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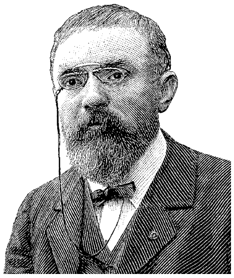

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

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Brownian motion in attenuated or renormalized inverse-square Poisson potential

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Abstract. We consider the parabolic Anderson problem with random potentials having inverse-square singularities around the points of a standard Poisson point process in $\mathbb{R}^d$, $d \ge 3$. The potentials we consider are obtained via superposition of translations over the points of the Poisson point process of a kernel $\mathfrak{K}$ behaving as $\mathfrak{K}(x) \approx \theta|x|^{-2}$ near the origin, where $\theta \in (0, (d-2)^2/16]$. In order to make sense of the corresponding path integrals, we require the potential to be either *attenuated* (meaning that $\mathfrak{K}$ is integrable at infinity) or, when $d = 3$, *renormalized*, as introduced by Chen and Kulik in (*Ann. Inst. Henri Poincaré Probab. Stat.* **48** (2012) 631–660). Our main results include existence and large-time asymptotics of non-negative solutions via Feynman–Kac representation. In particular, we settle for the renormalized potential in $d = 3$ the existence problem with critical parameter $\theta = 1/16$, left open by Chen and Rosinski in (Chen and Rosinski (2011)).

Résumé. Nous considérons le problème parabolique d'Anderson avec potentiels aléatoires ayant des singularités en carré inverse autour des points d'un processus de Poisson standard dans $\mathbb{R}^d$, $d \ge 3$. Les potentiels sont obtenus par superposition de translations par les points du processus de Poisson d'un noyau $\mathfrak{K}$ satisfaisant $\mathfrak{K}(x) \approx \theta|x|^{-2}$ près de l'origine, où $\theta \in (0, (d-2)^2/16]$. Afin de pouvoir définir les intégrales de chemin correspondantes, nous demandons que le noyau soit ou bien *atténué* (intégrable à l'infini), ou, en $d = 3$, *renormalisé* au sens de Chen et Kulik (*Ann. Inst. Henri Poincaré Probab. Stat.* **48** (2012) 631–660). Nous montrons l'existence et le comportement en temps long des solutions positives par représentation de Feynman–Kac, en particulier dans le cas critique $\theta = 1/16$ laissé ouvert par Chen et Rosinski (Chen et Rosinski (2011)).

MSC: 60J65; 60G55; 60K37; 35J10; 35P15

Keywords: Brownian motion in Poisson potential; Parabolic Anderson model; Inverse square potential; Multipolar Hardy inequality

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Nonparametric density estimation from observations with multiplicative measurement errors

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Abstract. In this paper we study the problem of pointwise density estimation from observations with multiplicative measurement errors. We elucidate the main feature of this problem: the influence of the estimation point on the estimation accuracy. In particular, we show that, depending on whether this point is separated away from zero or not, there are two different regimes in terms of the rates of convergence of the minimax risk. In both regimes we develop kernel-type density estimators and prove upper bounds on their maximal risk over suitable nonparametric classes of densities. We show that the proposed estimators are rate-optimal by establishing matching lower bounds on the minimax risk. Finally we test our estimation procedures on simulated data.

Résumé. Dans cet article, nous étudions le problème de l'estimation de densité ponctuelle à partir d'observations avec erreurs multiplicatives. Nous clarifions l'élément essentiel de ce problème: l'influence du point d'estimation sur la précision de l'estimation. En particulier, nous montrons que, selon que le point est éloigné de zéro ou pas, il y a deux régimes différents qui s'expriment en termes de la vitesse de convergence d'un risque minimax. Dans les deux régimes, nous développons des estimateurs de type noyau et prouvons des bornes supérieures sur leur risque maximal, ceci sur une classe convenable non paramétrique de densités. Nous montrons que les estimateurs proposés sont d'ordres optimaux en établissant des bornes inférieures correspondantes sur le risque minimax. Enfin, nous testons notre procédé d'estimation sur des données simulées.

MSC: 60G05; 60G20

Keywords: Multiplicative measurement errors; Scale mixtures; The Mellin transform; Multiplicative censoring; Density estimation

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Exponentially slow mixing in the mean-field Swendsen–Wang dynamics

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Abstract. Swendsen–Wang dynamics for the Potts model was proposed in the late 1980's as an alternative to single-site heat-bath dynamics, in which global updates allow this MCMC sampler to switch between metastable states and ideally mix faster. Gore and Jerrum (*J. Stat. Phys.* **97** (1999) 67–86) found that this dynamics may in fact exhibit slow mixing: they showed that, for the Potts model with $q \geq 3$ colors on the complete graph on n vertices at the critical point $\beta_c(q)$, Swendsen–Wang dynamics has $t_{\text{MIX}} \geq \exp(c\sqrt{n})$. Galanis *et al.* (In *Proc. of the 19th International Workshop on Randomization and Computation (RANDOM 2015)* (2015) 815–828) showed that $t_{\text{MIX}} \geq \exp(cn^{1/3})$ throughout the critical window (β_s, β_S) around β_c, and Blanca and Sinclair (In *Proc. of the 19th International Workshop on Randomization and Computation (RANDOM 2015)* (2015) 528–543) established that $t_{\text{MIX}} \geq \exp(c\sqrt{n})$ in the critical window for the corresponding mean-field FK model, which implied the same bound for Swendsen–Wang via known comparison estimates. In both cases, an upper bound of $t_{\text{MIX}} \leq \exp(c'n)$ was known. Here we show that the mixing time is truly exponential in n: namely, $t_{\text{MIX}} \geq \exp(cn)$ for Swendsen–Wang dynamics when $q \geq 3$ and $\beta \in (\beta_s, \beta_S)$, and the same bound holds for the related MCMC samplers for the mean-field FK model when $q > 2$.

