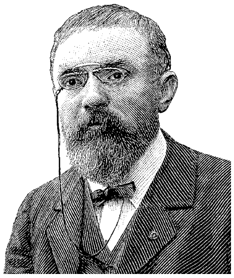

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Regular Dirichlet extensions of one-dimensional Brownian motion[1]

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Abstract. The regular Dirichlet extension is the dual concept of regular Dirichlet subspace. The main purpose of this paper is to characterize all the regular Dirichlet extensions of one-dimensional Brownian motion and to explore their structures. It is shown that every regular Dirichlet extension of one-dimensional Brownian motion may essentially decomposed into at most countable disjoint invariant intervals and an $\mathcal{E}$-polar set relative to this regular Dirichlet extension. On each invariant interval the regular Dirichlet extension is characterized uniquely by a scale function in a given class. To explore the structure of regular Dirichlet extension we apply the idea introduced in (*Ann. Probab.* **45** (2017) 857–872), we formulate the trace Dirichlet forms and attain the darning process associated with the restriction to each invariant interval of the orthogonal complement of $H^1_{\mathrm{e}}(\mathbb{R})$ in the extended Dirichlet space of the regular Dirichlet extension. As a result, we find an answer to a long-standing problem whether a pure jump Dirichlet form has proper regular Dirichlet subspaces.

Résumé. L'extension régulière de Dirichlet est la notion duale de celle de sous-espace de Dirichlet régulier. Le but principal de cet article est de caractériser toutes les extensions régulières de Dirichlet du mouvement brownien unidimensionnel et d'explorer leurs structures. On montre que chaque extension régulière du mouvement brownien unidimensionnel de Dirichlet peut essentiellement se décomposer en intervalles invariants disjoints au plus dénombrables et en un ensemble $\mathcal{E}$-polaire relatif à cette extension régulière de Dirichlet. Sur chaque intervalle invariant, l'extension régulière de Dirichlet est caractérisée de manière unique par une fonction d'échelle dans une classe donnée. Pour explorer la structure de l'extension régulière de Dirichlet, on applique l'idée introduite dans (*Ann. Probab.* **45** (2017) 857–872), on formule les formes de Dirichlet de trace et atteint le processus de reprise associé à la restriction à chaque intervalle invariant du complément orthogonal de $H^1_e(\mathbb{R})$ dans l'espace de Dirichlet étendu de l'extension régulière de Dirichlet. En conséquence, nous trouvons une réponse à un problème de longue date, à savoir si une forme pure de Dirichlet avec sauts comporte des sous-espaces propres de Dirichlet réguliers.

MSC: Primary 31C25; 60J55; secondary 60J60

Keywords: Regular Dirichlet extensions; Regular Dirichlet subspaces; Trace Dirichlet forms; Diffusion processes

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Metastability of one-dimensional, non-reversible diffusions with periodic boundary conditions

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Abstract. We consider small perturbations of a dynamical system on the one-dimensional torus. We derive sharp estimates for the pre-factor of the stationary state, we examine the asymptotic behavior of the solutions of the Hamilton–Jacobi equation for the pre-factor, we compute the capacities between disjoint sets, and we prove the metastable behavior of the process among the deepest wells following the martingale approach. We also present a bound for the probability that a Markov process hits a set before some fixed time in terms of the capacity of an enlarged process.

Résumé. Nous considérons de petites perturbations d'un système dynamique sur le tore unidimensionnel. Nous obtenons des estimations précises pour le pré-facteur de l'état stationnaire, nous examinons le comportement asymptotique des solutions de l'équation de Hamilton–Jacobi pour le pré-facteur, nous calculons les capacités entre des ensembles disjoints et nous prouvons le comportement métastable du processus parmi les puits les plus profonds en suivant l'approche martingale. Nous présentons également une borne pour la probabilité qu'un processus de Markov atteigne un ensemble avant un certain instant en termes de capacité d'un processus élargi.

