–5847. MR1936554
[13] A. Edelman. Stochastic differential equations and random matrices. In *SIAM Conference on Applied Linear Algebra*. The College of William and Mary, Williamsburg, 2003. Available at http://math.mit.edu/~edelman/homepage/talks/siam2003.ppt.
[14] A. Edelman and B. Sutton. From random matrices to stochastic operators. *J. Stat. Phys.* **127** (2007) 1121–1165. MR2331033
[15] P. Forrester. *Log-Gases and Random Matrices*. *London Mathematical Society Monographs Series* **34**. Princeton University Press, Princeton, 2010. MR2641363
[16] V. Gorin and M. Shkolnikov. Stochastic Airy semigroup through tridiagonal matrices. *Ann. Probab.* **46** (2018) 2287–2344. Available at arXiv:1601.06800v1. MR3813993
[17] Y. Hariya. A pathwise interpretation of the Gorin–Shkolnikov identity. *Electron. Commun. Probab.* **21** (2016) 1–6. MR3533284
[18] D. Holcomb and G. R. M. Flores. Edge scaling of the β-Jacobi ensemble. *J. Stat. Phys.* **149** (2012) 1136–1160. MR3006323
[19] S. Jacquot and B. Valko. Bulk scaling limit of the Laguerre ensemble. *Electron. J. Probab.* **11** (2011) 314–346. MR2771139
[20] A. Javanmarda, A. Montanari and F. Ricci-Tersenghi. Phase transitions in semidefinite relaxations. *Proc. Natl. Acad. Sci. USA* **113** (2016) E2218–E2223. MR3494080
[21] R. L. Karandikar. On pathwise stochastic integration. *Stochastic Process. Appl.* **57** (1995) 11–18. MR1327950
[22] R. Kozhan. Rank one non-Hermitian perturbations of Hermitian β-ensembles of random matrices. *J. Stat. Phys.* **168** (2017) 92–108. MR3659979
[23] M. Krishnapur, B. Rider and B. Virág. Universality of the stochastic Airy operator. *Comm. Pure Appl. Math.* **69** (2016) 145–199. MR3433632
[24] G. Lawler and V. Limic. *Random Walk: A Modern Introduction*. *Cambridge Studies in Advanced Mathematics* **123**. Cambridge University Press, Cambridge, 2010. MR2677157
[25] P. Lax. *Functional Analysis*. John Wiley & Sons, New York, 2002. MR1892228
[26] I. G. Macdonald. *Symmetric Functions and Hall Polynomials*, 2nd edition. The Clarendon Press, Oxford University Press, New York, 2015. MR3443860
[27] S. Péché. The largest eigenvalue of small rank perturbations of Hermitian random matrices. *Probab. Theory Related Fields* **134** (2006) 127–173. MR2221787
[28] J. Pitman. The distribution of local times of a Brownian bridge. In *Séminaire de Probabilités XXXIII* 388–394. *Lecture Notes in Math.* **1709**. Springer-Verlag, Berlin, 1999. MR1768012
[29] J. Pitman. The SDE solved by local times of a Brownian excursion or bridge derived from the height profile of a random tree or forest. *Ann. Probab.* **27** (1999) 261–283. MR1681110
[30] J. Pitman. *Combinatorial Stochastic Processes*. *Lecture Notes in Math.* **1875**. Springer-Verlag, Berlin, 2006. MR2245368
[31] J. Ramírez and B. Rider. Diffusion at the random matrix hard edge. *Comm. Math. Phys.* **288** (2009) 887–906. MR2504858
[32] J. Ramírez, B. Rider and B. Virág. Beta ensembles, stochastic Airy spectrum, and a diffusion. *J. Amer. Math. Soc.* **24** (2011) 919–944. MR2813333
[33] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Grundlehren der Mathematischen Wissenschaften* **293**. Springer-Verlag, Berlin, 1999. MR1725357
[34] B. Rider and P. Waters. Universality of the stochastic Bessel operator. Preprint, 2016. Available at arXiv:1610.01637.
[35] H. F. Trotter. A property of Brownian motion paths. *Illinois J. Math.* **2** (1958) 425–433. MR0096311
[36] B. Valkó and B. Virág. Continuum limits of random matrices and the Brownian carousel. *Invent. Math.* **177** (2009) 463–508. MR2534097
[37] B. Valkó and B. Virág. Operator limit of the circular beta ensemble, 2017. Available at arXiv:1710.06988v1.
[38] B. Valkó and B. Virág. The sine_β operator. *Invent. Math.* **209** (2017) 275–327. MR3660310

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1439–1476
https://doi.org/10.1214/18-AIHP924

Pathwise differentiability of reflected diffusions in convex polyhedral domains

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Abstract. Reflected diffusions in convex polyhedral domains arise in a variety of applications, including interacting particle systems, queueing networks, biochemical reaction networks and mathematical finance. Under suitable conditions on the data, we establish pathwise differentiability of such a reflected diffusion with respect to its defining parameters – namely, its initial condition, drift and diffusion coefficients, and (oblique) directions of reflection along the boundary of the domain. We characterize the right-continuous regularization of a pathwise derivative of the reflected diffusion as the pathwise unique solution to a constrained linear stochastic differential equation with jumps whose drift and diffusion coefficients, domain and directions of reflection depend on the state of the reflected diffusion. Previous work in the multidimensional context has been largely restricted to the study of differentiability of stochastic flows for (normally) reflected Brownian motions. A key difficulty is to identify a suitable linearization of the dynamics of the local time process, especially in the presence of a non-smooth boundary. We take a new approach that uses properties of directional derivatives of the associated extended Skorokhod map, and their characterization in terms of the so-called derivative problem. The proof involves establishing certain path properties of the reflected diffusion at nonsmooth parts of the boundary of the polyhedral domain, which may be independent interest, and proving that pathwise derivatives of reflected diffusions can be characterized in terms of directional derivatives of the extended Skorokhod map. As a corollary, we obtain a probabilistic representation for derivatives of expectations of functionals of reflected diffusions, which is useful for sensitivity analysis of reflected diffusions.

Résumé. Les diffusions réfléchies dans les domaines polyhédriques convexes apparaissent dans diverses applications, notamment les systèmes de particules, les réseaux de files d'attente, les réseaux de réaction biochimique et la finance mathématique. Dans des conditions appropriées sur les données, nous établissons la différentiabilité selon la trajectoire d'une telle diffusion réfléchie par rapport à ses paramètres de définition, tel que sa condition initiale, ses coefficients de dérive et de diffusion et ses directions (obliques) de réflexion le long des limites du domaine. Nous caractérisons la régularisation continue droite d'une dérivée selon la trajectorie de la diffusion réfléchie comme solution unique à une équation différentielle stochastique linéaire contrainte avec sauts dont les coefficients de dérive et de diffusion, le domaine et les directions de réflexion dépendent de l'état de la diffusion réfléchie. Les travaux antérieurs dans le contexte multidimensionnel ont été en grande partie limités à l'étude de la différentiabilité des flux stochastiques des mouvements browniens (normalement) réfléchis. Une difficulté essentielle consiste à identifier une linéarisation appropriée de la dynamique du processus de temps local, en particulier en présence d'une frontière non lisse. Nous adoptons une nouvelle approche qui utilise les propriétés des dérivées directionnelles de l'application de Skorokhod étendue associée et leur caractérisation en termes d'une problème dérivé. La preuve implique l'établissement de certaines propriétés des trajectoires de la diffusion réfléchie aux frontieres non lisses du domaine polyhédral, qui peut être un resultat d'intérêt indépendant, et de démontrer que les dérivées selon la trajectorie des diffusions réfléchies peuvent être caractérisées en termes de dérivées directionnelles de l'application de Skorokhod étendu. En corollaire, nous obtenons une représentation probabiliste pour les dérivées des valeurs d'attendues des fonctionnelles de diffusions réfléchies, ce qui est utile pour l'analyse de sensibilité des diffusions réfléchies.

MSC: Primary 60G17; 90C31; 93B35; secondary 60H07; 60H10; 65C30

Keywords: Reflected diffusion; Reflected Brownian motion; Boundary jitter property; Derivative process; Pathwise differentiability; Stochastic flow; Sensitivity analysis; Directional derivative of the extended Skorokhod map; Derivative problem

References

[1] S. Andres. Pathwise differentiability for SDEs in a convex polyhedron with oblique reflection. *Ann. Inst. Henri Poincaré Probab. Stat.* **45** (1) (2009) 104–116. MR2500230

[2] S. Andres. Pathwise differentiability for SDEs in a smooth domain with reflection. *Electron. J. Probab.* **16** (28) (2011) 845–879. MR2793243

[3] A. D. Banner, R. Fernholz and I. Karatzas. Atlas models of equity markets. *Ann. Appl. Probab.* **15** (4) (2005) 2296–2330. MR2187296

[4] M. Bossy, M. Cissé and D. Talay. Stochastic representations of derivatives of solutions of one-dimensional parabolic variational inequalities with Neumann boundary conditions. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** (2) (2011) 395–424. MR2814416

[5] A. Budhiraja and C. Lee. Long time asymptotics for constrained diffusions in polyhedral domains. *Stochastic Process. Appl.* **117** (8) (2007) 1014–1036. MR2340877

[6] K. Burdzy. Differentiability of stochastic flow of reflected Brownian motions. *Electron. J. Probab.* **14** (75) (2009) 2182–2240. MR2550297

[7] K. Burdzy and D. Nualart. Brownian motion reflected on Brownian motion. *Probab. Theory Related Fields* **122** (4) (2002) 471–493. MR1902187

[8] H. Chen and A. Mandelbaum. Leontief systems, RBVs and RBMs. In *Applied Stochastic Analysis (London, 1989)* 1–43. M. H. A. Davis and R. J. Elliott (Eds). Gordon and Breach, New York, 1991. MR1108415

[9] H. Chen and X. Shen. Strong approximations for multiclass feedforward queueing networks. *Ann. Probab.* **10** (3) (2000) 828–876. MR1789981

[10] C. Costantini, E. Gobet and N. El Karoui. Boundary sensitivities for diffusion processes in time dependent domains. *Appl. Math. Optim.* **54** (2006) 159–187. MR2239532

[11] J. D. Deuschel and L. Zambotti. Bismut-Elworthy's formula and random walk representation for SDEs with reflection. *Stochastic Process. Appl.* **115** (6) (2005) 907–925. MR2134484

[12] A. B. Dieker and X. Gao. Sensitivity analysis for diffusion processes constrained to an orthant. *Ann. Appl. Probab.* **24** (5) (2014) 1918–1945. MR3226168

[13] P. Dupuis and H. Ishii. On Lipschitz continuity of the solution mapping to the Skorokhod problem with applications. *Stoch. Stoch. Rep.* **35** (1991) 31–62. MR1110990

[14] P. Glasserman. *Monte Carlo Methods in Financial Engineering*. Springer, New York, 2003. MR1999614

[15] T. Ichiba, V. Papathanakos, A. Banner, I. Karatzas and R. Fernholz. Hybrid atlas models. *Ann. Appl. Probab.* **21** (2) (2011) 609–644. MR2807968

[16] W. Kang and K. Ramanan. A Dirichlet process characterization of a class of reflected diffusions. *Ann. Probab.* **38** (3) (2010) 1062–1105. MR2674994

[17] W. Kang and K. Ramanan. On the submartingale problem for reflected diffusions in domains with piecewise smooth boundaries. *Ann. Probab.* **45** (2017) 404–468. MR3601653

[18] W. Kang and R. J. Williams. An invariance principle for semimartingale reflecting Brownian motions in domains with piecewise smooth boundaries. *Ann. Appl. Probab.* **17** (2) (2007) 741–779. MR2308342

[19] H. Kunita. *Stochastic Flows and Stochastic Differential Equations*. *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, UK, 1997. MR1472487

[20] D. Lipshutz and K. Ramanan. A Monte Carlo method for estimating sensitivities of reflected diffusions in convex polyhedral domains. *Stoch. Syst.* **9** (2019) 101–140.