Résumé. La dynamique de Swendsen–Wang a été proposée à la fin des années 1980 comme une alternative à la dynamique du bain-de-chaleur à un site, dans laquelle des mises à jour globales permettent à cet algorithme MCMC de passer plus vite d'un état métastable à un état de mélange idéal. Gore et Jerrum (*J. Stat. Phys.* **97** (1999) 67–86) ont trouvé que cette dynamique peut en fait montrer un mélange lent: ils ont montré, pour le modèle de Potts à $q \geq 3$ couleurs sur le graphe complet sur n sommets au point critique $\beta_c(q)$, que la dynamique de Swendsen–Wang vérifie $t_{\text{MIX}} \geq \exp(c\sqrt{n})$. Galanis *et al.* (In *Proc. of the 19th International Workshop on Randomization and Computation (RANDOM 2015)* (2015) 815–828) a montré que $t_{\text{MIX}} \geq \exp(cn^{1/3})$ dans toute la fenêtre critique (β_s, β_S) autour de β_c, et Blanca et Sinclair (In *Proc. of the 19th International Workshop on Randomization and Computation (RANDOM 2015)* (2015) 528–543) ont établit que $t_{\text{MIX}} \geq \exp(c\sqrt{n})$ dans la fenêtre critique pour le modèle de champs moyen FK, ce qui implique la même borne pour Swendsen–Wang grâce des estimées de comparaison connues. Dans les deux cas, une borne supérieure de $t_{\text{MIX}} \leq \exp(c'n)$ était connue. Dans cet article, nous montrons que le temps de mélange est vraiment exponentiel en n: plus précisément, $t_{\text{MIX}} \geq \exp(cn)$ pour la dynamique de Swendsen–Wang quand $q \geq 3$ et $\beta \in (\beta_s, \beta_S)$, et la même borne est vraie pour l'algorithme MCMC associé pour le modèle de champs moyen FK quand $q > 2$.

MSC: 60K35; 82B20; 82B27; 82C20

Keywords: Potts model; Swendsen–Wang; Mixing time; FK model; Random graphs

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On the thin-shell conjecture for the Schatten classes

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Abstract. We study the thin-shell conjecture for the Schatten classes. In particular, we establish the conjecture for the operator norm, and we also improve on the best known bound for the Schatten classes, due to Barthe and Cordero-Erausquin (*Proc. Lond. Math. Soc.* **106** (2013) 33–64) or Lee and Vempala (2017), for a few more cases. We also show that a necessary condition for the conjecture to be true for any of the Schatten classes is a rather strong negative correlation property: as a consequence of this we obtain the validity of this negative correlation property for all the cases for which we already know the conjecture is true (as for example for the operator norm), but moreover also for all the cases for which we can get a better estimate than the one in (*Proc. Lond. Math. Soc.* **106** (2013) 33–64) or (Lee and Vempala (2017)). For the proofs, our starting point is techniques that were employed for the Schatten classes in (*Math. Ann.* **312** (1998) 773–783) and (*Ann. Inst. Henri Poincaré Probab. Stat.* **43** (2007) 87–99) with regard to other problems.

Résumé. Nous étudions la conjecture de la variance (ou autrement dit, de la concentration du volume d'un convexe dans une petite couronne euclidienne) pour les classes de Schatten. En particulier, nous établissons la conjecture pour la norme d'opérateur, et nous améliorons également le meilleur majorant connu, grâce à Barthe et Cordero-Erausquin (*Proc. Lond. Math. Soc.* **106** (2013) 33–64) ou Lee et Vempala (2017), dans quelques cas de plus.

Nous montrons aussi qu'une condition nécessaire pour que la conjecture soit vraie dans une des classes de Schatten est une propriété de corrélation négative qui doit être suffisamment forte: ceci implique que nous obtenons la validité de cette propriété dans tous les cas pour lesquels on peut démontrer la conjecture (comme par exemple pour la norme d'opérateur), mais aussi dans tous les cas pour lesquels on peut obtenir une meilleure estimation que celle dans (*Proc. Lond. Math. Soc.* **106** (2013) 33–64) ou (Lee and Vempala (2017)).

En ce qui concerne les démonstrations, notre point de départ consiste en des techniques qui ont été utilisées dans (*Math. Ann.* **312** (1998) 773–783) et (*Ann. Inst. Henri Poincaré Probab. Stat.* **43** (2007) 87–99) pour les classes de Schatten dans le contexte d'autres problèmes connexes.

MSC: 46B20; 47B10; 46B07

Keywords: Schatten-Von Neumann classes; Isotropic convex body; Concentration inequalities; Variance conjecture

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Spectral statistics of sparse Erdős–Rényi graph Laplacians

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Abstract. We consider the bulk eigenvalue statistics of Laplacian matrices of large Erdős–Rényi random graphs in the regime $p \geq N^{\delta}/N$ for any fixed $\delta > 0$. We prove a local law down to the optimal scale $\eta \gtrsim N^{-1}$ which implies that the eigenvectors are delocalized. We consider the local eigenvalue statistics and prove that both the gap statistics and averaged correlation functions coincide with the GOE in the bulk.

Résumé. Nous nous intéressons aux statistiques, dans l'intérieur du spectre, des valeurs propres de matrices laplaciennes de grands graphes d'Erdős–Rényi aléatoires dans le régime où $p \geq N^{\delta}/N$ pour un $\delta > 0$ fixé arbitraire. Nous montrons une loi locale jusqu'à l'échelle optimale $\eta \gtrsim N^{-1}$ qui implique que les vecteurs propres sont délocalisés. Nous considérons les statistiques locales des valeurs propres et montrons que les statistiques des intervalles et les fonctions de corrélation moyennées coïncident avec le GOE dans l'intérieur du spectre.

MSC: 15B52

Keywords: Random matrix theory; Universality

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Markovian integral equations

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Abstract. We analyze multidimensional Markovian integral equations that are formulated with a progressive time-inhomogeneous Markov process that has Borel measurable transition probabilities. In the case of a path process of a path-dependent diffusion, the solutions to these integral equations lead to the concept of mild solutions to path-dependent partial differential equations (PPDEs). Our goal is to establish uniqueness, stability, existence and non-extendibility of solutions among a certain class of maps. By requiring the Feller continuity of the Markov process, we give weak conditions under which solutions become continuous. Moreover, we provide a multidimensional Feynman–Kac formula and a one-dimensional global existence and uniqueness result.

Résumé. Nous analysons des équations intégrales Markoviennes multidimensionnelles qui sont formulées avec un processus de Markov progressif et non homogène dans le temps qui a des probabilités de transition Borel-mesurables. Dans les cas d'un processus de trajectoire d'une diffusion dépendante de trajectoire, les solutions à ces équations intégrales mènent au concept de solutions «mild » d'équations aux dérivées partielles dépendant de trajectoire. Notre objectif est d'établir unicité, stabilité, existence et non-extensibilité des solutions parmi une certaine classe de fonctions. En exigeant la continuité de Feller du processus de Markov, nous donnons des conditions faibles sous lesquelles les solutions deviennent continues. En outre, nous fournissons une formule multidimensionnelle de Feynmanc–Kac et un résultat unidimensionnel d'existence et d'unicité globales.