MSC: 60J60; 60F99

Keywords: Non-reversible diffusions; Potential theory; Metastability; Dirichlet principle; Thomson principle; Eyring–Kramers formula

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Optimal survival strategy for branching Brownian motion in a Poissonian trap field

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Abstract. We study a branching Brownian motion Z with a generic branching law, evolving in $\mathbb{R}^d$, where a field of Poissonian traps is present. Each trap is a ball with constant radius. The traps are hard in the sense that the process is killed instantly once it enters the trap field. We focus on two cases of Poissonian fields, a uniform field and a radially decaying field, and consider an annealed environment. Using classical results on the convergence of the speed of branching Brownian motion, we establish precise annealed results on the population size of Z, given that it avoids the trap field, while staying alive up to time t. The results are stated so that each gives an 'optimal survival strategy' for Z. As corollaries of the results concerning the population size, we prove several other optimal survival strategies concerning the range of Z, and the size and position of clearings in $\mathbb{R}^d$. We also prove a result about the hitting time of a single trap by a branching system (Lemma 1), which may be useful in a completely generic setting too.

Inter alia, we answer some open problems raised in (*Markov Process. Related Fields* **9** (2003) 363–389).

Résumé. Nous étudions un mouvement brownien branchant Z ayant une loi de branchement générique et évoluant dans $\mathbb{R}^d$, où se trouve un champ de pièges poissonniens. Chaque piège est constitué d'une boule de rayon constant. Les pièges sont durs, au sens où le processus est tué instantanément dès qu'il pénètre dans l'un des pièges. Nous nous concentrons sur deux cas particuliers de champs poissonniens, un champ uniforme et un champ décroissant radialement, et nous considérons un environnement *annealed*. En utilisant des résultats classiques sur la convergence de la vitesse du mouvement brownien branchant, nous établissons des résultats *annealed* précis sur la taille de la population décrite par Z, conditionnellement à ce qu'il évite l'ensemble des pièges et reste en vie jusqu'au temps t. Les résultats sont formulés de sorte que chacun d'entre eux donne une 'stratégie optimale de survie' pour Z. En corollaires de ces résultats, nous démontrons l'optimalité de plusieurs autres stratégies concernant le support de Z jusqu'au temps t et la taille et la position de clairières dans $\mathbb{R}^d$. Nous démontrons également un résultat sur le temps d'atteinte d'un seul piège par un système branchant (Lemme 1), qui pourra aussi être utile dans un cadre totalement générique.

Au passage, nous apportons une réponse à plusieurs questions ouvertes formulées dans (*Markov Process. Related Fields* **9** (2003) 363–389).

MSC: 60J80; 60K37; 60F10

Keywords: Branching Brownian motion; Poissonian traps; Random environment; Hard obstacles; Optimal survival strategy

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Adaptive density estimation on bounded domains

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Abstract. We study the estimation, in $\mathbb{L}_p$-norm, of density functions defined on $[0,1]^d$. We construct a new family of kernel density estimators that do not suffer from the so-called boundary bias problem and we propose a data-driven procedure based on the Goldenshluger and Lepski approach that jointly selects a kernel and a bandwidth. We derive two estimators that satisfy oracle-type inequalities. They are also proved to be adaptive over a scale of anisotropic or isotropic Sobolev–Slobodetskii classes (which are particular cases of Besov or Sobolev classical classes). The main interest of the isotropic procedure is to obtain adaptive results without any restriction on the smoothness parameter.

Résumé. Nous étudions l'estimation, en norme $\mathbb{L}_p$, d'une densité de probabilté définie sur $[0,1]^d$. Nous construisons une nouvelle famille d'estimateurs à noyaux qui ne sont pas biaisés au bord du domaine de définition et nous proposons une procédure de sélection simultanée d'un noyau et d'une fenêtre de lissage en adaptant la méthode développée par Goldenshluger et Lepski. Deux estimateurs différents, déduits de cette procédure générale, sont proposés et des inégalités oracles sont établies pour chacun d'eux. Ces inégalités permettent de prouver que les-dits estimateurs sont adaptatifs par rapport à des familles de classes de Sobolev–Slobodetskii anisotropes ou isotropes. Dans cette dernière situation aucune borne supérieure sur le paramètre de régularité n'est imposée.

