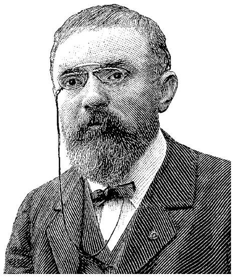
Volume 56, Number 2, 2020
ISSN 0246-0203



ANNALES DE L'INSTITUT HENRI POINCARÉ
PROBABILITÉS ET STATISTIQUES

56|2

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Vol. 56, No. 2 (May, 2020) 765–1513

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques (ISSN 0246-0203), Volume 56, Number 2, May 2020. Published quarterly by Association des Publications de l'Institut Henri Poincaré.

POSTMASTER: Send address changes to Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques, Dues and Subscriptions Office, 9650 Rockville Pike, Suite L 2310, Bethesda, Maryland 20814-3998 USA.

Président et directeur de la publication : Cédric Villani
Périodicité : 4 nos / an

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 765–781
https://doi.org/10.1214/19-AIHP981

Path-space moderate deviations for a Curie–Weiss model of self-organized criticality

Francesca Collet[a], Matthias Gorny[b] and Richard C. Kraaij[c]

[a]*Department of Mathematics "Tullio Levi-Civita", University of Padova, via Trieste 63, 35121 Padova, Italy. E-mail: fcollet@math.unipd.it*
[b]*Laboratoire de Mathématiques, Université Paris-Sud, Bâtiment 425, 91405 Orsay Cedex, France. E-mail: matthias.gorny@math.u-psud.fr*
[c]*Delft Institute of Applied Mathematics, Delft University of Technology, van Mourik Broekmanweg 6, 2628 XE Delft, The Netherlands. E-mail: r.c.kraaij@tudelft.nl*

Abstract. The dynamical Curie–Weiss model of self-organized criticality (SOC) was introduced in (*Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 658–678) and it is derived from the classical generalized Curie–Weiss by imposing a microscopic Markovian evolution having the distribution of the Curie–Weiss model of SOC (*Ann. Probab.* **44** (2016) 444–478) as unique invariant measure. In the case of Gaussian single-spin distribution, we analyze the dynamics of moderate fluctuations for the magnetization. We obtain a path-space moderate deviation principle via a general analytic approach based on convergence of non-linear generators and uniqueness of viscosity solutions for associated Hamilton–Jacobi equations. Our result shows that, under a peculiar moderate space-time scaling and without tuning external parameters, the typical behavior of the magnetization is critical.

Résumé. Le modèle de Curie–Weiss de criticalité auto-organisée dynamique a été construit dans (*Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 658–678) à partir du modèle de Curie–Weiss généralisé. Il s'agit d'un processus de Markov continu dont l'unique mesure invariante est la loi du modèle de Curie–Weiss de criticalité auto-organisée (*Ann. Probab.* **44** (2016) 444–478). Dans le cas Gaussien, nous étudions les fluctuations modérées de la magnétisation. Nous obtenons un principe de déviations modérées dans l'espace des chemins en utilisant une approche analytique basée sur la convergence de générateurs non-linéaires et sur l'unicité des solutions de viscosité pour des équations de Hamilton–Jacobi associées. Notre résultat montre que, dans une certaine échelle de temps modérée et sans intervention de paramètres extérieurs, le comportement critique de la magnétisation est critique.

MSC: 60F10; 60J60; 60K35

Keywords: Moderate deviations; Interacting particle systems; Mean-field interaction; Self-organized criticality; Hamilton–Jacobi equation; Perturbation theory for Markov processes

References

[1] M. J. Aschwanden (Ed.). *Self-Organized Criticality Systems*. Open Academic Press, Berlin, 2013.
[2] P. Bak. Complexity and criticality. In *How Nature Works: The Science of Self-Organized Criticality* 1–32. Springer Science + Business Media, New York, 1996. MR1417042 https://doi.org/10.1007/978-1-4757-5426-1
[3] P. Bak and K. Sneppen. Punctuated equilibrium and criticality in a simple model of evolution. *Phys. Rev. Lett.* **71** (1993) 4083–4086.
[4] P. Bak, C. Tang and K. Wiesenfeld. Self-organized criticality: An explanation of the $1/f$ noise. *Phys. Rev. Lett.* **59** (1987) 381–384. MR0949160 https://doi.org/10.1103/PhysRevA.38.364
[5] R. Cerf and M. Gorny. A Curie–Weiss model of self-organized criticality. *Ann. Probab.* **44** (2016) 444–478. MR3456343 https://doi.org/10.1214/14-AOP978
[6] F. Collet and R. C. Kraaij. Dynamical moderate deviations for the Curie–Weiss model. *Stochastic Process. Appl.* **127** (2017) 2900–2925. MR3682118 https://doi.org/10.1016/j.spa.2017.01.002
[7] F. Collet and R. C. Kraaij. Path-space moderate deviation principles for the random field Curie–Weiss model. *Electron. J. Probab.* **23** (2018) 1–45 (paper no. 21). MR3771758 https://doi.org/10.1214/17-EJP117
[8] X. Deng, J. Feng and Y. Liu. A singular 1-D Hamilton–Jacobi equation, with application to large deviation of diffusions. *Commun. Math. Sci.* **9** (2011) 289–300. MR2836847
[9] D. Dhar. Theoretical studies of self-organized criticality. *Phys. A* **369** (2006) 29–70. MR2246566 https://doi.org/10.1016/j.physa.2006.04.004
[10] R. S. Ellis and C. M. Newman. Limit theorems for sums of dependent random variables occurring in statistical mechanics. *Z. Wahrsch. Verw. Gebiete* **44** (1978) 117–139. MR0503333 https://doi.org/10.1007/BF00533049
[11] W. Feller. *An Introduction to Probability Theory and Its Applications. Volume I*, 3rd edition. John Wiley & Sons Inc., New York, 1968. MR0228020

[12] J. Feng, J.-P. Fouque and R. Kumar. Small-time asymptotics for fast mean-reverting stochastic volatility models. *Ann. Appl. Probab.* **22** (2012) 1541–1575. MR2985169 https://doi.org/10.1214/11-AAP801

[13] J. Feng and T. G. Kurtz. *Large Deviations for Stochastic Processes*. *Mathematical Surveys and Monographs* **131**. American Mathematical Society, Providence, RI, 2006. MR2260560 https://doi.org/10.1090/surv/131

[14] M. Gorny. A Curie–Weiss model of self-organized criticality: The Gaussian case. *Markov Process. Related Fields* **20** (2014) 563–576. MR3289133

[15] M. Gorny. A dynamical Curie–Weiss model of SOC: The Gaussian case. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 658–678. MR3634269 https://doi.org/10.1214/15-AIHP729

[16] R. C. Kraaij. Large deviations for finite state Markov jump processes with mean-field interaction via the comparison principle for an associated Hamilton–Jacobi equation. *J. Stat. Phys.* **164** (2016) 321–345. MR3513255 https://doi.org/10.1007/s10955-016-1542-8

[17] T. G. Kurtz. Convergence of sequences of semigroups of nonlinear operators with an application to gas kinetics. *Trans. Amer. Math. Soc.* **186** (1973) 259–272. MR0336482 https://doi.org/10.2307/1996565

[18] T. G. Kurtz. A limit theorem for perturbed operator semigroups with applications to random evolutions. *J. Funct. Anal.* **12** (1973) 55–67. MR0365224 https://doi.org/10.1016/0022-1236(73)90089-x

[19] R. Meester and A. Sarkar. Rigorous self-organised criticality in the modified Bak–Sneppen model. *J. Stat. Phys.* **149** (2012) 964–968. MR2999570 https://doi.org/10.1007/s10955-012-0627-2

[20] G. C. Papanicolaou, D. Stroock and S. R. S. Varadhan. Martingale approach to some limit theorems. In *Statistical Mechanics, Dynamical Systems and the Duke Turbulence Conference. Volume 3*. Duke University Series, Durham, NC, 1977. MR0461684

[21] G. Pruessner. *Self-Organised Criticality: Theory, Models and Characterisation*. Cambridge University Press, Cambridge, 2012.

[22] B. Ráth and B. Tóth. Erdős–Rényi random graphs + forest fires = self-organized criticality. *Electron. J. Probab.* **14** (2009) 1290–1327 (paper no. 45). MR2511285 https://doi.org/10.1214/EJP.v14-653

[23] D. Sornette. *Critical Phenomena in Natural Sciences. Chaos, Fractals, Selforganization and Disorder: Concepts and Tools*. Springer-Verlag, Berlin, 2006. MR2220576

[24] D. L. Turcotte. Self-organized criticality. *Rep. Progr. Phys.* **62** (1999) 1377–1429.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 782–791
https://doi.org/10.1214/19-AIHP982

Divergence of shape fluctuation for general distributions in first-passage percolation

Shuta Nakajima

Research Institute in Mathematical Sciences, Kyoto University, Kyoto, Japan. E-mail: njima@kurims.kyoto-u.ac.jp

Abstract. We study the shape fluctuation in the first-passage percolation on $\mathbb{Z}^d$. It is known that it diverges when the distribution obeys Bernoulli in Zhang (*Probab. Theory. Related. Fields.* **136** (2006) 298–320). In this paper, we extend the result to general distributions.

Résumé. Nous étudions les fluctuations de la forme limite pour la percolation de premier passage dans $\mathbb{Z}^d$. Il est connu que ces fluctuations divergent dans le cas des lois de Bernoulli [Zhang (*Probab. Theory. Related. Fields.* **136** (2006) 298–320)]. Dans cet article, nous étendons ce résultat à toutes les lois.

MSC: Primary 60K37; secondary 60K35; 82A51

Keywords: Random environment; First-passage percolation; Shape fluctuation

References

[1] K. S. Alexander. Approximation of subadditive functions and convergence rates in limiting-shape results. *Ann. Probab.* **25** (1997) 30–55. MR1428498 https://doi.org/10.1214/aop/1024404277
[2] J. Anderson and J. Johansson. The probability density function tail of the Kardar–Parisi–Zhang equation in the strongly non-linear regime. *J. Phys. A: Math. Theor.* **49** (50) (2016). MR3584402 https://doi.org/10.1088/1751-8113/49/50/505001
[3] A. Auffinger, M. Damron and J. Hanson. *50 Years of First-Passage Percolation*. *University Lecture Series* **68**. American Mathematical Society, Providence, RI, 2017. MR3729447
[4] S. Chatterjee. A general method for lower bounds on fluctuations of random variables. *Ann. Probab.* (2019). To appear. MR3980917 https://doi.org/10.1214/18-AOP1304
[5] J. Cox and R. Durrett. Some limit theorems for percolation processes with necessary and sufficient conditions. *Ann. Probab.* **9** (1981) 583–603. MR0624685
[6] H. Duminil-Copin, H. Kesten, F. Nazarov, Y. Peres and V. Sidoravicius. On the number of maximal paths in directed last-passage percolation. ArXiv e-print, 2018. Available at arXiv:1801.05777.
[7] H. Kesten. Aspects of first passage percolation. *Lecture Notes in Math.* **1180** (1986) 125–264. MR0876084 https://doi.org/10.1007/BFb0074919
[8] H. Kesten. On the speed of convergence in first-passage percolation. *Ann. Appl. Probab.* **3** (1993) 296–338. MR1221154
[9] C. Newman and M. Piza. Divergence of shape fluctuations in two dimensions. *Ann. Probab.* **23** (1995) 977–1005. MR1349159
[10] R. Pemantle and Y. Peres. Planar first-passage percolation times are not tight. In *Probability and Phase Transition* 261–264. *Cambridge*, *1993*. *NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci.* **420**. Kluwer Academic, Dordrecht, 1994. MR1283187 https://doi.org/10.1007/978-94-015-8326-8_16
[11] J. van den Berg and H. Kesten. Inequalities for the time constant in first-passage percolation. *Ann. Appl. Probab.* (1993) 56–80. MR1202515
[12] Y. Zhang. The divergence of fluctuations for shape in first passage percolation. *Probab. Theory Related Fields* **136** (2) (2006) 298–320. MR2240790 https://doi.org/10.1007/s00440-005-0488-6

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 792–825
https://doi.org/10.1214/19-AIHP983

Parabolic Anderson model with a fractional Gaussian noise that is rough in time[1]

Xia Chen

Department of Mathematics, University of Tennessee, Knoxville TN 37996, USA. E-mail: xchen@math.utk.edu

Abstract. This paper concerns the parabolic Anderson equation

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\frac{\partial^{d+1} W^{\mathbf{H}}}{\partial t\,\partial x_1\cdots\partial x_d}$$

generated by a $(d+1)$-dimensional fractional noise with the Hurst parameter $\mathbf{H} = (H_0, H_1, \dots, H_d)$ with special interest in the setting that some of $H_0, \dots, H_d$ are less than half. In the recent work (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 941–976), the case of the spatial roughness has been investigated. To put the last piece of the puzzle in place, this work investigates the case when $H_0 < 1/2$ with the concern on solvability, Feynman–Kac's moment formula and intermittency of the system.

Résumé. Cet article concerne l'équation d'Anderson parabolique

$$\frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + u\frac{\partial^{d+1} W^{\mathbf{H}}}{\partial t\,\partial x_1\cdots\partial x_d}$$

engendrée par un bruit fractionnaire de dimension $d+1$ avec un paramètre de Hurst $\mathbf{H} = (H_0, H_1, \dots, H_d)$, en portant une attention particulière au cas où certains des paramètres $H_0, \dots, H_d$ sont inférieurs à $1/2$. Le cas rugueux en espace avait fait l'objet du travail récent (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 941–976). Pour mettre en place la dernière pièce du puzzle, cet article examine le cas $H_0 < 1/2$ en se penchant sur les problèmes de résolution, de la formule des moments de Feynman–Kac et de l'intermittence du système.

MSC: 60F10; 60H15; 60H40; 60J65; 81U10

Keywords: Parabolic Anderson equation; Dalang's condition; Fractional; Rough and critical Gaussian noises; Feynman–Kac's representation; Brownian motion; Moment asymptotics

References

[1] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. Dover, New York, 1965. MR1225604

[2] R. Balan, M. Jolis and L. Quer-Sardanyons. SPDEs with fractional noise in space with index $H < 1/2$. *Statist. Probab. Lett.* **119** (2016) 310–316. MR3555303 https://doi.org/10.1016/j.spl.2016.09.003

[3] R. A. Carmona and S. A. Molchanov. Parabolic Anderson model and intermittency. *Mem. Amer. Math. Soc.* **108** (1994). MR1185878 https://doi.org/10.1090/memo/0518

[4] L. Chen, Y. Z. Hu, K. Kalbasi and D. Nualart. Intermittency for the stochastic heat equation driven by a rough time fractional Gaussian noise. *Probab. Theory Related Fields* **171** (2018) 431–457. MR3800837 https://doi.org/10.1007/s00440-017-0783-z

[5] X. Chen. Precise intermittency for the parabolic Anderson equation with an $(1+1)$-dimensional time-space white noise. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 1486–1499. MR3414455 https://doi.org/10.1214/15-AIHP673

[6] X. Chen. Spatial asymptotics for the parabolic Anderson models with generalized time-space Gaussian noise. *Ann. Probab.* **44** (2016) 1535–1598. MR3474477 https://doi.org/10.1214/15-AOP1006

[7] X. Chen. Moment asymptotics for parabolic Anderson equation with fractional time-space noise: In Skorokhod regime PDF. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (2017) 819–841. MR3634276 https://doi.org/10.1214/15-AIHP738

[8] X. Chen. Parabolic Anderson model with rough or critical Gaussian noise. *Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2) (2019) 941–976.

[9] X. Chen, Y. Z. Hu, D. Nualart and S. Tindel. Spatial asymptotics for the parabolic Anderson model driven by a Gaussian rough noise PDF. *Electron. J. Probab.* **22** (2017) 1–38. MR3690290 https://doi.org/10.1214/17-EJP83

[10] X. Chen, Y. Z. Hu, S. Song and F. Xing. Exponential asymptotics for time-space Hamiltonians. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 1529–1561. MR3414457 https://doi.org/10.1214/13-AIHP588

[11] X. Chen and T. V. Phan. Free energy in a mean field of Brownian particles. *Discrete Contin. Dyn. Syst.* **39** (2019) 747–769. MR3918193 https://doi.org/10.3934/dcds.2019031

[12] R. C. Dalang. Extending martingale measure stochastic integral with applications to spatially homogeneous S.P.D.E's. *Electron. J. Probab.* **4** (1999) 1–29. MR1684157 https://doi.org/10.1214/EJP.v4-43

[13] A. Deya. On a modelled rough heat equation. *Probab. Theory Related Fields* **166** (2016) 1–65. MR3547736 https://doi.org/10.1007/s00440-015-0650-8

[14] M. Hairer and C. Labbé. A simple construction of the continuum parabolic Anderson model on $\mathbb{R}^2$. *Electron. Commun. Probab.* **20** (2015) 43, 11 pp. MR3358965 https://doi.org/10.1214/ECP.v20-4038

[15] H. Hankel. Die Euler'schen Integrale bei unbeschränkter Variabilität des Arguments. *Z. Math. Phys.* **9** (1864) 1–21.

[16] Y. Z. Hu, J. Huang, K. Le, D. Nualart and S. Tindel. Stochastic heat equation with rough dependence in space. *Ann. Probab.* **45** (2017) 4561–4616. MR3737918 https://doi.org/10.1214/16-AOP1172

[17] Y. Z. Hu, J. Huang, D. Nualart and D. Tindel. Stochastic heat equations with general multiplicative Gaussian noise: Hölder continuity and intermittency. *Electron. J. Probab.* **20** (2015) 55, 50 pp. MR3354615 https://doi.org/10.1214/EJP.v20-3316

[18] Y. Z. Hu, J. Huang, D. Nualart and S. Tindel. Parabolic Anderson model with rough dependence in space. In *Proceedings of the Abel Conference*. To appear. MR3737918 https://doi.org/10.1214/16-AOP1172

[19] Y. Z. Hu and D. Nualart. Stochastic heat equation driven by fractional noise and local time. *Probab. Theory Related Fields* **143** (2009) 285–328. MR2449130 https://doi.org/10.1007/s00440-007-0127-5

[20] Y. Z. Hu, D. Nualart and J. Song. Feynman–Kac formula for heat equation driven by fractional white noise. *Ann. Probab.* **39** (2011) 291–326. MR2778803 https://doi.org/10.1214/10-AOP547

[21] J. Huang, K. Lê and D. Nualart. Large time asymptotics for the parabolic Anderson model driven by space and time correlated noise. *Stoch. Partial Differ. Equ. Anal. Comput.* **5** (2017) 614–651. MR3736656 https://doi.org/10.1007/s40072-017-0099-0

[22] K. Lê. A remark on a result of Xia Chen. *Statist. Probab. Lett.* **118** (2016) 124–126. MR3531492 https://doi.org/10.1016/j.spl.2016.06.004

[23] M. Ledoux and M. Talagrand. *Probability in Banach Spaces*. Springer, Berlin, 1991. MR1102015 https://doi.org/10.1007/978-3-642-20212-4

[24] H. Li and X. Chen. Precise moment asymptotics for the stochastic heat equation of a time-derivative Gaussian noise PDF. *Acta Math. Sci. Ser. B Engl. Ed.* **39B** (2019) 629–644.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 826–846
https://doi.org/10.1214/19-AIHP984

Stable matchings in high dimensions via the Poisson-weighted infinite tree

Alexander E. Holroyd[a], James B. Martin[b] and Yuval Peres[c]

[a]*E-mail: holroyd@uw.edu; url: http://aeholroyd.org*
[b]*Department of Statistics, University of Oxford. E-mail: martin@stats.ox.ac.uk; url: http://www.stats.ox.ac.uk/%7Emartin*
[c]*E-mail: yuval@yuvalperes.com; url: http://yuvalperes.com*

Abstract. We consider the stable matching of two independent Poisson processes in $\mathbb{R}^d$ under an asymmetric color restriction. Blue points can only match to red points, while red points can match to points of either color. It is unknown whether there exists a choice of intensities of the red and blue processes under which all points are matched. We prove that for any fixed intensities, there are unmatched blue points in sufficiently high dimension. Indeed, if the ratio of red to blue intensities is ρ then the intensity of unmatched blue points converges to $e^{-\rho}/(1+\rho)$ as $d \to \infty$. We also establish analogous results for certain multi-color variants. Our proof uses stable matching on the Poisson-weighted infinite tree (PWIT), which can be analyzed via differential equations. The PWIT has been used in many settings as a scaling limit for models involving complete graphs with independent edge weights, but as far as we are aware, this is the first presentation of a rigorous application to high-dimensional Euclidean space. Finally, we analyze the asymmetric matching problem under a hierarchical metric, and show that there are unmatched points for all intensities.

Résumé. Nous considérons l'appariement stable de deux processus de Poisson indépendants dans $\mathbb{R}^d$, sous une contrainte asymmetrique sur les couleurs. Un point bleu ne peut être apparié qu'à un point rouge, tandis qu'un point rouge peut être apparié soit à un point rouge soit à un point bleu. On ne sait pas s'il existe un choix d'intensités des processus bleu et rouge tel que tous les points sont appariés. Nous démontrons que pour toutes intensités fixes, il y a des points bleus non-appariés en dimension suffisamment elevée. En effet, si le rapport de l'intensité des points rouges à l'intensité des points bleus est ρ, l'intensité des points bleus non-appariés tend vers $e^{-\rho}/(1+\rho)$ quand $d \to \infty$. On établit aussi des résultats analogues pour certaines variantes à multiples couleurs. La preuve utilise un modèle d'appariement stable sur l'arbre infini pondéré de Poisson (PWIT), qui peut être analysé par des équations différentielles. Le PWIT a été utilisé dans de nombreux contextes comme limite d'échelle pour des modèles impliquant des graphes complets pondérés de variables aléatoires indépendants, mais à notre connaissance, celle-ci est la première présentation d'une application rigoureuse à l'espace euclidien en haute dimension. Finalement, on analyse le problème d'appariement asymétrique sous une métrique hiérarchique, et on démontre qu'il existe des points non-appariés pour tout choix d'intensités.

MSC: Primary 60D05; secondary 60G55; 05C70

Keywords: Poisson process; Point process; Stable matching; Poisson-weighted infinite tree

References

[1] L. Addario-Berry, S. Griffiths and R. J. Kang. Invasion percolation on the Poisson-weighted infinite tree. *Ann. Appl. Probab.* **22** (3) (2012) 931–970. MR2977982 https://doi.org/10.1214/11-AAP761
[2] D. Aldous. Asymptotics in the random assignment problem. *Probab. Theory Related Fields* **93** (4) (1992) 507–534. MR1183889 https://doi.org/10.1007/BF01192719
[3] D. Aldous and J. M. Steele. The objective method: Probabilistic combinatorial optimization and local weak convergence. In *Probability on Discrete Structures* 1–72. *Encyclopaedia Math. Sci.* **110**. Springer, Berlin, 2004. MR2023650 https://doi.org/10.1007/978-3-662-09444-0_1
[4] G. Amir, O. Angel and A. E. Holroyd. Multicolor Poisson matching, 2016. Available at arXiv:1605.06485.
[5] C. Bordenave, P. Caputo and D. Chafaï. Spectrum of large random reversible Markov chains: Heavy-tailed weights on the complete graph. *Ann. Probab.* **39** (4) (2011) 1544–1590. MR2857250 https://doi.org/10.1214/10-AOP587
[6] C. Bordenave, P. Caputo and D. Chafaï. Spectrum of non-Hermitian heavy tailed random matrices. *Comm. Math. Phys.* **307** (2) (2011) 513–560. MR2837123 https://doi.org/10.1007/s00220-011-1331-9
[7] C. Bordenave and D. Chafaï. Around the circular law. *Probab. Surv.* **9** (2012) 1–89. MR2908617 https://doi.org/10.1214/11-PS183
[8] D. J. Daley and G. Last. Descending chains, the lilypond model, and mutual-nearest-neighbour matching. *Adv. in Appl. Probab.* **37** (3) (2005) 604–628. MR2156551 https://doi.org/10.1239/aap/1127483738

[9] M. Deijfen, A. E. Holroyd and J. B. Martin. Friendly frogs, stable marriage, and the magic of invariance. *Amer. Math. Monthly* **124** (5) (2017) 387–402. MR3633970 https://doi.org/10.4169/amer.math.monthly.124.5.387

[10] D. Gale and L. S. Shapley. College admissions and the stability of marriage. *Amer. Math. Monthly* **69** (1) (1962) 9–15. MR1531503 https://doi.org/10.2307/2312726

[11] A. E. Holroyd, R. Pemantle, Y. Peres and O. Schramm. Poisson matching. *Ann. Inst. Henri Poincaré Probab. Stat.* **45** (1) (2009) 266–287. MR2500239 https://doi.org/10.1214/08-AIHP170

[12] O. Kallenberg. *Foundations of Modern Probability*, 2nd edition. *Probability and Its Applications (New York)*. Springer-Verlag, New York, 2002. MR1876169 https://doi.org/10.1007/978-1-4757-4015-8

[13] J. Salez and D. Shah. Belief propagation: An asymptotically optimal algorithm for the random assignment problem. *Math. Oper. Res.* **34** (2) (2009) 468–480. MR2554069 https://doi.org/10.1287/moor.1090.0380

[14] J. M. Steele. Minimal spanning trees for graphs with random edge lengths. In *Mathematics and Computer Science, II (Versailles, 2002), Trends Math* 223–245. Birkhäuser, Basel, 2002. MR1940139

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 847–883
https://doi.org/10.1214/19-AIHP986

Infinite rate symbiotic branching on the real line: The tired frogs model

Achim Klenke[a] and Leonid Mytnik[b]

[a]*Institut für Mathematik, Johannes Gutenberg-Universität Mainz, Staudingerweg 9, D-55099 Mainz, Germany. E-mail: math@aklenke.de*
[b]*Faculty of Industrial Engineering and Management, Technion – Israel Institute of Technology, Haifa 32000, Israel. E-mail: leonid@ie.technion.ac.il*

Abstract. Consider a population of infinitesimally small frogs on the real line. Initially the frogs on the positive half-line are dormant while those on the negative half-line are awake and move according to the heat flow. At the interface, the incoming wake frogs try to wake up the dormant frogs and succeed with a probability proportional to their amount among the total amount of involved frogs at the specific site. Otherwise, the incoming frogs also fall asleep. This frog model is a special case of the infinite rate symbiotic branching process on the real line with different motion speeds for the two types.

We construct this frog model as the limit of approximating processes and compute the structure of jumps. We show that our frog model can be described by a stochastic partial differential equation on the real line with a jump type noise.

Résumé. Considérons une population de grenouilles infinitésimales sur la droite réelle. Au début, toutes les grenouilles à droite de l'origine sont endormies tandis que les grenouilles à gauche sont éveillées et bougent comme un flux de chaleur. A l'interface, les grenouilles éveillées qui arrivent essaient de réveiller les grenouilles dormantes. Elles le font avec succès avec une probabilité proportionnelle à leur proportion par rapport à la population totale de grenouilles à cet endroit. Si elles échouent, les grenouilles arrivantes s'endorment aussi. Ce modèle de grenouilles est un modèle spécifique de branchement symbiotique sur la droite réelle où les populations bougent avec des vitesses différentes. Nous construisons le modèle par une procédure d'approximation et nous calculons la structure du processus de sauts. Nous montrons que notre modèle des grenouilles peut être décrit par une équation différentielle partielle stochastique avec sauts.