MSC: 60H30; 60J25; 35K40; 35K58; 45G15

Keywords: Integral equation; Markov process; Mild solution; Path-dependent PDE; Feynman–Kac formula; Log-Laplace equation; Superprocess

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Ergodicity of stochastic differential equations with jumps and singular coefficients

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Abstract. We show the strong well-posedness of SDEs driven by general multiplicative Lévy noises with Sobolev diffusion and jump coefficients and integrable drifts. Moreover, we also study the strong Feller property, irreducibility as well as the exponential ergodicity of the corresponding semigroup when the coefficients are time-independent and singular dissipative. In particular, the large jump is allowed in the equation. To achieve our main results, we present a general approach for treating the SDEs with jumps and singular coefficients so that one just needs to focus on Krylov's a priori estimates for SDEs.

Résumé. Nous montrons que les EDS dirigées par un bruit de Lévy multiplicatif général avec des coefficients de diffusion et de saut Sobolev, et une dérive intégrable, sont fortement bien posées. De plus, nous étudions la propriété forte de Feller, l'irréductibilité ainsi que l'ergodicité exponentielle des semi-groupes correspondants quand les coefficients sont indépendants du temps et singulièrement dissipatifs. En particulier, les grands sauts sont autorisés dans l'équation. Pour aboutir au résultat principal, nous présentons une approche générale pour traiter les EDS avec sauts et coefficients singuliers, de telle sorte que nous devons seulement nous intéresser aux estimées a priori de Krylov pour les EDS.

MSC: 60H10; 60J60

Keywords: Pathwise uniqueness; Krylov's estimate; Zvonkin's transformation; Ergodicity; Heat kernel

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Statistical limits of spiked tensor models

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Abstract. We study the statistical limits of both detecting and estimating a rank-one deformation of a symmetric random Gaussian tensor. We establish upper and lower bounds on the critical signal-to-noise ratio, under a variety of priors for the planted vector: (i) a uniformly sampled unit vector, (ii) i.i.d. ± 1 entries, and (iii) a sparse vector where a constant fraction ρ of entries are i.i.d. ± 1 and the rest are zero. For each of these cases, our upper and lower bounds match up to a $1 + o(1)$ factor as the order d of the tensor becomes large. For sparse signals (iii), our bounds are also asymptotically tight in the sparse limit $\rho \to 0$ for any fixed d (including the $d = 2$ case of sparse PCA). Our upper bounds for (i) demonstrate a phenomenon reminiscent of the work of Baik, Ben Arous and Péché: an 'eigenvalue' of a perturbed tensor emerges from the bulk at a strictly lower signal-to-noise ratio than when the perturbation itself exceeds the bulk; we quantify the size of this effect. We also provide some general results for larger classes of priors. In particular, the large d asymptotics of the threshold location differs between problems with discrete priors versus continuous priors. Finally, for priors (i) and (ii) we carry out the replica prediction from statistical physics, which is conjectured to give the exact information-theoretic threshold for any fixed d.

Of independent interest, we introduce a new improvement to the second moment method for contiguity, on which our lower bounds are based. Our technique conditions away from rare 'bad' events that depend on interactions between the signal and noise. This enables us to close $\sqrt{2}$-factor gaps present in several previous works.

Résumé. Nous étudions les limites statistiques pour détecter et estimer une déformation de rang un d'un tenseur Gaussien symétrique aléatoire. Nous établissons une borne inférieure et une borne supérieure sur le rapport signal-bruit critique, sous diverses lois a priori pour le vecteur planté: (i) un vecteur unité aléatoire uniforme, (ii) des entrées i.i.d. ± 1, et (iii) un vecteur creux où une fraction constante ρ d'entrées sont i.i.d. ± 1 et les autres nulles. Pour chacun de ces cas, nos bornes supérieures et inférieures coïncident à un facteur $1 + o(1)$ près quand l'ordre d du tenseur devient grand. Pour des signaux creux (iii), nos bornes sont aussi asymptotiquement tendues dans la limite $\rho \to 0$ pour tout d fixé (incluant le cas $d = 2$ du PCA creux). Notre borne supérieure pour (i) montre un phénomène rappelant le travail de Baik, Ben Arous et Péché: une valeur propre du tenseur perturbé émerge de l'intérieur du spectre à un rapport signal-bruit strictement inférieur que quand la perturbation elle-même sort de l'intérieur du spectre; nous quantifions la taille de cet effet. Nous donnons aussi des résultats généraux pour une grande classe de lois a priori. En particulier, l'asymptotique quand d devient grand de la valeur de seuil diffère entre les problèmes avec lois a priori discrètes et continues. Finalement, pour les lois a priori (i) et (ii), nous vérifions la prédiction issue de la méthode des répliques en physique statistique, qui est conjecturée donner l'information théorique exacte sur le seuil pour tout d fixé.

D'un intérêt indépendant, nous introduisons une amélioration à la méthode du second moment par contiguïté, sur laquelle notre borne inférieure est basée. Notre méthode conditionne loin les rares « mauvais » événements qui dépendent des interactions entre le signal et le bruit. Ceci nous permet de résoudre le trou de facteur $\sqrt{2}$ présent dans les articles précédents.

MSC: Primary 62F03; secondary 68Q87

Keywords: Spiked tensor model; Contiguity; Second moment method

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On laws of large numbers in L^2 for supercritical branching Markov processes beyond λ-positivity

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Abstract. We give necessary and sufficient conditions for laws of large numbers to hold in L^2 for the empirical measure of a large class of branching Markov processes, including λ-positive systems but also some λ-transient ones, such as the branching Brownian motion with drift and absorption at 0. This is a significant improvement over previous results on this matter, which had only dealt so far with λ-positive systems. Our approach is purely probabilistic and is based on spinal decompositions and many-to-few lemmas. In addition, we characterize when the limit in question is always strictly positive on the event of survival, and use this characterization to derive a simple method for simulating (quasi-)stationary distributions.

Résumé. Nous obtenons des conditions nécessaires et suffisantes pour des lois des grands nombres dans L^2 concernant les mesures empiriques d'une large classe de processus de Markov branchants, comme le mouvement Brownien branchant avec dérive et absorption en 0. Cela constitue un pas significatif pour ce genre de résultats qui étaient jusqu'à présent limités aux processus λ-positifs. Notre approche est purement probabiliste et est basée sur des décompositions en épine (spine) et des lemmes associés. De plus, nous caractérisons la stricte positivité de la limite quand le processus de branchement survit et utilisons cette caractérisation pour donner une méthode simple de simulation de distributions (quasi-)stationaires.