[21] D. Lipshutz and K. Ramanan. On directional derivatives of Skorokhod maps in convex polyhedral domains. *Ann. Appl. Probab.* **28** (2) (2018) 688–750. MR3784487

[22] P. Malliavin and A. Thalmaier. *Stochastic Calculus of Variations in Mathematical Finance*. Springer, Berlin, 2006. MR2189710

[23] A. Mandelbaum and W. A. Massey. Strong approximations for time-dependent queues. *Math. Oper. Res.* **20** (1) (1995) 33–64. MR1320446

[24] A. Mandelbaum and G. Pats. State-dependent stochastic networks. Part I. Approximations and applications with continuous diffusion limits. *Ann. Appl. Probab.* **8** (2) (1998) 569–646. MR1624965

[25] A. Mandelbaum and K. Ramanan. Directional derivatives of oblique reflection maps. *Math. Oper. Res.* **35** (3) (2010) 527–558. MR2724063

[26] M. Metivier. Pathwise differentiability with respect to a parameter of solutions of stochastic differential equations. *Sémin. Probab. Strasbg.* **16** (1982) 490–502. MR0658709

[27] W. P. Peterson. A heavy traffic limit theorem for networks of queues with multiple customer types. *Math. Oper. Res.* **16** (1) (1991) 90–118. MR1106792

[28] A. Pilipenko. Differentiability of stochastic reflecting flow with respect to starting point. *Commun. Stoch. Anal.* **7** (1) (2013) 17–37. MR3080985

[29] K. Ramanan. Reflected diffusions defined via the extended Skorokhod map. *Electron. J. Probab.* **11** (36) (2006) 934–992. MR2261058

[30] K. Ramanan and M. I. Reiman. Fluid and heavy traffic diffusion limits for a generalized processor sharing model. *Ann. Appl. Probab.* **13** (1) (2003) 100–139. MR1951995

[31] K. Ramanan and M. I. Reiman. The heavy traffic limit of an unbalanced generalized processor sharing model. *Ann. Appl. Probab.* **18** (1) (2008) 22–58. MR2380890

[32] M. I. Reiman. Open queueing networks in heavy traffic. *Math. Oper. Res.* **9** (3) (1984) 441–458. MR0757317

[33] L. C. G. Rogers and D. Williams. *Diffusions, Markov Processes and Martingales. Volume 2: Itô Calculus*, 2nd edition. Cambridge University Press, Cambridge, UK, 2000. MR1780932

[34] D. W. Stroock and S. R. S. Varadhan. On the support of diffusion processes with applications to the strong maximum principle. In *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability. Volume 3: Probability Theory* 333–359. University of California Press, Berkeley, CA, 1972. MR0400425

[35] S. R. S. Varadhan and R. J. Williams. Brownian motion in a wedge with oblique reflection. *Comm. Pure Appl. Math.* **38** (4) (1984) 405–443. MR0792398
[36] J. Warren. Dyson's Brownian motions, intertwining and interlacing. *Electron. J. Probab.* **12** (2007) 573–590. MR2299928
[37] W. Whitt. An introduction to stochastic-process limits and their application to queues. Internet supplement, 2002. Available at http://www.columbia.edu/~ww2040/supplement.html.
[38] J. Yang and H. J. Kushner. A Monte Carlo method for sensitivity analysis and parametric optimization of nonlinear stochastic systems. *SIAM J. Control Optim.* **29** (5) (1991) 1216–1249. MR1110093
[39] L. Zambotti. Fluctuations for a $\nabla\phi$ interface model with repulsion from a wall. *Probab. Theory Related Fields* **129** (3) (2004) 315–339. MR2128236

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1477–1508
https://doi.org/10.1214/18-AIHP925

Einstein relation and linear response in one-dimensional Mott variable-range hopping

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Abstract. We consider one-dimensional Mott variable-range hopping. This random walk is an effective model for the phonon-induced hopping of electrons in disordered solids within the regime of strong Anderson localization at low carrier density. We introduce a bias and prove the linear response as well as the Einstein relation, under an assumption on the exponential moments of the distances between neighboring points. In a previous paper (*Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2018) 1165–1203) we gave conditions on ballisticity, and proved that in the ballistic case the environment viewed from the particle approaches, for almost any initial environment, a given steady state which is absolutely continuous with respect to the original law of the environment. Here, we show that this bias-dependent steady state has a derivative at zero in terms of the bias (linear response), and use this result to get the Einstein relation. Our approach is new: instead of using e.g. perturbation theory or regeneration times, we show that the Radon–Nikodym derivative of the bias-dependent steady state with respect to the equilibrium state in the unbiased case satisfies an L^p-bound, $p > 2$, uniformly for small bias. This L^p-bound yields, by a general argument not involving our specific model, the statement about the linear response.

Résumé. Nous considérons le modèle « Mott variable-range hopping ». Cette marche aléatoire décrit la conduction des electrons dans des solides désordonnés dans le régime de localisation forte d'Anderson lorsque la densité des porteurs de charge est faible. En particulier, nous considérons une marche aléatoire de Mott unidimensionelle soumise à un champ extérieur. Sous une hypothèse à propos des moments exponentiels de la distance entre les points consécutifs, nous montrons la réponse linéaire et la relation d'Einstein. Dans un travail précedent, voir (*Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2018) 1165–1203), nous avons donné des conditions pour la ballisticité de la marche. En plus, nous avons montré que l'environnement vu de la particule converge en loi (pour presque tous les points de départ) vers une mesure invariante (état stationnaire) qui est absolument continue par rapport à la loi originale de l'environnement. Ici, nous montrons que cet état stationnaire a une dérivée en zéro par rapport au bias (réponse linéaire), et nous utilisons ce résultat pour démontrer la relation d'Einstein. Notre méthode est nouvelle : au lieu d'utiliser des arguments perturbatifs ou des temps de régéneration, nous donnons une borne en L^p, $p > 2$, pour la densité de l'état stationnaire par rapport à la mesure invariante sans biais. Cette borne est uniforme dans le biais pour des biais qui sont proches de zéro. L'argument pour déduire la réponse linéaire de cette borne est général et ne dépend pas des détails de notre modèle.

MSC: 60K37; 60J25; 60G55; 82D30

Keywords: Mott variable-range hopping; Random walk in random environment; Random conductance model; Environment seen from the particle; Steady states; Linear response; Einstein relation

References

[1] V. Ambegoakar, B. I. Halperin and J. S. Langer. Hopping conductivity in disordered systems. *Phys. Rev. B* **4** (1971) 2612–2620.
[2] L. Avena, O. Blondel and A. Faggionato. Analysis of random walks in dynamic random environments via L^2-perturbations. *Stochastic Process. Appl.* **128** (2018) 3490–3530. MR3849817

[3] G. Ben Arous, Y. Hu, S. Olla and O. Zeitouni. Einstein relation for biased random walk on Galton–Watson trees. *Ann. Inst. Henri Poincaré Probab. Stat.* **49** (2013) 698–721. MR3112431
[4] A. Bovier. Metastability: A potential theoretic approach. In *International Congress of Mathematicians. Vol. III* 499–518. Eur. Math. Soc., Zürich, 2006. MR2275693
[5] A. Bovier and F. den Hollander. *Metastability: A Potential Theoretic Approach*. Springer Verlag, Berlin, 2015. MR3445787
[6] P. Caputo and A. Faggionato. Diffusivity in one-dimensional generalized Mott variable-range hopping models. *Ann. Appl. Probab.* **19** (2009) 1459–1494. MR2538077
[7] F. Comets and S. Popov. Ballistic regime for random walks in random environment with unbounded jumps and Knudsen billiards. *Ann. Inst. Henri Poincaré Probab. Stat.* **48** (2012) 721–744. MR2976561
[8] D. J. Daley and D. Vere-Jones. *An Introduction to the Theory of Point Processes*. Springer Verlag, New York, 1988. MR0950166
[9] A. De Masi, P. A. Ferrari, S. Goldstein and W. D. Wick. An invariance principle for reversible Markov processes. Applications to random motions in random environments. *J. Stat. Phys.* **55** (1989) 787–855. MR1003538
[10] A. Faggionato, N. Gantert and M. Salvi. The velocity of 1d Mott variable range hopping with external field. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2018) 1165–1203. MR3825879
[11] A. Faggionato and M. Salvi. Regularity of biased 1D random walks in random environment. Preprint, 2018. Available at arXiv:1802.07874.
[12] A. Faggionato and F. Martinelli. Hydrodynamic limit of a disordered lattice gas. *Probab. Theory Related Fields* **127** (2003) 535–608. MR2021195
[13] A. Faggionato and P. Mathieu. Mott law as upper bound for a random walk in a random environment. *Comm. Math. Phys.* **281** (2008) 263–286. MR2403611
[14] A. Faggionato, H. Schulz-Baldes and D. Spehner. Mott law as lower bound for a random walk in a random environment. *Comm. Math. Phys.* **263** (2006) 21–64. MR2207323
[15] P. Franken, D. König, U. Arndt and V. Schmidt. *Queues and Point Processes*. John Wiley and Sons, Chichester, 1982. MR0691670
[16] N. Gantert, X. Guo and J. Nagel. Einstein relation and steady states for the random conductance model. *Ann. Probab.* **45** (2017) 2533–2567. MR3693969
[17] N. Gantert, P. Mathieu and A. Piatnitski. Einstein relation for reversible diffusions in a random environment. *Comm. Pure Appl. Math.* **65** (2012) 187–228. MR2855544
[18] X. Guo. Einstein relation for random walks in random environment. *Ann. Probab.* **44** (2016) 324–359. MR3456340
[19] C. Kipnis and S. R. S. Varadhan. Central limit theorem for additive functionals of reversible Markov processes and applications to simple exclusion. *Comm. Math. Phys.* **104** (1986) 1–19. MR0834478
[20] T. Komorowski, C. Landim and S. Olla. *Fluctuations in Markov Processes*. *Grundlehren der Mathematischen Wissenschaften* **345**. Springer Verlag, Berlin, 2012. MR2952852
[21] T. Komorowski and S. Olla. Einstein relation for random walks in random environments. *Stochastic Process. Appl.* **115** (2005) 1279–1301. MR2152375
[22] T. Komorowski and S. Olla. On mobility and Einstein relation for tracers in time-mixing random environments. *J. Stat. Phys.* **118** (2005) 407–435. MR2123642
[23] S. M. Kozlov. The method of averaging and walks in inhomogeneous environments. *Russian Math. Surveys* **40** (1985) 73–145. MR0786087
[24] J. Kurkijärvi. Hopping conductivity in one dimension. *Phys. Rev. B* **8** (1973) 922–924.
[25] H.-C. Lam and J. Depauw. Einstein relation for reversible random walks in random environment on $\mathbb{Z}$. *Stochastic Process. Appl.* **126** (2016) 983–996. MR3461187
[26] C. Landim, S. Olla and S. B. Volchan. Driven tracer particle in 1 dimensional symmetric simple exclusion. *Comm. Math. Phys.* **192** (1998) 287–307. MR1617558
[27] J. L. Lebowitz and H. Rost. The Einstein relation for the displacement of a test particle in a random environment. *Stochastic Process. Appl.* **54** (1994) 183–196. MR1307334
[28] M. Loulakis. Einstein relation for a tagged particle in simple exclusion processes. *Comm. Math. Phys.* **229** (2005) 347–367. MR1923179
[29] M. Loulakis. Mobility and Einstein relation for a tagged particle in asymmetric mean zero random walk with simple exclusion. *Ann. Inst. Henri Poincaré Probab. Stat.* **41** (2005) 237–254. MR2124642
[30] P. Maillard and O. Zeitouni. Performance of the Metropolis algorithm on a disordered tree: The Einstein relation. *Ann. Appl. Probab.* **24** (2014) 2070–2090. MR3226172
[31] P. Mathieu and A. Piatnitski. Quenched invariance principles for random walks on percolation clusters. *Proc. Roy. Soc. Edinburgh Sect. A* **463** (2007) 2287–2307. MR2345229
[32] P. Mathieu and A. Piatnitski. Steady states, fluctuation-dissipation theorems and homogenization for diffusions in a random environment with finite range of dependence. Available at arXiv:1601.02944. MR3840915
[33] R. E. Megginson. *An Introduction to Banach Space Theory*. Springer Verlag, New York, 1998. MR1650235
[34] A. Miller and E. Abrahams. Impurity conduction at low concentrations. *Phys. Rev.* **120** (1960) 745–755.
[35] N. F. Mott. On the transition to metallic conduction in semiconductors. *Can. J. Phys.* **34** (1956) 1356–1368.
[36] N. F. Mott. Conduction in glasses containing transition metal ions. *J. Non-Crystal. Solids* **1** (1968) 1–17.
[37] N. F. Mott. Conduction in non-crystalline materials III. Localized states in a pseudogap and near extremities of conduction and valence bands. *Philos. Mag.* **19** (1969) 835–852.
[38] N. F. Mott and E. A. Davis. *Electronic Processes in Non-Crystaline Materials*. Oxford University Press, New York, 1979.
[39] M. Pollak, M. Ortuño and A. Frydman. *The Electron Glass*. Cambridge University Press, Cambridge, 2013.
[40] J. Quastel. Bulk diffusion in a system with site disorder. *Ann. Probab.* **34** (2006) 1990–2036. MR2271489
[41] M. Reed and B. Simon. *Methods of Modern Mathematical Physics. Functional Analysis. Vol. I*. Academic Press, San Diego, 1980. MR0751959
[42] B. Shklovskii and A. L. Efros. *Electronic Properties of Doped Semiconductors*. Springer Verlag, Berlin, 1984.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1509–1530
https://doi.org/10.1214/18-AIHP926

Limit laws for self-loops and multiple edges in the configuration model

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Abstract. We consider self-loops and multiple edges in the configuration model as the size of the graph tends to infinity. The interest in these random variables is due to the fact that the configuration model, conditioned on being simple, is a uniform random graph with prescribed degrees. Simplicity corresponds to the absence of self-loops and multiple edges.