MSC: 60K35; 60J80; 60J68; 60J75; 60H15

Keywords: Symbiotic branching; Mutually catalytic branching; Infinite rate branching; Stochastic partial differential equation; Frog model

References

[1] J. Blath, M. Hammer and M. Ortgiese. The scaling limit of the interface of the continuous-space symbiotic branching model. *Ann. Probab.* **44** (2) (2016) 807–866. MR3474460 https://doi.org/10.1214/14-AOP989

[2] D. A. Dawson. Measure-valued Markov processes. In *École d'Été de Probabilités de Saint-Flour XXI – 1991* 1–260 P. L. Hennequin (Ed.). *Lecture Notes in Mathematics* **1541**. Springer, Berlin, 1993. MR1242575 https://doi.org/10.1007/BFb0084190

[3] D. A. Dawson and E. A. Perkins. Long-time behavior and coexistence in a mutually catalytic branching model. *Ann. Probab.* **26** (3) (1998) 1088–1138. MR1634416 https://doi.org/10.1214/aop/1022855746

[4] L. Döring and L. Mytnik. Mutually catalytic branching processes and voter processes with strength of opinion. *ALEA Lat. Am. J. Probab. Math. Stat.* **9** (2012) 1–51. MR2876839

[5] A. M. Etheridge and K. Fleischmann. Compact interface property for symbiotic branching. *Stochastic Process. Appl.* **114** (1) (2004) 127–160. MR2094150 https://doi.org/10.1016/j.spa.2004.05.006

[6] S. N. Ethier and T. G. Kurtz. *Markov Processes: Characterization and Convergence*. John Wiley & Sons Inc., New York, 1986. MR0838085 https://doi.org/10.1002/9780470316658

[7] L. R. Fontes, F. P. Machado and A. Sarkar. The critical probability for the frog model is not a monotonic function of the graph. *J. Appl. Probab.* **41** (1) (2004) 292–298. MR2036292 https://doi.org/10.1239/jap/1077134688

[8] N. Gantert and P. Schmidt. Recurrence for the frog model with drift on $\mathbb{Z}$. *Markov Process. Related Fields* **15** (1) (2009) 51–58. MR2509423

[9] N. Ikeda and S. Watanabe. *Stochastic Differential Equations and Diffusion Processes*, 2nd edition. *North-Holland Mathematical Library* **24**. North-Holland Publishing Co., Amsterdam, 1989. MR0637061

[10] C. G. J. Jacobi. Suite des notices sur les fonctions elliptiques. (V. p. 192). *J. Reine Angew. Math.* **3** (1828) 303–310. MR1577699 https://doi.org/10.1515/crll.1828.3.303

[11] J. Jacod and A. N. Shiryaev. *Limit Theorems for Stochastic Processes*, 2nd edition. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **288**. Springer-Verlag, Berlin, 2003. MR1943877 https://doi.org/10.1007/978-3-662-05265-5

[12] A. Jakubowski. On the Skorokhod topology. *Ann. Inst. Henri Poincaré Probab. Stat.* **22** (3) (1986) 263–285. MR0871083

[13] A. Klenke and L. Mytnik. Infinite rate mutually catalytic branching. *Ann. Probab.* **38** (4) (2010) 1690–1716. MR2663642 https://doi.org/10.1214/09-AOP520

[14] A. Klenke and L. Mytnik. Infinite rate mutually catalytic branching in infinitely many colonies: Construction, characterization and convergence. *Probab. Theory Related Fields* **154** (3–4) (2012) 533–584. MR3000554 https://doi.org/10.1007/s00440-011-0376-1

[15] A. Klenke and L. Mytnik. Infinite rate mutually catalytic branching in infinitely many colonies: The longtime behavior. *Ann. Probab.* **40** (1) (2012) 103–129. MR2917768 https://doi.org/10.1214/10-AOP621

[16] A. Klenke and M. Oeler. A Trotter type approach to infinite rate mutually catalytic branching. *Ann. Probab.* **38** (2) (2010) 479–497. MR2642883 https://doi.org/10.1214/09-AOP488

[17] E. Kosygina and M. P. W. Zerner. A zero-one law for recurrence and transience of frog processes. *Probab. Theory Related Fields* **168** (1–2) (2017) 317–346. MR3651054 https://doi.org/10.1007/s00440-016-0711-7

[18] L. Mytnik. Uniqueness for a mutually catalytic branching model. *Probab. Theory Related Fields* **112** (2) (1998) 245–253. MR1653845 https://doi.org/10.1007/s004400050189

[19] S. Yu. Popov. Frogs and some other interacting random walks models. In *Discrete Random Walks (Paris, 2003)* 277–288. *Discrete Math. Theor. Comput. Sci. Proc., AC*. Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2003. MR2042394

[20] J. Walsh. An introduction to stochastic partial differential equations. *Lecture Notes in Math.* **1180** (1986) 265–439. MR0876085 https://doi.org/10.1007/BFb0074920

[21] E. T. Whittaker and G. N. Watson. *A Course of Modern Analysis*, 4th edition. Cambridge University Press, Cambridge, 1966. MR1424469 https://doi.org/10.1017/CBO9780511608759

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 884–916
https://doi.org/10.1214/19-AIHP987

A perturbation analysis of stochastic matrix Riccati diffusions

Adrian N. Bishop[a], Pierre Del Moral[b] and Angèle Niclas[c]

[a]*University of Technology Sydney (UTS) and CSIRO, Ultimo, NSW 2007, Australia.*
[b]*INRIA, Bordeaux Research Center, France E-mail: pierre.del-moral@inria.fr*
[c]*École Normale Supérieure de Lyon, 69342, Lyon, France.*

Abstract. Matrix differential Riccati equations are central in filtering and optimal control theory. The purpose of this article is to develop a perturbation theory for a class of stochastic matrix Riccati diffusions. Diffusions of this type arise, for example, in the analysis of ensemble Kalman–Bucy filters since they describe the flow of certain sample covariance estimates. In this context, the random perturbations come from the fluctuations of a mean field particle interpretation of a class of nonlinear diffusions equipped with an interacting sample covariance matrix functional. The main purpose of this article is to derive non-asymptotic Taylor-type expansions of stochastic matrix Riccati flows with respect to some perturbation parameter. These expansions rely on an original combination of stochastic differential analysis and nonlinear semigroup techniques on matrix spaces. The results here quantify the fluctuation of the stochastic flow around the limiting deterministic Riccati equation, at any order. The convergence of the interacting sample covariance matrices to the deterministic Riccati flow is proven as the number of particles tends to infinity. Also presented are refined moment estimates and sharp bias and variance estimates. These expansions are also used to deduce a functional central limit theorem at the level of the diffusion process in matrix spaces.

Résumé. Les équations de Riccati matricielles jouent un rôle important dans la théorie du filtrage et du contrôle optimal. Cet article présente une théorie des perturbations d'une classe d'équations de Riccati matricielles stochastiques. Ces modèles probabilistes sont d'un usage courant dans la théorie des filtres de Kalman d'Ensemble. Ils représentent dans ce contexte l'évolution des matrices de covariance empiriques associées à un ensemble de diffusions en interaction. Les perturbations aléatoires résultent des fluctuations stochastiques d'un système de particules de type champ moyen interagissant avec la mesure empirique du système. Nous présentons dans cet article une formule de Taylor non asymptotique pour des flots stochastiques de diffusion de Riccati matricelles par rapport à un paramètre de fluctuation. Ces développements sont fondés sur un nouveau calcul différentiel stochastique et une analyse fine de semigroupes non linéaires dans des espaces de matrices. Ces résultats permettent de quantifier avec précision les fluctuations des flots de matrices stochastiques autour des systèmes limites à tout ordre. Nous illustrons ces résultats avec une preuve de la convergence des matrices empiriques de filtres de Kalman d'Ensemble vers la solution d'équations de Riccati déterministes lorsque le nombre de particules tends vers l'infini. Nous présentons dans ce cadre des estimations fines des biais et des variances, ainsi qu'un theorème de la limite centrale fonctionnel au niveau du processus matriciel.

MSC: 11M50; 60B20; 62M20; 60G35

Keywords: Riccati matrix differential equation; Covariance matrices; Filtering; Ensemble Kalman filters; Interacting particle systems; Perturbation theory

References

[1] A. Ahdida and A. Alfonsi. Exact and high order discretization schemes for Wishart processes and their affine extensions. *Ann. Appl. Probab.* **23** (3) (2013) 1025–1073. MR3076677 https://doi.org/10.1214/12-AAP863

[2] P. J. Antsaklis and A. N. Michel. *A Linear Systems Primer*. Birkhäuser, Boston, 2007. MR2375027

[3] M. Arnaudon and P. Del Moral A variational approach to nonlinear and interacting diffusions. arXiv preprint, 2018. Available at arXiv:1812.04269.

[4] O. E. Barndorff-Nielsen and R. Stelzer. Positive-definite matrix processes of finite variation. *Probab. Math. Statist.* **27** (1) (2007) 3–43. MR2353270

[5] K. Bergemann and S. Reich. An ensemble Kalman–Bucy filter for continuous data assimilation. *Meteorologische Zeitschrift.* **21** (3) (2012) 213–219.

[6] A. N. Bishop and P. Del Moral. On the stability of Kalman–Bucy diffusion processes. *SIAM J. Control Optim.* **55** (6) (2017) 4015–4047. Available at arXiv:1610.04686.

[7] A. N. Bishop and P. Del Moral. On the stability of matrix-valued Riccati diffusions, 2018. Available at arXiv:1808.00235.

[8] A. N. Bishop and P. Del Moral. Stability properties of systems of linear stochastic differential equations with random coefficients. *SIAM J. Control Optim.* **57** (2) (2019) 1023–1042. Available at arXiv:1804.09349. MR3924607 https://doi.org/10.1137/18M1182759

[9] A. N. Bishop and P. Del Moral. An explicit Floquet-type representation of Riccati aperiodic exponential semigroups. *International Journal of Control*. Available at arXiv:1805.02127. https://doi.org/10.1080/00207179.2019.1590647

[10] A. N. Bishop, P. Del Moral, K. Kamatani and B. Rémillard. On one-dimensional Riccati diffusions. *Ann. Appl. Probab.* **29** (2) (2019) 1127–1187. MR3910025 https://doi.org/10.1214/18-AAP1431

[11] A. N. Bishop, P. Del Moral and A. Niclas. An introduction to Wishart matrix moments. *Found. Trends Mach. Learn.* **11** (2) (2018) 97–218. Available at arXiv:1710.10864.

[12] A. N. Bishop, P. Del Moral and S. Pathiraja. Perturbations and projections of Kalman–Bucy semigroups. *Stochastic Process. Appl.* **128** (9) (2018) 2857–2904. MR3834846 https://doi.org/10.1016/j.spa.2017.10.006

[13] J. M. Bismut. Linear quadratic optimal stochastic control with random coefficients. *SIAM J. Control Optim.* **14** (3) (1976) 419–444. MR0406663 https://doi.org/10.1137/0314028

[14] M. W. Browne. Asymptotically distribution-free methods for the analysis of covariance structures. *Br. J. Math. Stat. Psychol.* **37** (1) (1984) 62–83. MR0783499 https://doi.org/10.1111/j.2044-8317.1984.tb00789.x

[15] M. F. Bru. Diffusions of perturbed principal component analysis. *J. Multivariate Anal.* **29** (1) (1989) 127–136. MR0991060 https://doi.org/10.1016/0047-259X(89)90080-8

[16] M. F. Bru. Wishart processes. *J. Theoret. Probab.* **4** (4) (1991) 725–751. MR1132135 https://doi.org/10.1007/BF01259552

[17] C. Cuchiero, D. Filipovic, E. Mayerhofer and J. Teichmann. Affine processes on positive semi-definite matrices. *Ann. Appl. Probab.* **21** (2) (2011) 397–463. MR2807963 https://doi.org/10.1214/10-AAP710

[18] J. de Wiljes, S. Reich and W. Stannat. Long-time stability and accuracy of the ensemble Kalman–Bucy filter for fully observed processes and small measurement noise. *SIAM J. Appl. Dyn. Syst.* **17** (2) (2018) 1152–1181. MR3787772 https://doi.org/10.1137/17M1119056

[19] P. Del Moral. *Mean Field Simulation for Monte Carlo Integration*. Chapman & Hall/CRC Press, London, 2013. MR3060209

[20] P. Del Moral, A. Kurtzmann and J. Tugaut. On the stability and the uniform propagation of chaos of a class of extended ensemble Kalman–Bucy filters. *SIAM J. Control Optim.* **55** (1) (2017) 119–155. MR3597159 https://doi.org/10.1137/16M1087497

[21] P. Del Moral and A. Niclas. A Taylor expansion of the square root matrix functional. *J. Math. Anal. Appl.* **1** (465) (2018) 259–266. Available at arXiv:1705.08561. MR3806701 https://doi.org/10.1016/j.jmaa.2018.05.005

[22] P. Del Moral and S. Penev. *Stochastic Processes: From Applications to Theory*. CRC Press-Chapman & Hall, London, 2016. MR3618157

[23] P. Del Moral and J. Tugaut. On the stability and the uniform propagation of chaos properties of ensemble Kalman–Bucy filters. *Ann. Appl. Probab.* **28** (2) (2018) 790–850. MR3784489 https://doi.org/10.1214/17-AAP1317

[24] J. L. Doob. *Stochastic Processes*. J. Wiley & Sons, New York, 1953. MR0058896

[25] G. Evensen. The ensemble Kalman filter: Theoretical formulation and practical implementation. *Ocean Dyn.* **53** (4) (2003) 343–367.

[26] C. Gourieroux, J. Jasiak and R. Sufana. The Wishart autoregressive process of multivariate stochastic volatility. *J. Econometrics* **150** (2) (2009) 167–181. MR2535514 https://doi.org/10.1016/j.jeconom.2008.12.016

[27] P. Graczyk and L. Vostrikova. Moments of Wishart processes via Ito calculus. *Theory Probab. Appl.* **51** (4) (2007) 609–625. MR2338064 https://doi.org/10.1137/S0040585X97982682

[28] A. Graham. *Kronecker Products and Matrix Calculus with Applications*. J. Wiley & Sons, New York, 1981. MR0640865

[29] N. J. Higham. *Functions of Matrices: Theory and Computation*. SIAM, Philadelphia, 2008. MR2396439 https://doi.org/10.1137/1.9780898717778

[30] P. L. Houtekamer and H. L. Mitchell. Data assimilation using an ensemble Kalman filter technique. *Mon. Weather Rev.* **126** (3) (1998) 796–811.

[31] Y. Hu and X. Y. Zhou. Indefinite stochastic Riccati equations. *SIAM J. Control Optim.* **42** (1) (2003) 123–137. MR1982738 https://doi.org/10.1137/S0363012901391330

[32] M. Hutzenthaler, A. Jentzen and P. E. Kloeden. Strong and weak divergence in finite time of Euler's method for stochastic differential equations with non-globally Lipschitz continuous coefficients. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **467** (2130) (2011) 1563–1576. MR2795791 https://doi.org/10.1098/rspa.2010.0348

[33] E. Kalnay. *Atmospheric Modelling, Data Assimilation, and Predictability*. Cambridge University Press, Cambridge, 2003.

[34] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*. Springer, New York, 1996. MR1121940 https://doi.org/10.1007/978-1-4612-0949-2

[35] M. Katori and H. Tanemura. Non-colliding Brownian motions and Harish-Chandra formula. *Electron. Commun. Probab.* **8** (2003) 112–121. MR2042750 https://doi.org/10.1214/ECP.v8-1076

[36] M. Katori and H. Tanemura. Symmetry of matrix-valued stochastic processes and non-colliding diffusion particle systems. *J. Math. Phys.* **45** (8) (2004) 3058–3085. MR2077500 https://doi.org/10.1063/1.1765215

[37] D. T. B. Kelly, K. J. H. Law and A. M. Stuart. Well-posedness and accuracy of the ensemble Kalman filter in discrete and continuous time. *Nonlinearity* **27** (10) (2014) 2579–2603. MR3265724 https://doi.org/10.1088/0951-7715/27/10/2579

[38] M. G. Kendall and A. Stuart. *The Advanced Theory of Statistics*. C. Griffin & Company, London, 1943. MR0019869

[39] M. Kohlmann and S. Tang. Multidimensional backward stochastic Riccati equations and applications. *SIAM J. Control Optim.* **41** (6) (2003) 1696–1721. MR1972529 https://doi.org/10.1137/S0363012900378760

[40] K. J. H. Law, H. Tembine and R. Tempone. Deterministic mean-field ensemble Kalman filtering. *SIAM J. Sci. Comput.* **38** (3) (2016) A1251–A1279. MR3493118 https://doi.org/10.1137/140984415

[41] F. Le Gland, V. Monbet and V. D. Tran. Large sample asymptotics for the ensemble Kalman filter. In *The Oxford Handbook of Nonlinear Filtering* 598–631, 2011. MR2884610

[42] A. J. Majda and X. T. Tong. Performance of ensemble Kalman filters in large dimensions, 2016. Available at arXiv:1606.09321. MR3794517 https://doi.org/10.1002/cpa.21722

[43] J. Mandel, L. Cobb and J. D. Beezley. On the convergence of the ensemble Kalman filter. *Appl. Math.* **56** (6) (2011) 533–541. MR2886236 https://doi.org/10.1007/s10492-011-0031-2

[44] E. Mayerhofer, O. Pfaffel and R. Stelzer. On strong solutions for positive definite jump-diffusions. In *Stochastic Processes and Their Applications* 2072–2086, **121**, 2011. MR2819242 https://doi.org/10.1016/j.spa.2011.05.006

[45] H. P. McKean. A class of Markov processes associated with nonlinear parabolic equations. *Proc. Natl. Acad. Sci. USA* **56** (6) (1966) 1907–1911. MR0221595 https://doi.org/10.1073/pnas.56.6.1907

[46] S. Méléard. Asymptotic behaviour of some interacting particle systems; McKean–Vlasov and Boltzmann models. Probabilistic models for nonlinear partial differential equations. In *Lecture Notes in Mathematics Book Series (LNM)* 42–95, **1627**. Springer, Berlin, 1996. MR1431299 https://doi.org/10.1007/BFb0093177

[47] S. Reich and C. J. Cotter. Ensemble filter techniques for intermittent data assimilation. In *Large Scale Inverse Problems: Computational Methods and Applications in the Earth Sciences* 91–134. M. Cullen, M. A. Freitag, S. Kindermann and R. Scheichl (Eds). De Gruyter, Berlin, 2013. Available at arXiv:1208.6572.

[48] P. Sakov and P. R. Oke. A deterministic formulation of the ensemble Kalman filter: An alternative to ensemble square root filters. *Tellus A.* **60** (2) (2008) 361–371.

[49] A. S. Sznitman Topics in Propagation of Chaos. In *Course Given at the Ecole d'Eté de Probabilités de Saint-Flour in 1989 Lecture Notes in Mathematics Book Series (LNM)* **1464** 164–251. Springer-Verlag, Berlin, 1991. MR1108185 https://doi.org/10.1007/BFb0085169

[50] A. Taghvaei and P. G. Mehta. An optimal transport formulation of the linear feedback particle filter. In *Proc. of the 2016 American Control Conference (ACC)*, *Boston, USA*, 2016.

[51] X. T. Tong, A. J. Majda and D. Kelly. Nonlinear stability and ergodicity of ensemble based Kalman filters. *Nonlinearity* **29** (2) (2016) 657–691. MR3461611 https://doi.org/10.1088/0951-7715/29/2/657

[52] X. T. Tong, A. J. Majda and D. Kelly. Nonlinear stability of the ensemble Kalman filter with adaptive covariance inflation. *Commun. Math. Sci.* **14** (5) (2016) 1283–1313. MR3506803 https://doi.org/10.4310/CMS.2016.v14.n5.a5

[53] J. L. van Hemmen and T. Ando. An inequality for trace ideals. *Comm. Math. Phys.* **76** (143) (1980) 143–148. MR0587507

[54] C. F. Van Loan. The ubiquitous Kronecker product. *J. Comput. Appl. Math.* **123** (1) (2000) 85–100. MR1798520 https://doi.org/10.1016/S0377-0427(00)00393-9

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 917–927
https://doi.org/10.1214/19-AIHP988

Indistinguishability of collections of trees in the uniform spanning forest

Tom Hutchcroft

Statslab, DPMMS, University of Cambridge, United Kingdom. E-mail: t.hutchcroft@maths.cam.ac.uk

Abstract. We prove the following indistinguishability theorem for k-tuples of trees in the uniform spanning forest of $\mathbb{Z}^d$: Suppose that $\mathscr{A}$ is a property of a k-tuple of components that is stable under finite modifications of the forest. Then either every k-tuple of distinct trees has property $\mathscr{A}$ almost surely, or no k-tuple of distinct trees has property $\mathscr{A}$ almost surely. This generalizes the indistinguishability theorem of the author and Nachmias (2016), which applied to individual trees. Our results apply more generally to any graph that has the Liouville property and for which every component of the USF is one-ended.

Résumé. Dans cet article, nous prouvons le théorème d'indistinguabilité suivant pour les k-uplets d'arbres dans le modèle de forêt couvrante uniforme sur $\mathbb{Z}^d$: supposons que $\mathscr{A}$ est une propriété d'un k-uplet de composantes connexes qui est stable par modification finie de la forêt, alors ou bien chaque k-uplet satisfait la propriété $\mathscr{A}$ presque sûrement, ou bien aucun ne la satisfait presque sûrement. Ce résultat étend le théorème d'indistinguabilité de l'auteur et de Nachmias (2016) qui ne couvrait que le cas d'arbres pris individuellement. Notre résultat s'applique plus généralement à tout graphe vérifiant la propriété de Liouville et pour lequel toutes les composantes connexes de la FCU ont un seul bout.

MSC: Primary 60B99; secondary 37A20

Keywords: Uniform spanning forest; Indistinguishability; Ergodicity; Liouville; Zero-one law

References

[1] D. Aldous and R. Lyons. Processes on unimodular random networks. *Electron. J. Probab.* **12** (54) (2007) 1454–1508. MR2354165 https://doi.org/10.1214/EJP.v12-463
[2] O. Angel, M. T. Barlow, O. Gurel-Gurevich and A. Nachmias. Boundaries of planar graphs, via circle packings. *Ann. Probab.* **44** (3) (2016) 1956–1984. MR3502598 https://doi.org/10.1214/15-AOP1014
[3] I. Benjamini, H. Kesten, Y. Peres and O. Schramm. Geometry of the uniform spanning forest: Transitions in dimensions $4, 8, 12, \dots$. *Ann. of Math. (2)* **160** (2) (2004) 465–491. MR2123930 https://doi.org/10.4007/annals.2004.160.465
[4] I. Benjamini, R. Lyons, Y. Peres and O. Schramm. Uniform spanning forests. *Ann. Probab.* **29** (1) (2001) 1–65. MR1825141 https://doi.org/10.1214/aop/1008956321
[5] I. Benjamini and O. Schramm. Harmonic functions on planar and almost planar graphs and manifolds, via circle packings. *Invent. Math.* **126** (3) (1996) 565–587. MR1419007 https://doi.org/10.1007/s002220050109
[6] D. Blackwell. On transient Markov processes with a countable number of states and stationary transition probabilities. *Ann. Math. Stat.* **26** (1955) 654–658. MR0075479 https://doi.org/10.1214/aoms/1177728425
[7] L. Bowen. Finitary random interlacements and the Gaboriau–Lyons problem. arXiv preprint, 2017. Available at arXiv:1707.09573. MR3962876 https://doi.org/10.1007/s00039-019-00494-4
[8] Y. Derriennic. Quelques applications du théorème ergodique sous-additif. In *Conference on Random Walks (Kleebach, 1979) (French)* 183–201. *Astérisque* **74**. Soc. Math. France, Paris, 1980. MR0588163
[9] D. Gaboriau. Orbit equivalence and measured group theory. In *Proceedings of the International Congress of Mathematicians. Volume III* 1501–1527. Hindustan Book Agency, New Delhi, 2010. MR2827853
[10] D. Gaboriau and R. Lyons. A measurable-group-theoretic solution to von Neumann's problem. *Invent. Math.* **177** (3) (2009) 533–540. MR2534099 https://doi.org/10.1007/s00222-009-0187-5
[11] H.-O. Georgii. *Gibbs Measures and Phase Transitions*, 2nd edition. *De Gruyter Studies in Mathematics* **9**. de Gruyter, Berlin, 2011. MR2807681 https://doi.org/10.1515/9783110250329
[12] M. Gheysens and N. Monod. Fixed points for bounded orbits in Hilbert spaces. *Ann. Sci. Éc. Norm. Supér. (4)* **50** (1) (2017) 131–156. MR3621428 https://doi.org/10.24033/asens.2317
[13] O. Häggström, Y. Peres and R. H. Schonmann. Percolation on transitive graphs as a coalescent process: Relentless merging followed by simultaneous uniqueness. In *Perplexing Problems in Probability* 69–90. *Progr. Probab.* **44**. Birkhäuser, Boston, MA, 1999. MR1703125

[14] T. Hutchcroft. Wired cycle-breaking dynamics for uniform spanning forests. *Ann. Probab.* **44** (6) (2016) 3879–3892. MR3572326 https://doi.org/10.1214/15-AOP1063
[15] T. Hutchcroft. Interlacements and the wired uniform spanning forest. *Ann. Probab.* **46** (2) (2018) 1170–1200. MR3773383 https://doi.org/10.1214/17-AOP1203
[16] T. Hutchcroft and A. Nachmias. Indistinguishability of trees in uniform spanning forests. *Probab. Theory Related Fields* (2016) 1–40. MR3733658 https://doi.org/10.1214/17-EJP116
[17] T. Hutchcroft and A. Nachmias. Uniform spanning forests of planar graphs. arXiv preprint, 2016. Available at arXiv:1603.07320.
[18] T. Hutchcroft and Y. Peres. The component graph of the uniform spanning forest: Transitions in dimensions 9, 10, 11, ... *Probab. Theory Related Fields* (2018). MR3773383 https://doi.org/10.1214/17-AOP1203
[19] A. S. Kechris and B. D. Miller. *Topics in Orbit Equivalence*. *Lecture Notes in Mathematics* **1852**. Springer-Verlag, Berlin, 2004. MR2095154 https://doi.org/10.1007/b99421
[20] R. Lyons, B. J. Morris and O. Schramm. Ends in uniform spanning forests. *Electron. J. Probab.* **13** (58) (2008) 1702–1725. MR2448128 https://doi.org/10.1214/EJP.v13-566
[21] R. Lyons and Y. Peres. *Probability on Trees and Networks*. *Cambridge Series in Statistical and Probabilistic Mathematics* **42**. Cambridge University Press, New York, 2016. Available at http://pages.iu.edu/~rdlyons/. MR3616205 https://doi.org/10.1017/9781316672815
[22] R. Lyons, Y. Peres and O. Schramm. Markov chain intersections and the loop-erased walk. *Ann. Inst. Henri Poincaré B, Probab. Stat.* **39** (2003) 779–791. MR1997212 https://doi.org/10.1016/S0246-0203(03)00033-5
[23] R. Lyons and O. Schramm. Indistinguishability of percolation clusters. *Ann. Probab.* **27** (4) (1999) 1809–1836. MR1742889 https://doi.org/10.1214/aop/1022677549
[24] S. Martineau. Ergodicity and indistinguishability in percolation theory. *Enseign. Math.* **61** (3–4) (2015) 285–319. MR3539840 https://doi.org/10.4171/LEM/61-3/4-2
[25] R. Pemantle. Choosing a spanning tree for the integer lattice uniformly. *Ann. Probab.* **19** (4) (1991) 1559–1574. MR1127715
[26] S. Rohde. Oded Schramm: From circle packing to SLE. *Ann. Probab.* **39** (2011) 1621–1667. MR2884870 https://doi.org/10.1214/10-AOP590
[27] W. Rudin. *Real and Complex Analysis*, 3rd edition. McGraw-Hill, New York, 1987. MR0924157
[28] P. Tang. Heavy Bernoulli-percolation clusters are indistinguishable. Preprint, 2018. Available at arXiv:1809.01284.
[29] A. Timár. Indistinguishability of the components of random spanning forests. *Ann. Probab.* **46** (4) (2018) 2221–2242. MR3813990 https://doi.org/10.1214/17-AOP1225
[30] R. Tucker-Drob. Ergodic hyperfinite subgraphs and one-ended subforests. In preparation.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 928–953
https://doi.org/10.1214/19-AIHP989

Sanov-type large deviations in Schatten classes

Zakhar Kabluchko[a], Joscha Prochno[b] and Christoph Thäle[c]

[a]*Institut für Mathematische Stochastik, Westfälische Wilhelms-Universität Münster, Germany. E-mail: zakhar.kabluchko@uni-muenster.de*
[b]*Institut für Mathematik und Wissenschaftliches Rechnen, Karl-Franzens-Universität Graz, Austria. E-mail: j.prochno@hull.ac.uk*
[c]*Fakultät für Mathematik, Ruhr-Universität Bochum, Germany. E-mail: christoph.thaele@rub.de*

Abstract. Denote by $\lambda_1(A), \dots, \lambda_n(A)$ the eigenvalues of an $(n \times n)$-matrix A. Let Z_n be an $(n \times n)$-matrix chosen uniformly at random from the matrix analogue to the classical ℓ_p^n-ball, defined as the set of all self-adjoint $(n \times n)$-matrices satisfying $\sum_{k=1}^n |\lambda_k(A)|^p \le 1$. We prove a large deviations principle for the (random) spectral measure of the matrix $n^{1/p} Z_n$. As a consequence, we obtain that the spectral measure of $n^{1/p} Z_n$ converges weakly almost surely to a non-random limiting measure given by the Ullman distribution, as $n \to \infty$. The corresponding results for random matrices in Schatten trace classes, where eigenvalues are replaced by the singular values, are also presented.