MSC: 60F99; 60J80

Keywords: Law of large numbers; Branching Markov processes; Spine decomposition; h-transform; λ-positivity

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Some properties of the free stable distributions

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Abstract. We investigate certain analytical properties of the free α-stable densities on the line. We prove that they are all classically infinitely divisible when $\alpha \le 1$ and that they belong to the extended Thorin class when $\alpha \le 3/4$. The Lévy measure is explicitly computed for $\alpha = 1$, showing that free 1-stable distributions are not in the Thorin class except in the drifted Cauchy case. In the symmetric case we show that the free stable densities are not infinitely divisible when $\alpha > 1$. In the one-sided case we prove, refining unimodality, that the densities are whale-shaped, that is their successive derivatives vanish exactly once on their support. We also derive several fine properties of spectrally one-sided free stable densities, including a detailed analysis of the Kanter random variable, complete asymptotic expansions at zero, and several intrinsic features of whale-shaped functions.

Résumé. Nous étudions certaines propriétés analytiques des densités α-stables libres sur la droite. Nous montrons qu'elles sont classiquement infiniment divisibles pour $\alpha \le 1$ et qu'elles appartiennent à la classe de Thorin étendue pour $\alpha \le 3/4$. La mesure de Lévy est calculée explicitement pour $\alpha = 1$ et ce calcul entraîne que les lois 1-stables libres n'appartiennent pas à la classe de Thorin, sauf dans le cas de la loi de Cauchy avec dérive. Dans le cas symétrique, nous montrons que les densités α-stables libres ne sont pas infiniment divisibles quand $\alpha > 1$. Dans le cas de signe constant nous montrons que les densités stables libres ont une courbe en baleine, autrement dit que leurs dérivées successives ne s'annulent qu'une seule fois sur leurs supports, ce qui constitue un raffinement de l'unimodalité. Nous établissons enfin plusieurs propriétés précises des densités stables libres spectralement de signe constant, parmi lesquelles une analyse détaillée de la variable aléatoire de Kanter, des expansions asymptotiques complètes en zéro, ainsi que plusieurs propriétés intrinsèques des courbes en baleine.

MSC: 33E20; 46L54; 60E07; 62E15

Keywords: Free stable distribution; Infinite divisibility; Shape of densities; Wright function

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Existence of densities for the dynamic Φ^4_3 model

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Abstract. We apply Malliavin calculus to the Φ^4_3 equation on the torus and prove existence of densities for the solution of the equation evaluated at regular enough test functions. We work in the framework of regularity structures and rely on Besov-type spaces of modelled distributions in order to prove Malliavin differentiability of the solution. Our result applies to a large family of Gaussian space–time noises including white noise, in particular the noise may be degenerate as long as it is sufficiently rough on small scales.

Résumé. Nous appliquons le calcul de Malliavin à l'équation Φ^4_3 sur le tore et prouvons l'existence des densités pour les évaluations de la solution contre des fonctions test suffisamment régulières. Nous travaillons dans le cadre des structures de régularité et utilisons les espaces de distributions modelées de type Besov afin de prouver la différentiabilité au sens de Malliavin de la solution. Notre résultat s'applique à une large classe de bruits gaussiens en espace-temps incluant le bruit blanc, en particulier le bruit peut être dégénéré tant qu'il est suffisamment irrégulier à petite échelle.

MSC: 60H07; 60H15

Keywords: Malliavin calculus; Regularity structures; Stochastic quantization equation; Singular SPDE

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The asymptotic equivalence of the sample trispectrum and the nodal length for random spherical harmonics

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Abstract. We study the asymptotic behaviour of the nodal length of random $2d$-spherical harmonics f_ℓ of high degree $\ell \to \infty$, i.e. the length of their zero set $f_\ell^{-1}(0)$. It is found that the nodal lengths are asymptotically equivalent, in the L^2-sense, to the "sample trispectrum", i.e., the integral of $H_4(f_\ell(x))$, the fourth-order Hermite polynomial of the values of f_ℓ. A particular by-product of this is a Quantitative Central Limit Theorem (in Wasserstein distance) for the nodal length, in the high energy limit.

Résumé. Nous étudions le comportement asymptotique de la longueur nodale de fonctions propres aléatoires f_ℓ du Laplacien sphérique pour valeurs propres très élevés $\ell \to +\infty$, c'est-à-dire la longueur de leur ensemble de niveau zéro $f_\ell^{-1}(0)$. Nous démontrons que la longueur nodale est asymptotiquement équivalente, au sens de L^2, au « sample trispectrum », c'est-à-dire l'intégral de $H_4(f_\ell(x))$, le polynôme de Hermite d'ordre quatre évalué en f_ℓ. Une conséquence de ce résultat est un Théorème Central Limite quantitatif (dans le sens de la distance de Wasserstein) pour la longueur nodale, quand l'énergie tend vers l'infini.

MSC: 60G60; 62M15; 53C65; 42C10; 33C55

Keywords: Nodal length; Spherical harmonics; Sample trispectrum; Berry's cancellation; Quantitative Central Limit Theorem

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Internal diffusion-limited aggregation with uniform starting points

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Abstract. We study internal diffusion-limited aggregation with uniform starting points on $\mathbb{Z}^d$. In this model, each new particle starts from a vertex chosen uniformly at random on the existing aggregate. We prove that the limiting shape of the aggregate is a Euclidean ball.

Résumé. Nous étudions le modèle d'agrégation limitée par diffusion interne avec points de départ uniformes sur $\mathbb{Z}^d$. Dans ce modèle, chaque nouvelle particule est ajoutée à un point choisi uniformément au hasard parmi ceux de l'agrégat existant. Nous prouvons que l'agrégat normalisé admet comme forme limite la boule euclidienne.