We show that the number of self-loops and multiple edges converges in distribution to two independent Poisson random variables when the second moment of the empirical degree distribution converges. We also provide estimations on the total variation distance between the numbers of self-loops and multiple edges and their limits, as well as between the sum of these values and the Poisson random variable to which this sum converges to. This revisits previous works of Bollobás, of Janson, of Wormald and others. The error estimates also imply sharp asymptotics for the number of simple graphs with prescribed degrees.

The error estimates follow from an application of the Stein–Chen method for Poisson convergence, which is a novel method for this problem. The asymptotic independence of self-loops and multiple edges follows from a Poisson version of the Cramér–Wold device using thinning, which is of independent interest.

When the degree distribution has infinite second moment, our general results break down. We can, however, prove a central limit theorem for the number of self-loops, and for the multiple edges between vertices of degrees much smaller than the square root of the size of the graph. Our results and proofs easily extend to directed and bipartite configuration models.

Résumé. Nous considérons les boucles et les arêtes multiples dans le modèle de configuration lorsque la taille du graphe tend vers l'infini. L'intérêt de ces variables aléatoires est dû au fait que le modèle de configuration, conditionné à la simplicité, est un graphe aléatoire uniforme avec des degrés prescrits. La simplicité correspond à l'absence des boucles et des arêtes multiples.

Nous montrons que le nombre des boucles et des arêtes multiples converge en loi vers deux variables aléatoires indépendantes qui suivent des lois de Poisson lorsque le moment d'ordre 2 de la loi empirique des degrés converge. Nous fournissons aussi des estimations des distances de variation totale entre les nombres des boucles et des arêtes multiples et leurs limites, ainsi qu'entre la somme de ces nombres et la variable aléatoire, qui suit une loi de Poisson, vers laquelle converge cette somme. Cela revisite les œuvres précédentes de Bollobás comme de Janson, de Wormald, et d'autres. Les estimations d'erreur impliquent également une asymptotique précise pour le nombre de graphes simples avec des degrés prescrits.

Les estimations d'erreur découlent d'une application de la méthode de Stein–Chen pour la convergence vers une loi de Poisson, qui est une nouvelle méthode pour ce problème. L'indépendance asymptotique des boucles et des arêtes multiples suit à partir d'une version Poisson du dispositif Cramér–Wold utilisant l'amincissement, qui est intéressant en lui-même.

Lorsque la loi des degrés a un moment d'ordre 2 infini, nos résultats généraux échouent. Nous pouvons, cependant, prouver un théorème de la limite centrale pour le nombre des boucles, et pour les arêtes multiples entre sommets avec degrés beaucoup plus petits que la racine carrée de la taille du graphe. Nos résultats et preuves peuvent facilement s'étendre aux modèles de configuration orientés et bipartis.

MSC: 60K35; 60K37; 82B43

Keywords: Configuration model; Self-loops; Multiple edges; Chen–Stein Poisson approximation

References

[1] R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. *Rev. Modern Phys.* **74** (1) (2002) 47–97. MR1895096

[2] N. Alon and J. Spencer. *The Probabilistic Method*, 2nd edition. *Wiley-Interscience Series in Discrete Mathematics and Optimization.* John Wiley & Sons, New York, 2000. MR1885388

[3] A. Barbour, L. Holst and S. Janson. *Poisson Approximation*. *Oxford Studies in Probability.* **2**. The Clarendon Press Oxford University Press, New York, 1992. MR1163825

[4] E. A. Bender and E. R. Canfield. The asymptotic number of labelled graphs with given degree sequences. *J. Combin. Theory Ser. A* **24** (1978) 296–307. MR0505796

[5] J. Blanchet and A. Stauffer. Characterizing optimal sampling of binary contingency tables via the configuration model. *Random Structures Algorithms* **42** (2) (2013) 159–184. MR3019396

[6] B. Bollobás. A probabilistic proof of an asymptotic formula for the number of labelled regular graphs. *European J. Combin.* **1** (4) (1980) 311–316. MR0595929

[7] B. Bollobás. *Random Graphs*, 2nd edition. *Cambridge Studies in Advanced Mathematics* **73**. Cambridge University Press, Cambridge, 2001. MR1864966

[8] T. Britton, M. Deijfen and A. Martin-Löf. Generating simple random graphs with prescribed degree distribution. *J. Stat. Phys.* **124** (6) (2006) 1377–1397. MR2266448

[9] C. Cooper and A. Frieze. The size of the largest strongly connected component of a random digraph with a given degree sequence. *Combin. Probab. Comput.* **13** (3) (2004) 319–337. MR2056402

[10] P. Gao and N. Wormald. Enumeration of graphs with a heavy-tailed degree sequence. *Adv. Math.* **287** (2016) 412–450. MR3422681

[11] C. Holmgren and S. Janson. Using Stein's method to show Poisson and normal limit laws for fringe subtrees. *Discrete Math. Theor. Comput. Sci.* **BA** (2014) 169–180. MR3525973

[12] C. Holmgren and S. Janson. Limit laws for functions of fringe trees for binary search trees and random recursive trees. *Electron. J. Probab.* **20** (4) (2015) 1–51. MR3311217

[13] S. Janson. The probability that a random multigraph is simple. *Combin. Probab. Comput.* **18** (1–2) (2009) 205–225. MR2497380

[14] S. Janson. The probability that a random multigraph is simple. II. *J. Appl. Probab.* **51A** (2014) 123–137. MR3317354

[15] S. Janson, M. Luczak and P. Windridge. Law of large numbers for the sir epidemic on a random graph with given degrees. *Random Structures Algorithms* **45** (4) (2014) 724–761. MR3275704

[16] B. D. McKay and N. C. Wormald. Asymptotic enumeration by degree sequence of graphs with degrees $o(n^{1/2})$. *Combinatorica* **11** (4) (1991) 369–382. MR1137769

[17] M. Molloy and B. Reed. A critical point for random graphs with a given degree sequence. *Random Structures Algorithms* **6** (2–3) (1995) 161–179. MR1370952

[18] M. Molloy and B. Reed. The size of the giant component of a random graph with a given degree sequence. *Combin. Probab. Comput.* **7** (3) (1998) 295–305. MR1664335

[19] M. E. J. Newman. The structure and function of complex networks. *SIAM Rev.* **45** (2) (2003) 167–256 (electronic). MR2010377

[20] M. E. J. Newman. Random graphs as models of networks. In *Handbook of Graphs and Networks* 35–68. Wiley-VCH, Weinheim, 2003. MR2016118

[21] M. E. J. Newman, S. Strogatz and D. Watts. Random graphs with arbitrary degree distribution and their application. *Phys. Rev. E* **64** (2000) 026118.

[22] R. van der Hofstad. *Random Graphs and Complex Networks*. *Cambridge Series in Statistical and Probabilistic Mathematics* **1**. Cambridge University Press, Cambridge, 2017. MR3617364

[23] R. van der Hofstad and J. Komjáthy. When is a scale-free graph ultra-small?. *Journ. Stat. Phys.* **169** (2017) 223–264. MR3704860

[24] P. van der Hoorn and N. Litvak. Upper bounds for number of removed edges in the erased configuration model. In *Algorithms and Models for the Web Graph* 54–65. *Lecture Notes in Comput. Sci.* **9479**. Springer, Cham, 2015. MR3500668

[25] N. C. Wormald. The asymptotic distribution of short cycles in random regular graphs. *J. Combin. Theory Ser. B* **31** (2) (1981) 168–182. MR0630980

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1531–1565
https://doi.org/10.1214/18-AIHP927

Gravitation versus Brownian motion

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Abstract. We investigate the motion of an inert (massive) particle being impinged from below by a particle performing (reflected) Brownian motion. The velocity of the inert particle increases in proportion to the local time of collisions and decreases according to a constant downward gravitational acceleration. We study fluctuations and strong laws of the motion of the particles. We further show that the joint distribution of the velocity of the inert particle and the gap between the two particles converges in total variation distance to a stationary distribution which has an explicit product form.

Résumé. Nous étudions le mouvement d'une particule inerte (massive) qui est frappé par en dessous par une particule effectuant un mouvement brownien (réfléchi). La vitesse de la particule inerte augmente proportionnellement au temps local des collisions et diminue en fonction d'une accélération gravitationnelle constante vers le bas. Nous étudions les fluctuations et les lois fortes du mouvement des particules. Enfin, nous montrons que la distribution conjointe de la vitesse de la particule inerte et de l'écart entre les deux particules converge pour la distance de variation totale vers une distribution stationnaire qui a une forme produit explicite.

MSC: Primary 60J65; secondary 60J55

Keywords: Brownian motion; Inert drift; Local time; Gravitation; Total variation distance

References

[1] S. Banerjee and D. Mukherjee. Join-the-shortest queue diffusion limit in Halfin–Whitt regime: Tail asymptotics and scaling of extrema, 2018. Available at arXiv:1803.03306.
[2] R. F. Bass, K. Burdzy, Z.-Q. Chen and M. Hairer. Stationary distributions for diffusions with inert drift. *Probab. Theory Related Fields* **146** (1–2) (2010) 1–47. MR2550357
[3] J. Bertoin. *Lévy Processes*. *Cambridge Tracts in Mathematics* **121**. Cambridge University Press, Cambridge, 1996. MR1406564
[4] K. Burdzy and D. White. Markov processes with product-form stationary distribution. *Electron. Commun. Probab.* **13** (2008) 614–627. MR2461535
[5] P. Eschenfeldt and D. Gamarnik. Join the shortest queue with many servers. The heavy-traffic asymptotics. *Math. Oper. Res.* **43** (3) (2018) 867–886.
[6] N. Ikeda and S. Watanabe. *Stochastic Differential Equations and Diffusion Processes*, 2nd edition. *North-Holland Mathematical Library* **24**. North-Holland, Amsterdam, 1989. MR0637061
[7] W. Kang and K. Ramanan. Characterization of stationary distributions of reflected diffusions. *Ann. Appl. Probab.* **24** (4) (2014) 1329–1374. MR3210998
[8] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*, 2nd edition. *Graduate Texts in Mathematics* **113**. Springer, New York, 1991. MR1121940
[9] F. B. Knight. On the path of an inert object impinged on one side by a Brownian particle. *Probab. Theory Related Fields* **121** (4) (2001) 577–598. MR1872429
[10] A. E. Kyprianou. *Introductory Lectures on Fluctuations of Lévy Processes with Applications*. Springer, Berlin, 2006. MR2250061
[11] S. M. Ross. *Introduction to Probability Models*. Academic Press, San Diego, 2014. MR3307945
[12] D. W. Stroock and S. R. S. Varadhan. Diffusion processes with boundary conditions. *Comm. Pure Appl. Math.* **24** (1971) 147–225. MR0277037