Résumé. Notons $\lambda_1(A), \dots, \lambda_n(A)$ les valeurs propres d'une matrice A de taille $n \times n$. Soit Z_n une matrice $n \times n$ choisie aléatoirement et uniformément dans l'équivalent matriciel de la boule unité de l'ensemble classique ℓ_p^n, défini comme l'ensemble des matrices $n \times n$ auto-adjointes satisfaisant $\sum_{k=1}^n |\lambda_k(A)|^p \le 1$. Nous prouvons un principe de grandes déviations pour la mesure spectrale aléatoire de la matrice $n^{1/p} Z_n$. Comme conséquence, nous obtenons que la mesure spectrale de $n^{1/p} Z_n$ converge faiblement presque sûrement vers une mesure déterministe limite décrite par la loi d'Ullman lorsque n tend vers l'infini. Nous présentons également les résultats correspondants pour les matrices aléatoires dans les classes de trace de Schatten, où les valeurs propres sont remplacées par les valeurs singulières.

MSC: Primary 47B10; 60B20; 60F10; secondary 46B07; 47B10; 52A21; 52A23

Keywords: Asymptotic geometric analysis; Coulomb gas; Eigenvalues; Gaussian ensembles; High dimensional convexity; Large deviations principles; Matrix unit balls; Random matrix theory; Schatten classes; Singular values

References

[1] D. Alonso-Gutiérrez, J. Prochno and C. Thäle. Large deviations for high-dimensional random projections of ℓ_p^n-balls. *Adv. in Appl. Math.* **99** (2018) 1–35. MR3806754 https://doi.org/10.1016/j.aam.2018.04.003
[2] D. Alonso-Gutierrez, J. Prochno and C. Thäle. Gaussian fluctuations for high-dimensional random projections of ℓ_p^n-balls. *Bernoulli*. To appear, 2019.
[3] G. W. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, 2010. MR2760897
[4] M. Anttila, K. Ball and I. Perissinaki. The central limit problem for convex bodies. *Trans. Amer. Math. Soc.* **355** (12) (2003) 4723–4735. MR1997580 https://doi.org/10.1090/S0002-9947-03-03085-X
[5] S. Artstein-Avidan, A. Giannopoulos and V. D. Milman. *Asymptotic Geometric Analysis. Part I*. *Mathematical Surveys and Monographs* **202**. American Mathematical Society, Providence, RI, 2015. MR3331351 https://doi.org/10.1090/surv/202
[6] F. Barthe and D. Cordero-Erausquin. Invariances in variance estimates. *Proc. Lond. Math. Soc. (3)* **106** (1) (2013) 33–64. MR3020738 https://doi.org/10.1112/plms/pds011
[7] G. Ben Arous and A. Guionnet. Large deviations for Wigner's law and Voiculescu's non-commutative entropy. *Probab. Theory Related Fields* **108** (4) (1997) 517–542. MR1465640 https://doi.org/10.1007/s004400050119
[8] S. Brazitikos, A. Giannopoulos, P. Valettas and B.-H. Vritsiou. *Geometry of Isotropic Convex Bodies*. *Mathematical Surveys and Monographs* **196**. American Mathematical Society, Providence, RI, 2014. MR3185453 https://doi.org/10.1090/surv/196
[9] A. Dembo and O. Zeitouni. *Large Deviations. Techniques and Applications*. *Stochastic Modelling and Applied Probability* **38**. Springer-Verlag, Berlin, 2010. Corrected reprint of the second (1998) edition. MR2571413 https://doi.org/10.1007/978-3-642-03311-7
[10] F. den Hollander. *Large Deviations*. *Fields Institute Monographs* **14**. American Mathematical Society, Providence, RI, 2000. MR1739680
[11] N. Gantert, S. S. Kim and K. Ramanan. Large deviations for random projections of ℓ^p balls. *Ann. Probab.* **45** (6B) (2017) 4419–4476. MR3737915 https://doi.org/10.1214/16-AOP1169

[12] Y. Gordon and D. R. Lewis. Absolutely summing operators and local unconditional structures. *Acta Math.* **133** (1974) 27–48. MR0410341 https://doi.org/10.1007/BF02392140

[13] O. Guédon. Concentration phenomena in high dimensional geometry. In *Journées MAS 2012* 47–60. *ESAIM Proc.* **44**. EDP Sci., Les Ulis, 2014. MR3178607 https://doi.org/10.1051/proc/201444002

[14] O. Guédon, P. Nayar and T. Tkocz. Concentration inequalities and geometry of convex bodies. In *Analytical and Probabilistic Methods in the Geometry of Convex Bodies* 9–86. *IMPAN Lect. Notes* **2**. Polish Acad. Sci. Inst. Math., Warsaw, 2014. MR3329056

[15] O. Guédon and G. Paouris. Concentration of mass on the Schatten classes. *Ann. Inst. Henri Poincaré Probab. Stat.* **43** (1) (2007) 87–99. MR2288271 https://doi.org/10.1016/j.anihpb.2006.01.002

[16] F. Hiai and D. Petz. Eigenvalue density of the Wishart matrix and large deviations. *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **1** (4) (1998) 633–646. MR1665279 https://doi.org/10.1142/S021902579800034X

[17] F. Hiai and D. Petz. A large deviation theorem for the empirical eigenvalue distribution of random unitary matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **36** (1) (2000) 71–85. MR1743092 https://doi.org/10.1016/S0246-0203(00)00116-3

[18] F. Hiai and D. Petz. *The Semicircle Law, Free Random Variables and Entropy*. *Mathematical Surveys and Monographs* **77**. American Mathematical Society, Providence, RI, 2000. MR1746976

[19] A. Hinrichs, J. Prochno and J. Vybíral. Entropy numbers of embeddings of Schatten classes. *J. Funct. Anal.* **273** (10) (2017) 3241–3261. MR3695893 https://doi.org/10.1016/j.jfa.2017.08.008

[20] Z. Kabluchko, J. Prochno and C. Thäle. Exact asymptotic volume and volume ratio of Schatten unit balls. Preprint, 2018. Available at arXiv:1804.03467.

[21] Z. Kabluchko, J. Prochno and C. Thäle. Intersection of unit balls in classical matrix ensembles. Preprint, 2018. Available at arXiv:1804.03466.

[22] Z. Kabluchko, J. Prochno and C. Thäle. High-dimensional limit theorems for random vectors in ℓ_p^n-balls. *Commun. Contemp. Math.* **21** (1) (2019) 1750092. MR3904638 https://doi.org/10.1142/S0219199717500924

[23] O. Kallenberg. *Foundations of Modern Probability*, 2nd edition. *Probability and Its Applications*. Springer-Verlag, New York, 2002. MR1876169 https://doi.org/10.1007/978-1-4757-4015-8

[24] S. S. Kim. Problems at the interface of probability and convex geometry: Random projections and constrained processes. Ph.D. thesis, Brown University, 2017.

[25] S. S. Kim and K. Ramanan. A conditional limit theorem for high-dimensional ℓ^p-spheres. *J. Appl. Probab.* **55** (4) (2018) 1060–1077. MR3899928 https://doi.org/10.1017/jpr.2018.71

[26] A. Klenke. *Probability Theory: A Comprehensive Course*, 2nd edition. *Universitext*. Springer, London, 2014. MR3112259 https://doi.org/10.1007/978-1-4471-5361-0

[27] H. König, M. Meyer and A. Pajor. The isotropy constants of the Schatten classes are bounded. *Math. Ann.* **312** (4) (1998) 773–783. MR1660231 https://doi.org/10.1007/s002080050245

[28] Y. V. Prohorov. Convergence of random processes and limit theorems in probability theory. *Teor. Veroyatn. Primen.* **1** (1956) 177–238. MR0084896

[29] J. Radke and B.-H. Vritsiou. On the thin-shell conjecture for the Schatten classes. Preprint, 2016. Available at arXiv:1602.06934.

[30] E. B. Saff and V. Totik. *Logarithmic Potentials with External Fields*. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **316**. Springer-Verlag, Berlin, 1997. Appendix B by Thomas Bloom. MR1485778 https://doi.org/10.1007/978-3-662-03329-6

[31] G. Schechtman and M. Schmuckenschläger. Another remark on the volume of the intersection of two L_p^n balls. In *Geometric Aspects of Functional Analysis (1989–90)* 174–178. *Lecture Notes in Math.* **1469**. Springer, Berlin, 1991. MR1122622 https://doi.org/10.1007/BFb0089224

[32] G. Schechtman and J. Zinn. On the volume of the intersection of two L_p^n balls. *Proc. Amer. Math. Soc.* **110** (1) (1990) 217–224. MR1015684 https://doi.org/10.2307/2048262

[33] S. Szarek and N. Tomczak-Jaegermann. On nearly Euclidean decomposition for some classes of Banach spaces. *Compos. Math.* **40** (3) (1980) 367–385. MR0571056

[34] N. Tomczak-Jaegermann. The moduli of smoothness and convexity and the Rademacher averages of trace classes S_p $(1 \le p < \infty)$. *Studia Math.* **50** (1974) 163–182. MR0355667

[35] D. Voiculescu. The analogues of entropy and of Fisher's information measure in free probability theory. I. *Comm. Math. Phys.* **155** (1) (1993) 71–92. MR1228526

[36] B.-H. Vritsiou. Selberg-type integrals and the variance conjecture for the operator norm. Preprint, 2018. Available at arXiv:1805.03129.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 954–982
https://doi.org/10.1214/19-AIHP990

A Central Limit Theorem for Wasserstein type distances between two distinct univariate distributions

Philippe Berthet[a], Jean-Claude Fort[b] and Thierry Klein[a,c]

[a]*Institut de Mathématiques de Toulouse; UMR5219, Université de Toulouse; CNRS, UPS IMT, F-31062 Toulouse Cedex 9, France. E-mail: philippe.berthet@math.univ-toulouse.fr; thierry.klein@math.univ-toulouse.fr*
[b]*MAP5; UMR 8145; Université Paris Descartes; CNRS, 45 rue des Saints Pères 75006 Paris, France. E-mail: jean-claude.fort@parisdescartes.fr*
[c]*ENAC – Ecole Nationale de l'Aviation Civile, Université de Toulouse, France. E-mail: thierry01.klein@enac.fr*

Abstract. In this article we study the natural nonparametric estimator of a Wasserstein type cost between two distinct continuous distributions F and G on $\mathbb{R}$. The estimator is based on the order statistics of a sample having marginals F, G and any joint distribution. We prove a central limit theorem under general conditions relating the tails and the cost function. In particular, these conditions are satisfied by Wasserstein distances of order $p > 1$ and compatible classical probability distributions.

Résumé. Dans cet article nous étudions l'estimateur non paramétrique naturel d'un coût de type Wasserstein entre deux lois F et G distinctes et continues sur $\mathbb{R}$. Cet estimateur est construit à partir des statistiques d'ordre d'un échantillon d'un couple quelconque de lois marginales F et G. Nous démontrons un théorème limite central sous des conditions générales reliant les queues de distribution à la fonction de coût. En particulier, ces conditions sont satisfaites par les distances de Wasserstein d'ordre $p > 1$ et les lois classiques compatibles.

MSC: 62G30; 62G20; 60F05; 60F17

Keywords: Central Limit Theorems; Generalized Wasserstein distances; Empirical processes; Strong approximation; Dependent samples

References

[1] P. C. Álvarez-Esteban, E. del Barrio, J. A. Cuesta-Albertos and C. Matrán. Uniqueness and approximate computation of optimal incomplete transportation plans. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** (2) (2011) 358–375. MR2814414 https://doi.org/10.1214/09-AIHP354
[2] R. R. Bahadur. A note on quantiles in large samples. *Ann. Math. Stat.* **37** (1966) 577–580. MR0189095 https://doi.org/10.1214/aoms/1177699450
[3] P. Berthet and D. M. Mason. Revisiting two strong approximation results of Dudley and Philipp. In *High Dimensional Probability* 155–172. *IMS Lecture Notes Monogr. Ser.* **51**. Inst. Math. Statist, Beachwood, OH, 2006. MR2387767 https://doi.org/10.1214/074921706000000824
[4] S. G. Bobkov and M. Ledoux. One-dimensional empirical measures, order statistics and kantorovich transport distances. Preprint, 2016. To appear in *Mem. Amer. Math. Soc.*
[5] S. Cambanis, G. Simons and W. Stout. Inequalities for $Ek(X, Y)$ when the marginals are fixed. *Z. Wahrsch. Verw. Gebiete* **36** (4) (1976) 285–294. MR0420778 https://doi.org/10.1007/BF00532695
[6] M. Csörgö and P. Révész. Strong approximations of the quantile process. *Ann. Statist.* **6** (4) (1978) 882–894. MR0501290
[7] E. del Barrio, E. Giné and C. Matrán. Central limit theorems for the Wasserstein distance between the empirical and the true distributions. *Ann. Probab.* **27** (2) (1999) 1009–1071. MR1698999 https://doi.org/10.1214/aop/1022677394
[8] E. del Barrio, E. Giné and C. Matrán. Correction: "Central limit theorems for the Wasserstein distance between the empirical and the true distributions" [Ann. Probab. **27** (1999), no. 2, 1009–1071; MR1698999 (2000g:60034)]. *Ann. Probab.* **31** (2) (2003) 1142–1143. MR1964961
[9] E. del Barrio, E. Giné and F. Utzet. Asymptotics for L_2 functionals of the empirical quantile process, with applications to tests of fit based on weighted Wasserstein distances. *Bernoulli* **11** (1) (2005) 131–189. MR2121458 https://doi.org/10.3150/bj/1110228245
[10] E. del Barrio and J.-M. Loubès. Central limit theorems for empirical transportation cost in general dimension. *Ann. Probab.* **47** (2) (2019) 926–951. MR3916938 https://doi.org/10.1214/18-AOP1275
[11] G. Freitag, C. Czado and A. Munk. A nonparametric test for similarity of marginals – with applications to the assessment of population bioequivalence. *J. Statist. Plann. Inference* **137** (3) (2007) 697–711. MR2301710 https://doi.org/10.1016/j.jspi.2006.06.003
[12] J. Kiefer. Deviations between the sample quantile process and the sample df. In *Nonparametric Techniques in Statistical Inference (Proc. Sympos., Indiana Univ., Bloomington, Ind., 1969)* 299–319. Cambridge Univ. Press, London, 1970. MR0277071
[13] T. Mikosch. Regular variation, subexponentiality and their applications in probability theory. Operations Research EURANDOM European Institute for Statistics, Probability, and their Applications. EURANDOM report, Eindhoven University of Technology, 1999.

[14] A. Munk and C. Czado. Nonparametric validation of similar distributions and assessment of goodness of fit. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **60** (1) (1998) 223–241. MR1625620 https://doi.org/10.1111/1467-9868.00121

[15] T. Rippl, A. Munk and A. Sturm. Limit laws of the empirical Wasserstein distance: Gaussian distributions. *J. Multivariate Anal.* **151** (2016) 90–109. MR3545279 https://doi.org/10.1016/j.jmva.2016.06.005

[16] E. Seneta. *Regularly Varying Functions*. *Lecture Notes in Mathematics* **508**. Springer-Verlag, Berlin–New York, 1976. MR0453936

[17] G. R. Shorack. *Probability for Statisticians*. Springer, 2000. MR1762415

[18] M. Sommerfeld and A. Munk. Inference for empirical Wasserstein distances on finite spaces. ArXiv e-prints, 2016. MR3744719 https://doi.org/10.1111/rssb.12236

[19] C. Villani. *Topics in Optimal Transportation*. American Mathematical Society, 2003. MR1964483 https://doi.org/10.1007/b12016

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 983–1001
https://doi.org/10.1214/19-AIHP991

On the mixing time of the Diaconis–Gangolli random walk on contingency tables over $\mathbb{Z}/q\mathbb{Z}$

Evita Nestoridi and Oanh Nguyen

Department of Mathematics, Princeton University, USA. E-mail: exn@princeton.edu; onguyen@princeton.edu

Abstract. The Diaconis–Gangolli random walk is an algorithm that generates an almost uniform random graph with prescribed degrees. In this paper, we study the mixing time of the Diaconis–Gangolli random walk restricted on $n \times n$ contingency tables over $\mathbb{Z}/q\mathbb{Z}$. We prove that the random walk exhibits cutoff at $\frac{n^2}{4(1-\cos\frac{2\pi}{q})}\log n$, when $\log q = o(\frac{\sqrt{\log n}}{\log\log n})$.

Résumé. La marche aléatoire de Diaconis–Gangolli est un algorithme qui génère un graphe aléatoire à degrés prescrits, de loi presque uniforme. Dans cet article, nous étudions le temps de mélange de cette marche aléatoire restreinte aux tableaux de contingence de taille $n \times n$ sur $\mathbb{Z}/q\mathbb{Z}$. Nous montrons que la marche aléatoire présente une transition abrupte (cutoff) à $\frac{n^2}{4(1-\cos(\frac{2\pi}{q}))}\log n$, où $\log q = o(\frac{\sqrt{\log n}}{\log\log n})$.

MSC: Primary 60J10; secondary 60C05; 60G42

Keywords: Contingency table; Random graphs; Mixing time; Cutoff

References

[1] W. Baldoni-Silva, J. A. De Loera and M. Vergne. Counting integer flows in networks. *Found. Comput. Math.* **4** (2004) 277–314. MR2078665 https://doi.org/10.1007/s10208-003-0088-8
[2] A. Barvinok. Brunn–Minkowski inequalities for contingency tables and integer flows. *Adv. Math.* **211** (2007) 105–122. MR2313530 https://doi.org/10.1016/j.aim.2006.07.012
[3] A. Barvinok. Enumerating contingency tables via random permanents. *Combin. Probab. Comput.* **17** (2008) 1–19. MR2376421 https://doi.org/10.1017/S0963548307008668
[4] A. Barvinok. Asymptotic estimates for the number of contingency tables, integer flows, and volumes of transportation polytopes. *Int. Math. Res. Not. IMRN* (2009) 348–385.
[5] A. Barvinok. What does a random contingency table look like? *Combin. Probab. Comput.* **19** (2010) 517–539. MR2647491 https://doi.org/10.1017/S0963548310000039
[6] A. Barvinok and J. A. Hartigan. Maximum entropy Gaussian approximations for the number of integer points and volumes of polytopes. *Adv. in Appl. Math.* **45** (2010) 252–289. MR2646125 https://doi.org/10.1016/j.aam.2010.01.004
[7] A. Barvinok and J. A. Hartigan. An asymptotic formula for the number of non-negative integer matrices with prescribed row and column sums. *Trans. Amer. Math. Soc.* **364** (2012) 4323–4368. MR2912457 https://doi.org/10.1090/S0002-9947-2012-05585-1
[8] A. Barvinok and J. A. Hartigan. The number of graphs and a random graph with a given degree sequence. *Random Structures Algorithms* **42** (2013) 301–348. MR3039682 https://doi.org/10.1002/rsa.20409
[9] A. Barvinok, Z. Luria, A. Samorodnitsky and A. Yong. An approximation algorithm for counting contingency tables. *Random Structures Algorithms* **37** (2010) 25–66. MR2674620 https://doi.org/10.1002/rsa.20301
[10] E. A. Bender. The asymptotic number of non-negative integer matrices with given row and column sums. *Discrete Math.* **10** (1974) 217–223. MR0389621 https://doi.org/10.1016/0012-365X(74)90118-6
[11] E. A. Bender and E. R. Canfield. The asymptotic number of labeled graphs with given degree sequences. *J. Combin. Theory Ser. A* **24** (1978) 296–307. MR0505796 https://doi.org/10.1016/0097-3165(78)90059-6
[12] I. Bezáková, N. Bhatnagar and E. Vigoda. Sampling binary contingency tables with a greedy start. In *Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithms* 414–423. ACM, New York, 2006. MR2368838 https://doi.org/10.1145/1109557.1109604
[13] I. Bezáková, N. Bhatnagar and E. Vigoda. Sampling binary contingency tables with a greedy start. *Random Structures Algorithms* **30** (2007) 168–205. MR2283227 https://doi.org/10.1002/rsa.20155
[14] I. Bezáková, A. Sinclair, D. Stefankovic and E. Vigoda. Negative examples for sequential importance sampling of binary contingency tables. In *Algorithms—ESA 2006* 136–147. *Lecture Notes in Comput. Sci.* **4168**. Springer, Berlin, 2006. MR2347138 https://doi.org/10.1007/11841036_15

[15] I. Bezáková, A. Sinclair, D. Stefankovic and E. Vigoda. Negative examples for sequential importance sampling of binary contingency tables. *Algorithmica* **64** (2012) 606–620. MR2989466 https://doi.org/10.1007/s00453-011-9569-3

[16] J. Blanchet and A. Stauffer. Characterizing optimal sampling of binary contingency tables via the configuration model. *Random Structures Algorithms* **42** (2013) 159–184. MR3019396 https://doi.org/10.1002/rsa.20403

[17] J. Blitzstein and P. Diaconis. A sequential importance sampling algorithm for generating random graphs with prescribed degrees. *Internet Math.* **6** (2010) 489–522. MR2809836 https://doi.org/10.1080/15427951.2010.557277

[18] B. Bollobás. A probabilistic proof of an asymptotic formula for the number of labelled regular graphs. *European J. Combin.* **1** (1980) 311–316. MR0595929 https://doi.org/10.1016/S0195-6698(80)80030-8

[19] E. R. Canfield, C. Greenhill and B. D. McKay. Asymptotic enumeration of dense 0–1 matrices with specified line sums. *J. Combin. Theory Ser. A* **115** (2008) 32–66. MR2378856 https://doi.org/10.1016/j.jcta.2007.03.009

[20] Y. Chen, P. Diaconis, S. P. Holmes and J. S. Liu. Sequential Monte Carlo methods for statistical analysis of tables. *J. Amer. Statist. Assoc.* **100** (2005) 109–120. MR2156822 https://doi.org/10.1198/016214504000001303

[21] Y. Chen, I. H. Dinwoodie and S. Sullivant. Sequential importance sampling for multiway tables. *Ann. Statist.* **34** (2006) 523–545. MR2275252 https://doi.org/10.1214/009053605000000822

[22] F. R. K. Chung, R. L. Graham and S.-T. Yau. On sampling with Markov chains. In *Proceedings of the Seventh International Conference on Random Structures and Algorithms* 55–77. *Atlanta, GA*, *1995*, **9**, 1996. MR1611732 https://doi.org/10.1002/(SICI)1098-2418(199608/09)9:1/2<55::AID-RSA4>3.3.CO;2-C

[23] M. Cryan and M. Dyer. A polynomial-time algorithm to approximately count contingency tables when the number of rows is constant. *J. Comput. System Sci.* **67** (2003) 291–310. Special issue on STOC2002 (Montreal, QC). MR2022833 https://doi.org/10.1016/S0022-0000(03)00014-X

[24] M. Cryan, M. Dyer, L. A. Goldberg, M. Jerrum and R. Martin. Rapidly mixing Markov chains for sampling contingency tables with a constant number of rows. *SIAM J. Comput.* **36** (2006) 247–278. MR2231648 https://doi.org/10.1137/S0097539703434243

[25] M. Cryan, M. Dyer and D. Randall. Approximately counting integral flows and cell-bounded contingency tables. *SIAM J. Comput.* **39** (2010) 2683–2703. MR2645886 https://doi.org/10.1137/060650544

[26] P. Diaconis. *Group Representations in Probability and Statistics*. *Institute of Mathematical Statistics Lecture Notes—Monograph Series* **11**. Institute of Mathematical Statistics, Hayward, CA, 1988. MR0964069

[27] P. Diaconis. Threads Through Group Theory. In *Character Theory of Finite Groups*. *Contemp. Math.* **524**, 33–47. Amer. Math. Soc., Providence, RI, 2010. MR2731916 https://doi.org/10.1090/conm/524/10343

[28] P. Diaconis and B. Efron. Testing for independence in a two-way table: New interpretations of the chi-square statistic. *Ann. Statist.* **13** (1985) 845–913. With discussions and with a reply by the authors. MR0803747 https://doi.org/10.1214/aos/1176349634

[29] P. Diaconis and B. Efron. Probabilistic-geometric theorems arising from the analysis of contingency tables. In *Contributions to the Theory and Application of Statistics* 103–125. Academic Press, Boston, MA, 1987.