MSC: Primary 82C24; secondary 60J45

Keywords: Growth model; Random walk; IDLA; Harmonic measure

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Empirical risk minimization as parameter choice rule for general linear regularization methods

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Abstract. We consider the statistical inverse problem to recover f from noisy measurements $Y = Tf + \sigma\xi$ where ξ is Gaussian white noise and T a compact operator between Hilbert spaces. Considering general reconstruction methods of the form $\hat{f}_\alpha = q_\alpha(T^*T)T^*Y$ with an ordered filter q_α, we investigate the choice of the regularization parameter α by minimizing an unbiased estimate of the predictive risk $\mathbb{E}[\|Tf - T\hat{f}_\alpha\|^2]$. The corresponding parameter α_{pred} and its usage are well-known in the literature, but oracle inequalities and optimality results in this general setting are unknown. We prove a (generalized) oracle inequality, which relates the direct risk $\mathbb{E}[\|f - \hat{f}_{\alpha_{\mathrm{pred}}}\|^2]$ with the oracle prediction risk $\inf_{\alpha>0}\mathbb{E}[\|Tf - T\hat{f}_\alpha\|^2]$. From this oracle inequality we are then able to conclude that the investigated parameter choice rule is of optimal order in the minimax sense.

Finally we also present numerical simulations, which support the order optimality of the method and the quality of the parameter choice in finite sample situations.

Résumé. Nous considérons le problème inverse stochastique de reconstruire f à partir de données bruitées $Y = Tf + \sigma\xi$ où ξ est un bruite blanc et T un opérateur compact entre espaces de Hilbert. Considérant des méthodes de reconstruction générales de la forme $\hat{f}_\alpha = q_\alpha(T^*T)T^*Y$ avec un filtre ordonné q_α, nous examinons le choix du paramètre de régularisation α en minimisant un estimateur non biaisé du risque prédictif $\mathbb{E}[\|Tf - T\hat{f}_\alpha\|^2]$. Le paramètre correspondant α_{pred} et son utilisation sont bien connus dans la littérature mais les inégalites oracles et les résultats d'optimalité dans ce cadre général sont inconnus. Nous prouvons une inégalité oracle (généralisée), qui relie le risque direct $\mathbb{E}[\|f - \hat{f}_{\alpha_{\mathrm{pred}}}\|^2]$ au risque de l'oracle de prédiction $\inf_{\alpha>0}\mathbb{E}[\|Tf - T\hat{f}_\alpha\|^2]$. A partir de cette inégalité oracle nous sommes alors capable de conclure que la règle de choix du paramètre examiné est d'ordre optimale au sens minimax.

Finalement nous présentons aussi des simulations numériques qui confirment l'optimalité de l'ordre de la méthode et la qualité du choix du paramètre dans des situations discrètes.

MSC: Primary 62G05; 62G20; secondary 65J22; 65J20

Keywords: Statistical inverse problem; Regularization method; Filter-based inversion; A-posteriori parameter choice rule; Order optimality; Exponential bounds; Oracle inequality

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Nonconventional moderate deviations theorems and exponential concentration inequalities

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Abstract. We obtain moderate deviations theorems and exponential (Bernstein type) concentration inequalities for "nonconventional" sums of the form $S_N = \sum_{n=1}^{N}(F(\xi_{q_1(n)}, \xi_{q_2(n)}, \dots, \xi_{q_\ell(n)}) - \bar{F})$, where most of the time we consider $q_i(n) = in$, but our results also hold true for more general $q_i(n)$'s such as polynomials. Here ξ_n, $n \geq 0$ is a sufficiently fast mixing vector process with some stationarity conditions, F is a function satisfying certain regularity conditions and $\bar{F}$ is a certain centralizing constant. When $\xi_n, n \geq 0$ are independent and identically distributed a large deviations theorem was obtained in (*Probab. Theory Related Fields* **158** (2014) 197–224) and one of the purposes of this paper is to obtain related results in the (weakly) dependent case. Several normal approximation type results will also be derived. In particular, two more proofs of the nonconventional central limit theorem are given and a Rosenthal type inequality is obtained. Our results hold true, for instance, when $\xi_n = (T^n f_i)_{i=1}^{\wp}$ where T is a topologically mixing subshift of finite type, a Gibbs–Markov map, a hyperbolic diffeomorphism, a Young tower or an expanding transformation taken with a Gibbs invariant measure, as well as in the case when ξ_n, $n \geq 0$ forms a stationary and (stretched) exponentially fast ϕ-mixing sequence, which, for instance, holds true when $\xi_n = (f_i(\Upsilon_n))_{i=1}^{\wp}$ where Υ_n is a Markov chain satisfying the Doeblin condition considered as a stationary process with respect to its invariant measure.

Résumé. Nous obtenons un théorème de déviations modérées et des inégalités de concentration exponentielles (du type de Bernstein) pour des sommes «non-conventionnelles» de la forme $S_N = \sum_{n=1}^{N}(F(\xi_{q_1(n)}, \xi_{q_2(n)}, \dots, \xi_{q_\ell(n)}) - \bar{F})$, où la plupart du temps nous considérons $q_i(n) = in$, mais nos résultats restent aussi vrais pour des $q_i(n)$ plus généraux tels que des polynômes. Ici, ξ_n, $n \geq 0$ est un processus vectoriel suffisamment mélangeant avec des conditions de stationnarité, F est une fonction satisfaisant certaines propriétés de régularité et $\bar{F}$ est une constante de centrage. Quand ξ_n, $n \geq 0$ sont indépendants et identiquement distribués, un principe de grande déviation a été obtenu dans (*Probab. Theory Related Fields* **158** (2014) 197–224) et un des objectifs de cet article est d'obtenir des résultats analogues dans le cas faiblement dépendant. Plusieurs résultats de type approximation normale sont aussi obtenus. En particulier, deux nouvelles preuves du théorème central limite non-conventionnel sont données et une inégalité de type Rosenthal est obtenue. Nos résultats sont vrais par exemple quand $\xi_n = (T^n f_i)_{i=1}^{\wp}$ où T est un sous-shift de type fini topologiquement mélangeant, une application Gibbs–Markov, un difféomorphisme hyperbolique, une tour de Young ou une transformation expansive pour une mesure invariante de Gibbs, tout comme dans le cas où ξ_n, $n \geq 0$ forme une suite stationnaire exponentiellement (ou streched exponentiellement) ϕ-mélangeante, ce qui, par exemple, et vrai lorsque $\xi_n = (f_i(\Upsilon_n))_{i=1}^{\wp}$ où Υ_n est une chaîne de Markov satisfaisant une condition de Doeblin, considérée comme un processus stationnaire par rapport à une mesure invariante.