MSC: 62G05; 62G20

Keywords: Multivariate kernel density estimation; Bounded data; Boundary bias; Adaptive estimation; Oracle inequality; Sobolev–Slobodetskii classes

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On intermediate level sets of two-dimensional discrete Gaussian free field

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Abstract. We consider the discrete Gaussian Free Field (DGFF) in scaled-up (square-lattice) versions of suitably regular continuum domains $D \subset \mathbb{C}$ and describe the scaling limit, including local structure, of the level sets at heights growing as a λ-multiple of the height of the absolute maximum, for any $\lambda \in (0, 1)$. We prove that, in the scaling limit, the scaled spatial position of a typical point x sampled from this level set is distributed according to a Liouville Quantum Gravity (LQG) measure in D at parameter equal λ-times its critical value, the field value at x has an exponential intensity measure and the configuration near x reduced by the value at x has the law of a pinned DGFF reduced by a suitable multiple of the potential kernel. In particular, the law of the total size of the level set, properly-normalized, converges to that of the total mass of the LQG measure. This sharpens considerably an earlier conclusion by Daviaud (*Ann. Probab.* **34** (2006) 962–986).

Résumé. Nous considérons le champs Gaussien libre discret (DGFF) sur des versions renormalisées sur le réseau carré de domaines continus suffisamment réguliers $D \subset \mathbb{C}$ et décrivons la limite d'échelle, incluant la structure locale, des lignes de niveau lorsque que la hauteur croît comme λ-fois la hauteur du maximum absolu, pour tout $\lambda \in (0, 1)$. Nous montons que, dans la limite d'échelle, la position normalisée d'un point typique x tiré aléatoirement sur cette ligne de niveau a la loi de la mesure de Gravité Quantique de Liouville (LQG) dans D avec paramètre égal à λ-fois sa valeur critique, la valeur du champs en x ayant une mesure d'intensité exponentielle et la configuration près de x, réduite par la valeur en x, ayant la loi d'un champ libre épinglé DGFF réduit par un multiple adéquat du noyau potentiel. En particulier, la loi de la taille totale de la ligne de niveau, proprement normalisée, converge vers celle de la masse totale de la mesure LQG. Ceci améliore considérablement les résultats précédents de Daviaud (*Ann. Probab.* **34** (2006) 962–986).

MSC: 60G15; 82B41; 60G70; 62G30; 60G55; 60G57

Keywords: Gaussian Free Field; Level set; Point process; Liouville Quantum Gravity; Scaling limit; Conformal invariance

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On the martingale decompositions of Gundy, Meyer, and Yoeurp in infinite dimensions

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Abstract. We show that the canonical decomposition (comprising both the Meyer–Yoeurp and the Yoeurp decompositions) of a general X-valued local martingale is possible if and only if X has the UMD property. More precisely, X is a UMD Banach space if and only if for any X-valued local martingale M there exist a continuous local martingale M^c, a purely discontinuous quasi-left continuous local martingale M^q, and a purely discontinuous local martingale M^a with accessible jumps such that $M = M^c + M^q + M^a$. The corresponding weak L^1-estimates are provided. Important tools used in the proof are a new version of Gundy's decomposition of continuous-time martingales and weak L^1-bounds for a certain class of vector-valued continuous-time martingale transforms.

Résumé. Nous montons que la décomposition canonique (comprenant à la fois la décomposition de Meyer–Yoeurp et celle de Yoeurp) d'une martingale locale générale à valeurs dans X est possible si et seulement si X a la propriété UMD. Plus précisément, X est un espace de Banach UMD si et seulement si pour toute martingale M il existe une martingale locale continue M^c, une martingale locale purement discontinue et quasi-continue à gauche M^q et une martingale locale purement discontinue M^a à sauts accessibles, telles que $M = M^c + M^q + M^a$. Les estimées faibles L^1 correspondantes sont fournies. Les outils importants utilisés dans cette preuve sont une nouvelle version de la décomposition de Gundy d'une martingale à temps continu, et des bornes faibles dans L^1 pour une classe de transformations de martingales vectorielles à temps continu.