[30] P. Diaconis and A. Gangolli. Rectangular arrays with fixed margins. In *Discrete Probability and Algorithms* 15–41. *Minneapolis, MN*, *1993*. *IMA Vol. Math. Appl.* **72**. Springer, New York, 1995. MR1380519 https://doi.org/10.1007/978-1-4612-0801-3_3

[31] P. Diaconis and L. Saloff-Coste. Comparison techniques for random walk on finite groups. *Ann. Probab.* **21** (1993) 2131–2156. MR1245303

[32] P. Diaconis and L. Saloff-Coste. Comparison theorems for reversible Markov chains. *Ann. Appl. Probab.* **3** (1993) 696–730. MR1233621

[33] P. Diaconis and M. Shahshahani. Generating a random permutation with random transpositions. *Z. Wahrsch. Verw. Gebiete* **57** (1981) 159–179. MR0626813 https://doi.org/10.1007/BF00535487

[34] P. Diaconis and B. Sturmfels. Algebraic algorithms for sampling from conditional distributions. *Ann. Statist.* **26** (1998) 363–397. MR1608156 https://doi.org/10.1214/aos/1030563990

[35] M. Dyer and C. Greenhill. Polynomial-time counting and sampling of two-rowed contingency tables. *Theoret. Comput. Sci.* **246** (2000) 265–278. MR1780242 https://doi.org/10.1016/S0304-3975(99)00136-X

[36] M. Dyer, R. Kannan and J. Mount. Sampling contingency tables. *Random Structures Algorithms* **10** (1997) 487–506. MR1608222 https://doi.org/10.1002/(SICI)1098-2418(199707)10:4<487::AID-RSA4>3.0.CO;2-Q

[37] C. Greenhill and B. D. McKay. Asymptotic enumeration of sparse nonnegative integer matrices with specified row and column sums. *Adv. in Appl. Math.* **41** (2008) 459–481. MR2459445 https://doi.org/10.1016/j.aam.2008.01.002

[38] D. Hernek. Random generation of $2 \times n$ contingency tables. *Random Structures Algorithms* **13** (1998) 71–79. MR1634343 https://doi.org/10.1002/(SICI)1098-2418(199808)13:1<71::AID-RSA4>3.3.CO;2-O

[39] A. Liebenau and N. Wormald. Asymptotic enumeration of graphs by degree sequence, and the degree sequence of a random graph, 2017. Preprint. Available at arXiv:1702.08373.

[40] T. Matsui, Y. Matsui and Y. Ono. Random generation of $2 \times 2 \times \cdots \times 2 \times J$ contingency tables. *Theoret. Comput. Sci.* **326** (2004) 117–135. MR2094245 https://doi.org/10.1016/j.tcs.2004.06.014

[41] B. D. McKay and N. C. Wormald. Asymptotic enumeration by degree sequence of graphs of high degree. *European J. Combin.* **11** (1990) 565–580. MR1078713 https://doi.org/10.1016/S0195-6698(13)80042-X

[42] B. D. McKay and N. C. Wormald. Asymptotic enumeration by degree sequence of graphs with degrees $o(n^{1/2})$. *Combinatorica* **11** (1991) 369–382. MR1137769 https://doi.org/10.1007/BF01275671

[43] B. D. McKay and N. C. Wormald. The degree sequence of a random graph. I. The models. *Random Structures Algorithms* **11** (1997) 97–117. MR1610253 https://doi.org/10.1002/(SICI)1098-2418(199709)11:2<97::AID-RSA1>3.3.CO;2-E

[44] B. Morris. Improved bounds for sampling contingency tables. In *Randomization, Approximation, and Combinatorial Optimization* 121–129. *Berkeley, CA*, *1999*. *Lecture Notes in Comput. Sci.* **1671**. Springer, Berlin, 1999. MR1775514 https://doi.org/10.1007/978-3-540-48413-4_12

[45] G. W. Soules. New permanental upper bounds for nonnegative matrices. *Linear Multilinear Algebra* **51** (2003) 319–337. MR2010711 https://doi.org/10.1080/0308108031000098450

[46] D. B. Wilson. Mixing times of Lozenge tiling and card shuffling Markov chains. *Ann. Appl. Probab.* **14** (2004) 274–325. MR2023023 https://doi.org/10.1214/aoap/1075828054

[47] N. C. Wormald. *Models of Random Regular Graphs*. *London Mathematical Society Lecture Note Series*, 1999. MR1725006

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1002–1040
https://doi.org/10.1214/19-AIHP992

Weak uniqueness and density estimates for SDEs with coefficients depending on some path-functionals

Noufel Frikha[a] and Libo Li[b]

[a]*Sorbonne Université, Sorbonne Paris Cité, CNRS, Laboratoire de Probabilité Statistique et Modélisation, LPSM, F-75005, Paris, France. E-mail: frikha@math.univ-paris-diderot.fr*

[b]*Libo Li, School of Mathematics and Statistics, University of New South Wales, Sydney, Australia. E-mail: libo.li@unsw.edu.au*

Abstract. In this article, we develop a methodology to prove weak uniqueness for stochastic differential equations with coefficients depending on some path-functionals of the process. As an extension of the technique developed by Bass and Perkins (In *From Probability to Geometry (I): Volume in Honor of the 60th Birthday of Jean-Michel Bismut* (2009) 47–53) in the standard diffusion case, the proposed methodology allows one to deal with process whose probability laws is singular with respect to the Lebesgue measure. To illustrate our methodology, we prove weak existence and uniqueness in the two following examples: a diffusion process with coefficients depending on its running local time and a diffusion process with coefficients depending on its running maximum. In each example, we also prove the existence of the associated transition density and establish some Gaussian upper-estimates.

Résumé. Dans cet article, nous développons une méthodologie permettant de prouver l'unicité faible pour des équations différentielles stochastiques dont les coefficients dépendent de certaines fonctionnelles de la trajectoire du processus. Dans le prolongement de la technique développée par Bass & Perkins (In *From Probability to Geometry (I): Volume in Honor of the 60th Birthday of Jean-Michel Bismut* (2009) 47–53) dans le cas des processus de diffusions standards, la méthologie proposée permet de traiter le cas de processus dont la loi est singulière par rapport à la mesure de Lebesgue. Afin d'illustrer notre méthodologie, nous prouvons l'existence et l'unicité faible dans les deux exemples suivants: un processus de diffusion dont les coefficients dépendent du temps local en zéro courant et un processus de diffusion dont les coefficients dépendent du maximum courant. Dans chaque exemple, nous prouvons également l'existence d'une densité de transition associée et établissons des estimées Gaussiennes.

MSC: Primary 60H10; 60G46; secondary 60H30; 35K65

Keywords: Weak uniqueness; Martingale problem; Parametrix expansion; Local time; Maximum; Density estimates

References

[1] T. Appuhamillage, V. Bokil, E. Thomann, E. Waymire and B. Wood. Occupation and local times for skew Brownian motion with applications to dispersion across an interface. *Ann. Appl. Probab.* **21** (1) (2011) 183–214. MR2759199 https://doi.org/10.1214/10-AAP691
[2] E. Barucci, S. Polidoro and V. Vespri. Some results on partial differential equations and Asian options. *Math. Models Methods Appl. Sci.* **11** (3) (2001) 475–497. MR1830951 https://doi.org/10.1142/S0218202501000945
[3] R. F. Bass. *Diffusions and Elliptic Operators*. Springer, Berlin, 1997. MR1483890
[4] R. F. Bass and E. A. Perkins. A new technique for proving uniqueness for martingale problems. In *From Probability to Geometry (I): Volume in Honor of the 60th Birthday of Jean-Michel Bismut* 47–53, 2009.
[5] F. Delarue and S. Menozzi. Density estimates for a random noise propagating through a chain of differential equations. *J. Funct. Anal.* **259** (6) (2010) 1577–1630. MR2659772 https://doi.org/10.1016/j.jfa.2010.05.002
[6] K. D. Elworthy, A. Truman and H. Zhao. Generalized Itô formulae and space-time Lebesgue–Stieltjes integrals of local times. In *Séminaire de Probabilités XL* 117–136. *Lecture Notes in Math.* **1899**. Springer, Berlin, 2007. MR2409002 https://doi.org/10.1007/978-3-540-71189-6_5
[7] M. Forde. A diffusion-type process with a given joint law for the terminal level and supremum at an independent exponential time. *Stochastic Process. Appl.* **121** (12) (2011) 2802–2817. MR2844541 https://doi.org/10.1016/j.spa.2011.07.009
[8] A. Friedman. *Partial Differential Equations of Parabolic Type*. Prentice-Hall, New York, 1964. MR0181836
[9] N. Frikha, A. Kohatsu-Higa and L. Li. Integration by parts formula for killed processes: A point of view of approximation theory. Preprint. 2018. *Electron. J. Probab.* To appear.
[10] M. G. Garroni and J.-L. Menaldi. *Green Functions for Second Order Parabolic Integro-Differential Problems*. *Pitman Research Notes in Mathematics Series* **275**. Longman Scientific & Technical, Harlow, 1992. (Copublished in the United States with, John Wiley & Sons, Inc., New York.) MR1202037

[11] I. Karatzas and S. E. Shreve. Trivariate density of Brownian motion, its local and occupation times, with application to stochastic control. *Ann. Probab.* **12** (3) (1984) 819–828. MR0744236

[12] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*, 2nd edition. Springer Verlag, Berlin, 1991. MR1121940 https://doi.org/10.1007/978-1-4612-0949-2

[13] H. Kestelman. *Modern Theories of Integration*, 2nd revised edition. Dover Publications, Inc., New York, 1960. MR0122951

[14] S. Menozzi. Parametrix techniques and martingale problems for some degenerate Kolmogorov equations. *Electron. Commun. Probab.* **16** (2011) 234–250. MR2802040 https://doi.org/10.1214/ECP.v16-1619

[15] G. Peskir. A change-of-variable formula with local time on curves. *J. Theoret. Probab.* **18** (13) (2005) 499–535. MR2167640 https://doi.org/10.1007/s10959-005-3517-6

[16] C. Soize. *The Fokker–Planck Equation for Stochastic Dynamical Systems and Its Explicit Steady State Solutions*. *Series on Advances in Mathematics for Applied Sciences.* **17**. World Scientific Publishing Co., Inc., River Edge, NJ, 1994. MR1287386 https://doi.org/10.1142/9789814354110

[17] D. W. Stroock and S. R. S. Varadhan. *Multidimensional Diffusion Processes*. Springer-Verlag, Berlin Heidelberg New-York, 1979. MR0532498

[18] D. Talay. Stochastic Hamiltonian systems: Exponential convergence to the invariant measure, and discretization by the implicit Euler scheme. *Markov Process. Related Fields* **8** (2) (2002) 163–198. MR1924934

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1041–1071
https://doi.org/10.1214/19-AIHP993

On a toy network of neurons interacting through their dendrites

Nicolas Fournier[a], Etienne Tanré[b] and Romain Veltz[b]

[a]*LPMA, UMR 7599, UPMC, Case 188, 4 place Jussieu, 75252 Paris Cedex 5, France. E-mail: nicolas.fournier@upmc.fr*
[b]*Université Côte d'Azur, Inria, France. E-mail: Etienne.Tanre@inria.fr; Romain.Velz@inria.fr*

Abstract. Consider a large number n of neurons, each being connected to approximately N other ones, chosen at random. When a neuron spikes, which occurs randomly at some rate depending on its electric potential, its potential is set to a minimum value $v_{\min}$, and this initiates, after a small delay, two fronts on the (linear) dendrites of all the neurons to which it is connected. Fronts move at constant speed. When two fronts (on the dendrite of the same neuron) collide, they annihilate. When a front hits the soma of a neuron, its potential is increased by a small value w_n. Between jumps, the potentials of the neurons are assumed to drift in $[v_{\min}, \infty)$, according to some well-posed ODE. We prove the existence and uniqueness of a heuristically derived mean-field limit of the system when $n, N \to \infty$ with $w_n \simeq N^{-1/2}$. We make use of some recent versions of the results of Deuschel and Zeitouni (*Ann. Probab.* **23** (1995) 852–878) concerning the size of the longest increasing subsequence of an i.i.d. collection of points in the plan. We also study, in a very particular case, a slightly different model where the neurons spike when their potential reach some maximum value $v_{\max}$, and find an explicit formula for the (heuristic) mean-field limit.

Résumé. Considérons un grand nombre n de neurones, chacun étant connecté à environ N autres, choisis au hasard. Quand un neurone décharge, ce qui se produit au hasard à un certain taux en fonction de son potentiel électrique, son potentiel est remis à une valeur minimale $v_{\min}$, ce qui déclenche, après un petit délai, deux fronts sur les dendrites (linéaires) de tous les neurones auxquels il est connecté. Les fronts se déplacent à vitesse constante. Lorsque deux fronts (sur la dendrite du même neurone) entrent en collision, ils s'annihilent. Lorsqu'un front touche le soma d'un neurone, son potentiel est augmenté d'une petite valeur w_n. Entre les sauts, les potentiels des neurones évoluent dans $[v_{\min}, \infty)$, suivant une EDO. Nous prouvons l'existence et l'unicité d'une limite champ moyen du système lorsque $n, N \to \infty$ avec $w_n \simeq N^{-1/2}$ obtenue de manière heuristique. Nous utilisons certaines versions récentes des résultats de Deuschel et Zeitouni (*Ann. Probab.* **23** (1995) 852–878) concernant la taille de la sous-suite croissante la plus longue d'une suite i.i.d. de points du plan. Nous étudions également, dans un cas très particulier, un modèle légèrement différent où les neurones déchargent quand leur potentiel atteint une valeur maximale $v_{\max}$. Nous trouvons heuristiquement une expression explicite pour la limite de champ moyen.

MSC: 60K35; 60J75; 92C20

Keywords: Mean-field limit; Propagation of chaos; Nonlinear stochastic differential equations; Ulam's problem; Longest increasing subsequence; Biological neural networks

References

[1] D. Aldous and P. Diaconis. Hammersley's interacting particle process and longest increasing subsequences. *Probab. Theory Related Fields* **103** (1995) 199–213. MR1355056 https://doi.org/10.1007/BF01204214
[2] A. L. Basdevant, L. Gerin, J. B. Gouéré and A. Singh. From Hammersley's lines to Hammersley's trees. *Probab. Theory Related Fields* **171** (2018) 1–51. MR3800829 https://doi.org/10.1007/s00440-017-0772-2
[3] B. Bollobás and P. Winkler. The longest chain among random points in Euclidean space. *Proc. Amer. Math. Soc.* **103** (1988) 347–353. MR0943043 https://doi.org/10.2307/2047138
[4] M. Bossy, O. Faugeras and D. Talay. Clarification and complement to "Mean-field description and propagation of chaos in networks of Hodgkin–Huxley and FitzHugh–Nagumo neurons". *J. Math. Neurosci.* **5** (2015) 23. MR3392551 https://doi.org/10.1186/s13408-015-0031-8
[5] M. Cáceres, J. Carrillo and B. Perthame. Analysis of nonlinear noisy integrate & fire neuron models: Blow-up and steady states. *J. Math. Neurosci.* **1** (2011) 33. MR2853216 https://doi.org/10.1186/2190-8567-1-7
[6] J. Calder, S. Esedoḡlu and A. O. Hero. A Hamilton–Jacobi equation for the continuum limit of nondominated sorting. *SIAM J. Math. Anal.* **46** (2014) 603–638. MR3163240 https://doi.org/10.1137/13092842X
[7] J. Carrillo, B. Perthame, D. Salort and D. Smets. Qualitative properties of solutions for the noisy integrate and fire model in computational neuroscience. *Nonlinearity* **28** (2015) 3365–3388. MR3403402 https://doi.org/10.1088/0951-7715/28/9/3365

[8] E. Cator and P. Groeneboom. Hammersley's process with sources and sinks. *Ann. Probab.* **33** (2005) 879–903. MR2135307 https://doi.org/10.1214/009117905000000053

[9] J. Chevallier. Mean-field limit of generalized Hawkes processes. *Stochastic Process. Appl.* **127** (2017) 3870–3912. MR3718099 https://doi.org/10.1016/j.spa.2017.02.012

[10] J. Chevallier, M. Cáceres, M. Doumic and P. Reynaud-Bouret. Microscopic approach of a time elapsed neural model. *Math. Models Methods Appl. Sci.* **25** (2015) 2669–2719. MR3411353 https://doi.org/10.1142/S021820251550058X

[11] J. Chevallier, A. Duarte, E. Löcherbach and G. Ost. Mean field limits for nonlinear spatially extended Hawkes processes with exponential memory kernels. *Stochastic Process. Appl.* **129** (2019) 1–27. MR3906989 https://doi.org/10.1016/j.spa.2018.02.007

[12] A. De Masi, A. Galves, E. Löcherbach and E. Presutti. Hydrodynamic limit for interacting neurons. *J. Stat. Phys.* **158** (2015) 866–902. MR3311484 https://doi.org/10.1007/s10955-014-1145-1

[13] F. Delarue, J. Inglis, S. Rubenthaler and E. Tanré. Global solvability of a networked integrate-and-fire model of McKean–Vlasov type. *Ann. Appl. Probab.* **25** (2015) 2096–2133. MR3349003 https://doi.org/10.1214/14-AAP1044

[14] F. Delarue, J. Inglis, S. Rubenthaler and E. Tanré. Particle systems with a singular mean-field self-excitation. Application to neuronal networks. *Stochastic Process. Appl.* **125** (2015) 2451–2492. MR3322871 https://doi.org/10.1016/j.spa.2015.01.007

[15] J. D. Deuschel and O. Zeitouni. Limiting curves for i.i.d. records. *Ann. Probab.* **23** (1995) 852–878. MR1334175

[16] S. Ditlevsen and E. Löcherbach. Multi-class oscillating systems of interacting neurons. *Stochastic Process. Appl.* **127** (2017) 1840–1869. MR3646433 https://doi.org/10.1016/j.spa.2016.09.013

[17] N. Fournier and E. Löcherbach. On a toy model of interacting neurons. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 1844–1876. MR3573298 https://doi.org/10.1214/15-AIHP701

[18] T. Górski, R. Veltz, M. Galtier, H. Fragnaud, B. Telenczuk and A. Destexhe. Dendritic sodium spikes endow neurons with inverse firing rate response to correlated synaptic activity. *J. Comput. Neurosci.* **45** (2018) 223–234.

[19] J. M. Hammersley. A few seedlings of research. In *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* 345–394. *Univ. California, Berkeley, Calif., 1970/1971. Vol. I: Theory of Statistics*. Univ. California Press, Berkeley, CA, 1972. MR0405665

[20] A. L. Hodgkin and A. F. Huxley. A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol.* **117** (1952) 500–544.

[21] J. Inglis and D. Talay. Mean-field limit of a stochastic particle system smoothly interacting through threshold hitting-times and applications to neural networks with dendritic component. *SIAM J. Math. Anal.* **47** (2015) 3884–3916. MR3411723 https://doi.org/10.1137/140989042

[22] M. Kac. Foundations of kinetic theory. In *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, 1954–1955, Vol. III (Berkeley and Los Angeles, 1956)* 171–197. University of California Press. MR0084985

[23] E. R. Kandel. *Principles of Neural Science*, 5th edition. McGraw-Hill, New York, 2013.

[24] C. Koch. *Biophysics of Computation: Information Processing in Single Neurons Oxford Univ. Press Paperback. Computational Neuroscience*. Oxford Univ. Press, New York, 2004.

[25] B. F. Logan and L. A. Shepp. A variational problem for random Young tableaux. *Adv. Math.* **26** (1977) 206–222. MR1417317 https://doi.org/10.1016/0001-8708(77)90030-5

[26] E. Luçon. Quenched limits and fluctuations of the empirical measure for plane rotators in random media. *Electron. J. Probab.* **16** (2011) 792–829. MR2793244 https://doi.org/10.1214/EJP.v16-874

[27] E. Luçon and W. Stannat. Mean field limit for disordered diffusions with singular interactions. *Ann. Appl. Probab.* **24** (2014) 1946–1993. MR3226169 https://doi.org/10.1214/13-AAP968

[28] E. Luçon and W. Stannat. Transition from Gaussian to non-Gaussian fluctuations for mean-field diffusions in spatial interaction. *Ann. Appl. Probab.* **26** (2016) 3840–3909. MR3582819 https://doi.org/10.1214/16-AAP1194

[29] H. P. McKean. Propagation of chaos for a class of non-linear parabolic equations. In *Lecture Series in Differential equations 7* 41–57. Catholic University, Washington, 1967. MR0233437

[30] S. Méléard. Asymptotic behaviour of some interacting particle systems; McKean–Vlasov and Boltzmann models. In *Probabilistic Models for Nonlinear Partial Differential Equations* 42–95. *Lecture Notes in Math.* **1627**. Fond. CIME, Springer, Berlin, 1996. MR1431299 https://doi.org/10.1007/BFb0093177

[31] S. Ostojic, N. Brunel and V. Hakim. Synchronization properties of networks of electrically coupled neurons in the presence of noise and heterogeneities. *J. Comput. Neurosci.* **26** (2009) 369–392. MR2505036 https://doi.org/10.1007/s10827-008-0117-3

[32] K. Pakdaman, M. Thieullen and G. Wainrib. Fluid limit theorems for stochastic hybrid systems with application to neuron models. *Adv. in Appl. Probab.* **42** (2010) 761–794. MR2779558 https://doi.org/10.1239/aap/1282924062

[33] A. Renart, N. Brunel and X.-J. Wang. Mean-field theory of irregularly spiking neuronal populations and working memory in recurrent cortical networks. In *Computational Neuroscience: A Comprehensive Approach* 431–490. *Chapman & Hall/CRC Mathematical Biology and Medicine Series*, 2004. MR2029670

[34] M. Riedler, M. Thieullen and G. Wainrib. Limit theorems for infinite-dimensional piecewise deterministic Markov processes. Applications to stochastic excitable membrane models. *Electron. J. Probab.* **17** (2012) 48. MR2955047 https://doi.org/10.1214/EJP.v17-1946

[35] G. Stuart, N. Spruston and M. Häusser. *Dendrites*, 3rd edition. Oxford University Pres, New York, 2015.

[36] A.-S. Sznitman. Topics in propagation of chaos. In *École d'Été de Probabilités de Saint-Flour XIX—1989* 165–251. *Lecture Notes in Math.* **1464**. Springer, Berlin, 1991. MR1108185 https://doi.org/10.1007/BFb0085169

[37] S. M. Ulam. Monte Carlo calculations in problems of mathematical physics. In *Modern mathematics for the engineer: Second series* 261–281. McGraw-Hill, New York, 1961.

[38] A. M. Versik and S. V. Kerov. Asymptotic behavior of the Plancherel measure of the symmetric group and the limit form of Young tableaux. *Dokl. Akad. Nauk SSSR* **233** (1977) 1024–1027. MR0480398

[39] I. Yakupov and M. Buzdalov. Incremental non-dominated sorting with O(N) insertion for the two-dimensional case. In *IEEE Congress on Evolutionary Computation (CEC)* 1853–1860, 2015.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1072–1098
https://doi.org/10.1214/19-AIHP994

Asymptotics of free fermions in a quadratic well at finite temperature and the Moshe–Neuberger–Shapiro random matrix model

Karl Liechty[a,1] and Dong Wang[b,2]

[a]*Department of Mathematical Sciences, DePaul University, Chicago, IL, 60614 USA. E-mail: kliechty@depaul.edu*
[b]*Department of Mathematics, National University of Singapore, Singapore, 119076. E-mail: matwd@nus.edu.sg*

Abstract. We derive the local statistics of the canonical ensemble of free fermions in a quadratic potential well at finite temperature, as the particle number approaches infinity. This free fermion model is equivalent to a random matrix model proposed by Moshe, Neuberger and Shapiro. Limiting behaviors obtained before for the grand canonical ensemble are observed in the canonical ensemble: We have at the edge the phase transition from the Tracy–Widom distribution to the Gumbel distribution via the Kardar–Parisi–Zhang (KPZ) crossover distribution, and in the bulk the phase transition from the sine point process to the Poisson point process.

Résumé. Nous décrivons les statistiques locales de l'ensemble canonique de fermions libres dans un puits de potentiel quadratique à température finie, dans la limite où le nombre de particules tend vers l'infini. Ce modèle de fermions libres est équivalent à un modèle matriciel aléatoire proposé par Moshe, Neuberger et Shapiro. Les comportements à la limite précédemment obtenus pour l'ensemble grand-canonique sont observés dans l'ensemble canonique: Nous avons, au bord de l'ensemble, une transition de phase de la distribution de Tracy–Widom à la distribution de Gumbel via la distribution croisée de Kardar–Parisi–Zhang (KPZ), et dans l'ensemble, une transition de phase du processus ponctuel sinus au processus ponctuel de Poisson.

MSC: Primary 15B52; secondary 82B23

Keywords: Determinantal point process; Poisson process; Random matrices; Free fermions; Tracy–Widom distribution; KPZ universality

References

[1] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. *National Bureau of Standards Applied Mathematics Series* **55**. For sale by the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C., 1964. MR0167642

[2] G. Amir, I. Corwin and J. Quastel. Probability distribution of the free energy of the continuum directed random polymer in $1+1$ dimensions. *Comm. Pure Appl. Math.* **64** (4) (2011) 466–537. MR2796514 https://doi.org/10.1002/cpa.20347

[3] G. W. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, 2010. MR2760897

[4] C. Andréief. Note sur une relation entre les intégrales définines des produits des fonctions. *Mém. Soc. Sci. Phys. Nat. Bordeaux (3)* **2** (1886) 1–14.

[5] G. E. Andrews, R. Askey and R. Roy. *Special Functions*. *Encyclopedia of Mathematics and Its Applications.* **71**. Cambridge University Press, Cambridge, 1999. MR1688958 https://doi.org/10.1017/CBO9781107325937

[6] G. Barraquand. A phase transition for q-TASEP with a few slower particles. *Stochastic Process. Appl.* **125** (7) (2015) 2674–2699. MR3332851 https://doi.org/10.1016/j.spa.2015.01.009

[7] A. Borodin and I. Corwin. Macdonald processes. *Probab. Theory Related Fields* **158** (1–2) (2014) 225–400. MR3152785 https://doi.org/10.1007/s00440-013-0482-3

[8] A. Borodin, I. Corwin and T. Sasamoto. From duality to determinants for q-TASEP and ASEP. *Ann. Probab.* **42** (6) (2014) 2314–2382. MR3265169 https://doi.org/10.1214/13-AOP868

[9] I. Corwin. The Kardar–Parisi–Zhang equation and universality class. *Random Matrices Theory Appl.* **1** (1) (2012) 1130001. MR2930377 https://doi.org/10.1142/S2010326311300014

[10] D. S. Dean, P. Le Doussal, S. N. Majumdar and G. Schehr. Finite-temperature free fermions and the Kardar–Parisi–Zhang equation at finite time. *Phys. Rev. Lett.* **114** (2015) 110402.

[11] D. S. Dean, P. Le Doussal, S. N. Majumdar and G. Schehr. Noninteracting fermions at finite temperature in a d-dimensional trap: Universal correlations. *Phys. Rev. A* **94** (2016) 063622.