MSC: Primary 60F10; secondary 60F05; 37D20; 37D25; 37A25

Keywords: Nonconventional setup; Mixing; Large deviations; Moderate deviations; Exponential concentration inequalities; The method of cumulants; Martingale approximation; Central limit theorem; Berry–Esseen theorem

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Renewal theorems and mixing for non Markov flows with infinite measure

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Abstract. We obtain results on mixing for a large class of (not necessarily Markov) infinite measure semiflows and flows. Erickson proved, amongst other things, a strong renewal theorem in the corresponding i.i.d. setting. Using operator renewal theory, we extend Erickson's methods to the deterministic (i.e. non-i.i.d.) continuous time setting and obtain results on mixing as a consequence.

Our results apply to intermittent semiflows and flows of Pomeau–Manneville type (both Markov and nonMarkov), and to semiflows and flows over Collet–Eckmann maps with nonintegrable roof function.

Résumé. Nous obtenons des résultats de mélange pour une large classe de flots et de semi-flots préservant une mesure de masse infinie (et qui ne sont pas necessairement markoviens). Erickson a prouvé, entre autres choses, un théorème de renouvellement fort dans le contexte de variables aleatoires indépendantes et identiquement distribuées. En utilisant la théorie des opérateurs de renouvellement, nous étendons les méthodes d'Erickson au cas du temps continu déterministe (et donc on i.i.d.) et en déduisons des résultats sur le mélange.

Nos résultats s'appliquent a des semi-flots et flots intermittents de type Pomeau–Manneville (à la fois de type markoviens ou de type non-markoviens) ainsi qu'à des suspensions au-dessus de transformations de Collet–Eckmann pour lesquelles la fonction toit est non-intégrable.

MSC: 37A25; 37A40; 37A50; 60K05

Keywords: Flows; Renewal theorems; Operator renewal theory; Krickeberg mixing

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On a non-linear 2D fractional wave equation

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Abstract. We investigate the following non-linear stochastic wave equation model:

$$\begin{cases} \partial_t^2 u - \Delta u = u^2 + \dot{B}, & t \in [0,T], x \in \mathbb{R}^2, \\ u(0,\cdot) = \phi_0, & \partial_t u(0,\cdot) = \phi_1, \end{cases}$$

where ϕ_0, ϕ_1 are deterministic initial conditions in an appropriate Sobolev space and $\dot{B}$ stands for a space–time fractional noise. In this two-dimensional situation, we develop a strategy based on a third-order expansion of the equation, which, combined with a Wick-renormalization procedure, allows us to extend the results of Deya (2019) to a rougher noise.

We also point out the limits of this specific strategy when considering a highly rough noise.

Résumé. Nous nous intéressons au modèle d'équation des ondes stochastique non-linéaire suivant:

$$\begin{cases} \partial_t^2 u - \Delta u = u^2 + \dot{B}, & t \in [0,T], x \in \mathbb{R}^2, \\ u(0,\cdot) = \phi_0, & \partial_t u(0,\cdot) = \phi_1, \end{cases}$$

où ϕ_0, ϕ_1 sont des conditions initiales déterministes dans un espace de Sobolev approprié et $\dot{B}$ représente un bruit fractionnaire espace-temps. Dans cette situation bi-dimensionnelle, notre stratégie est basée sur un développement de l'équation à l'ordre trois, qui, combiné à une procédure de renormalisation de type Wick, nous permet d'étendre les résultats de Deya (2019) à des bruits plus rugueux.

Nous mettons également en avant les limites de cette stratégie particulière en présence de processus très irréguliers.

MSC: 60H15; 60G22; 35L71

Keywords: Stochastic wave equation; Fractional noise; Wick renormalization

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Scaling limits of discrete snakes with stable branching

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Abstract. We consider so-called discrete snakes obtained from size-conditioned critical Bienaymé–Galton–Watson trees by assigning to each node a random spatial position in such a way that the increments along each edge are i.i.d. When the offspring distribution belongs to the domain of attraction of a stable law with index $\alpha \in (1, 2]$, we give a necessary and sufficient condition on the tail distribution of the spatial increments for this spatial tree to converge, in a functional sense, towards the Brownian snake driven by the α-stable Lévy tree. We also study the case of heavier tails, and apply our result to study the number of inversions of a uniformly random permutation indexed by the tree.

Résumé. Nous considérons des « serpents stables » obtenus à partir d'arbres de Bienaymé–Galton–Watson critiques conditionnés par la taille en assignant à chaque nœud une position spatiale de sorte que les accroissements le long des arêtes sont i.i.d. Lorsque la loi de reproduction appartient au bassin d'attraction d'une loi stable d'indice $\alpha \in]1, 2]$, nous donnons une condition nécessaire et suffisante sur la queue de distribution des accroissements spatiaux pour que cet arbre converge – en un sens fonctionnel – vers le serpent brownien sur l'arbre de Lévy α-stable. Nous étudions également le cas de queues plus lourdes et nous appliquons nos résultats au nombre d'inversions d'une permutation aléatoire indexée par l'arbre.

MSC: 60J80; 60F17; 60G50

Keywords: Brownian snake; Discrete snakes; Invariance principles; Branching random walk

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Quasi-static large deviations

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Abstract. We consider the symmetric simple exclusion with open boundaries that are in contact with particle reservoirs at different densities. The reservoir densities changes at a slower time scale with respect to the natural time scale the system reaches the stationary state.This gives rise to the quasi static hydrodynamic limit proven in (*Journal of Statistical Physics* **161** (5) (2015) 1037–1058). We study here the large deviations with respect to this limit for the particle density field and the total current. We identify explicitely the large deviation functional and prove that it satisfies a fluctuation relation.

Résumé. Nous considérons l'exclusion simple symétrique avec des frontières ouvertes en contact avec des réservoirs de particules à différentes densités. Les densités des réservoirs changent plus lentement par rapport à l'échelle de temps naturelle où le système atteint l'état stationnaire. Cela donne lieu à la limite hydrodynamique quasi-statique. Nous étudions ici les grandes déviations par rapport à cette limite pour le champ de densité de particules et le courant total. Nous identifions explicitement le fonctionnelle de grandes déviations et nous prouvons qu'il satisfait une relation de fluctuation.

MSC: 60F10; 82C22; 82C70

Keywords: Quasi-static thermodynamic; Hydrodynamic limits; Large deviations

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Couplings in L^p distance of two Brownian motions and their Lévy area

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Abstract. We study co-adapted couplings of (canonical hypoelliptic) diffusions on the (subRiemannian) Heisenberg group, that we call (Heisenberg) Brownian motions and are the joint laws of a planar Brownian motion with its Lévy area. We show that contrary to the situation observed on Riemannian manifolds of non-negative Ricci curvature, for any co-adapted coupling, two Heisenberg Brownian motions starting at two given points can not stay at bounded distance for all time $t \geq 0$. Actually, we prove the stronger result that they can not stay bounded in L^p for $p \geq 2$.