MSC: Primary 60G44; secondary 60G07; 60G57; 60H99; 46N30

Keywords: Gundy's decomposition; Continuous-time martingales; UMD spaces; Canonical decomposition; Meyer–Yoeurp decomposition; Yoeurp decomposition; Weak estimates; Weak differential subordination

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Transportation inequalities for non-globally dissipative SDEs with jumps via Malliavin calculus and coupling

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Abstract. By using the mirror coupling for solutions of SDEs driven by pure jump Lévy processes, we extend some transportation and concentration inequalities, which were previously known only in the case where the coefficients in the equation satisfy a global dissipativity condition. Furthermore, by using the mirror coupling for the jump part and the coupling by reflection for the Brownian part, we extend analogous results for jump diffusions. To this end, we improve some previous results concerning such couplings and show how to combine the jump and the Brownian case. As a crucial step in our proof, we develop a novel method of bounding Malliavin derivatives of solutions of SDEs with both jump and Gaussian noise, which involves the coupling technique and which might be of independent interest. The bounds we obtain are new even in the case of diffusions without jumps.

Résumé. En utilisant le couplage miroir pour les solutions d'EDS dirigées par un processus de Lévy de saut pur, nous généralisons des inégalités de transport et de concentration, qui étaient précédemment connues seulement dans le cas où les coefficients de l'équation satisfont une condition dissipative globale. De plus, en utilisant un couplage miroir pour la partie de sauts et le couplage par réflexion pour la partie Brownienne, nous étendons des résultats analogues pour les diffusions à sauts. A cette fin, nous améliorons des résultats précédents concernant le couplage et montrons comment combiner les cas à sauts et le cas brownien. Dans une étape cruciale de la preuve, nous développons une méthode nouvelle pour borner la dérivée de Malliavin des solutions d'EDS avec à la fois sauts et bruit Gaussien, ce qui utilise le couplage et peut être d'un intérêt indépendant. Les bornes que nous obtenons sont nouvelles même dans le cas de diffusions sans saut.

MSC: 60G51; 60H10; 60H07; 60E15

Keywords: Stochastic differential equations; Lévy processes; Transportation inequalities; Couplings; Wasserstein distances; Malliavin calculus

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Paracontrolled distributions on Bravais lattices and weak universality of the 2d parabolic Anderson model

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Abstract. We develop a discrete version of paracontrolled distributions as a tool for deriving scaling limits of lattice systems, and we provide a formulation of paracontrolled distributions in weighted Besov spaces. Moreover, we develop a systematic martingale approach to control the moments of polynomials of i.i.d. random variables and to derive their scaling limits. As an application, we prove a weak universality result for the parabolic Anderson model: We study a nonlinear population model in a small random potential and show that under weak assumptions it scales to the linear parabolic Anderson model.

Résumé. Nous développons une version discrète de la théorie des distributions paracontrôlées comme outil pour déduire les limites d'échelles des modèles discrets, et nous proposons une formulation des distributions paracontrôlées dans les espaces de Besov avec poids. De plus, nous obtenons une approche martingale pour contrôler systématiquement les moments des polynômes des variables aléatoires i.i.d., et pour déduire leurs limites d'échelles. Comme application, un résultat d'universalité faible pour le modèle parabolique d'Anderson est obtenu : nous étudions un modèle non linéaire d'une population dans un potentiel aléatoire, et démontrons, sous des hypothèses faibles, que le modèle converge vers le modèle parabolique d'Anderson linéaire.

MSC: 60H15; 60F05; 30H25

Keywords: Paracontrolled distributions; Scaling limits; Weak universality; Bravais lattices; Besov spaces; Parabolic Anderson model

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The Circular Law for random regular digraphs

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Abstract. Let $\log^C n \leq d \leq n/2$ for a sufficiently large constant $C > 0$ and let A_n denote the adjacency matrix of a uniform random d-regular directed graph on n vertices. We prove that as n tends to infinity, the empirical spectral distribution of A_n, suitably rescaled, is governed by the Circular Law. A key step is to obtain quantitative lower tail bounds for the smallest singular value of additive perturbations of A_n.