[12] F. Deelan Cunden, F. Mezzadri and N. O'Connell. Free fermions and the classical compact groups. *J. Stat. Phys.* **171** (5) (2018) 768–801. MR3800894 https://doi.org/10.1007/s10955-018-2029-6
[13] P. L. Ferrari and B. Vető. Tracy-Widom asymptotics for q-TASEP. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (4) (2015) 1465–1485. MR3414454 https://doi.org/10.1214/14-AIHP614
[14] T. Imamura and T. Sasamoto. Determinantal structures in the O'Connell–Yor directed random polymer model. *J. Stat. Phys.* **163** (4) (2016) 675–713. MR3488569 https://doi.org/10.1007/s10955-016-1492-1
[15] T. Imamura and T. Sasamoto. Fluctuations for stationary q-TASEP. *Probab. Theory Related Fields* **174** (1–2) (2019) 647–730. MR3947332 https://doi.org/10.1007/s00440-018-0868-3
[16] K. Johansson. From Gumbel to Tracy-Widom. *Probab. Theory Related Fields* **138** (1–2) (2007) 75–112. MR2288065 https://doi.org/10.1007/s00440-006-0012-7
[17] K. Johansson and G. Lambert. Gaussian and non-Gaussian fluctuations for mesoscopic linear statistics in determinantal processes. *Ann. Probab.* **46** (3) (2018) 1201–1278. MR3785588 https://doi.org/10.1214/17-AOP1178
[18] M. Korhonen and E. Lee. The transition probability and the probability for the left-most particle's position of the q-totally asymmetric zero range process. *J. Math. Phys.* **55** (1) (2014) 013301. MR3390432 https://doi.org/10.1063/1.4851758
[19] P. Le Doussal, S. N. Majumdar, A. Rosso and G. Schehr. Exact short-time height distribution in the one-dimensional Kardar–Parisi–Zhang equation and edge fermions at high temperature. *Phys. Rev. Lett.* **117** (2016) 070403.
[20] P. Le Doussal, S. N. Majumdar and G. Schehr. Periodic airy process and equilibrium dynamics of edge fermions in a trap. *Ann. Physics* **383** (2017) 312–345. MR3682029 https://doi.org/10.1016/j.aop.2017.05.018
[21] E. Lee and D. Wang. Distributions of a particle's position and their asymptotics in the q-deformed totally asymmetric zero range process with site dependent jumping rates. *Stochastic Process. Appl.* **129** (5) (2019) 1795–1828. MR3944785 https://doi.org/10.1016/j.spa.2018.06.005
[22] K. Liechty and D. Wang. Nonintersecting Brownian motions on the unit circle. *Ann. Probab.* **44** (2) (2016) 1134–1211. MR3474469 https://doi.org/10.1214/14-AOP998
[23] K. Liechty and D. Wang. Asymptotics of free fermions in a quadratic well at finite temperature and the Moshe–Neuberger–Shapiro random matrix model, 2017. Available at arXiv:1706.06653v3.
[24] M. Moshe, H. Neuberger and B. Shapiro. Generalized ensemble of random matrices. *Phys. Rev. Lett.* **73** (11) (1994) 1497–1500. MR1291352 https://doi.org/10.1103/PhysRevLett.73.1497
[25] F. W. J. Olver. *Asymptotics and Special Functions. AKP Classics*. A K Peters, Ltd., Wellesley, MA, 1997. Reprint of the 1974 original, Academic Press, New York, MR0435697 (55 #8655). MR1429619
[26] F. W. J. Olver, D. W. Lozier, R. F. Boisvert and C. W. Clark (Eds). *NIST Handbook of Mathematical Functions*. U.S. Department of Commerce, National Institute of Standards and Technology, Washington, DC; Cambridge University Press, Cambridge, 2010. With 1 CD-ROM (Windows, Macintosh and UNIX). MR2723248
[27] R. K. Pathria and P. D. Beale. *Statistical Mechanics*, 3rd edition. Elsevier/Academic Press, Amsterdam, 2011.
[28] J. Quastel. Introduction to KPZ. In *Current Developments in Mathematics, 2011* 125–194. Int. Press, Somerville, MA, 2012. MR3098078
[29] T. Sasamoto and M. Wadati. Exact results for one-dimensional totally asymmetric diffusion models. *J. Phys. A* **31** (28) (1998) 6057–6071. MR1633078 https://doi.org/10.1088/0305-4470/31/28/019
[30] B. Simon. *Trace Ideals and Their Applications*, 2nd edition. *Mathematical Surveys and Monographs* **120**. American Mathematical Society, Providence, RI, 2005. MR2154153
[31] G. Szegő. *Orthogonal Polynomials*, 4th edition. *American Mathematical Society, Colloquium Publications* **XXIII**. American Mathematical Society, Providence, RI, 1975. MR0372517
[32] C. A. Tracy and H. Widom. A Fredholm determinant representation in ASEP. *J. Stat. Phys.* **132** (2) (2008) 291–300. MR2415104 https://doi.org/10.1007/s10955-008-9562-7
[33] C. A. Tracy and H. Widom. On ASEP with step Bernoulli initial condition. *J. Stat. Phys.* **137** (5–6) (2009) 825–838. MR2570751 https://doi.org/10.1007/s10955-009-9867-1
[34] D. Wang and D. Waugh. The transition probability of the q-TAZRP (q-bosons) with inhomogeneous jump rates. *SIGMA Symmetry Integrability Geom. Methods Appl.* **12** (2016) 037. MR3485981 https://doi.org/10.3842/SIGMA.2016.037
[35] D. Wood. The computation of polylogarithms. Technical Report 15-92*, University of Kent, Computing Laboratory, University of Kent, Canterbury, UK, 1992.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1099–1128
https://doi.org/10.1214/19-AIHP995

Non-equilibrium fluctuations for the SSEP with a slow bond

D. Erhard[a], T. Franco[a], P. Gonçalves[b], A. Neumann[c] and M. Tavares[a]

[a]*UFBA Instituto de Matemática, Campus de Ondina, Av. Adhemar de Barros, S/N. CEP 40170-110 Salvador, Brazil.*
E-mail: erharddirk@gmail.com; tertu@ufba.br; tavaresaguiar57@gmail.com
[b]*Center for Mathematical Analysis, Geometry and Dynamical Systems, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal. E-mail: patricia.goncalves@math.tecnico.ulisboa.pt*
[c]*UFRGS, Instituto de Matemática e Estatística, Campus do Vale, Av. Bento Gonçalves, 9500. CEP 91509-900, Porto Alegre, Brasil.*
E-mail: aneumann@mat.ufrgs.br

Abstract. We prove the non-equilibrium fluctuations for the one-dimensional symmetric simple exclusion process with a slow bond. This generalizes a result of [*Stochastic Process. Appl.* **123** (2013) 4156–4185; *Stochastic Process. Appl.* **126** (2016) 3235–3242], which dealt with the equilibrium fluctuations. The foundation stone of our proof is a precise estimate on the correlations of the system, and that is by itself one of the main novelties of this paper. To obtain these estimates, we first deduce a spatially discrete PDE for the covariance function and we relate it to the local times of a random walk in a non-homogeneous environment via Duhamel's principle. Projection techniques and coupling arguments reduce the analysis to the problem of studying the local times of the classical random walk. We think that the method developed here can be applied to a variety of models, and we provide a discussion on this matter.

Résumé. Nous décrivons les fluctuations hors-équilibre du processus d'exclusion simple symétrique en dimension 1 avec des liens lents. Ceci étend un résultat de [*Stochastic Process. Appl.* **123** (2013) 4156–4185; *Stochastic Process. Appl.* **126** (2016) 3235–3242], qui traitait des fluctuations à l'équilibre. La pierre de touche de notre preuve est une estimée précise des corrélations du système, qui est en elle-même une des nouveautés principales de cet article. Pour obtenir ces estimées, nous obtenons dans un premier temps une EDP discrète en espace pour la fonction de covariance et nous la relions aux temps locaux d'une marche aléatoire dans un environnement non-homogène, par le principe de Duhamel. Des techniques de projection et des arguments de couplage permettent de réduire l'analyse à l'étude des temps locaux de la marche aléatoire classique. Nous pensons que cette méthode peut être appliquée à une variété de modèles, et nous argumentons ce point.

MSC: 60K35

Keywords: Non-equilibrium fluctuations; Slowed exclusion; Local times of random walks; Two point correlation function

References

[1] P. Billingsley. *Convergence of Probability Measures*, 2nd edition. Wiley, New York, 1999. MR1700749 https://doi.org/10.1002/9780470316962
[2] A. De Masi, N. Ianiro, A. Pellegrinotti and E. Presutti. A survey of the hydrodynamical behavior of many-particle systems. In *Nonequilibrium Phenomena, II* 123–294. *Stud. Statist. Mech., XI*. North-Holland, Amsterdam, 1984. MR0757003
[3] T. Franco, P. Gonçalves and A. Neumann. Hydrodynamical behavior of symmetric exclusion with slow bonds. *Ann. Inst. Henri Poincaré Probab. Stat.* **49** (2) (2013) 402–427. MR3088375 https://doi.org/10.1214/11-AIHP445
[4] T. Franco, P. Gonçalves and A. Neumann. Phase transition in equilibrium fluctuations of symmetric slowed exclusion. *Stochastic Process. Appl.* **123** (12) (2013) 4156–4185. MR3096351 https://doi.org/10.1016/j.spa.2013.06.016
[5] T. Franco, P. Gonçalves and A. Neumann. Phase transition of a heat equation with Robin's boundary conditions and exclusion process. *Trans. Amer. Math. Soc.* **367** (2015) 6131–6158. MR3356932 https://doi.org/10.1090/S0002-9947-2014-06260-0
[6] T. Franco, P. Gonçalves and A. Neumann. Corrigendum to: Phase transition in equilibrium fluctuations of symmetric slowed exclusion. *Stochastic Process. Appl.* **126** (10) (2016) 3235–3242. MR3542633 https://doi.org/10.1016/j.spa.2016.06.003
[7] T. Franco, P. Gonçalves and A. Neumann. Non-equilibrium and stationary fluctuations of a slowed boundary symmetric exclusion. *Stochastic Process. Appl.* **129** (2019) 1413–1442. MR3926561 https://doi.org/10.1016/j.spa.2018.05.005
[8] T. Franco and C. Landim. Hydrodynamic limit of gradient exclusion processes with conductances. *Arch. Ration. Mech. Anal.* **195** (2) (2010) 409–439. MR2592282 https://doi.org/10.1007/s00205-008-0206-5
[9] R. A. Holley and D. W. Stroock. Generalized Ornstein–Uhlenbeck processes and infinite particle branching Brownian motions. *Publ. Res. Inst. Math. Sci.* **14** (3) (1978) 741–788. MR0527199 https://doi.org/10.2977/prims/1195188837

[10] J. Jacod and A. N. Shiryaev. Limit theorems for stochastic processes. *Grundlehren Math. Wiss.* **147** (288) (2003) 519–528. MR1943877 https://doi.org/10.1007/978-3-662-05265-5

[11] M. D. Jara and C. Landim. Nonequilibrium central limit theorem for a tagged particle in symmetric simple exclusion. *Ann. Inst. Henri Poincaré Probab. Stat.* **42** (5) (2006) 567–577. MR2259975 https://doi.org/10.1016/j.anihpb.2005.04.007

[12] M. D. Jara and C. Landim. Quenched non-equilibrium central limit theorem for a tagged particle in the exclusion process with bond disorder. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** (2) (2008) 341–361. MR2446327 https://doi.org/10.1214/07-AIHP112

[13] C. Kipnis and C. Landim. *Scaling Limits of Interacting Particle Systems*, 1st edition. *Grundlehren der Mathematischen Wissenschaften.* **320**. Springer-Verlag, Berlin Heidelberg, 1999. MR1707314 https://doi.org/10.1007/978-3-662-03752-2

[14] G. Lawler and V. Limic. *Random Walk: A Modern Introduction*. *Cambridge Studies in Advanced Mathematics* **123**. Cambridge University Press, Cambridge, 2010. MR2677157 https://doi.org/10.1017/CBO9780511750854

[15] D. A. Levin and Y. Peres. *Markov Chains and Mixing Times*. American Mathematical Society, Providence, RI, 2017. MR3726904

[16] I. Mitoma. Tightness of probabilities on $C([0,1];\mathcal{Y}')$ and $D([0,1];\mathcal{Y}')$. *Ann. Probab.* **11** (4) (1983) 989–999. MR0714961

[17] K. Ravishankar. Fluctuations from the hydrodynamical limit for the symmetric simple exclusion in $\mathbb{Z}^d$. *Stochastic Process. Appl.* **42** (1) (1992) 31–37. MR1172505 https://doi.org/10.1016/0304-4149(92)90024-K

[18] M. Reed and B. Simon. *Methods of Modern Mathematical Physics I: Functional Analysis*, 1st edition. Academic Press, San Diego, 1981. MR0493419

[19] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **293**. Springer-Verlag, Berlin, 1999. MR1725357 https://doi.org/10.1007/978-3-662-06400-9

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1129–1161
https://doi.org/10.1214/19-AIHP996

Infinite geodesics in hyperbolic random triangulations

Thomas Budzinski[a,b]

[a] *ENS Paris, Paris, France. E-mail: thomas.budzinski@ens.fr*
[b] *Université Paris-Saclay, Paris, France*

Abstract. We study the structure of infinite geodesics in the Planar Stochastic Hyperbolic Triangulations $\mathbb{T}_\lambda$ introduced in (*Probab. Theory Related Fields* **165** (2016) 509–540). We prove that these geodesics form a supercritical Galton–Watson tree with geometric offspring distribution. The tree of infinite geodesics in $\mathbb{T}_\lambda$ provides a new notion of boundary, which is a realization of the Poisson boundary. By scaling limit arguments, we also obtain a description of the tree of infinite geodesics in the hyperbolic Brownian plane. Finally, by combining our main result with a forthcoming paper (Budzinski (2018)), we obtain new hyperbolicity properties of $\mathbb{T}_\lambda$: they satisfy a weaker form of Gromov-hyperbolicity and admit bi-infinite geodesics.

Résumé. Nous étudions la structure des géodésiques infinies dans les Triangulations Planaires Stochastiques Hyperboliques (PSHT) $\mathbb{T}_\lambda$ introduites dans (*Probab. Theory Related Fields* **165** (2016) 509–540). Nous montrons que ces géodésiques forment un arbre de Galton–Watson surcritique de loi de reproduction géométrique. L'arbre des géodésiques infinies de $\mathbb{T}_\lambda$ fournit une nouvelle notion de bord, qui est une réalisation de la frontière de Poisson. Par des arguments de limites d'échelle, on en déduit une description de l'arbre des géodésiques infinies du plan brownien hyperbolique. Enfin, en combinant notre résultat principal avec ceux de (Budzinski (2018)), nous obtenons de nouvelles propriétés d'hyperbolicité de $\mathbb{T}_\lambda$: ces triangulations vérifient une forme faible d'hyperbolicité à la Gromov, et admettent des géodésiques bi-infinies.

MSC: 60C05; 05C80

Keywords: Random planar maps; Hyperbolicity; Infinite geodesics; Poisson boundary

References

[1] C. Abraham. Rescaled bipartite planar maps converge to the Brownian map. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2) (2016) 575–595. MR3498001 https://doi.org/10.1214/14-AIHP657
[2] L. Addario-Berry and M. Albenque. The scaling limit of random simple triangulations and random simple quadrangulations. *Ann. Probab.* **45** (5) (2017) 2767–2825. MR3706731 https://doi.org/10.1214/16-AOP1124
[3] O. Angel. Growth and percolation on the uniform infinite planar triangulation. *Geom. Funct. Anal.* **13** (5) (2003) 935–974. MR2024412 https://doi.org/10.1007/s00039-003-0436-5
[4] O. Angel, T. Hutchcroft, A. Nachmias and G. Ray. Unimodular hyperbolic triangulations: Circle packing and random walk. *Invent. Math.* **206** (1) (2016) 229–268. MR3556528 https://doi.org/10.1007/s00222-016-0653-9
[5] O. Angel, B. Kolesnik and G. Miermont. Stability of geodesics in the Brownian map. *Ann. Probab.* **45** (5) (2017) 3451–3479. MR3706747 https://doi.org/10.1214/16-AOP1140
[6] O. Angel, A. Nachmias and G. Ray. Random walks on stochastic hyperbolic half planar triangulations. *Random Structures Algorithms* **49** (2) (2016) 213–234. MR3536537 https://doi.org/10.1002/rsa.20625
[7] O. Angel and G. Ray. Classification of half planar maps. *Ann. Probab.* **43** (3) (2015) 1315–1349. MR3342664 https://doi.org/10.1214/13-AOP891
[8] O. Angel and G. Ray. The half plane UIPT is recurrent. *Probab. Theory Related Fields* **170** (3) (2018) 657–683. MR3773797 https://doi.org/10.1007/s00440-017-0767-z
[9] O. Angel and O. Schramm. Uniform infinite planar triangulations. *Comm. Math. Phys.* **241** (2–3) (2003) 191–213. MR2013797 https://doi.org/10.1007/978-1-4419-9675-6_16
[10] K. B. Athreya and P. E. Ney. *Branching Processes*. *Die Grundlehren der Mathematischen Wissenschaften* **196**. Springer-Verlag, Berlin, 1972. MR0373040
[11] J. Beltran and J.-F. Le Gall. Quadrangulations with no pending vertices. *Bernoulli* **19** (2013) 1150–1175. MR3102547 https://doi.org/10.3150/12-BEJSP13
[12] I. Benjamini and R. Tessera. First passage percolation on a hyperbolic graph admits bi-infinite geodesics. *Electron. Commun. Probab.* **22** (2017) paper no. 14. MR3615665 https://doi.org/10.1214/17-ECP44
[13] J. Bettinelli, E. Jacob and G. Miermont. The scaling limit of uniform random plane maps, via the Ambjørn–Budd bijection. *Electron. J. Probab.* **19** (2014) paper no. 74. MR3256874 https://doi.org/10.1214/EJP.v19-3213

[14] T. Budzinski. The hyperbolic Brownian plane. *Probab. Theory Related Fields* **171** (1) (2018) 503–541. MR3800839 https://doi.org/10.1007/s00440-017-0785-x
[15] T. Budzinski. Supercritical causal maps: Geodesics and simple random walk, 2018. Available at arXiv:1806.10588.
[16] N. Curien. Planar stochastic hyperbolic triangulations. *Probab. Theory Related Fields* **165** (3) (2016) 509–540. MR3520011 https://doi.org/10.1007/s00440-015-0638-4
[17] N. Curien and J.-F. Le Gall. The Brownian plane. *J. Theoret. Probab.* **27** (4) (2014) 1249–1291. MR3278940 https://doi.org/10.1007/s10959-013-0485-0
[18] N. Curien and J.-F. Le Gall. First-passage percolation and local modifications of distances in random triangulations, 2015. Available at arXiv:1511.04264.
[19] N. Curien and J.-F. Le Gall. Scaling limits for the peeling process on random maps. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (1) (2017) 322–357. MR3606744 https://doi.org/10.1214/15-AIHP718
[20] N. Curien and L. Ménard. The skeleton of the UIPT, seen from infinity. *Ann. Henri Lebesgue* **1** (2018) 87–125.
[21] N. Curien, L. Ménard and G. Miermont. A view from infinity of the uniform infinite planar quadrangulation. *ALEA Lat. Am. J. Probab. Math. Stat.* **10** (1) (2013) 45–88. MR3083919
[22] W. Esty. The reverse Galton–Watson process. *J. Appl. Probab.* **12** (1975) 574–580. MR0375490 https://doi.org/10.1017/s0021900200048397
[23] M. Gromov. Hyperbolic manifolds, groups and actions. In *Riemann Surfaces and Related Topics: Proceedings of the 1978 Stony Brook Conference (State Univ. New York, Stony Brook, N.Y., 1978)* 183–213. *Ann. of Math. Stud.* **97**. Princeton Univ. Press, Princeton, N.J., 1981. MR0624814
[24] T. Hutchcroft and Y. Peres. Boundaries of planar graphs: A unified approach. *Electron. J. Probab.* **22** (2017) paper no. 100. MR3733658 https://doi.org/10.1214/17-EJP116
[25] H. Kesten. Aspects of first passage percolation. In *École d'été de probabilités de Saint-Flour, XIV—1984* 125–264. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876084 https://doi.org/10.1007/BFb0074919
[26] H. Kesten. Subdiffusive behavior of random walk on a random cluster. *Ann. Inst. Henri Poincaré Probab. Stat.* **22** (4) (1986) 425–487. MR0871905
[27] M. Krikun. A uniformly distributed infinite planar triangulation and a related branching process. *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI)* **307**(Teor. Predst. Din. Sist. Komb. i Algoritm. Metody. 10) (2004) 141–174, 282–283. MR2050691 https://doi.org/10.1007/s10958-005-0424-4
[28] M. Krikun. Explicit enumeration of triangulations with multiple boundaries. *Electron. J. Combin.* **14** (1) (2007) paper no. 61 (electronic). MR2336338
[29] M. Krikun. Local structure of random quadrangulations. Available at arXiv:0512304.
[30] J.-F. Le Gall. Geodesics in large planar maps and in the Brownian map. *Acta Math.* **205** (2010) 287–360. MR2746349 https://doi.org/10.1007/s11511-010-0056-5
[31] J.-F. Le Gall. Uniqueness and universality of the Brownian map. *Ann. Probab.* **41** (2013) 2880–2960. MR3112934 https://doi.org/10.1214/12-AOP792
[32] J.-F. Le Gall and F. Paulin. Scaling limits of bipartite planar maps are homeomorphic to the 2-sphere. *Geom. Funct. Anal.* **18** (3) (2008) 893–918. MR2438999 https://doi.org/10.1007/s00039-008-0671-x
[33] C. Marzouk. Scaling limits of random bipartite planar maps with a prescribed degree sequence. *Random Structures Algorithms* **53** (2018) 448–503. MR3854042 https://doi.org/10.1002/rsa.20773
[34] L. Ménard. Volumes in the uniform infinite planar triangulation: From skeletons to generating functions. *Combin. Probab. Comput.* **27** (6) (2018) 946–973. MR3872310 https://doi.org/10.1017/S0963548318000093
[35] G. Miermont. The Brownian map is the scaling limit of uniform random plane quadrangulations. *Acta Math.* **210** (2) (2013) 319–401. MR3070569 https://doi.org/10.1007/s11511-013-0096-8
[36] R. Stephenson. Local convergence of large critical multi-type Galton–Watson trees and applications to random maps. *J. Theoret. Probab.* **31** (1) (2018) 159–205. MR3769811 https://doi.org/10.1007/s10959-016-0707-3

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1162–1178
https://doi.org/10.1214/19-AIHP997

On the performance of the Euler–Maruyama scheme for SDEs with discontinuous drift coefficient

Thomas Müller-Gronbach[a] and Larisa Yaroslavtseva[b]

[a]*Faculty of Computer Science and Mathematics, University of Passau, Innstrasse 33, 94032 Passau, Germany.*
E-mail: thomas.mueller-gronbach@uni-passau.de
[b]*Seminar for Applied Mathematics, Department of Mathematics, ETH Zürich, HG G 53.2, Rämistrasse 101, 8092 Zurich, Switzerland.*
E-mail: larisa.yaroslavtseva@sam.math.ethz.ch

Abstract. Recently a lot of effort has been invested to analyze the L_p-error of the Euler–Maruyama scheme in the case of stochastic differential equations (SDEs) with a drift coefficient that may have discontinuities in space. For scalar SDEs with a piecewise Lipschitz drift coefficient and a Lipschitz diffusion coefficient that is non-zero at the discontinuity points of the drift coefficient so far only an L_p-error rate of at least $1/(2p)$ – has been proven. In the present paper we show that under the latter conditions on the coefficients of the SDE the Euler–Maruyama scheme in fact achieves an L_p-error rate of at least $1/2$ for all $p \in [1, \infty)$ as in the case of SDEs with Lipschitz coefficients. The proof of this result is based on a detailed analysis of appropriate occupation times for the Euler–Maruyama scheme.

Résumé. De nombreux efforts ont été consacrés récemment à l'analyse de l'erreur L_p de schéma d'Euler–Maruyama pour des équations différentielles stochastiques (EDS) avec un coefficient de dérive pouvant avoir des discontinuités en espace. Jusqu'à présent, pour des EDS scalaires avec un coefficient de dérive Lipschitz par morceaux et un coefficient de diffusion Lipschitz qui est non nul aux points de discontinuité du coefficient de dérive, seule une borne d'erreur L_p avec un taux d'au moins $1/(2p)$ – a été obtenue. Dans cet article, nous montrons que sous les hypothèses précédentes, le schéma d'Euler–Maruyama réalise un taux d'erreur L_p d'au moins $1/2$ pour tout $p \in [1, \infty)$, comme dans le cas d'EDS avec coefficients Lipschitz. La preuve de ce résultat se fonde sur une analyse détaillée de temps d'occupation bien choisis pour le schéma d'Euler–Maruyama.

MSC: 65C30; 60H35; 60H10

Keywords: Stochastic differential equations; Discontinuous drift coefficient; Strong approximation; Euler–Maruyama scheme

References

[1] O. Faure. Simulation du mouvement brownien et des diffusions. Thesis, ENPC, 1992.
[2] M. Gerencsér, A. Jentzen and D. Salimova. On stochastic differential equations with arbitrarily slow convergence rates for strong approximation in two space dimensions. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **473** (2017) 20170104, 16 pp. MR3741822 https://doi.org/10.1098/rspa.2017.0104
[3] S. Göttlich, K. Lux and A. Neuenkirch. The Euler scheme for stochastic differential equations with discontinuous drift coefficient: A numerical study of the convergence rate, 2017, arXiv:1705.04562.
[4] I. Gyöngy. A note on Euler's approximations. *Potential Anal.* **8** (1998) 205–216. MR1625576 https://doi.org/10.1023/A:1008605221617
[5] I. Gyöngy and N. Krylov. Existence of strong solutions for Itô's stochastic equations via approximations. *Probab. Theory Related Fields* **105** (1996) 143–158. MR1392450 https://doi.org/10.1007/BF01203833
[6] M. Hairer, M. Hutzenthaler and A. Jentzen. Loss of regularity for Kolmogorov equations. *Ann. Probab.* **43** (2015) 468–527. MR3305998 https://doi.org/10.1214/13-AOP838
[7] N. Halidias and P. E. Kloeden. A note on the Euler–Maruyama scheme for stochastic differential equations with a discontinuous monotone drift coefficient. *BIT* **48** (2008) 51–59.
[8] M. Hefter, A. Herzwurm and T. Müller-Gronbach. Lower error bounds for strong approximation of scalar SDEs with non-Lipschitzian coefficients. *Ann. Appl. Probab.* **29** (2019) 178–216. MR3910003 https://doi.org/10.1214/18-AAP1411
[9] M. Hefter and A. Jentzen. On arbitrarily slow convergence rates for strong numerical approximations of Cox–Ingersoll–Ross processes and squared Bessel processes. *Finance Stoch.* **23** (2019) 139–172. MR3898403 https://doi.org/10.1007/s00780-018-0375-5
[10] N. Hofmann, T. Müller-Gronbach and K. Ritter. The optimal discretization of stochastic differential equations. *J. Complexity* **17** (2001) 117–153. MR1817611 https://doi.org/10.1006/jcom.2000.0570

[11] M. Hutzenthaler, A. Jentzen and P. E. Kloeden. Strong and weak divergence in finite time of Euler's method for stochastic differential equations with non-globally Lipschitz continuous coefficients. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* **467** (2011) 1563–1576. MR2795791 https://doi.org/10.1098/rspa.2010.0348

[12] A. Jentzen, T. Müller-Gronbach and L. Yaroslavtseva. On stochastic differential equations with arbitrary slow convergence rates for strong approximation. *Commun. Math. Sci.* **14** (2016) 1477–1500. MR3538358 https://doi.org/10.4310/CMS.2016.v14.n6.a1

[13] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*, 2nd edition. *Graduate Texts in Mathematics* **113**. Springer-Verlag, New York, 1991. MR1121940 https://doi.org/10.1007/978-1-4612-0949-2

[14] G. Leobacher and M. Szölgyenyi. A numerical method for SDEs with discontinuous drift. *BIT* **56** (2016) 151–162. MR3486457 https://doi.org/10.1007/s10543-015-0549-x

[15] G. Leobacher and M. Szölgyenyi. A strong order 1/2 method for multidimensional SDEs with discontinuous drift. *Ann. Appl. Probab.* **27** (2017) 2383–2418. MR3693529 https://doi.org/10.1214/16-AAP1262

[16] G. Leobacher and M. Szölgyenyi. Convergence of the Euler–Maruyama method for multidimensional SDEs with discontinuous drift and degenerate diffusion coefficient. *Numer. Math.* **138** (2018) 219–239. MR3745015 https://doi.org/10.1007/s00211-017-0903-9

[17] G. Leobacher, S. Thonhauser and M. Szölgyenyi. On the existence of solutions of a class of SDEs with discontinuous drift and singular diffusion. *Electron. Commun. Probab.* **20** (2015) 14 pp. MR3304412 https://doi.org/10.1214/ECP.v20-3149

[18] X. Mao. *Stochastic Differential Equations and Applications*, 2nd edition. Horwood Publishing Limited, Chichester, 2008. MR2380366 https://doi.org/10.1533/9780857099402

[19] T. Müller-Gronbach. The optimal uniform approximation of systems of stochastic differential equations. *Ann. Appl. Probab.* **12** (2002) 664–690.