We also prove two positive results. We first study the coupling by reflection and show that it stays bounded in L^p for $0 \leq p < 1$. Secondly, we construct an explicit static (and in particular non co-adapted) coupling between the laws of two Brownian motions, which provides L^1-Wasserstein control uniformly in time.

Finally, we explain how the results generalise to the Heisenberg groups of higher dimension.

Résumé. Dans cet article, nous étudions les couplages co-adaptés de diffusions hypoelliptiques sur le groupe de Heisenberg (sous-riemannien). Ces diffusions canoniques, que nous appelons mouvement brownien sur le groupe de Heisenberg, sont constituées d'un mouvement brownien planaire et de son aire de Lévy. Nous prouvons que, contrairement au cas des variétés riemanniennes à courbure de Ricci positive ou nulle, pour tout couplage co-adapté, deux mouvements browniens partant de deux points donnés ne peuvent rester à distance bornée pour tout temps $t \geq 0$. Nous prouvons en fait le résultat plus fort qu'ils ne peuvent rester bornés dans L^p pour $p \geq 2$.

Nous établissons également deux résultats positifs. D'une part, nous étudions le couplage par réflexion et montrons qu'il reste borné dans L^p pour $0 < p < 1$. D'autre part, nous construisons un couplage explicite statique (et en particulier non-adapté) entre les lois de deux mouvements browniens qui donne un contrôle en distance L^1 de Wasserstein uniforme en temps.

Enfin, nous expliquons comment nos résultats se généralisent aux groupes de Heisenberg de dimension supérieure.

MSC: 60H10; 60J60; 60J65; 53C17; 22E25

Keywords: Heisenberg group; Co-adapted coupling; Wasserstein distance; Hypoelliptic diffusion

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Bounds on the Poincaré constant for convolution measures

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Abstract. We establish a Shearer-type inequality for the Poincaré constant, showing that the Poincaré constant corresponding to the convolution of a collection of measures can be nontrivially controlled by the Poincaré constants corresponding to convolutions of subsets of measures. This implies, for example, that the sequence of Poincaré constants corresponding to successive convolutions in the central limit theorem is non-increasing. We also establish a dimension-free stability estimate for subadditivity of the Poincaré constant on convolutions which uniformly improves an earlier one-dimensional estimate of a similar nature by Johnson (*Teor. Veroyatn. Primen.* **48** (2003) 615–620). As a byproduct of our arguments, we find that the various monotone properties of entropy, Fisher information and the Poincaré constant on convolutions have a common, simple root in Shearer's inequality.

Résumé. Nous démontrons une inégalité de type Shearer pour les constantes de Poincaré, selon laquelle la constante correspondant à la convolution d'une famille de mesures peut être contrôlée de manière non-triviale par celles de convolutions de sous-familles. Ceci implique, par exemple, que les constantes de Poincaré décroissent de manière monotone le long du théorème central limite. Nous démontrons également une estimée de stabilité indépendante de la dimension pour la sous additivité des constantes de Poincaré de convolutions, améliorant un résultat unidimensionnel similaire dû à Johnson (*Teor. Veroyatn. Primen.* **48** (2003) 615–620). Comme conséquence de nos arguments, nous montrons que les diverses propriétés de monotonie de l'entropie, de l'information de Fisher et de la constantes de Poincaré pour les convolutions trouvent une même source en l'inégalité de Shearer.

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Keywords: Functional inequalities; Poincaré inequalities; Stability; Convolution measures

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A growth-fragmentation model related to Ornstein–Uhlenbeck type processes

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Abstract. Growth-fragmentation processes describe systems of particles in which each particle may grow larger or smaller, and divide into smaller ones as time proceeds. Unlike previous studies, which have focused mainly on the self-similar case, we introduce a new type of growth-fragmentation which is closely related to Lévy driven Ornstein–Uhlenbeck type processes. Our model can be viewed as a generalization of compensated fragmentation processes introduced by Bertoin, or the stochastic counterpart of a family of growth-fragmentation equations. We establish a convergence criterion for a sequence of such growth-fragmentations. We also prove that, under certain conditions, this system fulfills a law of large numbers.

Résumé. Les processus de croissance-fragmentation étudient l'évolution au cours du temps de systèmes de particules, dans lesquels la taille de chaque particule peut croître et décroître, les particules pouvant parfois se fragmenter. Contrairement aux études précédentes, qui se sont concentrées principalement sur les cas auto-similaires, nous introduisons un nouveau modèle qui est associé aux processus d'Ornstein–Uhlenbeck liés aux processus de Lévy. Notre modèle peut être vu comme une généralisation des processus de fragmentation compensés introduits par Bertoin, ou la contrepartie stochastique d'une famille d'équations de croissance-fragmentation. Nous établissons un critère de convergence pour une suite de telles croissance-fragmentations, et une loi des grands nombres dans un cas particulier.

MSC: 60G51; 60J80

Keywords: Growth-fragmentation; Ornstein–Uhlenbeck type process; Branching particle system; Law of large numbers

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Gradient bounds for Kolmogorov type diffusions

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Abstract. We study gradient bounds and other functional inequalities for the diffusion semigroup generated by Kolmogorov type operators. The focus is on two different methods: coupling techniques and generalized Γ-calculus techniques. The advantages and drawbacks of each of these methods are discussed.

Résumé. Nous étudions des bornes de gradient et autres inégalités fonctionnelles pour des semigroupes de diffusion engendrés par des opérateurs de type Kolmogorov. Nous nous concentrons sur deux méthodes : couplage et Γ-calculus. Les avantages et inconvénients de ces méthodes sont discutés.

MSC: Primary 60J60; secondary 60J45; 58J65; 35H10

Keywords: Coupling; Hypoelliptic diffusion; Kolmogorov diffusion; Curvature-dimension inequality; Gradient estimates

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A central limit theorem for Fleming–Viot particle systems[1]

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Abstract. Fleming–Viot type particle systems represent a classical way to approximate the distribution of a Markov process with killing, given that it is still alive at a final deterministic time. In this context, each particle evolves independently according to the law of the underlying Markov process until its killing, and then branches instantaneously from the state of another randomly chosen particle. While the consistency of this algorithm in the large population limit has been recently studied in several articles, our purpose here is to prove Central Limit Theorems under very general assumptions. For this, the key suppositions are that the particle system does not explode in finite time, and that the jump and killing times have atomless distributions. In particular, this includes the case of elliptic diffusions with hard killing.