[13] H. Thorisson. *Coupling, Stationarity, and Regeneration*. Springer, New York, 2000. MR1741181

[14] S. Watanabe. On stochastic differential equations for multi-dimensional diffusion processes with boundary conditions. *J. Math. Kyoto Univ.* **11** (1971) 169–180. MR0275537

[15] D. White. Processes with inert drift. *Electron. J. Probab.* **12** (55) (2007) 1509–1546. MR2365876

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1566–1598
https://doi.org/10.1214/18-AIHP928

On the roughness of the paths of RBM in a wedge

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Abstract. Reflected Brownian motion (RBM) in a wedge is a 2-dimensional stochastic process Z whose state space in $\mathbb{R}^2$ is given in polar coordinates by $S = \{(r,\theta) : r \geq 0, 0 \leq \theta \leq \xi\}$ for some $0 < \xi < 2\pi$. Let $\alpha = (\theta_1 + \theta_2)/\xi$, where $-\pi/2 < \theta_1, \theta_2 < \pi/2$ are the directions of reflection of Z off each of the two edges of the wedge as measured from the corresponding inward facing normal. We prove that in the case of $1 < \alpha < 2$, RBM in a wedge is a Dirichlet process. Specifically, its unique Doob-Meyer type decomposition is given by $Z = X + Y$, where X is a two-dimensional Brownian motion and Y is a continuous process of zero energy. Furthermore, we show that for $p > \alpha$, the strong p-variation of the sample paths of Y is finite on compact intervals, and, for $0 < p \leq \alpha$, the strong p-variation of Y is infinite on $[0, T]$ whenever Z has been started from the origin. We also show that on excursion intervals of Z away from the origin, (Z, Y) satisfies the standard Skorokhod problem for X. However, on the entire time horizon (Z, Y) does not satisfy the standard Skorokhod problem for X, but nevertheless we show that it satisfies the extended Skorkohod problem.

Résumé. Le mouvement Brownien réfléchi (RBM) dans un coin est un processus stochastique 2-dimensionnel Z dont l'espace d'états dans $\mathbb{R}^2$ est donné en coordonnées polaires par $S = \{(r,\theta) : r \geq 0, 0 \leq \theta \leq \xi\}$ pour un $0 < \xi < 2\pi$. Soit $\alpha = (\theta_1 + \theta_2)/\xi$, où $-\pi/2 < \theta_1, \theta_2 < \pi/2$ sont les angles de réflexion de Z sur chacun des côtés du cône, mesurés à partir des normales rentrantes correspondantes. Nous montrons que dans le cas $1 < \alpha < 2$, le RBM dans un coin est un processus de Dirichlet. Plus précisément, son unique décomposition de Doob-Meyer est donnée par $Z = X + Y$, où X est un mouvement brownien 2-dimensionnel et Y est un processus continu d'énergie zéro. De plus, nous montrons que pour $p > \alpha$, la p-variation forte des trajectoires de Y est finie sur les intervalles compacts, et, pour $0 < p \leq \alpha$, la p-variation forte de Y est infinie sur $[0, T]$ dès que Z est issu de l'origine. Nous montrons aussi que sur les intervalles d'excursions de Z en dehors de l'origine, (Z, Y) satisfait le problème de Skorokhod standard pour X. Sur l'intervalle de temps infini, (Z, Y) ne satisfait pas le problème de Skorokhod standard pour X, mais satisfait néanmoins le problème de Skorokhod étendu.

MSC: 60J65; 60J27; 60G17; 60G52; 60J55

Keywords: Reflected Brownian motion; Extended Skorokhod problem; Dirichlet process; p-variation

References

[1] J. Bertoin. Subordinators: Examples and applications. In *Lectures on Probability Theory and Statistics (Saint-Flour, 1997)* 1–91. *Lecture Notes in Math.* **1717**. Springer, Berlin, 1999. MR1746300
[2] R. Blumenthal and R. Getoor. *Markov Processes and Potential Theory*. Academic Press, New York, 1968. MR0264757
[3] J. Bretagnolle. p-variation de fonctions aléatoires lère partie: Séries de Rademacher. In *Séminaire de Probabilités VI, Université de Strasbourg* 51–63. Springer, Berlin, 1972. MR0373019
[4] A. Brown and C. Pearcy. *Introduction to Operator Theory*. Springer, New York, Heidelberg, Berlin, 1977. MR0511596
[5] K. Burdzy and E. Toby. A Skorohod-type lemma and a decomposition of reflected Brownian motion. *Ann. Probab.* **23** (1995) 586–604. MR1334162
[6] H. Chen and D. D. Yao. *Fundamentals of Queueing Networks: Performance, Asymptotics, and Optimization*. *Stochastic Modelling and Applied Probability* **46**. Springer Science & Business Media, New York, 2013. MR1835969

[7] V. Chistyakov and O. Galkin. On maps of bounded p-variation with $p > 1$. *Positivity* **2** (1998) 19–45. MR1655755

[8] E. Çinlar. *Probability and Stochastics*. *Graduate Texts in Mathematics* **261**. Springer, New York, Dordrecht, Heidelberg, 2011. MR2767184

[9] E. Çinlar, J. Jacod, P. Protter and M. Sharpe. Semimartingales and Markov processes. *Probab. Theory Related Fields* **54** (1980) 161–219. MR0597337

[10] J. W. Cohen and O. J. Boxma. *Boundary Value Problems in Queueing System Analysis*, **79**. Elsevier, New York, 2000. MR0703000

[11] F. Coquet, A. Jakubowski, J. Mémin and L. Słominski. Natural decomposition of processes and weak Dirichlet processes. In *In Memoriam Paul-André Meyer* 81–116. Springer, Berlin, 2006. MR2276891

[12] J. Dai. Steady-state analysis of reflected Brownian motions: Characterization, numerical methods and queueing applications, Stanford University, 1990. MR2685786

[13] A. Dieker and J. Moriarty. Reflected Brownian motion in a wedge: Sum-of-exponential stationary densities. *Electron. Commun. Probab.* **15** (2009) 1, 1–16. MR2472171

[14] S. Ethier and T. Kurtz. *Markov Processes: Characterization and Convergence*. John Wiley & Sons, New York, 1986. MR0838085

[15] G. Fayolle and R. Iasnogorodski. Two coupled processors: The reduction to a Riemann–Hilbert problem. *Z. Wahrsch. Verw. Gebiete* **47** (1979) 325–351. MR0525314

[16] M. Foddy. Analysis of Brownian motion with drift, confined to a quadrant by oblique reflection, Stanford University, 1983. MR2633392

[17] H. Föllmer. Calcul d'Itô sans probabilités. *Sémin. Probab. Strasbourg* **15** (1981) 143–150. MR0622559

[18] H. Föllmer. Dirichlet processes. In *Stochastic Integrals* 476–478. Springer, Berlin, 1981.

[19] W. Kang and K. Ramanan. A Dirichlet process characterization of a class of reflected diffusions. *Ann. Probab.* **38** (2010) 1062–1105. MR2674994

[20] W. Kang and K. Ramanan. On the submartingale problem for reflected diffusions in domains with piecewise smooth boundaries. Preprint, 2014. Available at arXiv:1412.0729. MR3601653

[21] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer Science & Business Media, New York, 1998. MR1121940

[22] E. Lieb and M. Loss. *Analysis*. American Mathematical Society, Providence, 2001. MR1817225

[23] K. Ramanan. Reflected diffusions defined via the extended Skorokhod map. *Electron. J. Probab.* **11** (2006) 934–992. MR2261058

[24] K. Ramanan and M. I. Reiman. Fluid and heavy traffic diffusion limits for a generalized processor sharing model. *Ann. Appl. Probab.* **13** (2003) 100–139. MR1951995

[25] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*. Springer, New York, 1999. MR1725357

[26] K. Sato. *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press, Cambridge, 1999. MR1739520

[27] T. Simon. Small ball estimates in p-variation for stable processes. *J. Theoret. Probab.* **17** (2004) 979–1002. MR2105744

[28] S. Varadhan and R. Williams. Brownian motion in a wedge with oblique reflection. *Comm. Pure Appl. Math.* **38** (1985) 405–443. MR0792398

[29] R. Williams. Recurrence classification and invariant measure for reflected Brownian motion in a wedge. *Ann. Probab.* **13** (1985) 758–778. MR0799421

[30] R. Williams. Reflected Brownian motion in a wedge: Semimartingale property. *Z. Wahrsch. Verw. Gebiete* **69** (1985) 161–176. MR0779455

[31] R. Williams. Local time and excursions of reflected Brownian motion in a wedge. *Publ. Res. Inst. Math. Sci.* **23** (1987) 297–319. MR0890921

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1599–1648
https://doi.org/10.1214/18-AIHP929

Diffusion limit for the random walk Metropolis algorithm out of stationarity

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Abstract. The Random Walk Metropolis (RWM) algorithm is a Metropolis–Hastings Markov Chain Monte Carlo algorithm designed to sample from a given target distribution π^N with Lebesgue density on $\mathbb{R}^N$. Like any other Metropolis–Hastings algorithm, RWM constructs a Markov chain by randomly proposing a new position (the "proposal move"), which is then accepted or rejected according to a rule which makes the chain reversible with respect to π^N. When the dimension N is large, a key question is to determine the optimal scaling with N of the proposal variance: if the proposal variance is too large, the algorithm will reject the proposed moves too often; if it is too small, the algorithm will explore the state space too slowly. Determining the optimal scaling of the proposal variance gives a measure of the cost of the algorithm as well. One approach to tackle this issue, which we adopt here, is to derive diffusion limits for the algorithm. Such an approach has been proposed in the seminal papers (*Ann. Appl. Probab.* **7** (1) (1997) 110–120; *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **60** (1) (1998) 255–268). In particular, in (*Ann. Appl. Probab.* **7** (1) (1997) 110–120) the authors derive a diffusion limit for the RWM algorithm under the two following assumptions: (i) the algorithm is started in stationarity; (ii) the target measure π^N is in product form. The present paper considers the situation of practical interest in which both assumptions (i) and (ii) are removed. That is (a) we study the case (which occurs in practice) in which the algorithm is started out of stationarity and (b) we consider target measures which are in non-product form. Roughly speaking, we consider target measures that admit a density with respect to Gaussian; such measures arise in Bayesian nonparametric statistics and in the study of conditioned diffusions. We prove that, out of stationarity, the optimal scaling for the proposal variance is $O(N^{-1})$, as it is in stationarity. In this optimal scaling, a diffusion limit is obtained and the cost of reaching and exploring the invariant measure scales as $O(N)$. Notice that the optimal scaling in and out of stationatity need not be the same in general, and indeed they differ e.g. in the case of the MALA algorithm (*Stoch. Partial Differ. Equ. Anal Comput.* **6** (3) (2018) 446–499). More importantly, our diffusion limit is given by a stochastic PDE, coupled to a scalar ordinary differential equation; such an ODE gives a measure of how far from stationarity the process is and can therefore be taken as an indicator of convergence. In this sense, this paper contributes understanding to the old-standing problem of monitoring convergence of MCMC algorithms.