[20] T. Müller-Gronbach. Optimal pointwise approximation of SDEs based on Brownian motion at discrete points. *Ann. Appl. Probab.* **14** (2004) 1605–1642. MR2099646 https://doi.org/10.1214/105051604000000954

[21] T. Müller-Gronbach. Strong approximation of systems of stochastic differential equations. Habilitation thesis, TU Darmstadt 2002, iv+161. MR1910644 https://doi.org/10.1214/aoap/1026915620

[22] T. Müller-Gronbach and L. Yaroslavtseva. A note on strong approximation of SDEs with smooth coefficients that have at most linearly growing derivatives. *J. Math. Anal. Appl.* **467** (2018) 1013–1031. MR3842417 https://doi.org/10.1016/j.jmaa.2018.07.041

[23] A. Neuenkirch, M. Szölgyenyi and L. Szpruch. An adaptive Euler–Maruyama scheme for stochastic differential equations with discontinuous drift and its convergence analysis. *SIAM J. Numer. Anal.* **57** (2019) 378–403. MR3914573 https://doi.org/10.1137/18M1170017

[24] H.-L. Ngo and D. Taguchi. Strong rate of convergence for the Euler–Maruyama approximation of stochastic differential equations with irregular coefficients. *Math. Comp.* **85** (300) (2016) 1793–1819. MR3471108 https://doi.org/10.1090/mcom3042

[25] H.-L. Ngo and D. Taguchi. On the Euler–Maruyama approximation for one-dimensional stochastic differential equations with irregular coefficients. *IMA J. Numer. Anal.* **37** (4) (2017) 1864–1883. MR3712177 https://doi.org/10.1093/imanum/drw058

[26] H.-L. Ngo and D. Taguchi. Strong convergence for the Euler–Maruyama approximation of stochastic differential equations with discontinuous coefficients. *Statist. Probab. Lett.* **125** (2017) 55–63. MR3626068 https://doi.org/10.1016/j.spl.2017.01.027

[27] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. Springer-Verlag, Berlin, 1999. MR1725357 https://doi.org/10.1007/978-3-662-06400-9

[28] P. Speckman. L_p-Approximation of Autoregressive Gaussian Processes. Report, Dep. of Statistics, University of Oregon, 1979.

[29] L. Yaroslavtseva. On non-polynomial lower error bounds for adaptive strong approximation of SDEs. *J. Complexity* **42** (2017) 1–18. MR3683255 https://doi.org/10.1016/j.jco.2017.04.002

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1179–1202
https://doi.org/10.1214/19-AIHP998

Absence of percolation for Poisson outdegree-one graphs

David Coupier[a], David Dereudre[b] and Simon Le Stum[b]

[a]*Université Polytechnique des Hauts-de-France, LAMAV EA 4015, Valenciennes, France. E-mail: David.Coupier@univ-valenciennes.fr*
[b]*Université de Lille, Laboratoire Paul Painlevé, UMR CNRS 8524, Villeneuve d'Ascq, France*

Abstract. A Poisson outdegree-one graph is an oriented graph based on a Poisson point process such that each vertex has only one outgoing edge. The paper focuses on the absence of percolation for such graphs. Our main result is based on two assumptions. The Shield assumption ensures that the graph is locally determined with possible random horizons. The Loop assumption ensures that any forward branch of the graph merges on a loop provided that the Poisson point process is augmented with a finite collection of well-chosen points. Several models satisfy these general assumptions and inherit in consequence the absence of percolation. In particular, we solve in Theorem 3.1 a conjecture by Daley et al. on the absence of percolation for the line-segment model (Conjecture 7.1 of (*Probab. Math. Statist.* **36** (2016) 221–246), discussed in (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 127–145) as well). In this planar model, a segment is growing from any point of the Poisson process and stops its growth whenever it hits another segment. The random directions are picked independently and uniformly on the unit sphere. Another model of geometric navigation is presented and also fulfills the Shield and Loop assumptions.

Résumé. Nous considérons des graphes orientés dont l'ensemble des sommets est donné par un processus ponctuel de Poisson et tels que chaque sommet admette une et une seule arête sortante. Le résultat principal de ce papier est l'absence de percolation pour de tels graphes satisfaisant deux hypothèses. L'hypothèse Shield stipule que l'état du graphe localement ne dépend que de son voisinage, tandis que l'hypothèse Loop prétend que toute branche orientée du graphe échoue sur un cycle dès qu'un ensemble (fini) de sommets bien choisis est ajouté au processus de Poisson. Plusieurs modèles intéressants satisfont ces deux hypothèses générales et, par conséquent, ne percolent pas. Nous résolvons ainsi dans Theorem 3.1 une conjecture de Daley et al. sur l'absence de percolation pour le "line-segment model" (Conjecture 7.1 de (*Probab. Math. Statist.* **36** (2016) 221–246), également discutée dans (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 127–145)). Dans ce modèle bidimensionnel, un segment pousse depuis chaque point du processus de Poisson jusqu'à ce qu'il heurte un autre segment, stoppant ainsi sa croissance. Les directions dans lesquelles poussent les segments sont choisies uniformément sur la sphère et indépendamment les unes des autres. Enfin, un autre modèle dit de navigation est présenté et satisfait aussi les hypothèses Shield et Loop.

MSC: 60D05; 60K35

Keywords: Stochastic geometry; Geometric random graphs; Percolation; Mass transport principle; Lilypond model; Line-segment model

References

[1] I. Benjamini, R. Lyons, Y. Peres and O. Schramm. Critical percolation on any nonamenable group has no infinite clusters. *Ann. Probab.* **27** (3) (1999) 1347–1356. MR1733151 https://doi.org/10.1214/aop/1022677450
[2] N. Bonichon and J.-F. Marckert. Asymptotics of geometrical navigation on a random set of points in the plane. *Adv. in Appl. Probab.* **43** (4) (2011) 899–942. MR2867939 https://doi.org/10.1239/aap/1324045692
[3] D. Coupier and V. C. Tran. The 2D-directed spanning forest is almost surely a tree. *Random Structures Algorithms* **42** (1) (2013) 59–72. MR2999212 https://doi.org/10.1002/rsa.20400
[4] D. J. Daley, S. Ebert and G. Last. Two lilypond systems of finite line-segments. *Probab. Math. Statist.* **36** (2) (2016) 221–246. MR3593022
[5] D. J. Daley and G. Last. Descending chains, the lilypond model, and mutual-nearest-neighbour matching. *Adv. in Appl. Probab.* **37** (3) (2005) 604–628. MR2156551 https://doi.org/10.1239/aap/1127483738
[6] O. Häggström and R. Meester. Nearest neighbor and hard sphere models in continuum percolation. *Random Structures Algorithms* **9** (3) (1996) 295–315. MR1606845 https://doi.org/10.1002/(SICI)1098-2418(199610)9:3<295::AID-RSA3>3.3.CO;2-3
[7] C. Hirsch. On the absence of percolation in a line-segment based lilypond model. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (1) (2016) 127–145. MR3449297 https://doi.org/10.1214/14-AIHP638
[8] G. Last and S. Ebert. On a class of growth-maximal hard-core processes. *Stoch. Models* **31** (2013) 153–184. MR3306148 https://doi.org/10.1080/15326349.2015.973722

[9] G. Last and M. Penrose. Percolation and limit theory for the Poisson lilypond model. *Random Structures Algorithms* **42** (2) (2013) 226–249. MR3019399 https://doi.org/10.1002/rsa.20410

[10] G. Last and M. Penrose. *Lectures on the Poisson Process*. *IMS Textbook*. Cambridge University Press, 2017. MR3791470

[11] T. Liggett, R. Schonmann and A. Stacey. Domination by product measures. *Ann. Probab.* **25** (1) (1997) 71–95. MR1428500 https://doi.org/10.1214/aop/1024404279

[12] T. Schreiber and N. Soja. Limit theory for planar Gilbert tessellations. *Probab. Math. Statist.* **31** (1) (2011) 149–160. MR2804981

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1203–1229
https://doi.org/10.1214/19-AIHP999

Trees within trees II: Nested fragmentations

Jean-Jil Duchamps

Sorbonne Université, CNRS, Laboratoire de Probabilités, Statistiques et Modélisations (LPSM), 4 place Jussieu, F-75005 Paris, France.
E-mail: jean-jil.duchamps@normalesup.org

Abstract. Similarly as in (*Electron. J. Probab.* **23** (2018)) where nested coalescent processes are studied, we generalize the definition of partition-valued homogeneous Markov fragmentation processes to the setting of nested partitions, i.e. pairs of partitions (ζ, ξ) where ζ is finer than ξ. As in the classical univariate setting, under exchangeability and branching assumptions, we characterize the jump measure of nested fragmentation processes, in terms of erosion coefficients and dislocation measures. Among the possible jumps of a nested fragmentation, three forms of erosion and two forms of dislocation are identified – one being specific to the nested setting and relating to a bivariate paintbox process.

Résumé. Poursuivant l'idée de (*Electron. J. Probab.* **23** (2018)) où les processus de coalescence emboîtés sont étudiés, nous étendons ici la définition des processus de fragmentation markoviens homogènes aux processus de fragmentation à valeurs dans les partitions emboîtées, c'est-à-dire les paires de partitions (ζ, ξ) telles que ζ soit plus fine que ξ. Comme dans le contexte classique (dit univarié), sous des hypothèses d'échangeabilité et de branchement, nous caractérisons la mesure de saut des processus de fragmentation emboîtés en termes de coefficients d'érosion et de mesures de dislocation. Les sauts d'une fragmentation emboîtée peuvent être de plusieurs natures différentes : nous distinguons trois formes d'érosions et deux formes de dislocations, l'une d'elles étant spécifique au contexte des partitions emboîtées et étant générée par un processus de pots de peinture bivarié.

MSC: 60G09; 60G57; 60J25; 60J35; 60J75; 92D15

Keywords: Fragmentations; Exchangeable; Partition; Random tree; Coalescent; Population genetics; Gene tree; Species tree; Phylogenetics; Evolution

References

[1] D. Aldous. Probability distributions on cladograms. In *Random Discrete Structures* 1–18. D. Aldous and R. Pemantle (Eds). Springer, New York, 1996. MR1395604 https://doi.org/10.1007/978-1-4612-0719-1_1
[2] N. Berestycki. Recent progress in coalescent theory. *Ensaios Matemáticos* **16** (1) (2009) 1–193. Available at http://www.emis.ams.org/journals/em/docs/ensaio_matematico/em_16_berestycki.pdf. MR2574323
[3] J. Bertoin. *Random Fragmentation and Coagulation Processes*. Cambridge University Press, 2006. MR2253162 https://doi.org/10.1017/CBO9780511617768
[4] A. Blancas, J.-J. Duchamps, A. Lambert and A. Siri-Jégousse. Trees within trees: Simple nested coalescents. *Electron. J. Probab.* **23** (2018). MR3858922 https://doi.org/10.1214/18-EJP219
[5] A. Blancas, T. Rogers, J. Schweinsberg and A. Siri-Jégousse. The nested Kingman coalescent: Speed of coming down from infinity. *Ann. Appl. Probab.* **29** (3) (2019) 1808–1836. MR3914557 https://doi.org/10.1214/18-AAP1440
[6] B. Chen, D. Ford and M. Winkel. A new family of Markov branching trees: The alpha-gamma model. *Electron. J. Probab.* **14** (2009) 400–430. MR2480547 https://doi.org/10.1214/EJP.v14-616
[7] H. Crane. Generalized Markov branching trees. *Adv. in Appl. Probab.* **49** (01) (2017) 108–133. MR3631218 https://doi.org/10.1017/apr.2016.81
[8] H. Crane and H. Towsner. The structure of combinatorial Markov processes, 2016. Available at arXiv:1603.05954.
[9] J. J. Doyle. Trees within trees: Genes and species, molecules and morphology. *Syst. Biol.* **46** (3) (1997) 537–553.
[10] A. Etheridge. *Some Mathematical Models from Population Genetics: École d'ete de Probabilités de Saint-Flour XXXIX-2009*. *Lecture Notes in Mathematics* **2012**. Springer, 2011. ISBN 978-3-642-16631-0 978-3-642-16632-7. MR2759587 https://doi.org/10.1007/978-3-642-16632-7
[11] D. J. Ford. Probabilities on cladograms: Introduction to the alpha model. Ph.D. thesis, Stanford University, 2006. Available at arXiv:math/0511246. MR2708802
[12] C. Foucart. Distinguished exchangeable coalescents and generalized Fleming–Viot processes with immigration. *Adv. in Appl. Probab.* **43** (02) (2011) 348–374. MR2848380 https://doi.org/10.1239/aap/1308662483
[13] B. Haas, G. Miermont, J. Pitman and M. Winkel. Continuum tree asymptotics of discrete fragmentations and applications to phylogenetic models. *Ann. Probab.* **36** (5) (2008) 1790–1837. MR2440924 https://doi.org/10.1214/07-AOP377
[14] J. Kingman. The coalescent. *Stochastic Process. Appl.* **13** (3) (1982) 235–248. MR0671034 https://doi.org/10.1016/0304-4149(82)90011-4

[15] A. Lambert. Population dynamics and random genealogies. *Stoch. Models* **24**(sup1) (2008) 45–163. MR2466449 https://doi.org/10.1080/15326340802437728

[16] A. Lambert. Probabilistic models for the (sub)tree(s) of life. *Braz. J. Probab. Stat.* **31** (3) (2017) 415–475. MR3693976 https://doi.org/10.1214/16-BJPS320

[17] A. Lambert and E. Schertzer. Coagulation-transport equations and the nested coalescents. In *Probability Theory and Related Fields*, 2019.

[18] W. P. Maddison. Gene trees in species trees. *Syst. Biol.* **46** (3) (1997) 523–536.

[19] R. D. Page and M. A. Charleston. From gene to organismal phylogeny: Reconciled trees and the gene tree/species tree problem. *Molecular Phylogenetics and Evolution* **7** (2) (1997) 231–240.

[20] R. D. Page and M. A. Charleston. Trees within trees: Phylogeny and historical associations. *Trends Ecol. Evol.* **13** (9) (1998) 356–359.

[21] J. Pitman. Coalescents with multiple collisions. *Ann. Probab.* **27** (4) (1999) 1870–1902. MR1742892 https://doi.org/10.1214/aop/1022677552

[22] S. Sagitov. The general coalescent with asynchronous mergers of ancestral lines. *J. Appl. Probab.* **36** (4) (1999) 1116–1125. MR1742154 https://doi.org/10.1017/s0021900200017903

[23] C. Semple and M. Steel. *Phylogenetics*. *Oxford Lecture Series in Mathematics and Its Applications* **24**. Oxford University Press, Oxford, New York, 2003. ISBN 978-0-19-850942-4. MR2060009

[24] S. M. Srivastava. *A Course on Borel Sets*. *Graduate Texts in Mathematics* **180**. Springer, Berlin Heidelberg, 1998. MR1619545 https://doi.org/10.1007/978-3-642-85473-6

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1230–1250
https://doi.org/10.1214/19-AIHP1000

Stochastic Hölder continuity of random fields governed by a system of stochastic PDEs

Kai Du[a,1], Jiakun Liu[b,2] and Fu Zhang[c,3]

[a]*Shanghai Center for Mathematical Sciences, Fudan University, Shanghai 200438, China. E-mail: kdu@fudan.edu.cn*
[b]*School of Mathematics and Applied Statistics, University of Wollongong, Wollongong NSW 2522, Australia. E-mail: jiakunl@uow.edu.au*
[c]*College of Science, University of Shanghai for Science and Technology, Shanghai 200093, China. E-mail: fuzhang82@gmail.com*

Abstract. This paper constructs a solvability theory for a system of stochastic partial differential equations. On account of the Kolmogorov continuity theorem, solutions are looked for in certain Hölder-type classes in which a random field is treated as a space-time function taking values in L^p-space of random variables. A modified stochastic parabolicity condition involving p is proposed to ensure the finiteness of the associated norm of the solution, which is showed to be sharp by examples. The Schauder-type estimates and the solvability theorem are proved.

Résumé. Cet article construit une théorie sur la solvabilité d'un système d'équations différentielles partielles stochastiques. En raison du théorème de continuité de Kolmogorov, les solutions sont recherchées dans certaines classes de Hölder, dans lesquelles un champ aléatoire est considéré comme une fonction spatio-temporelle prenant des valeurs dans l'espace L^p des variables aléatoires. Une condition de parabolicité stochastique modifiée impliquant p est proposée afin d'assurer la finitude de la norme associée de la solution. En étudiant des exemples, cette condition est montrée être optimale. Les estimations de type de Schauder et la solvabilité de l'équation sont démontrées.

MSC: Primary 60H15; 35R60; secondary 35K45

Keywords: Stochastic partial differential system; Stochastic parabolicity condition; Schauder estimate; Stochastic continuity

References

[1] R. A. Adams and J. J. F. Fournier. *Sobolev Spaces*, 2nd edition. *Pure and Applied Mathematics (Amsterdam)* **140**, xiv+305. Elsevier/Academic Press, Amsterdam, 2003. MR2424078
[2] V. Bally, A. Millet and M. Sanz-Solé. Approximation and support theorem in Hölder norm for parabolic stochastic partial differential equations. *Ann. Probab.* **23** (1) (1995) 178–222. MR1330767
[3] Z. Brzeźniak, B. Goldys and T. Jegaraj. Weak solutions of a stochastic Landau–Lifshitz–Gilbert equation. *Appl. Math. Res. Express. AMRX* **2013** (1) (2013) 1–33. MR3040886
[4] Z. Brzeźniak and M. Veraar. Is the stochastic parabolicity condition dependent on p and q? *Electron. J. Probab.* **17** (56) (2012) 1–24. MR2955048 https://doi.org/10.1214/EJP.v17-2186
[5] Z. Q. Chen and K. H. Kim. An L^p-theory for non-divergence form SPDEs driven by Lévy processes. *Forum Math.* **26** (2014) 1381–1411. MR3334033 https://doi.org/10.1515/forum-2011-0123
[6] P. L. Chow and J.-L. Jiang. Stochastic partial differential equations in Hölder spaces. *Probab. Theory Related Fields* **99** (1) (1994) 1–27. MR1273740 https://doi.org/10.1007/BF01199588
[7] G. Da Prato and J. Zabczyk. *Stochastic Equations in Infinite Dimensions*, 2nd edition. *Encyclopedia of Mathematics and Its Applications* **152**, xviii+493. Cambridge University Press, Cambridge, 2014. MR3236753 https://doi.org/10.1017/CBO9781107295513
[8] R. Dalang, D. Khoshnevisan and E. Nualart. Hitting probabilities for systems of non-linear stochastic heat equations with additive noise. *ALEA Lat. Am. J. Probab. Math. Stat.* **3** (2007) 231–271. MR2365643
[9] H. Dong and H. Zhang. Schauder estimates for higher-order parabolic systems with time irregular coefficients. *Calc. Var. Partial Differential Equations* **54** (1) (2015) 47–74. MR3385152 https://doi.org/10.1007/s00526-014-0777-y
[10] K. Du and J. Liu. A Schauder estimate for stochastic PDEs. *C. R. Math. Acad. Sci. Paris* **354** (4) (2016) 371–375. MR3473551 https://doi.org/10.1016/j.crma.2016.01.010
[11] K. Du and J. Liu. On the Cauchy problem for stochastic parabolic equations in Hölder spaces. *Trans. Amer. Math. Soc.* **371** (4) (2019) 2643–2664. MR3896092 https://doi.org/10.1090/tran/7533
[12] F. Flandoli. Dirichlet boundary value problem for stochastic parabolic equations: Compatibility relations and regularity of solutions. *Stochastics* **29** (3) (1990) 331–357. MR1042066 https://doi.org/10.1080/17442509008833620

[13] T. Funaki. Random motion of strings and related stochastic evolution equations. *Nagoya Math. J.* **89** (1983) 129–193. MR0692348 https://doi.org/10.1017/S0027763000020298
[14] M. Giaquinta. *Introduction to Regularity Theory for Nonlinear Elliptic Systems*. Birkhauser, 1993. MR1239172
[15] D. Gilbarg and N. S. Trudinger. *Elliptic Partial Differential Equations of Second Order*. *Classics in Mathematics*. Springer-Verlag, Berlin, 2001. MR1814364
[16] M. Hairer and J. C. Mattingly. Ergodicity of the 2D Navier–Stokes equations with degenerate stochastic forcing. *Ann. of Math. (2)* (2006) 993–1032. MR2259251 https://doi.org/10.4007/annals.2006.164.993
[17] E. Hille and R. S. Phillips. *Functional Analysis and Semi-Groups*. *American Mathematical Society Colloquium Publications* **31**. American Mathematical Society, Providence, RI, 1957. MR0089373 https://doi.org/10.4007/annals.2006.164.993
[18] K. H. Kim and K. Lee. A W_2^n-theory of stochastic parabolic partial differential systems on C^1-domains. *Potential Anal.* **38** (3) (2013) 951–984. MR3034605 https://doi.org/10.1007/s11118-012-9302-0
[19] K. H. Kim and K. Lee. A note on W_p^γ-theory of linear stochastic parabolic partial differential systems. *Stochastic Process. Appl.* **123** (1) (2013) 76–90. MR2988110 https://doi.org/10.1016/j.spa.2012.08.016
[20] N. V. Krylov. On L_p-theory of stochastic partial differential equations in the whole space. *SIAM J. Math. Anal.* **27** (2) (1996) 313–340. MR1377477 https://doi.org/10.1137/S0036141094263317
[21] N. V. Krylov. On SPDE's and superdiffusions. *Ann. Probab.* **25** (4) (1997) 1789–1809. MR1487436 https://doi.org/10.1214/aop/1023481111
[22] N. V. Krylov. An analytic approach to SPDEs. In *Stochastic Partial Differential Equations: Six Perspectives* 185–242. *Math. Surveys Monogr.* **64**. Amer. Math. Soc., Providence, RI, 1999. MR1661766 https://doi.org/10.1090/surv/064/05
[23] N. V. Krylov and B. L. Rozovsky. On the Cauchy problem for linear stochastic partial differential equations. *Izv. Ross. Akad. Nauk Ser. Mat.* **41** (6) (1977) 1267–1284. MR0501350 https://doi.org/10.4007/annals.2006.164.993
[24] G. M. Lieberman. *Second Order Parabolic Differential Equations*, **68**. World Scientific, 1996. MR1465184 https://doi.org/10.1142/3302
[25] R. Mikulevicius. On the Cauchy problem for parabolic SPDEs in Hölder classes. *Ann. Probab.* **28** (1) (2000) 74–103. MR1755998 https://doi.org/10.1214/aop/1019160112
[26] R. Mikulevicius and H. Pragarauskas. On Cauchy–Dirichlet problem in half-space for parabolic SPDEs in weighted Hölder spaces. *Stochastic Process. Appl.* **106** (2) (2003) 185–222. MR1989627 https://doi.org/10.1016/S0304-4149(03)00042-5
[27] R. Mikulevicius and H. Pragarauskas. On Cauchy–Dirichlet problem for parabolic quasilinear SPDEs. *Potential Anal.* **25** (1) (2006) 37–75. MR2238936 https://doi.org/10.1007/s11118-005-9006-9
[28] R. Mikulevicius and B. Rozovsky. A note on Krylov's L_p-theory for systems of SPDEs. *Electron. J. Probab.* **6** (12) (2001) 1–35. MR1831807 https://doi.org/10.1214/EJP.v6-85
[29] R. Mikulevicius and B. Rozovsky. Stochastic Navier–Stokes equations for turbulent flows. *SIAM J. Math. Anal.* **35** (5) (2004) 1250–1310. MR2050201 https://doi.org/10.1137/S0036141002409167
[30] R. Mikulevicius and B. Rozovsky. On unbiased stochastic Navier–Stokes equations. *Probab. Theory Related Fields* **154** (3–4) (2012) 787–834. MR3000562 https://doi.org/10.1007/s00440-011-0384-1
[31] C. Mueller and R. Tribe. Hitting properties of a random string. *Electron. J. Probab.* **7** (10) (2002) 1–29. MR1902843 https://doi.org/10.1214/EJP.v7-109
[32] E. Pardoux. Equations aux dérivées partielles stochastiques non linéaires monotones; Etude de solutions fortes de type Itô. PhD thesis, Univ. Paris-Sud, Orsay, 1975.
[33] P. Portal and M. Veraar. Stochastic maximal regularity for rough time-dependent problems. *Stoch. Partial Differ. Equ. Anal. Comput.* **7** (2019) 541–597. MR4022283 https://doi.org/10.1007/s40072-019-00134-w
[34] B. L. Rozovsky. Stochastic partial differential equations. *Mat. Sb.* **96** (138) (1975) 314–341, 344. MR0370764 https://doi.org/10.4007/annals.2006.164.993
[35] W. Schlag. Schauder and L^p estimates for parabolic systems via Campanato spaces. *Comm. Partial Differential Equations* **21** (7–8) (1996) 1141–1175. MR1399194 https://doi.org/10.1080/03605309608821221
[36] S. Tang and W. Wei. On the Cauchy problem for backward stochastic partial differential equations in Hölder spaces. *Ann. Probab.* **44** (1) (2016) 360–398. MR3456341 https://doi.org/10.1214/14-AOP976
[37] J. van Neerven, M. Veraar and L. Weis. Stochastic maximal L^p-regularity. *Ann. Probab.* **40** (2) (2012) 788–812. MR2952092 https://doi.org/10.1214/10-AOP626
[38] J. B. Walsh. An introduction to stochastic partial differential equations. In *École d'été de probabilités de Saint-Flour, XIV—1984* 265–439. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876085 https://doi.org/10.1007/BFb0074920
[39] M. Zakai. On the optimal filtering of diffusion processes. *Z. Wahrsch. Verw. Gebiete* **11** (3) (1969) 230–243. MR0242552 https://doi.org/10.1007/BF00536382

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1251–1283
https://doi.org/10.1214/19-AIHP1001

Hua–Pickrell diffusions and Feller processes on the boundary of the graph of spectra

Theodoros Assiotis

School of Mathematics, University of Bristol, U.K. E-mail: T.Assiotis@bristol.ac.uk

Abstract. We consider consistent diffusion dynamics, leaving the celebrated Hua–Pickrell measures, depending on a complex parameter s, invariant. These, give rise to Feller–Markov processes on the infinite dimensional boundary Ω of the "graph of spectra", the continuum analogue of the Gelfand–Tsetlin graph, via the method of intertwiners of Borodin and Olshanski. In the particular case of $s = 0$, this stochastic process is closely related to the Sine_2 point process on $\mathbb{R}$ that describes the spectrum in the bulk of large random matrices. Equivalently, these coherent dynamics are associated to interlacing diffusions in Gelfand–Tsetlin patterns having certain *Gibbs* invariant measures. Moreover, under an application of the Cayley transform when $s = 0$ we obtain processes on the circle leaving invariant the *multilevel* Circular Unitary Ensemble. We finally prove that the Feller processes on Ω corresponding to Dyson's Brownian motion and its stationary analogue are given by explicit and very simple deterministic dynamical systems.