Résumé. Soit $\log^C n \leq d \leq n/2$ pour une constante suffisamment grande $C > 0$. Notons A_n la matrice d'adjacence d'un graphe dirigé aléatoire d-régulier sur n sommets. Nous montrons que lorsque n tend vers l'infini, la distribution empirique des valeurs propres de A_n, convenablement normalisée, suit la loi du cercle. Une étape cruciale consiste à obtenir une borne inférieure quantitative asymptotique pour la plus petite valeur singulière de perturbations additives de A_n.

MSC: Primary 15B52; secondary 60B20; 05C80

Keywords: Random matrix; Directed graph; Logarithmic potential; Singular values; Non-normal matrix; Universality

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Erratum: Qualitative properties of certain piecewise deterministic Markov processes

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Limit fluctuations for density of asymmetric simple exclusion processes with open boundaries

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Abstract. We investigate the fluctuations of cumulative density of particles in the asymmetric simple exclusion process with respect to the stationary distribution (also known as the steady state), as a stochastic process indexed by [0, 1]. In three phases of the model and their boundaries within the fan region, we establish a complete picture of the scaling limits of the fluctuations of the density as the number of sites goes to infinity. In the maximal current phase, the limit fluctuation is the sum of two independent processes, a Brownian motion and a Brownian excursion. This extends an earlier result by Derrida et al. (*J. Statist. Phys.* **115** (2004) 365–382) for totally asymmetric simple exclusion process in the same phase. In the low/high density phases, the limit fluctuations are Brownian motion. Most interestingly, at the boundary of the maximal current phase, the limit fluctuation is the sum of two independent processes, a Brownian motion and a Brownian meander (or a time-reversal of the latter, depending on the side of the boundary). Our proofs rely on a representation of the joint generating function of the asymmetric simple exclusion process with respect to the stationary distribution in terms of joint moments of a Markov processes, which is constructed from orthogonality measures of the Askey–Wilson polynomials.

Résumé. Nous étudions les fluctuations de la densité de particules dans un processus d'exclusion simple asymétrique sous la distribution stationnaire (ou état stable), vues comme un processus stochastique indexé par [0, 1]. Pour trois des phases du modèle et à leurs frontières nous obtenons une description complète des limites d'échelles de ces fluctuations lorsque le nombre de sites tend vers l'infini. Dans la phase de courant maximal, la limite est la somme de deux processus indépendants : un mouvement brownien et une excursion brownienne. Ce résultat étend celui obtenu précédemment par Derrida et al. (*J. Statist. Phys.* **115** (2004) 365–382) pour le processus d'exclusion simple totalement asymétrique et dans la même phase. Dans les phases de fortes et faibles densités, les limites sont des mouvements browniens. De façon plus intéressante, à la frontière de la phase de courant maximal, la limite est la somme de deux processus indépendants : un mouvement brownien et un méandre brownien (ou, selon la partie de la frontière, un méandre brownien renversé en temps). Nos démonstrations reposent sur une représentation des fonctions génératrices des lois fini-dimensionnelles du processus d'exclusion simple asymétrique en termes de moments joints d'un processus de Markov construit à partir de mesures rendant orthogonaux les polynômes d'Askey–Wilson.

MSC: 60F05; 60K35

Keywords: Asymmetric simple exclusion process; Scaling limit; Phase transition; Askey–Wilson process; Brownian excursion; Brownian meander; Laplace transform; Tangent process

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Asymptotic nonequivalence of density estimation and Gaussian white noise for small densities[1]

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Abstract. It is well-known that density estimation on the unit interval is asymptotically equivalent to a Gaussian white noise experiment, provided the densities are sufficiently smooth and uniformly bounded away from zero. We show that a uniform lower bound, whose size we sharply characterize, is in general necessary for asymptotic equivalence to hold.

Résumé. Il est bien connu que l'estimation de densité sur l'intervalle [0, 1] est asymptotiquement équivalente à une expérience de bruit blanc, à condition que les densités soient suffisamment régulières et uniformément bornées loin de 0. Nous montrons qu'une borne inférieure uniforme, dont on caractérise précisément la valeur, est en général nécessaire pour que cette équivalence asymptotique ait lieu.