Résumé. L'algorithme *Random Walk Metropolis* (RWM) est un algorithme de *Markov Chain Monte Carlo* de type Metropolis–Hastings, conçu pour échantillonner une variable aléatoire de loi cible π^N ayant une densité par rapport à la mesure de Lebesgue sur $\mathbb{R}^N$. Comme tout algorithme de Metropolis–Hastings, RWM construit une chaîne de Markov en proposant une nouvelle position au hasard (le « pas proposé »), qui est ensuit accepté ou rejeté selon une règle choisie de sorte à rendre la chaîne réversible par rapport à π^N. Lorsque la dimension N est grande, une question cruciale est de déterminer l'échelle optimale (dépendant de N) de la variance du pas proposé : si cette variance est trop grande, l'algorithme rejettera les pas proposés trop souvent ; si elle est top petite, l'algorithme explorera l'espace d'états trop lentement. Déterminer l'échelle optimale de la variance donne également une mesure du coût de l'algorithme. Notre approche à ce problème est de déterminer des limites de diffusion pour l'algorithme. Une telle approche a été proposée dans les travaux fondateurs (*Ann. Appl. Probab.* **7** (1) (1997) 110–120; *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **60** (1) (1998) 255–268); en particulier, dans (*Ann. Appl. Probab.* **7** (1) (1997) 110–120), les auteurs déterminent une limite de diffusion pour l'algorithme RWM en supposant : (i) que l'algorithme démarre de la mesure stationnaire ; (ii) que la mesure cible π^N ait une forme produit. Le présent travail étudie la situation d'intérêt pratique où ces deux suppositions n'ont pas lieu. Ainsi (a) nous étudions le cas (qui a lieu en pratique) où l'algorithme commence dans un état non-stationnaire, et (b) nous considérons des mesures cibles qui n'ont ps une forme produit : en gros, les mesures que nous considérons ont une densité par rapport à la mesure gaussienne, et qui interviennent en statistique bayesienne non-paramétrique et dans l'étude des diffusions conditionnées. Nous montrons que, dans l'état non-stationnaire, l'échelle optimale de la variance du pas proposé est $O(N^{-1})$, c'est-à-dire la même que dans l'état stationnaire. À cette échelle optimale, nous obtenons une limite de diffusion et le coût pour atteindre et explorer la mesure invariante est d'ordre $O(N)$. Notons que les échelles optimales dans les cas stationnaires et non-stationnaires ne sont en générales pas les mêmes, et diffèrent par exemple dans le cas de l'algorithme MALA (*Stoch. Partial Differ. Equ. Anal Comput.* **6** (3) (2018) 446–499). De façon plus importante, notre limite de diffusion est donnée par une EDP stochastique couplée à une équation différentielle ordinaire scalaire. Une telle équation donne une mesure de la distance du processus à l'état stationnaire, et peut donc être vue comme un indicateur de convergence. En ce sens, ce travail contribue à comprendre le problème ancien de contrôler la convergence des algorithmes MCMC.

MSC: Primary 60J22; secondary 60J20; 60H10

Keywords: Markov Chain Monte Carlo; Random Walk Metropolis algorithm; Diffusion limit; Optimal scaling

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References

[1] E. Berger. Asymptotic behaviour of a class of stochastic approximation procedures. *Probab. Theory Related Fields* **71** (4) (1986) 517–552. MR0833268

[2] O. F. Christensen, G. O. Roberts and J. S. Rosenthal. Scaling limits for the transient phase of local Metropolis–Hastings algorithms. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **67** (2) (2005) 253–268. MR2137324

[3] M. Hairer, A. M. Stuart and J. Voss. Analysis of SPDEs arising in path sampling. Part II: The nonlinear case. *Ann. Appl. Probab.* **17** (5–6) (2007) 1657–1706. MR2358638

[4] M. Hairer, A. M. Stuart, J. Voss and P. Wiberg. Analysis of SPDEs arising in path sampling. Part I: The Gaussian case. *Commun. Math. Sci.* **3** (2005) 587–603. MR2188686

[5] I. S. Helland. Central limit theorems for martingales with discrete or continuous time. *Scand. J. Stat.* **9** (2) (1982) 79–94. MR0668684

[6] B. Jourdain, T. Lelièvre and B. Miasojedow. Optimal scaling for the transient phase of Metropolis Hastings algorithms: The longtime behavior. *Bernoulli* **20** (4) (2014) 1930–1978. MR3263094

[7] B. Jourdain, T. Lelièvre and B. Miasojedow. Optimal scaling for the transient phase of the random walk Metropolis algorithm: The mean-field limit. *Ann. Appl. Probab.* **25** (4) (2015) 2263–2300. MR3349007

[8] J. Kuntz, M. Ottobre and A. M. Stuart. Non-stationary phase of the MALA algorithm. *Stoch. Partial Differ. Equ. Anal Comput.* **6** (3) (2018) 446–499. MRMR3844656

[9] J. C. Mattingly, N. S. Pillai and A. M. Stuart. Diffusion limits of the random walk Metropolis algorithm in high dimensions. *Ann. Appl. Probab.* **22** (3) (2012) 881–930. MR2977981

[10] M. Ottobre, N. S. Pillai, F. J. Pinski and A. M. Stuart. A function space HMC algorithm with second order Langevin diffusion limit. *Bernoulli* **22** (1) (2016) 60–106. MR3449777

[11] N. S. Pillai, A. M. Stuart and A. H. Thiéry. Optimal scaling and diffusion limits for the Langevin algorithm in high dimensions. *Ann. Appl. Probab.* **22** (6) (2012) 2320–2356. MR3024970

[12] N. S. Pillai, A. M. Stuart and A. H. Thiéry. Noisy gradient flow from a random walk in Hilbert space. In *Stochastic Partial Differential Equations: Analysis and Computations* 196–232, **2**, 2014. MR3249584

[13] C. Prévôt and M. Röckner. *A Concise Course on Stochastic Partial Differential Equations*. *Lecture Notes in Mathematics* **1905**. Springer-Verlag, Berlin, 2007. MR2329435

[14] G. O. Roberts, A. Gelman and W. R. Gilks. Weak convergence and optimal scaling of random walk Metropolis algorithms. *Ann. Appl. Probab.* **7** (1) (1997) 110–120. MR1428751

[15] G. O. Roberts and J. S. Rosenthal. Optimal scaling of discrete approximations to Langevin diffusions. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **60** (1) (1998) 255–268. MR1625691

[16] A. M. Stuart. Inverse problems: A Bayesian perspective. *Acta Numer.* **19** (2010) 451–559. MR2652785

[17] L. Tierney. A note on Metropolis–Hastings kernels for general state spaces. *Ann. Appl. Probab.* **8** (1) (1998) 1–9. MR1620401

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1649–1678
https://doi.org/10.1214/18-AIHP930

Mixing times for a constrained Ising process on the two-dimensional torus at low density

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Abstract. We study a kinetically constrained Ising process (KCIP) associated with a graph G and density parameter p; this process is an interacting particle system with state space $\{0,1\}^G$, the locations of the particles. The 'constraint' in the name of the process refers to the rule that a vertex cannot change its state unless it has at least one neighbour in state '1'. The KCIP has been proposed by statistical physicists as a model for the glass transition. In this note, we study the mixing time of a KCIP on the 2-dimensional torus $G=\mathbb{Z}_L^2$ in the low-density regime $p=\frac{c}{L^2}$ for arbitrary $0<c<\infty$, extending our previous results for the analogous process on the torus $\mathbb{Z}_L^d$ in dimension $d\geq 3$. Our general approach is similar, but the extension requires more delicate bounds on the behaviour of the process at intermediate densities.

Résumé. Nous étudions un processus d'Ising avec contraintes cinétiques (PICC) associé à un graphe G et un paramètre de densité p. Ce processus est un système de particules en interaction avec espace d'états $\Omega=\{0,1\}^G$, décrivant les positions des particules. Les « contraintes » apparaissant dans le nom de ce processus réfèrent à la règle suivante: un sommet ne peut pas changer son état à moins qu'il ait un voisin dans l'état « 1 ». Le PICC a été proposé par des physiciens comme un modèle pour la transition vitreuse. Dans ce travail, nous analysons le temps de mélange d'un PICC sur le tore de dimension 2 $G=\mathbb{Z}_L^2$ dans le régime de faible densité $p=\frac{c}{L^2}$, où $0<c<\infty$. Ceci prolonge nos résultats au processus analogue sur le tore $G=\mathbb{Z}_L^d$, $d\geq 3$. Notre approche générale est similaire, mais cette extension requiert des bornes plus délicates sur le comportement du processus aux densités intermédiaires.

MSC: Primary 60J10; secondary 60J20

Keywords: Kinetically constrained process; Interacting particle systems; Markov chain; Mixing time; Glass transition

References

[1] D. Aldous. Open problems, 2017. Available at https://www.stat.berkeley.edu/~aldous/Research/OP/index.html; last accessed 7-June-2017.
[2] M. Ben and Y. Peres. Evolving sets, mixing and heat kernel bounds. *Probab. Theory Related Fields* **133** (2) (2005) 245–266. MR2198701
[3] O. Blondel, N. Cancrini, F. Martinelli, C. Roberto and C. Toninelli. Fredrickson–Andersen one spin facilitated model out of equilibrium. *Markov Process. Related Fields* **19** (2013) 383–406. MR3156958
[4] N. Cancrini, F. Martinelli, C. Roberto and C. Toninelli. Kinetically constrained models. In *New Trends in Mathematical Physics* 741–752, 2009.
[5] P. Chleboun, A. Faggionato and F. Martinelli. Time scale separation and dynamic heterogeneity in the low temperature East model. *Comm. Math. Phys.* **328** (3) (2014) 955–993. MR3201217
[6] P. Chleboun, A. Faggionato and F. Martinelli. Mixing time and local exponential ergodicity of the East-like process in $\mathbb{Z}^d$. *Ann. Fac. Sci. Toulouse Math. (6)* **24** (4) (2015) 717–743. MR3434253
[7] P. Chleboun, A. Faggionato and F. Martinelli. Relaxation to equilibrium of generalized East processes on $\mathbb{Z}^d$: Renormalization group analysis and energy-entropy competition. *Ann. Probab.* **44** (3) (2016) 1817–1863. MR3502595
[8] P. Chleboun and F. Martinelli. Mixing time bounds for oriented kinetically constrained spin models. *Electron. J. Probab.* **18** (2013) 1–9. MR3084571
[9] P. Chleboun and A. Smith. Mixing of the square plaquette model on a critical length scale. Preprint, 2018. Available at arXiv:1807.00634.

[10] P. Clifford and A. Sudbury. A model for spatial conflict. *Biometrika* **60** (3) (1973) 581–588. MR0343950
[11] M. Dyer, L. Goldberg, M. Jerrum and R. Martin. Markov chain comparison. *Probab. Surv.* **3** (2006) 89–111. MR2216963
[12] P. Flajolet, D. Gardy and L. Thimonier. Birthday paradox, coupon collectors, caching algorithms and self-organizing search. *Discrete Appl. Math.* **39** (3) (1992) 207–229. MR1189469
[13] G. Fredrickson and H. Andersen. Kinetic Ising model of the glass transition. *Phys. Rev. Lett.* **53** (1984) 1244–1247.
[14] G. Fredrickson and H. Andersen. Facilitated kinetic Ising models and the glass transition. *J. Chem. Phys.* **83** (1985) 5822–5831.
[15] J. Garrahan, P. Sollich and C. Toninelli. *Dynamical Heterogeneities in Glasses, Colloids and Granular Media*, Chapter 10. Oxford University Press, London, 2011.
[16] N. Hamamuki. A discrete isoperimetric inequality on lattices. *Discrete Comput. Geom.* **52** (2) (2014) 221–239. MR3249380
[17] R. Holley and T. Liggett. Ergodic theorems for weakly interacting infinite systems and the voter model. *Ann. Probab.* **3** (4) (1975) 643–663. MR0402985
[18] N. Jain and S. Orey. On the range of random walk. *Israel J. Math.* **6** (4) (1968) 373–380. MR0243623
[19] M. Jerrum, J.-B. Son, P. Tetali and E. Vigoda. Elementary bounds on Poincare and log-Sobolev constants for decomposable Markov chains. *Ann. Appl. Probab.* **14** (4) (2004) 1741–1765. MR2099650
[20] G. Kordzakhia and S. Lalley. Ergodicity and mixing properties of the Northeast model. *J. Appl. Probab.* **43** (3) (2006) 782–792. MR2274800
[21] D. Levin, Y. Peres and E. Wilmer. *Markov Chains and Mixing Times*. American Mathematical Society, Providence, RI, 2009. MR2466937
[22] N. Madras and D. Randall. Markov chain decomposition for convergence rate analysis. *Ann. Appl. Probab.* **12** (2002) 581–606. MR1910641
[23] F. Martinelli and C. Toninelli. Kinetically constrained spin models on trees. *Ann. Appl. Probab.* **23** (5) (2013) 1967–1987. MR3134727
[24] F. Martinelli and C. Toninelli. Towards a universality picture for the relaxation to equilibrium of kinetically constrained models. *Ann. Probab.* **47** (1) (2019) 324–361. MR3909971
[25] R. Oliveira. Mixing and hitting times for finite Markov chains. *Electron. J. Probab.* **17** (70) (2012) 1–12. MR2968677
[26] R. Oliveira. On the coalescence time of reversible random walks. *Trans. Amer. Math. Soc.* **364** (4) (2012) 2109–2128. MR2869200
[27] J. Palacios and P. Tetali. A note on expected hitting times for birth and death chains. *Statist. Probab. Lett.* **30** (1996) 119–125. MR1416997
[28] Y. Peres and P. Sousi. Mixing times are hitting times of large sets. *J. Theoret. Probab.* **9** (2013) 459–510. MR3370663
[29] D. Persi and L. Saloff-Coste. Comparison theorems for reversible Markov chains. *Ann. Appl. Probab.* **3** (3) (1993) 696–730. MR1233621
[30] D. Persi and L. Saloff-Coste. Logarithmic Sobolev inequalities for finite Markov chains. *Ann. Appl. Probab.* **6** (1996) 695–750. MR1410112
[31] N. Pillai and A. Smith. Elementary bounds on mixing times for decomposable Markov chains. *Stochastic Process. Appl.* **127** (9) (2017) 3068–3109. MR3682124
[32] N. S. Pillai and A. Smith. Mixing times for a constrained Ising process on the torus at low density. *Ann. Probab.* **45** (2) (2017) 1003–1070. MR3630292
[33] A. Smith. Comparison theory for Markov chains on different state spaces and application to random walk on derangements. *J. Theoret. Probab.* **28** (4) (2015) 1406–1430. MR3422936
[34] J. Theodore Cox. Coalescing random walks and voter model consensus times on the torus in $\mathbb{Z}^d$. *Ann. Probab.* **17** (4) (1989) 1333–1366. MR1048930