Résumé. Nous considérons des dynamiques de diffusions cohérentes, laissant les fameuses mesures de Hua–Pickrell, dépendant d'un paramètre complexe s, invariantes. Celles-ci donnent lieu à des processus de Feller–Markov sur la frontière infini-dimensionnelle Ω du «graphe de spectres», l'analogue continu du graphe de Gelfand–Tsetlin, par la méthode des entrelacements de Borodin et Olshanski. Dans le cas particulier de $s = 0$, ce processus stochastique est étroitement relié au processus ponctuel Sine2 sur R qui décrit l'intérieur du spectre des grandes matrices aléatoires. De manière équivalente, ces dynamiques cohérentes sont associées à des diffusions entrelacées dans des modèles de Gelfand–Tsetlin ayant certaines mesures invariantes de *Gibbs*. De plus, par une application de la transformation de Cayley lorsque $s = 0$, nous obtenons des processus sur le cercle laissant invariant l'ensemble circulaire unitaire *multiniveaux*. Nous prouvons enfin que les processus de Feller sur Ω correspondant au mouvement brownien de Dyson et à son analogue stationnaire sont donnés par des systèmes dynamiques déterministes très simples et explicites.

MSC: 60J60

Keywords: Feller processes; Diffusions; Dyson Brownian motion; Hua–Pickrell measures; Intertwinings

References

[1] T. Assiotis, N. O'Connell and J. Warren. Interlacing diffusions, 2016. Available at http://arxiv.org/abs/1607.07182.
[2] Y. Baryshnikov. GUEs and queues. *Probab. Theory Related Fields* **119** (2001) 256–274. MR1818248 https://doi.org/10.1007/PL00008760
[3] A. Borodin and G. Olshanski. Infinite random matrices and ergodic measures. *Comm. Math. Phys.* **223** (1) (2001) 87–123. MR1860761 https://doi.org/10.1007/s002200100529
[4] A. Borodin and G. Olshanski. Markov processes on the path space of the Gelfand–Tsetlin graph and on its boundary. *J. Funct. Anal.* **263** (2012) 248–303. MR2920848 https://doi.org/10.1016/j.jfa.2012.03.018
[5] A. Borodin and G. Olshanski. Markov dynamics on the Thoma cone: A model of time dependent determinantal processes with infinitely many particles. *Electron. J. Probab.* **13** (75) (2013) 1–43. MR3091721 https://doi.org/10.1214/EJP.v18-2729
[6] A. Bufetov. Ergodic decomposition for measures quasi-invariant under a Borel action of an inductively compact group. *Mat. Sb.* **205** (2) (2014) 39–70. MR3204667 https://doi.org/10.1070/sm2014v205n02abeh004371
[7] A. Bufetov and Y. Qiu. The explicit formulae for scaling limits in the ergodic decomposition of infinite Pickrell measures. *Ark. Mat.* **54** (2) (2016) 403–435. MR3546359 https://doi.org/10.1007/s11512-016-0230-x
[8] C. Cuenca. Markov processes on the duals to infinite-dimensional classical lie groups, 2016. Available at http://arxiv.org/abs/1608.02281. MR3825885 https://doi.org/10.1214/17-AIHP842
[9] M. Defosseux. Orbit measures, random matrix theory and interlaced determinantal processes. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** (1) (2010) 209–249. MR2641777 https://doi.org/10.1214/09-AIHP314
[10] A. Eberle. Markov processes, Lecture Notes 2016-17, 2017. Available at https://wt.iam.uni-bonn.de/eberle/skripten/.
[11] J. Faraut. *Infinite Dimensional Harmonic Analysis and Probability. Probability Measures on Groups: Recent Directions and Trends*. Tata Inst. Fund. Res, Mumbai, 2006. MR2213479

[12] P. Graczyk and J. Malecki. Multidimensional Yamada–Watanabe theorem and its applications to particle systems. *J. Math. Phys.* **54** (2) (2013). MR3076363 https://doi.org/10.1063/1.4790507

[13] P. Graczyk and J. Malecki. Strong solutions of non-colliding particle systems. *Electron. J. Probab.* **19** (2014) 1–21. MR3296535 https://doi.org/10.1214/EJP.v19-3842

[14] L. K. Hua. *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains*, Chinese edition. Peking, Science Press, 1958. English edition: *Transl. Math. Monographs* **6**, RI Providence, American Mathematical Society (1963). MR0598469

[15] N. Ikeda and S. Watanabe. *Stochastic Differential Equations and Diffusion Processes*, 2nd edition. *North Holland Mathematical Library* **24**. North-Holland publishing Co., Amsterdam, 1989. MR0637061

[16] K. Ito and H. P. McKean. *Diffusion Processes and Their Sample Paths, Second Printing*. Springer-Verlag, Berlin, 1974. MR0345224

[17] S. Karlin. *Total Positivity*, **1**. Stanford University Press, Stanford, 1968. MR0230102

[18] S. Karlin and J. McGregor. Coincidence probabilities. *Pacific J. Math.* **9** (4) (1959) 1141–1164. MR0114248

[19] G. W. Mackey. Borel structures on groups and their duals. *Trans. Amer. Math. Soc.* **85** (1957) 134–165. MR0089999 https://doi.org/10.2307/1992966

[20] A. Metcalfe, N. O'Connell and J. Warren. Interlaced processes on the circle. In *Annales de l'Institut Henri Poincare, Probabilites et Statistiques* 1165–1184, **45**, 2009. MR2572170 https://doi.org/10.1214/08-AIHP305

[21] Y. Neretin. Hua type integrals over unitary groups and over projective limits of unitary groups. *Duke Math. J.* **114** (2) (2002) 239–266. MR1920189 https://doi.org/10.1215/S0012-7094-02-11423-9

[22] G. Olshanski. The problem of harmonic analysis on the infinite dimensional unitary group. *J. Funct. Anal.* **205** (2003) 464–524. MR2018416 https://doi.org/10.1016/S0022-1236(02)00022-8

[23] G. Olshanski. Projections of orbital measures, Gelfand–Tsetlin polytopes and splines. *J. Lie Theory* **23** (4) (2013) 1011–1022. MR3185209

[24] G. Olshanski and A. Vershik. Ergodic unitarily invariant measures on the space of infinite Hermitian matrices. In *Contemporary Mathematical Physics* 137–175. *American Mathematical Society Ser. 2* 1996. MR1402920 https://doi.org/10.1090/trans2/175/09

[25] H. Osada and H. Tanemura. Infinite-dimensional stochastic differential equations and tail σ-fields, 2016. Available at https://arxiv.org/abs/1412.8674.

[26] V. Perez-Abreu and C. Tudor. Functional limit theorems for trace processes in a Dyson Brownian motion. *Commun. Stoch. Anal.* **1** (1) (2007) 415–428. MR2403859 https://doi.org/10.31390/cosa.1.3.05

[27] D. Pickrell. Measures on infinite dimensional Grassmann manifolds. *J. Funct. Anal.* **70** (2) (1987) 323–356. MR0874060 https://doi.org/10.1016/0022-1236(87)90116-9

[28] D. Pickrell. Mackey analysis of infinite classical motion groups. *Pacific J. Math.* **150** (1) (1991) 139–166. MR1120717

[29] Y. Qiu. Infinite random matrices & ergodic decomposition of finite and infinite Hua–Pickrell measures. *Adv. Math.* **308** (2017) 1209–1268. MR3600086 https://doi.org/10.1016/j.aim.2017.01.003

[30] K. Ramanan and M. Shkolnikov. Intertwinings of β-Dyson Brownian motions of different dimensions, 2016. Available at https://arxiv.org/abs/1608.01597. MR3795080 https://doi.org/10.1214/17-AIHP835

[31] R. Schiling and L. Partzsch. *Brownian Motion – An Introduction to Stochastic Processes*. De Gruyter Textbook, Berlin, 2012. MR2962168 https://doi.org/10.1515/9783110278989

[32] D. Stroock and S. R. S. Varadhan. *Multidimensional Diffusion Processes*. *A Series of Comprehensive Studies in Mathematics* **233**. Springer, Berlin, 2006. Reprint of the 1997 edition. MR2190038

[33] D. W. Stroock. *Partial Differential Equations for Probabilists*. *Cambridge Studies in Advanced Mathematics* **112**, 2008. MR2410225 https://doi.org/10.1017/CBO9780511755255

[34] Y. Sun. Laguerre and Jacobi analogues of the Warren process, 2016. Available at https://arxiv.org/abs/1610.01635.

[35] L. C. Tsai. Infinite dimensional stochastic differential equations for Dyson's model. *Probab. Theory Related Fields* **166** (3) (2016) 801–850. MR3568040 https://doi.org/10.1007/s00440-015-0672-2

[36] A. Vershik. Description of invariant measures for the actions of some infinite-dimensional groups. *Dokl. Akad. Nauk SSSR* **218** (1974) 749–752. English translation, *Soviet Math. Dokl.* **15**, 1396–1400. MR0372161

[37] J. Warren. Dyson's Brownian motions, intertwining and interlacing. *Electron. J. Probab.* **12** (2007) 573–590. MR2299928 https://doi.org/10.1214/EJP.v12-406

[38] T. Wihler. On the Holder continuity of matrix functions for normal matrices. *JIPAM. J. Inequal. Pure Appl. Math.* **10** (4) (2009). MR2577861

[39] G. Winkler. *Choquet Order and Simplices. With Applications in Probabilistic Models*. *Springer Lecture Notes in Mathematics* **1145**, 1985. MR0808401 https://doi.org/10.1007/BFb0075051

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1284–1320
https://doi.org/10.1214/19-AIHP1002

Outliers in the spectrum for products of independent random matrices

Natalie Coston[a], Sean O'Rourke[a] and Philip Matchett Wood[b]

[a]*Department of Mathematics, University of Colorado at Boulder, Boulder, CO 80309, USA.*
E-mail: natalie.coston@colorado.edu; sean.d.orourke@colorado.edu
[b]*Department of Mathematics, University of Wisconsin-Madison, 480 Lincoln Dr., Madison, WI 53706, USA. E-mail: pmwood@math.wisc.edu*

Abstract. For fixed $m \geq 1$, we consider the product of m independent $n \times n$ random matrices with iid entries as $n \to \infty$. Under suitable assumptions on the entries of each matrix, it is known that the limiting empirical distribution of the eigenvalues is described by the mth power of the circular law. Moreover, this same limiting distribution continues to hold if each iid random matrix is additively perturbed by a bounded rank deterministic error. However, the bounded rank perturbations may create one or more outlier eigenvalues. We describe the asymptotic location of the outlier eigenvalues, which extends a result of Tao (*Probab. Theory Related Fields* **155** (2013) 231–263) for the case of a single iid matrix. Our methods also allow us to consider several other types of perturbations, including multiplicative perturbations.

Résumé. Pour un $m \geq 1$ fixé, nous considérons le produit de m matrices aléatoires indépendantes de taille $n \times n$, à coefficients i.i.d., lorsque $n \to \infty$. Sous certaines hypothèses sur les coefficients de chaque matrice, il est connu que la loi empirique limite des valeurs propres est décrite par la puissance m-ième de la loi circulaire. De plus, cette même loi limite apparaît toujours si chacune des matrices i.i.d. est perturbée additivement par une erreur déterministe de rang borné. Néanmoins, les perturbations de rang borné peuvent créer quelques valeurs propres atypiques (outliers). Nous décrivons la localisation asymptotique de ces valeurs propres atypiques, ce qui généralise un résultat de Tao (*Probab. Theory Related Fields* **155** (2013) 231–263) dans la cas d'une seule matrice i.i.d. Nos méthodes nous permettent également de considérer d'autres types de perturbations, dont des perturbations multiplicatives.

MSC: 60B20

Keywords: Product random matrices; Outlier eigenvalues; Perturbed products; Isotropic limit law

References

[1] G. Akemann and Z. Burda. Universal microscopic correlation functions for products of independent Ginibre matrices. *J. Phys. A: Math. Theor.* **45** (2012). MR2993423 https://doi.org/10.1088/1751-8113/45/46/465201
[2] G. Akemann, Z. Burda and M. Kieburg. Universal distribution of Lyapunov exponents for products of Ginibre matrices. *J. Phys. A: Math. Theor.* **47** (39) (2014). MR3262164 https://doi.org/10.1088/1751-8113/47/39/395202
[3] G. Akemann, J. R. Ipsen and M. Kieburg. Products of rectangular random matrices: Singular values and progressive scattering. *Phys. Rev. E* **88** 052118 (2013). https://doi.org/10.1103/PhysRevE.88.052118
[4] G. Akemann, J. R. Ipsen and E. Strahov. Permanental processes from products of complex and quaternionic induced Ginibre ensembles. *Random Matrices Theory Appl.* **3** (4) (2014) 1450014. MR3279619 https://doi.org/10.1142/S2010326314500142
[5] G. Akemann, M. Kieburg and L. Wei. Singular value correlation functions for products of Wishart random matrices. *J. Phys. A: Math. Theor.* **46** (2013). MR3081917 https://doi.org/10.1088/1751-8113/46/27/275205
[6] G. Akemann and E. Strahov. Hole probabilities and overcrowding estimates for products of complex Gaussian matrices. *J. Stat. Phys.* **151** (6) (2013) 987–1003. MR3063493 https://doi.org/10.1007/s10955-013-0750-8
[7] G. Anderson. Convergence of the largest singular value of a polynomial in independent Wigner matrices. *Ann. Probab.* **41** (3B) (2013) 2103–2181. MR3098069 https://doi.org/10.1214/11-AOP739
[8] P. Arbenz, W. Gander and G. Golub. Restricted rank modification of the symmetric eigenvalue problem: Theoretical considerations. *Linear Algebra Appl.* **104** (1988) 75–95. MR0944010 https://doi.org/10.1016/0024-3795(88)90309-6
[9] Z. D. Bai. Circular law. *Ann. Probab.* **25** (1997) 494–529. MR1428519 https://doi.org/10.1214/aop/1024404298
[10] Z. D. Bai and G. M. Pan. Limiting behavior of eigenvectors of large Wigner matrices. *J. Stat. Phys.* **146** (2012) 519–549. MR2880031 https://doi.org/10.1007/s10955-011-0407-4
[11] Z. D. Bai and J. Silverstein. No eigenvalues outside the support of the limiting spectral distribution of large-dimensional sample covariance matrices. *Ann. Probab.* **26** (1) (1998) 316–345. MR1617051 https://doi.org/10.1214/aop/1022855421

[12] Z. D. Bai and J. Silverstein. *Spectral Analysis of Large Dimensional Random Matrices*. *Mathematics Monograph Series* **2**. Science Press, Beijing, 2006. MR2567175 https://doi.org/10.1007/978-1-4419-0661-8

[13] J. Baik, G. Ben Arous and S. Péché. Phase transition of the largest eigenvalue for nonnull complex sample covariance matrices. *Ann. Probab.* **33** (5) (2005) 1643–1697. MR2165575 https://doi.org/10.1214/009117905000000233

[14] S. Belinschi, H. Bercovivi and M. Capitaine. On the outlying eigenvalues of a polynomial in large independent random matrices. Available at arXiv:1703.08102 [math.PR].

[15] S. T. Belinschi and M. Capitaine. Spectral properties of polynomials in independent Wigner and deterministic matrices. *J. Funct. Anal.* **273** (12) (2017) 3901–3963. MR3711884 https://doi.org/10.1016/j.jfa.2017.07.010

[16] F. Benaych-Georges, G. Cébron and J. Rochet. Fluctuation of matrix entries and application to outliers of elliptic matrices. *Canad. J. Math.* **70** (1) (2018) 3–25. MR3744883 https://doi.org/10.4153/CJM-2017-024-8

[17] F. Benaych-Georges, A. Guionnet and M. Maida. Fluctuations of the extreme eigenvalues of finite rank deformations of random matrices. *Electron. J. Probab.* **16** (2011) 1621–1662. MR2835249 https://doi.org/10.1214/EJP.v16-929

[18] F. Benaych-Georges, A. Guionnet and M. Maida. Large deviations of the extreme eigenvalues of random deformations of matrices. *Probab. Theory Related Fields* **154** (3–4) (2012) 703–751. MR3000560 https://doi.org/10.1007/s00440-011-0382-3

[19] F. Benaych-Georges and R. N. Rao. The eigenvalues and eigenvectors of finite, low rank perturbations of large random matrices. *Adv. Math.* **227** (1) (2011) 494–521. MR2782201 https://doi.org/10.1016/j.aim.2011.02.007

[20] F. Benaych-Georges and R. N. Rao. The singular values and vectors of low rank perturbations of large rectangular random matrices. *J. Multivariate Anal.* **111** (2012) 120–135. MR2944410 https://doi.org/10.1016/j.jmva.2012.04.019

[21] F. Benaych-Georges and J. Rochet. Outliers in the single ring theorem. *Probab. Theory Related Fields* **165** (2016) 313–363. MR3500273 https://doi.org/10.1007/s00440-015-0632-x

[22] R. Bhatia. *Matrix Analysis*. *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1997. MR1477662 https://doi.org/10.1007/978-1-4612-0653-8

[23] C. Bordenave. On the spectrum of sum and product of non-Hermitian random matrices. *Electron. Commun. Probab.* **16** (2011) 104–113. MR2772389 https://doi.org/10.1214/ECP.v16-1606

[24] C. Bordenave and M. Capitaine. Outlier eigenvalues for deformed i.i.d. random matrices. *Comm. Pure Appl. Math.* **69** (11) (2016) 2131–2194. MR3552011 https://doi.org/10.1002/cpa.21629

[25] C. Bordenave, P. Caputo, D. Chafaï and K. Tikhomirov. On the spectral radius of a random matrix: an upper bound without fourth moment. *Ann. Probab.* **46** (4) (2018) 2268–2286. MR3813992 https://doi.org/10.1214/17-AOP1228

[26] C. Bordenave and D. Chafaï. Around the circular law. *Probab. Surv.* **9** (2012) 1–89. MR2908617 https://doi.org/10.1214/11-PS183

[27] Z. Burda. Free products of large random matrices – a short review of recent developments. *J. Phys., Conf. Ser.* **473** (2013) 012002. https://doi.org/10.1088/1742-6596/473/1/012002

[28] Z. Burda, R. A. Janik and B. Waclaw. Spectrum of the product of independent random Gaussian matrices. *Phys. Rev. E* **81** (2010). MR2736204 https://doi.org/10.1103/PhysRevE.81.041132

[29] Z. Burda, A. Jarosz, G. Livan, M. A. Nowak and A. Swiech. Eigenvalues and singular values of products of rectangular Gaussian random matrices. *Phys. Rev. E* **82** (2010). MR2787469 https://doi.org/10.1103/PhysRevE.82.061114

[30] D. L. Burkholder. Distribution function inequalities for martingales. *Ann. Probab.* **1** (1973) 19–42. MR0365692 https://doi.org/10.1214/aop/1176997023

[31] M. Capitaine, C. Donati-Martin and D. Féral. The largest eigenvalue of finite rank deformation of large Wigner matrices: Convergence and non universality of the fluctuations. *Ann. Probab.* **37** (1) (2009) 1–47. MR2489158 https://doi.org/10.1214/08-AOP394

[32] M. Capitaine, C. Donati-Martin and D. Féral. Central limit theorems for eigenvalues of deformations of Wigner matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **48** (1) (2012) 107–133. MR2919200 https://doi.org/10.1214/10-AIHP410

[33] M. Capitaine, C. Donati-Martin, D. Féral and M. Février. Free convolution with a semi-circular distribution and eigenvalues of spiked deformations of Wigner matrices. *Electron. J. Probab.* **16** (64) (2011) 1750–1792. MR2835253 https://doi.org/10.1214/EJP.v16-934

[34] N. Coston, S. O'Rourke and P. Wood. Outliers in the spectrum for products of independent random matrices. Available at arXiv:1711.07420 [math.PR].

[35] D. Féral and S. Péché. The largest eigenvalue of rank one deformation of large Wigner matrices. *Comm. Math. Phys.* **272** (2007) 185–228. MR2291807 https://doi.org/10.1007/s00220-007-0209-3

[36] P. J. Forrester. Lyapunov exponents for products of complex Gaussian random matrices. *J. Stat. Phys.* **151** (2013) 796–808. MR3055376 https://doi.org/10.1007/s10955-013-0735-7

[37] P. J. Forrester. Probability of all eigenvalues real for products of standard Gaussian matrices. *J. Phys. A* **47** (2014) 065202. MR3159513 https://doi.org/10.1088/1751-8113/47/6/065202

[38] F. Götze and T. Tikhomirov. On the asymptotic spectrum of products of independent random matrices. Available at arXiv:1012.2710.

[39] U. Haagerup and S. Thorbjørnsen. A new application of random matrices: $\mathrm{Ext}(C^*_{\mathrm{red}}(F_2))$ is not a group. *Ann. of Math.* **162** (2005) 711–775. MR2183281 https://doi.org/10.4007/annals.2005.162.711

[40] R. A. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge Univ. Press, 1991. MR1084815

[41] A. Knowles and J. Yin. The isotropic semicircle law and deformation of Wigner matrices. *Comm. Pure Appl. Math.* **LXVI** (2013) 1663–1749. MR3103909 https://doi.org/10.1002/cpa.21450

[42] A. Knowles and J. Yin. The outliers of a deformed Wigner matrix. *Ann. Probab.* **42** (5) (2014) 1980–2031. MR3262497 https://doi.org/10.1214/13-AOP855

[43] A. B. J. Kuijlaars and L. Zhang. Singular values of products of Ginibre random matrices, multiple orthogonal polynomials and hard edge scaling limits. *Comm. Math. Phys.* **332** (2) (2014) 759–781. MR3257662 https://doi.org/10.1007/s00220-014-2064-3

[44] Y. Nemish. No outliers in the spectrum of the product of independent non-Hermitian random matrices with independent entries. *J. Theoret. Probab.* (2016) 1–43. MR3769819

[45] Y. Nemish. Local law for the product of independent non-Hermitian random matrices with independent entries. *Electron. J. Probab.* **22** (2017), paper no. 22, 35. MR3622892 https://doi.org/10.1214/17-EJP38

[46] S. O'Rourke and D. Renfrew. Low rank perturbations of large elliptic random matrices. *Electron. J. Probab.* **19** (43) (2014) 1–65. MR3210544 https://doi.org/10.1214/EJP.v19-3057

[47] S. O'Rourke, D. Renfrew, A. Soshnikov and V. Vu. Products of independent elliptic random matrices. *J. Stat. Phys.* **160** (1) (2015) 89–119. MR3357969 https://doi.org/10.1007/s10955-015-1246-5

[48] S. O'Rourke and A. Soshnikov. Products of independent non-Hermitian random matrices. *Electron. J. Probab.* **16, Art. 81** (2011) 2219–2245. MR2861673 https://doi.org/10.1214/EJP.v16-954

[49] S. O'Rourke and P. Wood. Spectra of nearly Hermitian random matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (3) (2017) 1241–1279. MR3689967 https://doi.org/10.1214/16-AIHP754

[50] P. Pagacz and M. Wojtylak. Random perturbations of matrix polynomials. Available at arXiv:1703.01858 [math.PR].

[51] S. Péché. The largest eigenvalue of small rank perturbations of Hermitian random matrices. *Probab. Theory Related Fields* **134** (1) (2006) 127–173. MR2221787 https://doi.org/10.1007/s00440-005-0466-z

[52] A. Pizzo, D. Renfrew and A. Soshnikov. On finite rank deformations of Wigner matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **49** (1) (2013) 64–94. MR3060148 https://doi.org/10.1214/11-AIHP459

[53] A. B. Rajagopalan. Outlier eigenvalue fluctuations of perturbed iid matrices. Available at arXiv:1507.01441 [math.PR]. MR3389123

[54] D. Renfrew and A. Soshnikov. On finite rank deformations of Wigner matrices II: Delocalized perturbations. *Random Matrices Theory Appl.* **02** (2013) 1250015. MR3039820 https://doi.org/10.1142/S2010326312500153

[55] J. Rochet. Complex outliers of Hermitian random matrices. *J. Theoret. Probab.* **30** (4) (2017) 1624–1654. MR3736185 https://doi.org/10.1007/s10959-016-0686-4

[56] E. Strahov. Differential equations for singular values of products of Ginibre random matrices. *J. Phys. A: Math. Theor.* **47** (2014) 325203. MR3240458 https://doi.org/10.1088/1751-8113/47/32/325203

[57] T. Tao. Outliers in the spectrum of iid matrices with bounded rank perturbations. *Probab. Theory Related Fields* **155** (2013) 231–263. MR3010398 https://doi.org/10.1007/s00440-011-0397-9

[58] T. Tao and V. Vu. Random matrices: Universality of ESDs and the circular law. *Ann. Probab.* **38** (5) (2010) 2023–2065. MR2722794 https://doi.org/10.1214/10-AOP534

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1321–1335
https://doi.org/10.1214/19-AIHP1003

Interacting self-avoiding polygons

Volker Betz, Helge Schäfer and Lorenzo Taggi

FB Mathematik, TU Darmstadt, Darmstadt, Germany. E-mail: betz@mathematik.tu-darmstadt.de; url: http://www.mathematik.tu-darmstadt.de/~betz/; E-mail: hschaefer@mathematik.tu-darmstadt.de; url: https://www.mathematik.tu-darmstadt.de/fb/personal/details/helge_schaefer.de.jsp; E-mail: taggi@mathematik.tu-darmstadt.de; url: https://sites.google.com/site/lorenzotaggiswebpage2

Abstract. We consider a system of self-avoiding polygons interacting through a potential that penalizes or rewards the number of mutual touchings and we provide an exact computation of the critical curve separating a regime of long polygons from a regime of localized polygons. Moreover, we prove the existence of a sub-region of the phase diagram where the self-avoiding polygons are space filling and we provide a non-trivial characterization of the regime where the polygon length admits uniformly bounded exponential moments.

Résumé. Dans cet article, nous considérons un système de polygones auto-évitants interagissant au moyen d'un potentiel qui pénalise ou récompense le nombre de points de contacts. Nous calculons la forme exacte de la courbe critique séparant un régime de polygones longs d'un régime de polygones localisés. En outre, nous prouvons l'existence d'un sous-domaine du diagramme de phase dans lequel les polygones remplissent l'espace dans un sens faible et donnons une caractérisation non-triviale du sous-régime où les longueurs des polygones ont des moments exponentiels bornés.