MSC: Primary 62B15; secondary 62G10; 62G20

Keywords: Asymptotic equivalence; Density estimation; Gaussian white noise model; Small densities

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Discretisation of regularity structures

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Abstract. We introduce a general framework allowing to apply the theory of regularity structures to discretisations of stochastic PDEs. The approach pursued in this article is that we do not focus on any one specific discretisation procedure. Instead, we assume that we are given a scale $\varepsilon > 0$ and a "black box" describing the behaviour of our discretised objects at scales below ε.

Résumé. Nous introduisons un cadre général permettant d'appliquer la théorie des structures de régularité à des discrétisations d'EDP stochastiques. L'approche suivie dans cet article est que, au lieu de nous focaliser sur un type d'approximation spécifique, nous supposons donnée une échelle $\varepsilon > 0$ et une "boîte noire" décrivant le comportement des objets discrétisés aux échelles plus petites.

MSC: Primary 60H15; 60F17

Keywords: Regularity structures; Discretisation; Singular stochastic partial differential equations

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On time scales and quasi-stationary distributions for multitype birth-and-death processes

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Abstract. We consider a class of birth-and-death processes describing a population made of d sub-populations of different types which interact with one another. The state space is $\mathbb{Z}_+^d$ (unbounded). We assume that the population goes almost surely to extinction, so that the unique stationary distribution is the Dirac measure at the origin. These processes are parametrized by a scaling parameter K which can be thought as the order of magnitude of the total size of the population at time 0. For any fixed finite time span, it is well-known that such processes, when renormalized by K, are close, in the limit $K \to +\infty$, to the solutions of a certain differential equation in $\mathbb{R}_+^d$ whose vector field is determined by the birth and death rates. We consider the case where there is a unique attractive fixed point (off the boundary of the positive orthant) for the vector field (while the origin is repulsive). What is expected is that, for K large, the process will stay in the vicinity of the fixed point for a very long time before being absorbed at the origin. To precisely describe this behavior, we prove the existence of a quasi-stationary distribution (qsd, for short). In fact, we establish a bound for the total variation distance between the process conditioned to non-extinction before time t and the qsd. This bound is exponentially small in t, for $t \gg \log K$. As a by-product, we obtain an estimate for the mean time to extinction in the qsd. We also quantify how close is the law of the process (not conditioned to non-extinction) either to the Dirac measure at the origin or to the qsd, for times much larger than $\log K$ and much smaller than the mean time to extinction, which is exponentially large as a function of K. Let us stress that we are interested in what happens for finite K. We obtain results much beyond what large deviation techniques could provide.

Résumé. Nous considérons une classe de processus de naissance-et-mort décrivant une population constituée de d sous-populations de types différents qui intéragissent entre elles. L'espace d'état est $\mathbb{Z}_+^d$ (il est donc non borné). Nous supposons que la population s'éteint presque sûrement, de sorte que l'unique distribution de probabilité stationnaire est la masse de Dirac à l'origine. Nous faisons dépendre ces processus d'un paramètre d'échelle K qu'on peut interpréter comme l'ordre de grandeur de la taille totale de la population au temps 0. Etant donné un intervalle de temps, il est bien connu que de tels processus, normalisés par K, sont proches, dans la limite $K \to +\infty$, des solutions d'une certaine équation différentielle dans $\mathbb{R}_+^d$ dont le champ de vecteurs est déterminé par les taux de naissance et de mort du processus. Nous considérons le cas où le champ de vecteurs possède un unique point fixe attractif à l'intérieur de l'orthant positif, tandis que l'origine est un point fixe répulsif. On s'attend à ce que, pour K grand, le processus reste dans le voisinage du point fixe attractif pendant très longtemps avant d'être absorbé à l'origine. Afin de décrire précisément ce comportement, nous démontrons l'existence d'une distribution quasi-stationnaire (dqs, en abrégé). Nous établissons une borne pour la distance en variation totale entre le processus conditionné à ne pas s'éteindre avant le temps t et la dqs. Cette borne est exponentiellement petite en t pour $t \gg \log K$. En particulier, nous obtenons une estimation du temps moyen d'extinction dans la dqs. Nous quantifions également la distance entre le processus (non conditionné à la non-extinction) et une certaine combinaison convexe de la masse de Dirac à l'origine et de la dqs, ceci pour des temps beaucoup plus grands que $\log K$ et beaucoup plus petits que le temps moyen d'extinction, qui est exponentiellement grand en K. Insistons sur le fait que nous sommes intéressés par ce qui se passe pour K fini. Nous obtenons ainsi des résultats hors de portée des techniques de grandes déviations.