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1679–1711
https://doi.org/10.1214/18-AIHP931

Quasi-independence for nodal lines

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Abstract. We prove a quasi-independence result for level sets of a planar centered stationary Gaussian field with covariance $(x, y) \mapsto \kappa(x - y)$, with only mild conditions on the regularity of κ. As a first application, we study percolation for nodal lines in the spirit of (*Publ. Math. Inst. Hautes Études Sci.* **126** (2017) 131–176). In the said article, Beffara and Gayet rely on Tassion's method (*Ann. Probab.* **44** (2016) 3385–3398) to prove that, under some assumptions on κ, most notably that $\kappa \geq 0$ and $\kappa(x) = O(|x|^{-325})$, the nodal set satisfies a box-crossing property. The decay exponent was then lowered to $16 + \varepsilon$ by Beliaev and Muirhead in (*Comm. Math. Phys.* **359** (2018) 869–913). In the present work we lower this exponent to $4 + \varepsilon$ thanks to a new approach towards quasi-independence for crossing events. This approach does not rely on quantitative discretization. Our quasi-independence result also applies to events counting nodal components and we obtain a lower concentration result for the density of nodal components around the Nazarov and Sodin constant from (*Zh. Mat. Fiz. Anal. Geom.* **12** (2016) 205–278).

Résumé. On démontre un résultat de quasi-indépendance pour les lignes de niveau de champs gaussiens planaires stationnaires centrés de covariance $(x, y) \mapsto \kappa(x - y)$, sous de faibles conditions sur la régularité de κ. On applique d'abord ce résultat à l'étude de la percolation des lignes nodales dans l'esprit de (*Publ. Math. Inst. Hautes Études Sci.* **126** (2017) 131–176). Dans ledit article, Beffara et Gayet s'appuyent sur la méthode de Tassion (*Ann. Probab.* **44** (2016) 3385–3398) pour démontrer que sous certaines hypothèses sur κ, notamment que $\kappa \geq 0$ et $\kappa(x) = O(|x|^{-325})$, l'ensemble nodal satisfait une propriété de croisement de boîtes. L'exposant de décroissance a plus tard été réduit à $16 + \varepsilon$ par Beliaev et Muirhead dans (*Comm. Math. Phys.* **359** (2018) 869–913). Dans le présent article nous baissons cet exposant jusqu'à $4 + \varepsilon$ grâce à une nouvelle approche pour la quasi-indépendance d'événements de croisement. Cette approche ne s'appuie pas sur une discrétisation quantitative. Notre résultat de quasi-indépendance s'applique aussi à des événements de comptage de composantes nodales et nous obtenons un résultat de concentration par en dessous de la densité de composantes nodales autour de la constante de Nazarov et Sodin de (*Zh. Mat. Fiz. Anal. Geom.* **12** (2016) 205–278).

MSC: 60K35; 60D05

Keywords: Gaussian fields; Percolation; Quasi-independence; Influences

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References

[1] R. J. Adler and J. E. Taylor. *Random Fields and Geometry*. Springer, New York, 2007. MR2319516
[2] D. Ahlberg, V. Tassion and A. Teixeira. Sharpness of the phase transition for continuum percolation in $\mathbb{R}^2$. *Probab. Theory Related Fields* **172** (1–2) (2017) 525–581. MR3851838
[3] K. S. Alexander. Boundedness of level lines for two-dimensional random fields. *Ann. Probab.* **24** (4) (1996) 1653–1674. MR1415224
[4] J.-M. Azaïs and M. Wschebor. *Level Sets and Extrema of Random Processes and Fields*. Wiley, Hoboken, NJ, 2009. MR2478201
[5] D. Basu and A. Sapozhnikov. Crossing probabilities for critical Bernoulli percolation on slabs. Preprint, 2015. Available at arXiv:1512.05178.
[6] V. Beffara and D. Gayet. Percolation of random nodal lines. *Publ. Math. Inst. Hautes Études Sci.* **126** (2017) 131–176. MR3735866
[7] V. Beffara and D. Gayet. Percolation without FKG. Preprint, 2017. Available at arXiv:1710.10644.
[8] D. Beliaev and S. Muirhead. Discretisation schemes for level sets of planar Gaussian fields. *Comm. Math. Phys.* **359** (3) (2018) 869–913. MR3784534

[9] D. Beliaev, S. Muirhead and I. Wigman. Russo–Seymour–Welsh estimates for the Kostlan ensemble of random polynomials. Preprint, 2017. Available at arXiv:1709.08961.
[10] E. Bogomolny and C. Schmit. Random wavefunctions and percolation. *J. Phys. A: Math. Theor.* **40** (47) (2007) 14033–14043. MR2438110
[11] B. Bollobás and O. Riordan. The critical probability for random Voronoi percolation in the plane is $1/2$. *Probab. Theory Related Fields* **136** (3) (2006) 417–468. MR2257131
[12] B. Bollobás and O. Riordan. *Percolation*. Cambridge University Press, Cambridge, 2006. MR2283880
[13] E. W. Cheney and W. A. Light. *A Course in Approximation Theory*. *Graduate Studies in Mathematics* **101**. American Mathematical Society, Providence, RI, 2009. MR2474372
[14] H. Duminil-Copin, C. Hongler and P. Nolin. Connection probabilities and RSW-type bounds for the two-dimensional FK Ising model. *Comm. Pure Appl. Math.* **64** (9) (2011) 1165–1198. MR2839298
[15] H. Duminil-Copin, V. Tassion and A. Teixeira. The box-crossing property for critical two-dimensional oriented percolation. *Probab. Theory Related Fields* **171** (3–4) (2018) 685–708. MR3827220
[16] D. Gayet and J.-Y. Welschinger. Exponential rarefaction of real curves with many components. *Publ. Math. Inst. Hautes Études Sci.* **113** (2011) 69–96. MR2805598
[17] G. Grimmett. *Probability on Graphs. Random Processes on Graphs and Lattices*. Cambridge University Press, Cambridge, 2010. MR2723356
[18] G. R. Grimmett. *Percolation*. *Grundlehren der mathematischen Wissenschaften*. Springer, Berlin, 1999. MR1707339
[19] T. E. Harris. A lower bound for the critical probability in a certain percolation process. *Proc. Camb. Philos. Soc.* **56** (1960) 13–20. MR0115221
[20] H. Kesten. The critical probability of bond percolation on the square lattice equals $\frac{1}{2}$. *Comm. Math. Phys.* **74** (1) (1980) 41–59. MR0575895
[21] T. Letendre. Variance of the volume of random real algebraic submanifolds. Preprint, 2016. Available at arXiv:1608.05658.
[22] S. A. Molchanov and A. K. Stepanov. Percolation in random fields. I. *Theoret. Math. Phys.* **55** (2) (1983) 478–484. MR0734878
[23] S. A. Molchanov and A. K. Stepanov. Percolation in random fields. II. *Theoret. Math. Phys.* **55** (3) (1983) 592–599. MR0711007
[24] S. A. Molchanov and A. K. Stepanov. Percolation in random fields. III. *Theoret. Math. Phys.* **67** (2) (1986) 434–439. MR0851557
[25] F. Nazarov and M. Sodin. On the number of nodal domains of random spherical harmonics. *Amer. J. Math.* **131** (5) (2009) 1337–1357. MR2555843
[26] F. Nazarov and M. Sodin. Fluctuations in random complex zeroes: Asymptotic normality revisited. *Int. Math. Res. Not. IMRN* **24** (2011) 5720–5759. MR2863379
[27] F. Nazarov and M. Sodin. Asymptotic laws for the spatial distribution and the number of connected components of zero sets of Gaussian random functions. *Zh. Mat. Fiz. Anal. Geom.* **12** (3) (2016) 205–278. MR3522141
[28] F. Nazarov, M. Sodin and A. Volberg. Transportation to random zeroes by the gradient flow. *Geom. Funct. Anal.* **17** (3) (2007) 887–935. MR2346279
[29] F. Nazarov, M. Sodin and A. Volberg. The Jancovici–Lebowitz–Manificat law for large fluctuations of random complex zeroes. *Comm. Math. Phys.* **284** (3) (2008) 833–865. MR2452596
[30] C. Newman, V. Tassion and W. Wu. Critical percolation and the minimal spanning tree in slabs. *Comm. Pure Appl. Math.* **70** (11) (2017) 2084–2120. MR3707489
[31] V. I. Piterbarg. *Asymptotic Methods in the Theory of Gaussian Processes and Fields*. American Mathematical Society, Providence, RI, 1996. Transl. from the Russian by V. V. Piterbarg. Transl. ed. by Simeon Ivanov. MR1361884
[32] L. D. Pitt. Positively correlated normal variables are associated. *Ann. Probab.* **10** (1982) 496–499. MR0665603
[33] A. Rivera and H. Vanneuville. The critical threshold for Bargmann–Fock percolation. Preprint, 2017. Available at arXiv:1711.05012.
[34] L. Russo. A note on percolation. *Probab. Theory Related Fields* **43** (1) (1978) 39–48.
[35] P. D. Seymour and D. J. A. Welsh. Percolation probabilities on the square lattice. *Ann. Discrete Math.* **3** (1978) 227–245. MR0494572
[36] D. Slepian. The one-sided barrier problem for Gaussian noise. *Bell Labs Tech. J.* **41** (2) (1962) 463–501. MR0133183
[37] V. Tassion. Crossing probabilities for Voronoi percolation. *Ann. Probab.* **44** (5) (2016) 3385–3398. MR3551200

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1712–1750
https://doi.org/10.1214/18-AIHP932

Liouville quantum gravity spheres as matings of finite-diameter trees

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Abstract. We show that the unit area Liouville quantum gravity sphere can be constructed in two equivalent ways. The first, which was introduced by the authors and Duplantier in (Liouville quantum gravity as a mating of trees (2014) Preprint), uses a Bessel excursion measure to produce a Gaussian free field variant on the cylinder. The second uses a correlated Brownian loop and a "mating of trees" to produce a Liouville quantum gravity sphere decorated by a space-filling path. In the special case that $\gamma = \sqrt{8/3}$, we present a third equivalent construction, which uses the excursion measure of a $3/2$-stable Lévy process (with only upward jumps) to produce a pair of trees of quantum disks that can be mated to produce a sphere decorated by SLE_6. This construction is relevant to a program for showing that the $\gamma = \sqrt{8/3}$ Liouville quantum gravity sphere is equivalent to the Brownian map.