Résumé. Les systèmes de particules de type Fleming–Viot représentent une façon classique d'approximer la distribution d'un processus de Markov avec mort, sachant qu'il est encore vivant à un temps final déterministe. Dans ce contexte, chaque particule évolue indépendamment suivant la loi du processus de Markov sous-jacent jusqu'à sa mort, et branche instantanément à partir de la position d'une autre particule, choisie aléatoirement. Alors que la consistance en grande population de cet algorithme a été récemment étudiée dans quelques articles, notre but ici est de prouver un Théorème Central Limite sous des hypothèses très générales. Pour cela, deux hypothèses clefs sont que le système de particules n'explose pas en temps fini, et que les instants de sauts et de morts ont des lois sans atomes. En particulier, cela inclut le cas des diffusions elliptiques avec obstacles durs.

MSC: 82C22; 82C80; 65C05; 60J25; 60K35; 60K37

Keywords: Sequential Monte Carlo; Interacting particle systems; Process with killing

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Hydrodynamic limit for a facilitated exclusion process

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Abstract. We study the hydrodynamic limit for a periodic 1-dimensional exclusion process with a dynamical constraint, which prevents a particle at site x from jumping to site $x \pm 1$ unless site $x \mp 1$ is occupied. This process with degenerate jump rates admits transient states, which it eventually leaves to reach an ergodic component, assuming that the initial macroscopic density is larger than $\frac{1}{2}$, or one of its absorbing states if this is not the case. It belongs to the class of conserved lattice gases (CLG) which have been introduced in the physics literature as systems with active-absorbing phase transition in the presence of a conserved field. We show that, for initial profiles smooth enough and uniformly larger than the critical density $\frac{1}{2}$, the macroscopic density profile for our dynamics evolves under the diffusive time scaling according to a fast diffusion equation (FDE). The first step in the proof is to show that the system typically reaches an ergodic component in subdiffusive time.

Résumé. Nous étudions la limite hydrodynamique d'un système d'exclusion unidimensionnel avec une contrainte dynamique, qui empêche une particule en x de sauter en $x \pm 1$ à moins que $x \mp 1$ soit occupé. Ce processus à taux de sauts dégénérés admet des états transients, qu'il finit par quitter pour atteindre une composante ergodique dans le cas où la densité initiale macroscopique est supérieure à $\frac{1}{2}$, ou un de ses états absorbants dans l'autre cas. Ce processus fait partie des gaz conservatifs sur réseau, qui ont été introduits dans la litérature physique comme systèmes présentant une transition de phase active-absorbante en présence d'un champ conservé. Nous montrons que pour des profils initiaux de densité suffisamment réguliers et strictement supérieurs à $\frac{1}{2}$, le profil de densité macroscopique évolue à l'échelle diffusive suivant une équation de diffusion rapide (FDE). La première étape de la preuve consiste à montrer que, typiquement, le système atteint une composante ergodique en temps sous-diffusif.

MSC: 60K35; 35R35; 60J27

Keywords: Facilitated exclusion process; Hydrodynamic limit; Active-absorbing phase transition; Conserved lattice gases; Fast diffusion equation

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The existence phase transition for scale invariant Poisson random fractal models

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Abstract. In this paper we study the existence phase transition of scale invariant random fractal models. We determine the exact value of the critical point of this phase transition for all models satisfying some weak assumptions. In addition, we show that for a large subclass, the fractal model is in the empty phase at the critical point. This subclass of models includes the scale invariant Poisson Boolean model and the Brownian loop soup. In contrast to earlier results in the literature, we do not need to restrict our attention to random fractal models generated by open sets.

Résumé. Nous étudions l'existence d'une transition de phase pour des modèles de fractales aléatoires invariants d'échelle. Nous déterminons la valeur exacte du point critique de cette transition de phase pour une classe générale de modèles vérifiant quelques hypothèses faibles. De plus, nous montrons que pour une grande sous-classe, incluant le modèle de Poisson Booléen invariant d'échelle et la soupe de boucles brownienne, le modèle est dans la phase vide au point critique. Par contraste avec des résultats antérieurs dans la littérature, nous ne restreignons pas notre attention à des modèles de fractales aléatoires générés par des ensembles ouverts.

MSC: 60K35

Keywords: Poissonian random fractals; Phase transition

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Recurrence of Markov chain traces

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Abstract. It is shown that transient graphs for the simple random walk do not admit a nearest neighbor transient Markov chain (not necessarily a reversible one), that crosses all edges with positive probability, while there is such chain for the square grid $\mathbb{Z}^2$. In particular, the d-dimensional grid $\mathbb{Z}^d$ admits such a Markov chain only when $d = 2$. For $d = 2$ we present a relevant example due to Gady Kozma, while the general statement for transient graphs is obtained by proving that for every transient irreducible Markov chain on a countable state space which admits a stationary measure, its trace is almost surely recurrent for simple random walk. The case that the Markov chain is reversible is due to Gurel-Gurevich, Lyons and the first named author (2007). We exploit recent results in potential theory of non-reversible Markov chains in order to extend their result to the non-reversible setup.

Résumé. Nous montrons que les graphes pour lesquels la marche aléatoire simple est transiente n'admettent pas de chaîne de Markov transiente aux plus proches voisins (même non réversible) visitant toutes les arêtes avec probabilité positive, tandis qu'il en existe une pour le réseau carré $\mathbb{Z}^2$. En particulier, le réseau d-dimensionnel $\mathbb{Z}^d$ admet une telle chaîne de Markov seulement lorsque $d = 2$. Lorsque $d = 2$ nous présentons un exemple de Gady Kozma, et le résultat général est obtenu en prouvant que la trace de toute chaîne de Markov sur un espace d'état dénombrable qui admet une mesure stationnaire est presque sûrement récurrente pour la marche simple. Le cas où la chaîne est réversible a été traité par Gurel-Gurevich, Lyons et le premier auteur. Nous exploitons des résultats récents de théorie du potentiel sur les chaînes de Markov non réversibles pour étendre leur résultat au cas non réversible.

MSC: Primary 60J05; secondary 60D05

Keywords: Recurrence; Trace; Capacity

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Errata for *Perturbation by non-local operators*

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