MSC: Primary 60J75; secondary 37C10; 92D40

Keywords: Markov jump process; Differential equations; Competition models; Population ecology; Mean time to extinction; Lyapunov functions

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Reversibility of the non-backtracking random walk

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Abstract. Let G be a connected graph of uniformly bounded degree. A k non-backtracking random walk (k-NBRW) $(X_n)_{n=0}^{\infty}$ on G evolves according to the following rule: Given $(X_n)_{n=0}^{s}$, at time $s+1$ the walk picks at random some edge which is incident to X_s that was not crossed in the last k steps and moves to its other end-point. If no such edge exists then it makes a simple random walk step. Assume that for some $R>0$ every ball of radius R in G contains a cycle. We show that under some "nice" random time change the 1-NBRW becomes reversible. This is used to prove that it is recurrent iff the simple random walk is recurrent. A similar result is proved for every k under stronger assumptions in general, and with no assumptions for Cayley graphs of finitely generated Abelian groups.

Résumé. Soit G un graphe connexe dont les degrés sont uniformément bornés. La marche aléatoire k non-rebroussante (k-NBRW) sur G évolue de la manière suivante: sachant $(X_n)_{n=0}^{s}$, l'état X_{s+1} est obtenu en choisissant uniformément au hasard une arête incidente à X_s qui n'a pas été empruntée durant les k mouvements précédents, et en l'empruntant. Si toutes les arêtes incidentes à X_s ont été empruntées durant les k mouvements précédents, on choisit une arête incidente à X_s uniformément au hasard. Supposons que toute boule de rayon R dans G contient un cycle. Alors nous montrons que, sous un changement de temps judicieux, la 1-NBRW devient réversible. Nous en déduisons que cette marche est récurrente si et seulement si la marche aléatoire simple l'est aussi. Un résultat similaire est établi pour tout k, sous des hypothèses supplémentaires en général, mais sans aucune hypothèse dans le cas des graphes de Cayley de groupes abéliens finiment engendrés.

MSC: 60J10

Keywords: Non-backtracking random walk; Recurrence; Transience

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Tube estimates for diffusions under a local strong Hörmander condition

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Abstract. We study lower and upper bounds for the probability that a diffusion process in $\mathbb{R}^n$ remains in a tube around a deterministic skeleton path up to a fixed time. The diffusion coefficients $\sigma_1, \dots, \sigma_d$ may degenerate, but we assume that they satisfy a strong Hörmander condition involving the first order Lie brackets around the skeleton of interest. The tube is written in terms of a norm which accounts for the non-isotropic structure of the problem: in a small time δ, the diffusion process propagates with speed $\sqrt{\delta}$ in the direction of the diffusion vector fields σ_j and with speed δ in the direction of $[\sigma_i, \sigma_j]$. We first prove short-time (non-asymptotic) lower and upper bounds for the density of the diffusion. Then, we prove the tube estimate using a concatenation of these short-time density estimates.

Résumé. On étudie des bornes inférieures et supérieures pour la probabilité qu'un processus de diffusion dans R^n reste dans un petit tube autour d'un squelette déterministe jusqu'à un temps fixé. Les coefficients de diffusion $\sigma_1, \dots, \sigma_d$ peuvent dégénérer, mais on suppose qu'ils satisfont à une condition d'Hörmander forte sur les coefficients et leurs crochets de Lie de premier ordre autour du squelette d'intérêt. Le tube est écrit en termes d'une norme qui prend en compte la structure non isotrope du problème: en temps δ petit, le processus de diffusion se propage avec vitesse $\sqrt{\delta}$ dans la direction des vecteurs de diffusion σ_j et avec vitesse δ dans la direction de $[\sigma_i, \sigma_j]$. On prouve d'abord des bornes inférieures et supérieures en temps court (non asymptotiques) pour la densité de la diffusion. Ensuite, on prouve l'estimée de tube en utilisant une concaténation de ces estimées de densité en temps court.