Résumé. Nous montrons que la sphère de gravité quantique de Liouville d'aire unité peut être construite de deux façons équivalentes. La première, introduite par les auteurs et Duplantier dans (Liouville quantum gravity as a mating of trees (2014) Preprint), utilise la mesure d'excursion d'un processus de Bessel pour définir une variante du champ libre gaussien sur le cylindre. La seconde utilise une boucle d'un mouvement brownien corrélé et un « accouplement d'arbres » pour produire une sphère de gravité quantique de Liouville décorée par un chemin remplissant l'espace.

Dans le cas particulier où $\gamma = \sqrt{8/3}$, nous présentons une troisième construction équivalente, utilisant la mesure d'excursion d'un processus de Lévy stable d'exposant $3/2$ (sans sauts négatifs) pour produire une paire d'arbres de disques quantiques que l'on peut accoupler pour obtenir une sphère décorée par un SLE_6. Cette construction intervient dans un programme ayant pour but de montrer que la sphère de gravité quantique de Liouville pour $\gamma = \sqrt{8/3}$ est équivalente à la carte brownienne.

MSC: 60J67; 28C20

Keywords: Gaussian free field; Liouville quantum gravity; Schramm-Loewner evolution; Continuum random tree; Conformal welding

References

[1] J. Aru, Y. Huang and X. Sun. Two perspectives of the 2D unit area quantum sphere and their equivalence. *Comm. Math. Phys.* **356** (1) (2017) 261–283. MR3694028
[2] J. Barral, X. Jin, R. Rhodes and V. Vargas. Gaussian multiplicative chaos and KPZ duality. *Comm. Math. Phys.* **323** (2) (2013) 451–485. MR3096527
[3] J. Bertoin. *Lévy Processes*. *Cambridge Tracts in Mathematics* **121**. Cambridge University Press, Cambridge, MA, 1996. MR1406564
[4] J. Bettinelli and G. Miermont. Compact Brownian surfaces II. The general case. To appear, 2019.
[5] F. David, A. Kupiainen, R. Rhodes and V. Vargas. Liouville quantum gravity on the Riemann sphere. *Comm. Math. Phys.* **342** (3) (2016) 869–907. MR3465434
[6] F. David, R. Rhodes and V. Vargas. Liouville quantum gravity on complex tori. *J. Math. Phys.* **57** (2) (2016) 022302. MR3450564
[7] B. Duplantier, J. Miller and S. Sheffield. Liouville quantum gravity as a mating of trees. ArXiv e-prints, 2014. Available at arXiv:1409.7055. MR3729063
[8] B. Duplantier, R. Rhodes, S. Sheffield and V. Vargas. Critical Gaussian multiplicative chaos: Convergence of the derivative martingale. *Ann. Probab.* **42** (5) (2014) 1769–1808. MR3262492

[9] B. Duplantier, R. Rhodes, S. Sheffield and V. Vargas. Renormalization of critical Gaussian multiplicative chaos and KPZ relation. *Comm. Math. Phys.* **330** (1) (2014) 283–330. MR3215583
[10] B. Duplantier and S. Sheffield. Duality and the Knizhnik–Polyakov–Zamolodchikov relation in Liouville quantum gravity. *Phys. Rev. Lett.* **102** (15) (2009) 150603. MR2501276
[11] B. Duplantier and S. Sheffield. Liouville quantum gravity and KPZ. *Invent. Math.* **185** (2) (2011) 333–393. MR2819163
[12] H.-O. Georgii. *Gibbs Measures and Phase Transitions*, 2nd edition. *De Gruyter Studies in Mathematics* **9**. Walter de Gruyter & Co., Berlin, 2011. MR2807681
[13] C. Guillarmou, R. Rhodes and V. Vargas. Liouville quantum gravity on compact surfaces. ArXiv e-prints, 2016. Available at arXiv:1607.08467.
[14] E. Gwynne, N. Holden, J. Miller and X. Sun. Brownian motion correlation in the peanosphere for $\kappa > 8$. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (4) (2017) 1866–1889. MR3729638
[15] E. Gwynne, N. Holden and X. Sun. Joint scaling limit of a bipolar-oriented triangulation and its dual in the peanosphere sense. ArXiv e-prints, 2016. Available at arXiv:1603.01194.
[16] E. Gwynne and J. Miller. Convergence of the topology of critical Fortuin–Kasteleyn planar maps to that of CLE_κ on a Liouville quantum surface. To appear, 2019.
[17] E. Gwynne, J. Miller and S. Sheffield. The Tutte embedding of the mated-CRT map converges to Liouville quantum gravity. ArXiv e-prints, 2017. Available at arXiv:1705.11161.
[18] E. Gwynne, J. Miller and S. Sheffield. The Tutte embedding of the Poisson–Voronoi tessellation of the Brownian disk converges to $\sqrt{8/3}$-Liouville quantum gravity. ArXiv e-prints, 2018. Available at arXiv:1809.02091.
[19] E. Gwynne and X. Sun. Scaling limits for the critical Fortuin–Kastelyn model on a random planar map III: Finite volume case. ArXiv e-prints, 2015. Available at arXiv:1510.06346.
[20] E. Gwynne and X. Sun. Scaling limits for the critical Fortuin–Kasteleyn model on a random planar map II: Local estimates and empty reduced word exponent. *Electron. J. Probab.* **22** (2017), Paper No. 45. MR3661659
[21] Y. Huang, R. Rhodes and V. Vargas. Liouville Quantum Gravity on the unit disk. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (3) (2018) 1694–1730. MR3825895
[22] R. Kenyon, J. Miller, S. Sheffield and D. B. Wilson. Bipolar orientations on planar maps and SLE_{12}. ArXiv e-prints, 2015. Available at arxiv:1511.04068. To appear in Ann. Probab.
[23] R. Kenyon, J. Miller, S. Sheffield and D. B. Wilson. Six-vertex model and Schramm–Loewner evolution. *Phys. Rev. E* **95** (2017), 052146. DOI:10.1103/PhysRevE.95.052146.
[24] G. Lawler, O. Schramm and W. Werner. Conformal restriction: The chordal case. *J. Amer. Math. Soc.* **16** (4) (2003) 917–955. MR1992830
[25] G. F. Lawler. *Conformally Invariant Processes in the Plane*. *Mathematical Surveys and Monographs* **114**. American Mathematical Society, Providence, RI, 2005. MR2129588
[26] G. F. Lawler and S. Sheffield. A natural parametrization for the Schramm–Loewner evolution. *Ann. Probab.* **39** (5) (2011) 1896–1937. MR2884877
[27] J.-F. Le Gall. Uniqueness and universality of the Brownian map. *Ann. Probab.* **41** (4) (2013) 2880–2960. MR3112934
[28] G. Miermont. The Brownian map is the scaling limit of uniform random plane quadrangulations. *Acta Math.* **210** (2) (2013) 319–401. MR3070569
[29] J. Miller and S. Sheffield. An axiomatic characterization of the Brownian map. ArXiv e-prints, 2015. Available at arXiv:1506.03806.
[30] J. Miller and S. Sheffield. Liouville quantum gravity and the Brownian map I: The QLE(8/3, 0) metric. ArXiv e-prints, 2015. Available at arXiv:1507.00719.
[31] J. Miller and S. Sheffield. Liouville quantum gravity and the Brownian map II: Geodesics and continuity of the embedding. ArXiv e-prints, 2016. Available at arXiv:1605.03563.
[32] J. Miller and S. Sheffield. Liouville quantum gravity and the Brownian map III: The conformal structure is determined. ArXiv e-prints, 2016. Available at arXiv:1608.05391.
[33] J. Miller and S. Sheffield. Imaginary geometry I: Interacting SLEs. *Probab. Theory Related Fields* **164** (3–4) (2016) 553–705. MR3477777
[34] J. Miller and S. Sheffield. Imaginary geometry II: Reversibility of $\mathrm{SLE}_\kappa(\rho_1;\rho_2)$ for $\kappa \in (0,4)$. *Ann. Probab.* **44** (3) (2016) 1647–1722. MR3502592
[35] J. Miller and S. Sheffield. Imaginary geometry III: Reversibility of SLE_κ for $\kappa \in (4,8)$. *Ann. of Math. (2)* **184** (2) (2016) 455–486. MR3548530
[36] J. Miller and S. Sheffield. Quantum Loewner evolution. *Duke Math. J.* **165** (17) (2016) 3241–3378. MR3572845
[37] J. Miller and S. Sheffield. Imaginary geometry IV: Interior rays, whole-plane reversibility, and space-filling trees. *Probab. Theory Related Fields* **169** (3–4) (2017) 729–869. MR3719057
[38] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **293**. Springer-Verlag, Berlin, 1999. MR2000H:60050
[39] O. Schramm. Scaling limits of loop-erased random walks and uniform spanning trees. *Israel J. Math.* **118** (2000) 221–288. MR1776084
[40] O. Schramm and D. B. Wilson. SLE coordinate changes. *New York J. Math.* **11** (2005) 659–669. MR2188260
[41] S. Sheffield. Gaussian free fields for mathematicians. *Probab. Theory Related Fields* **139** (3–4) (2007) 521–541. MR2322706
[42] S. Sheffield. Conformal weldings of random surfaces: SLE and the quantum gravity zipper. *Ann. Probab.* **44** (5) (2016) 3474–3545. MR3551203
[43] S. Sheffield. Quantum gravity and inventory accumulation. *Ann. Probab.* **44** (6) (2016) 3804–3848. MR3572324
[44] M. Shimura. Excursions in a cone for two-dimensional Brownian motion. *J. Math. Kyoto Univ.* **25** (3) (1985) 433–443. MR0807490
[45] W. Werner. Random planar curves and Schramm–Loewner evolutions. In *Lectures on Probability Theory and Statistics* 107–195. *Lecture Notes in Math.* **1840**. Springer, Berlin, 2004. MR2079672

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1751–1764
https://doi.org/10.1214/18-AIHP933

Active phase for activated random walks on $\mathbb{Z}^d$, $d \geq 3$, with density less than one and arbitrary sleeping rate

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Abstract. It has been conjectured that the critical density of the Activated Random Walk model is strictly less than one for any value of the sleeping rate. We prove this conjecture on $\mathbb{Z}^d$ when $d \geq 3$ and, more generally, on graphs where the random walk is transient. Moreover, we establish the occurrence of a phase transition on non-amenable graphs, extending previous results which require that the graph is amenable or a regular tree.

Résumé. Il a été conjecturé que la densité critique pour le modèle de marches aléatoires activées était strictement inférieur à 1 pour toute valeur du taux d'endormissement. Nous démontrons cette conjecture pour $\mathbb{Z}^d$ quand $d \geq 3$ et, plus généralement, pour les graphes sur lesquels la marche aléatoire est transitoire. De plus, nous montrons l'existence d'une transition de phase pour les graphes non moyennables, généralisant ainsi des résultats antérieurs qui demandaient que le graphe soit moyennable ou un arbre régulier.

MSC: Primary 82C22; secondary 60K35; 82C26

Keywords: Interacting particle systems; Abelian networks; Absorbing-state phase transition; Self-organized criticality

References

[1] G. Amir and O. Gurel-Gurevich. On fixation of activated random walks. *Electron. Commun. Probab.* **15** (2010) 119–123. MR2643591
[2] R. Basu, S. Ganguly and C. Hoffman. Non-fixation of symmetric activated random walk on the line for small sleep rate. *Comm. Math. Phys.* **358** (2018) 1151–1185. MR3778354
[3] R. Dickman, L. T. Rolla and V. Sidoravicius. Activated random walkers: Facts, conjectures and challenges. *J. Stat. Phys.* **138** (2010) 126–142. MR2594894
[4] L. T. Rolla and V. Sidoravicius. Absorbing-state phase transition for driven-dissipative stochastic dynamics on $\mathbb{Z}$. *Invent. Math.* **188** (2012) 127–150. MR2897694
[5] L. T. Rolla, V. Sidoravicius and O. Zindy. Critical density for activated random walks, 2017. Available at arXiv:1707.06081.
[6] L. T. Rolla and L. Tournier. Sustained activity for biased activated random walks at arbitrarily low density. *Ann. Inst. Henri Poincaré* **54** (2018) 938–951. MR3795072
[7] E. Shellef. Nonfixation for activated random walk. *ALEA* **7** (2010) 137–149. MR2651824
[8] V. Sidoravicius and A. Teixeira. Absorbing-state transitions for stochastic sandpiles and activated random walk. *Electron. J. Probab.* **22** (2017) paper no. 33. MR3646059
[9] A. Stauffer and L. Taggi. Critical density of activated random walks on transitive graphs. *Ann. Probab.* **46** (2018) 2190–2220. MR3813989
[10] L. Taggi. Absorbing-state phase transition in biased activated random walk. *Electron. J. Probab.* **21** (2016) paper no. 13. MR3485355

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1765–1796
https://doi.org/10.1214/18-AIHP934

Supremum estimates for degenerate, quasilinear stochastic partial differential equations

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Abstract. We prove a priori estimates in L_∞ for a class of quasilinear stochastic partial differential equations. The estimates are obtained independently of the ellipticity constant ε and thus imply analogous estimates for degenerate quasilinear stochastic partial differential equations, such as the stochastic porous medium equation.

Résumé. Nous montrons une estimée a priori dans L_∞ pour une classe d'équations différentielles partielles stochastiques quasi-linéaires. Les estimées sont obtenues indépendamment de la constante d'ellipticité ε et impliquent par conséquent une estimée analogue pour les équations différentielles partielles stochastiques quasi-linéaires dégénérées, telles que l'équation stochastique des milieux poreux.

MSC: 60H15; 60G46

Keywords: Degenerate SPDEs; Stochastic porous medium; Moser's iteration

References

[1] V. Barbu, G. Da Prato and M. Röckner. *Michael Stochastic Porous Media Equations*. *Lecture Notes in Mathematics* **2163**. Springer, Cham, 2016. MR3560817
[2] V. Barbu and M. Röckner. An operatorial approach to stochastic partial differential equations driven by linear multiplicative noise. *J. Eur. Math. Soc. (JEMS)* **17** (7) (2015) 1789–1815. MR3361729
[3] L. A. Caffarelli and L. C. Evans. Continuity of the temperature in the two-phase Stefan problem. *Arch. Ration. Mech. Anal.* **81** (3) (1983) 199–220. MR0683353
[4] K. Dareiotis and M. Gerencsér. On the boundedness of solutions of SPDEs. *Stoch. Partial Differ. Equ. Anal. Comput.* **3** (1) (2015) 84–102. MR3312593
[5] K. Dareiotis and M. Gerencsér. Local L_∞-estimates, weak Harnack inequality, and stochastic continuity of solutions of SPDEs. *J. Differential Equations* **262** (1) (2017) 615–632. MR3567496
[6] A. Debussche, S. de Moor and M. Hofmanová. A regularity result for quasilinear stochastic partial differential equations of parabolic type. *SIAM J. Math. Anal.* **47** (2) (2015) 1590–1614. MR3340199
[7] L. Denis, A. Matoussi and L. Stoica. L_p estimates for the uniform norm of solutions of quasilinear SPDE's. *Probab. Theory Related Fields* **133** (4) (2005) 437–463. MR2197109
[8] E. DiBenedetto. Continuity of weak solutions to a general porous medium equation. *Indiana Univ. Math. J.* **32** (1) (1983) 83–118. MR0684758
[9] E. DiBenedetto. *Degenerate Parabolic Equations*. *Universitext*. Springer-Verlag, New York, 1993. MR1230384
[10] M. Gerencsér. Boundary regularity of stochastic PDEs. *Arxiv Preprint, arXiv:1705.05364, to appear in Annals of Probability.*
[11] M. Gerencsér, I. Gyöngy and N. Krylov. On the solvability of degenerate stochastic partial differential equations in Sobolev spaces. *Stoch. Partial Differ. Equ. Anal. Comput.* **3** (1) (2015) 52–83. MR3312592
[12] B. Gess. Strong solutions for stochastic partial differential equations of gradient type. *J. Funct. Anal.* **263** (8) (2012) 2355–2383. MR2964686
[13] B. Gess. Random attractors for stochastic porous media equations perturbed by space-time linear multiplicative noise. *Ann. Probab.* **42** (2) (2014) 818–864. MR3178475

[14] B. Gess and M. Hofmanová. Well-posedness and regularity for quasilinear degenerate parabolic-hyperbolic SPDE. *Ann. Probab.* **46** (5) (2018) 2495–2544. MR3846832

[15] B. Gess and M. Röckner. Singular-degenerate multivalued stochastic fast diffusion equations. *SIAM J. Math. Anal.* **47** (5) (2015) 4058–4090. MR3505171

[16] P. Grisvard. *Elliptic Problems in Nonsmooth Domains*. *Monographs and Studies in Mathematics* **24**. Pitman (Advanced Publishing Program), Boston, MA, 1985. MR0775683

[17] E. P. Hsu, Y. Wang and Z. Wang. Stochastic De Giorgi iteration and regularity of stochastic partial differential equations. *Ann. Probab.* **45** (5) (2017) 2855–2866. MR3706733

[18] N. V. Krylov. A relatively short proof of Itô's formula for SPDEs and its applications. *Stoch. Partial Differ. Equ. Anal. Comput.* **1** (1) (2013) 152–174. MR3327504

[19] N. V. Krylov and B. L. Rozovskii. Stochastic evolution equations. In *Stochastic Differential Equations: Theory and Applications* 1–69. *Interdiscip. Math. Sci.* **2**. World Sci. Publ., Hackensack, NJ, 2007. MR2391779

[20] O. A. Ladyženskaja, V. A. Solonnikov and N. N. Ural'ceva. *Linear and Quasilinear Equations of Parabolic Type*. *Translations of Mathematical Monographs* **23**. American Mathematical Society, Providence, RI, 1968 (Russian). Translated from the Russian by S. Smith. MR0241822

[21] J. Moser. A Harnack inequality for parabolic differential equations. *Comm. Pure Appl. Math.* **17** (1964) 101–134. MR0159139

[22] E. Pardoux. Sur des équations aux dérivées partielles stochastiques monotones. *C. R. Acad. Sci. Paris Sér. A–B* **275** (1972) A101–A103 (French). MR0312572

[23] C. Prévôt and M. Röckner. *A Concise Course on Stochastic Partial Differential Equations*. *Lecture Notes in Mathematics* **1905**. Springer, Berlin, 2007. MR2329435

[24] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Grundlehren der Mathematischen Wissenschaften* **293**. Springer-Verlag, Berlin, 1999. MR1725357

[25] B. L. Rozovskiĭ. *Stochastic Evolution Systems. Linear Theory and Applications to Nonlinear Filtering*. *Mathematics and Its Applications (Soviet Series)* **35**. Kluwer Academic Publishers Group, Dordrecht, 1990. Translated from the Russian by A. Yarkho. MR1135324

[26] P. E. Sacks. The initial and boundary value problem for a class of degenerate parabolic equations. *Comm. Partial Differential Equations* **8** (7) (1983) 693–733. MR0700733

[27] J. L. Vázquez. *The Porous Medium Equation. Mathematical Theory*. *Oxford Mathematical Monographs*. The Clarendon Press, Oxford University Press, Oxford, 2007. MR2286292

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2019, Vol. 55, No. 3, 1797–1813
https://doi.org/10.1214/19-AIHP1005

On thin local sets of the Gaussian free field

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Abstract. We study how small a local set of the continuum Gaussian free field (GFF) in dimension d has to be to ensure that this set is thin, which loosely speaking means that it captures no GFF mass on itself, in other words, that the field restricted to it is zero. We provide a criterion on the size of the local set for this to happen, and on the other hand, we show that this criterion is sharp by constructing small local sets that are not thin.

Résumé. Nous étudions à quel point un ensemble local du champ libre Gaussien (GFF) en dimension d doit être petit pour être sûr que l'ensemble est fin, ce qui signifie informellement que le GFF ne place pas de masse sur l'ensemble, i.e., que le champ restreint a l'ensemble vaut zéro. Nous donnons des critères portant sur la taille de l'ensemble local qui impliquent cette propriété, et par ailleurs nous montrons que ce critère est optimal en construisant des ensembles locaux petits qui ne sont pas fins.

MSC: 60D05; 60K35

Keywords: Gaussian free field; Local sets; Thin local sets

References

[1] J. Aru. The geometry of the Gaussian free field combined with SLE processes and the KPZ relation. 2015.
[2] J. Aru. KPZ relation does not hold for the level lines and SLE_κ flow lines of the Gaussian free field. *Probab. Theory Related Fields* **163** (2015) 465–526. MR3418748
[3] J. Aru, T. Lupu and A. Sepúlveda. First passage sets of the 2d continuum Gaussian free field. Preprint, 2017. Available at arXiv:1706.07737. MR3827968
[4] J. Aru, T. Lupu and A. Sepúlveda. First passage sets of the 2d continuum Gaussian free field. Preprint, 2018. Available at arXiv:1805.09204. MR3827968
[5] J. Aru, E. Powell and A. Sepúlveda. Approximating Liouville measure using local sets of the Gaussian free field. Preprint, 2017. Available at arXiv:1701.05872. To appear in *Ann. Inst. Fourier (Grenoble)*.
[6] J. Aru and A. Sepúlveda. Survey in preparation, 2018.
[7] J. Aru, A. Sepúlveda and W. Werner. On bounded-type thin local sets of the two-dimensional Gaussian free field. *J. Inst. Math. Jussieu* **18** (3) (2019) 591–618. MR3936643
[8] E. Bolthausen, J.-D. Deuschel and G. Giacomin. Entropic repulsion and the maximum of the two-dimensional harmonic crystal. *Ann. Probab.* **29** (4) (2001) 1670–1692. MR1880237
[9] J. D. Cook. Upper and lower bounds for the normal distribution function, 2009.
[10] J. Dubédat. SLE and the free field: Partition functions and couplings. *J. Amer. Math. Soc.* **22** (2009) 995–1054. MR2525778
[11] B. Duplantier and S. Sheffield. Liouville quantum gravity and KPZ. *Invent. Math.* **185** (2011) 333–393. MR2819163
[12] X. Hu, J. Miller and Y. Peres. Thick points of the Gaussian free field. *Ann. Probab.* **38** (2010) 896–926. MR2642894
[13] Y. Le Jan. *Markov Paths, Loops and Fields: École d'Été de Probabilités de Saint-Flour XXXVIII–2008*, **2026**. Springer, Berlin, 2011. MR2815763
[14] T. Lupu. From loop clusters and random interlacements to the free field. *Ann. Probab.* **44** (2016) 2117–2146. MR3502602
[15] J. Miller and S. Sheffield. The GFF and CLE(4). Slides, talks and private communications, 2011.
[16] J. Miller and S. Sheffield. Liouville quantum gravity and the Brownian map I: The QLE$(8/3, 0)$ metric. Preprint, 2015. Available at arXiv:1507.00719.
[17] J. Miller and S. Sheffield. Imaginary geometry I: Interacting SLEs. *Probab. Theory Related Fields* **164** (2016) 553–705. MR3477777
[18] J. Miller and S. Sheffield. Quantum Loewner evolution. *Duke Math. J.* **165** (2016) 3241–3378. MR3572845

[19] P. Mörters and Y. Peres. *Brownian Motion*, **30**. Cambridge University Press, Cambridge, 2010. MR2604525

[20] Ş. Nacu and W. Werner. Random soups, carpets and fractal dimensions. *J. Lond. Math. Soc.* **83** (2011) 789–809. MR2802511

[21] W. Qian and W. Werner. Decomposition of Brownian loop-soup clusters. Preprint, 2015. Available at arXiv:1509.01180. To appear in *J. Eur. Math. Soc. (JEMS)*.

[22] Y. A. Rozanov. *Markov Random Fields*. Springer, Berlin, 1982. MR0676644

[23] O. Schramm and S. Sheffield. A contour line of the continuum Gaussian free field. *Probab. Theory Related Fields* **157** (2013) 47–80. MR3101840

[24] O. Schramm, S. Sheffield and D. B. Wilson. Conformal radii for conformal loop ensembles. *Comm. Math. Phys.* **288** (2009) 43–53. MR2491617

[25] S. Sheffield. Gaussian free fields for mathematicians. *Probab. Theory Related Fields* **139** (2007) 521–541. MR2322706

[26] W. Werner. Topics on the GFF and CLE(4), 2016.

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