MSC: Primary 60H07; 60H10; secondary 60H30

Keywords: Tube estimates; Short-time density estimates; Hypoellipticity; Strong Hörmander condition; Malliavin calculus

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Sticky couplings of multidimensional diffusions with different drifts

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Abstract. We present a novel approach of coupling two multi-dimensional and non-degenerate Itô processes (X_t) and (Y_t) which follow dynamics with different drifts. Our coupling is *sticky* in the sense that there is a stochastic process (r_t), which solves a one-dimensional stochastic differential equation with a *sticky boundary* behavior at zero, such that almost surely $|X_t - Y_t| \le r_t$ for all $t \ge 0$. The coupling is constructed as a weak limit of Markovian couplings. We provide explicit, non-asymptotic and long-time stable bounds for the probability of the event $\{X_t = Y_t\}$.

Résumé. On présente une nouvelle approche de couplage de deux processus de Itô (X_t) et (Y_t) multi dimensionnels et non dégénérés qui suivent une dynamique avec des drifts différents. Le couplage est collant dans le sens qu'il existe un processus stochastique (r_t), qui résout une équation différentielle stochastique en dimension un avec un comportement collant à zéro, de sorte que presque sûrement, $|X_t - Y_t| \le r_t$ pour tous $t \ge 0$. Le couplage est construit comme une limite faible de couplages markoviens. On fournit des bornes explicites, non asymptotiques et stables à long terme pour la probabilité de l'événement $\{X_t = Y_t\}$.

MSC: 60J60; 60H10

Keywords: Diffusion process; Reflection coupling; Sticky boundary conditions; Stochastic stability; Perturbations of Markov processes; Total variation bounds; McKean–Vlasov

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On the boundary of the zero set of super-Brownian motion and its local time

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Abstract. If $X(t,x)$ is the density of one-dimensional super-Brownian motion, we prove that

$$\dim\big(\partial\{x : X(t,x) > 0\}\big) = 2 - 2\lambda_0 \in (0,1) \quad \text{a.s. on } \{X_t \neq 0\},$$

where $-\lambda_0 \in (-1,-1/2)$ is the lead eigenvalue of a killed Ornstein–Uhlenbeck process. This confirms a conjecture of Mueller, Mytnik and Perkins (*Ann. Probab.* **45** (2017) 3481–3543) who proved the above with positive probability. To establish this result we derive some new basic properties of a boundary local time recently introduced by one of us (Hughes), and analyze the behaviour of $X(t,\cdot)$ near the upper edge of its support. Numerical estimates of λ_0 suggest that the above Hausdorff dimension is approximately 0.224.

Résumé. Si l'on note $X(t,x)$ la densité du super-mouvement brownien de dimension 1, nous montrons que

$$\dim\big(\partial\{x : X(t,x) > 0\}\big) = 2 - 2\lambda_0 \in (0,1) \quad \text{p.s. sur } \{X_t \neq 0\},$$

ou $-\lambda_0 \in (-1,-1/2)$ est la valeur propre dominante d'un processus d'Ornstein–Uhlenbeck tué. Ceci confirme une conjecture de Mueller, Mytnik et Perkins (*Ann. Probab.* **45** (2017) 3481–3543), qui avaient montré que cette propriété a lieu avec probabilité strictement positive. Pour démontrer ce résultat, nous établissons quelques propriétés de base d'un temps local de bord introduit récemment par T. Hughes, et nous analysons le comportement de $X(t,\cdot)$ près de la borne supérieure de son support. Des simulations numériques de λ_0 suggèrent que la dimension de Hausdorff ci-dessus est approximativement 0,224.

MSC: Primary 60J68; secondary 60J55; 60H15; 28A78

Keywords: Super-Brownian motion; Hausdorff dimension; Stochastic pde; Zero-one law

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