MSC: 60K35; 82B27; 82B20

Keywords: Phase transition; Weakly space filling loops; Spatial random permutations

References

[1] V. Betz and L. Taggi. Scaling limit of a self-avoiding walk interacting with spatial random permutations, 2017. Available at arXiv:1612.07234.
[2] V. Betz and D. Ueltschi. Spatial random permutations and infinite cycles. *Comm. Math. Phys.* **285** (2009) 469–501. MR2461985 https://doi.org/10.1007/s00220-008-0584-4
[3] V. Betz and D. Ueltschi. Critical temperature of dilute Bose gases. *Phys. Rev. A* **81** (2010) 023611.
[4] F. den Hollander. *Random Polymers. École d'Été de Probabilités de Saint-Flour XXXVII – 2007*, *Lecture Notes in Mathematics*. Springer, ISSN. print edition: 0075-8434. MR2504175 https://doi.org/10.1007/978-3-642-00333-2
[5] H. Duminil-Copin, G. Kozma and A. Yadin. Supercritical self-avoiding walks are space-filling. *Ann. Inst. Henri Poincaré B, Probab. Stat.* **50** (2) (2014) 315–326. MR3189073 https://doi.org/10.1214/12-AIHP528
[6] R. P. Feynman. Atomic theory of the λ transition in helium. *Phys. Rev.* **91** (1953) 1291–1301.
[7] R. Kenyon. Conformal invariance of loops in the double dimer model. *Comm. Math. Phys.* **2** (2014). MR3165463 https://doi.org/10.1007/s00220-013-1881-0
[8] R. Kikuchi. λ transition of liquid helium. *Phys. Rev.* **96** (1954) 563–568.
[9] N. Madras and G. Slade. *The Self-Avoiding Walk*. Birkhäuser, 2013. Reprint of the 1996 Edition. MR2986656 https://doi.org/10.1007/978-1-4614-6025-1

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1336–1357
https://doi.org/10.1214/19-AIHP1004

Lower bounds for fluctuations in first-passage percolation for general distributions

Michael Damron[a], Jack Hanson[b], Christian Houdré[a] and Chen Xu[c]

[a]School of Mathematics, Georgia Institute of Technology, 686 Cherry St., Atlanta, GA 30332, USA. E-mail: mdamron6@gatech.edu; houdre@math.gatech.edu
[b]Department of Mathematics, The City College of NY, 160 Convent Ave, New York, NY 10031, USA. E-mail: jhanson@ccny.cuny.edu
[c]Uber Technologies Inc, 1455 Market St., San Francisco, CA 94103, USA. E-mail: xcabel@gmail.com

Abstract. In first-passage percolation (FPP), one assigns i.i.d. weights to the edges of the cubic lattice $\mathbb{Z}^d$ and analyzes the induced weighted graph metric. If $T(x, y)$ is the distance between vertices x and y, then a primary question in the model is: what is the order of the fluctuations of $T(0, x)$? It is expected that the variance of $T(0, x)$ grows like the norm of x to a power strictly less than 1, but the best lower bounds available are (only in two dimensions) of order $\log \|x\|$. This result was found in the '90s and there has not been any improvement since. In this paper, we address the problem of getting stronger fluctuation bounds: to show that $T(0, x)$ is with high probability not contained in an interval of size $o(\log \|x\|)^{1/2}$, and similar statements for FPP in thin cylinders. Such statements have been proved for special edge-weight distributions, and here we obtain such bounds for general edge-weight distributions. The methods involve inducing a fluctuation in the number of edges in a box whose weights are of "hi-mode" (large).

Résumé. En percolation de premier passage (PPP), on attribue des poids i.i.d. aux arêtes du réseau cubique $\mathbb{Z}^d$ et analyse la métrique de graphe induite. Si $T(x, y)$ dénote la distance entre les sommets x et y, une question fondamentale est de trouver l'ordre des fluctuations de $T(0, x)$. Il est escompté que la variance de $T(0, x)$ croît comme la norme, $\|x\|$, de x à une puissance strictement inférieure à 1, mais les meilleures bornes inférieures disponibles à ce jour (et seulement pour $d = 2$) sont d'ordres logarithmiques. Ce résultat a été démontré dans les années 90 et il a connu peu d'amélioration depuis. Dans ce papier, nous abordons le problème d'obtenir des bornes inférieures plus strictes, en montrant qu'avec très grande probabilité, la distance $T(0, y)$ n'est pas contenue dans un interval de taille $o(\log \|x\|)^{1/2}$. Un résultat similaire est aussi valide en percolation de dernier passage (PDP) dans des cylindres minces. Ce type de résultats qui n'avait été obtenu que pour des classes particulières de poids est ici démontré en toute généralité. Les (nouvelles) méthodes développées ici consistent à induire une fluctuation du nombre d'arêtes dans une boîte dont les poids sont en "mode-haut."

MSC: 60K35

Keywords: First-passage percolation; Fluctuations; Small-ball probability

References

[1] M. Aizenman, F. Germinet, A. Klein and S. Warzel. On Bernoulli decompositions for random variables, concentration bounds, and spectral localization. *Probab. Theory Related Fields* **143** (2009) 219–238. MR2449128 https://doi.org/10.1007/s00440-007-0125-7
[2] K. Alexander. Approximation of subadditive functions and convergence rates in limiting-shape results. *Ann. Probab.* **25** (1997) 30–55. MR1428498 https://doi.org/10.1214/aop/1024404277
[3] A. Auffinger and M. Damron. Differentiability at the edge of the percolation cone and related results in first-passage percolation. *Probab. Theory Related Fields* **156** (2013) 193–227. MR3055257 https://doi.org/10.1007/s00440-012-0425-4
[4] A. Auffinger, M. Damron and J. Hanson. Rate of convergence of the mean for sub-additive ergodic sequences. *Adv. Math.* **285** (2014) 138–181. MR3406498 https://doi.org/10.1016/j.aim.2015.07.028
[5] A. Auffinger, M. Damron and J. Hanson. *50 Years of First-Passage Percolation*. *University Lecture Series* **68**. American Mathematical Society, Providence, RI, 2017. v+161 pp. MR3729447
[6] E. Bates and S. Chatterjee. Fluctuation lower bounds in planar random growth models, 2018. Available at arXiv:1810.03656.
[7] M. Benaïm and R. Rossignol. Exponential concentration for first passage percolation through modified Poincaré inequalities. *Ann. Inst. Henri Poincaré B, Probab. Stat.* **44** (2008) 544–573. MR2451057 https://doi.org/10.1214/07-AIHP124
[8] I. Benjamini, G. Kalai and O. Schramm. First passage percolation has sublinear distance variance. *Ann. Probab.* **31** (2003) 1970–1978. MR2016607 https://doi.org/10.1214/aop/1068646373

[9] R. Cerf and M. Théret. Weak shape theorem in first passage percolation with infinite passage times. *Ann. Inst. Henri Poincaré B, Probab. Stat.* **52** (2016) 1351–1381. MR3531712 https://doi.org/10.1214/15-AIHP686
[10] S. Chatterjee. The universal relation between scaling exponents in first-passage percolation. *Ann. of Math. (2)* **177** (2013) 1–35. MR3010809 https://doi.org/10.4007/annals.2013.177.2.7
[11] S. Chatterjee. A general method for lower bounds on fluctuations of random variables. *Ann. Probab.* (2017). To appear. MR3980917 https://doi.org/10.1214/18-AOP1304
[12] S. Chatterjee and P. Dey. Central limit theorem for first-passage percolation time across thin cylinders. *Probab. Theory Related Fields* **156** (2013) 613–663. MR3078282 https://doi.org/10.1007/s00440-012-0438-z
[13] M. Damron, J. Hanson and P. Sosoe. Sublinear variance in first-passage percolation for general distributions. *Probab. Theory Related Fields* **163** (2015) 223–258. MR3405617 https://doi.org/10.1007/s00440-014-0591-7
[14] M. Damron, W.-K. Lam and X. Wang. Asymptotics for $2D$ critical first-passage percolation. *Ann. Probab.* **45** (2017) 2941–2970. MR3706736 https://doi.org/10.1214/16-AOP1129
[15] R. Durrett. Oriented percolation in two dimensions. *Ann. Probab.* **12** (1984) 999–1040. MR0757768
[16] R. Gong, C. Houdré and J. Lember. Lower bounds on the generalized central moments of the optimal alignments score of random sequences. *J. Theoret. Probab.* **31** (2018) 643–683. MR3803910 https://doi.org/10.1007/s10959-016-0730-4
[17] G. Grimmett *Percolation*, 2nd edition. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **321**. Springer-Verlag, Berlin, 1999. xiv+444 pp. MR1707339 https://doi.org/10.1007/978-3-662-03981-6
[18] C. Houdré and J. Ma. On the order of the central moments of the length of the longest common subsequence in random words. In *High Dimensional Probability VII* 105–136. *Progr. Probab.* **71**. Springer, Cham, 2016. MR3565261 https://doi.org/10.1007/978-3-319-40519-3_5
[19] C. Houdré and C. Xu. Concentration of geodesics in directed Bernoulli percolation, 2018. Available at arXiv:1607.02219.
[20] C. Houdré and C. Xu. Power low bounds for the central moments of the last passage time for directed percolation in a thin rectangle, 2018. Available at arXiv:1905.10034.
[21] K. Johansson. Shape fluctuations and random matrices. *Comm. Math. Phys.* **209** (2000) 437–476. MR1737991 https://doi.org/10.1007/s002200050027
[22] H. Kesten. Aspects of first passage percoclation. In *École d'Été de Probabilités de Saint Flour XIV* 125–264. *Lecture Notes in Mathematics* **1180**, 1986. MR0876084 https://doi.org/10.1007/BFb0074919
[23] H. Kesten. On the speed of convergence in first-passage percolation. *Ann. Appl. Probab.* **3** (1993) 296–338. MR1221154
[24] J. Lember and H. Matzinger. Standard deviation of the longest common subsequence. *Ann. Probab.* **37** (2009) 1192–1235. MR2537552 https://doi.org/10.1214/08-AOP436
[25] C. Licea, C. M. Newman and M. Piza. Superdiffusivity in first-passage percolation. *Probab. Theory Related Fields* **106** (1996) 559–591. MR1421992 https://doi.org/10.1007/s004400050075
[26] R. Marchand. Strict inequalities for the time constant in first passage percolation. *Ann. Appl. Probab.* **12** (2002) 1001–1038. MR1925450 https://doi.org/10.1214/aoap/1031863179
[27] C. M. Newman and M. Piza. Divergence of shape fluctuations in two dimensions. *Ann. Probab.* **23** (1995) 977–1005. MR1349159
[28] R. Pemantle and Y. Peres. Planar first-passage percolation times are not tight. In *Probability and Phase Transition (Cambridge, 1993)* 261–264. *NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci.* **420**. Kluwer Acad. Publ., Dordrecht, 1994. MR1283187 https://doi.org/10.1007/978-94-015-8326-8_16
[29] E. Sperner. Ein satz über Untermengen einer endlichen Menge. *Math. Z.* **27** (1928) 544–548. MR1544925 https://doi.org/10.1007/BF01171114
[30] R. Tessera. Speed of convergence in first passage percolation and geodesicity of the average distance. *Ann. Inst. Henri Poincaré B, Probab. Stat.* **54** (2014) 569–586. MR3795059 https://doi.org/10.1214/16-AIHP810
[31] C. Xu. Topics in percolation and sequence analysis. Ph. D. Thesis, Georgia Institute of Technology, 2018.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1358–1390
https://doi.org/10.1214/19-AIHP1006

The sharp phase transition for level set percolation of smooth planar Gaussian fields

Stephen Muirhead[a] and Hugo Vanneuville[b]

[a]*Department of Mathematics, King's College London. Current address: School of Mathematical Sciences, Queen Mary University of London. E-mail: s.muirhead@qmul.ac.uk*

[b]*Univ. Lyon 1, UMR5208, Institut Camille Jordan, 69100 Villeurbanne, France. E-mail: vanneuville@math.univ-lyon1.fr*

Abstract. We prove that the connectivity of the level sets of a wide class of smooth centred planar Gaussian fields exhibits a phase transition at the zero level that is analogous to the phase transition in Bernoulli percolation. In addition to symmetry, positivity and regularity conditions, we assume only that correlations decay polynomially with exponent larger than two – roughly equivalent to the integrability of the covariance kernel – whereas previously the phase transition was only known in the case of the Bargmann–Fock covariance kernel which decays super-exponentially. We also prove that the phase transition is *sharp*, demonstrating, without any further assumption on the decay of correlations, that in the sub-critical regime crossing probabilities decay exponentially.

Key to our methods is the white-noise representation of a Gaussian field; we use this on the one hand to prove new quasi-independence results, inspired by the notion of influence from Boolean functions, and on the other hand to establish sharp thresholds via the OSSS inequality for i.i.d. random variables, following the recent approach of Duminil-Copin, Raoufi and Tassion.

Résumé. Nous démontrons l'existence d'un phénomène de transition de phase pour les propriétés de connexion de lignes de niveau d'une grande classe de champs gaussiens planaires. En plus d'hypothèses de symétrie, régularié et positivité des corrélations, nous supposons que la covariance de ces champs décroît à vitesse polynomiale avec un exposant strictement plus grand que 2. Nous montrons par ailleurs que la transition de phase est "nette" dans le sens que, dans la phase sous-critique, les probabilités de connexion convergent vers 0 à vitesse exponentielle. Dans nos preuves, nous utilisons de façon centrale l'écriture des champs gaussiens lisses à l'aide d'un bruit blanc planaire. Nous utilisons cette représentation pour prouver de nouveaux résultats de quasi-indépendance spatiale, inspirés par la notion d'influence en théorie des fonctions booléennes. Par ailleurs, la structure produit du buit blanc nous permet d'utiliser l'inégalité d'OSSS, qui est une inégalité clef dans la récente approche d'étude des transitions de phase par Duminil-Copin, Raoufi et Tassion.

MSC: 60G60; 60K35

Keywords: Gaussian fields; Percolation; Sharp phase transition

References

[1] R. Adler and J. Taylor. *Random Fields and Geometry*. Springer, New York, 2007. MR2319516
[2] D. Ahlberg and R. Baldasso. Noise sensitivity and Voronoi percolation. *Electron. J. Probab.* **23** (2018) 108.
[3] K. Alexander. Boundedness of level lines for two-dimensional random fields. *Ann. Probab.* **24** (4) (1996) 1653–1674. MR1415224 https://doi.org/10.1214/aop/1041903201
[4] J. Azaïs and M. Wschebor. *Level Sets and Extrema of Random Processes and Fields*. John Wiley & Sons, Inc., Hoboken, NJ, 2009. MR2478201 https://doi.org/10.1002/9780470434642
[5] V. Beffara and D. Gayet. Percolation of random nodal lines. *Publ. Math.* **126** (2017) 131–176. MR3735866 https://doi.org/10.1007/s10240-017-0093-0
[6] D. Beliaev and S. Muirhead. Discretisation schemes for level sets of planar Gaussian fields. *Comm. Math. Phys.* **359** (2018) 869–913. MR3784534 https://doi.org/10.1007/s00220-018-3084-1
[7] D. Beliaev, S. Muirhead and I. Wigman. Russo–Seymour–Welsh estimates for the Kostlan ensemble of random polynomials. Preprint, 2017. Available at arXiv:1709.08961.
[8] I. Benjamini, G. Kalai and O. Schramm. Noise sensitivity of Boolean functions and applications to percolation. *Publ. Math.* **90** (1) (1999) 5–43. MR1813223
[9] I. Benjamini and O. Schramm. Conformal invariance of Voronoi percolation. *Comm. Math. Phys.* **197** (1) (1998) 75–107. MR1646475 https://doi.org/10.1007/s002200050443

[10] A. Berlinet and C. Thomas-Agnan. *Reproducing Kernel Hilbert Spaces in Probability and Statistics*. Springer, 2004. MR2239907 https://doi.org/10.1007/978-1-4419-9096-9

[11] E. D. Bernardino, A. Estrade and J. León. A test of gaussianity based on the Euler characteristic of excursion sets. *Electron. J. Stat.* **11** (1) (2017) 843–890. MR3629017 https://doi.org/10.1214/17-EJS1248

[12] E. Bogomolny, R. Dubertrand and C. Schmit. SLE description of the nodal lines of random wavefunctions. *J. Phys. A: Math. Theor.* **40** (2007) 381–395. MR2304899 https://doi.org/10.1088/1751-8113/40/3/003

[13] E. Bogomolny and C. Schmit. Random wavefunctions and percolation. *J. Phys. A: Math. Theor.* **40** (2007) 14033–14043. MR2438110 https://doi.org/10.1088/1751-8113/40/47/001

[14] B. Bollobás, O. Riordan. *Percolation*. Cambridge University Press, Cambridge, 2006. MR2283880 https://doi.org/10.1017/CBO9781139167383

[15] F. Camia and C. M. Newman. Critical percolation exploration path and SLE_6: A proof of convergence. *Probab. Theory Related Fields* **139** (3–4) (2007) 473–519. MR2322705 https://doi.org/10.1007/s00440-006-0049-7

[16] J. Cuzick. A central limit theorem for the number of zeros of a stationary Gaussian process. *Ann. Probab.* **4** (1976) 547–556. MR0420809 https://doi.org/10.1214/aop/1176996026

[17] H. Duminil-Copin, A. Raoufi and V. Tassion Subcritical phase of d-dimensional Poisson–Boolean percolation and its vacant set. Preprint, 2018. Available at arXiv:1805.00695.

[18] H. Duminil-Copin, A. Raoufi and V. Tassion. Exponential decay of connection probabilities for subcritical Voronoi percolation in $\mathbb{R}^d$. *Probab. Theory Related Fields* **173** (1–2) (2019) 479–490. MR3916112 https://doi.org/10.1007/s00440-018-0838-9

[19] H. Duminil-Copin, A. Raoufi and V. Tassion. Sharp phase transition for the random-cluster and Potts models via decision trees. *Ann. of Math.* **189** (1) (2019) 75–99. MR3898174 https://doi.org/10.4007/annals.2019.189.1.2

[20] C. Garban and J. Steif. *Noise Sensitivity of Boolean Functions and Percolation*. Cambridge University Press, Cambridge, 2014. MR3468568 https://doi.org/10.1017/CBO9781139924160

[21] B. Graham and G. Grimmett. Influence and sharp-threshold theorems for monotonic measures. *Ann. Probab.* **34** (2006) 1726–1745. MR2271479 https://doi.org/10.1214/009117906000000278

[22] G. Grimmett. *Percolation*. Springer, Berlin, Germany, 1999. MR1707339 https://doi.org/10.1007/978-3-662-03981-6

[23] T. Harris. A lower bound for the critical probability in a certain percolation process. *Proc. Camb. Philos. Soc.* **56** (1960) 13–20. MR0115221

[24] D. Higdon. Space and space-time modeling using process convolutions. In *Quantitative Methods for Current Environmental Issues*, C. Anderson, V. Barnett, P. Chatwin and A. El-Shaarawi (Eds). Spring, London, 2002. MR2059819

[25] S. Janson. *Gaussian Hilbert Spaces*. Cambridge University Press, Cambridge, 1997. MR1474726 https://doi.org/10.1017/CBO9780511526169

[26] H. Kesten. The critical probability of bond percolation on the square lattice equals $1/2$. *Comm. Math. Phys.* **74** (1980) 41–59. MR0575895

[27] H. Kesten. Scaling relations for 2d-percolation. *Comm. Math. Phys.* **109** (1) (1987) 109–156. MR0879034

[28] G. Kimeldorf and G. Wahba. A correspondence between Bayesian estimation on stochastic processes and smoothing by splines. *Ann. Math. Stat.* **41** (2) (1970) 495–502. MR0254999 https://doi.org/10.1214/aoms/1177697089

[29] T. Malevich. Asymptotic normality of the number of crossing of level zero by a Gaussian process. *Theory Probab. Appl.* **14** (2) (1969) 287–295. MR0268952

[30] S. Molchanov and A. Stepanov. Percolation in random fields. I. *Theoret. Math. Phys.* **55** (2) (1983) 478–484. MR0734878

[31] S. Molchanov and A. Stepanov. Percolation in random fields. II. *Theoret. Math. Phys.* **55** (3) (1983) 592–599. MR0734878

[32] S. Molchanov and A. Stepanov. Percolation in random fields. III. *Theoret. Math. Phys.* **67** (2) (1986) 434–439. MR0851557

[33] F. Nazarov and M. Sodin. Fluctuations in random complex zeroes: Asymptotic normality revisited. *Int. Math. Res. Not.* **2011** (24) (2011) 5720–5759. MR2863379 https://doi.org/10.1093/imrn/rnr007

[34] F. Nazarov and M. Sodin. Asymptotic laws for the spatial distribution and the number of connected components of zero sets of Gaussian random functions. *Math. Phys. Anal. Geom.* **12** (3) (2016) 205–278. MR3522141 https://doi.org/10.15407/mag12.03.205

[35] F. Nazarov, M. Sodin and A. Volberg. Transportation to random zeroes by the gradient flow. *Geom. Funct. Anal.* **17** (3) (2007) 887–935. MR2346279 https://doi.org/10.1007/s00039-007-0613-z

[36] F. Nazarov, M. Sodin and A. Volberg. The Jancovici–Lebowitz–Manificat law for large fluctuations of random complex zeroes. *Comm. Math. Phys.* **284** (3) (2008) 833–865. MR2452596 https://doi.org/10.1007/s00220-008-0646-7

[37] R. O'Donnell, M. Saks, O. Schramm and R. Servedio. Every decision tree has an influential variable. In *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)* 31–39, 2005.

[38] L. Pitt. Positively correlated normal variables are associated. *Ann. Probab.* **10** (2) (1982) 496–499. MR0665603

[39] A. Poularikas. *The Handbook of Formulas and Tables for Signal Processing*. CRC Press, Boca Raton, 1999.

[40] C. Rasmussen and C. Williams. *Gaussian Processes for Machine Learning*. MIT Press, Cambridge, 2006. MR2514435

[41] A. Rivera and H. Vanneuville. The critical threshold for Bargmann–Fock percolation. *Ann. H. Lebesgue* To appear, 2019. Available at arXiv:1711.05012.

[42] A. Rivera and H. Vanneuville. Quasi-independence for nodal lines. *Ann. Inst. H. Poincaré Probab. Statist.* To appear, 2019. Available at arXiv:1711.05009. MR4010948 https://doi.org/10.1214/18-aihp931

[43] P. Rodriguez. A 0-1 law for the massive Gaussian free field. *Probab. Theory Related Fields* **169** (2017) 901–930. MR3719059 https://doi.org/10.1007/s00440-016-0743-z

[44] O. Schramm and S. Sheffield. Contour lines of the two-dimensional discrete Gaussian free field. *Acta Math.* **202** (1) (2009) 21–137. MR2486487 https://doi.org/10.1007/s11511-009-0034-y

[45] O. Schramm and J. Steif. Quantitative noise sensitivity and exceptional times for percolation. *Ann. of Math.* **171** (2) (2010) 619–672. MR2630053 https://doi.org/10.4007/annals.2010.171.619

[46] S. Sheffield. Exploration trees and conformal loop ensembles. *Duke Math. J.* **147** (1) (2009) 79–129. MR2494457 https://doi.org/10.1215/00127094-2009-007

[47] S. Smirnov. Towards conformal invariance of $2d$ lattice models. In *Proceedings of the ICM*, 2007. MR2275653

[48] S. Smirnov and W. Werner. Critical exponents for two-dimensional percolation. *Math. Res. Lett.* **8** (5–6) (2001) 729–744. MR1879816 https://doi.org/10.4310/MRL.2001.v8.n6.a4

[49] V. Tassion. Crossing probabilities for Voronoi percolation. *Ann. Probab.* **44** (5) (2016) 3385–3398. MR3551200 https://doi.org/10.1214/15-AOP1052

[50] A. Weinrib. Percolation threshold of a two-dimensional continuum system. *Phys. Rev. B* **26** (3) (1982) 1352–1361. MR0667953 https://doi.org/10.1103/physrevb.26.1352
[51] A. Weinrib. Long-range correlated percolation. *Phys. Rev. B* **29** (1) (1984) 387–395. MR0729982 https://doi.org/10.1103/physrevb.29.387
[52] H. Wendland. *Scattered Data Approximation*. *Cambridge Monographs on Applied and Computational Mathematics*. Cambridge University Press, Cambridge, 2005. MR2131724

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1391–1418
https://doi.org/10.1214/19-AIHP1007

Parameter recovery in two-component contamination mixtures: The L^2 strategy

Sébastien Gadat[a], Jonas Kahn[b], Clément Marteau[c] and Cathy Maugis-Rabusseau[d]

[a] *Toulouse School of Economics, Université Toulouse 1 – Capitole, 21 allées de Brienne, 31000 Toulouse, France. E-mail: sebastien.gadat@tse-fr.eu*
[b] *Institut de Mathématiques de Toulouse; UMR5219, Université de Toulouse; CNRS, UPS, F-31062 Toulouse Cedex 9, France. E-mail: jonas.kahn@math.univ-toulouse.fr*
[c] *Univ Lyon, Université Claude Bernard Lyon 1, CNRS UMR 5208, Institut Camille Jordan, F-69622 Villeurbanne, France. E-mail: marteau@math.univ-lyon1.fr*
[d] *Institut de Mathématiques de Toulouse; UMR5219, Université de Toulouse; CNRS, INSA, F-31077 Toulouse Cedex 4, France. E-mail: cathy.maugis@insa-toulouse.fr*

Abstract. In this paper, we consider a parametric density contamination model. We work with a sample of i.i.d. data with a common density, $f^\star = (1-\lambda^\star)\phi + \lambda^\star\phi(\cdot - \mu^\star)$, where the shape ϕ is assumed to be known. We establish the optimal rates of convergence for the estimation of the mixture parameters $(\lambda^\star, \mu^\star) \in (0,1) \times \mathbb{R}^d$. In particular, we prove that the classical parametric rate $1/\sqrt{n}$ cannot be reached when at least one of these parameters is allowed to tend to 0 with n.

Résumé. Dans cet article, nous étudions un modèle de contamination paramétrique. Nous considérons un échantillon i.i.d de densité $f^\star = (1-\lambda^\star)\phi + \lambda^\star\phi(\cdot - \mu^\star)$, où la fonction ϕ est supposée connue. Nous établissons des vitesses de convergence optimales pour l'estimation des paramètres de mélange $(\lambda^\star, \mu^\star) \in (0,1) \times \mathbb{R}^d$. En particulier, nous prouvons que la vitesse paramétrique usuelle $1/\sqrt{n}$ ne peut pas être atteinte quand au moins un de ces paramètres est amené à tendre vers 0 avec n.

MSC: Primary 62G05; 62F15; secondary 62G20

Keywords: $\mathbb{L}^2$ contrast; Parameter estimation; Rate of convergence; Two-component contamination mixture model

References

[1] A. Azzalini. A class of distributions which includes the normal ones. *Scand. J. Stat.* (1985) 171–178. MR0808153
[2] S. Balakrishnan, M. Wainwright and B. Yu. Statistical guarantees for the EM algorithm: From population to sample-based analysis. *Ann. Statist.* **45** (2017) 77–120. MR3611487 https://doi.org/10.1214/16-AOS1435
[3] D. Bontemps and S. Gadat. Bayesian methods for the shape invariant model. *Electron. J. Stat.* **8** (1) (2014) 1522–1568. MR3263130 https://doi.org/10.1214/14-EJS933
[4] L. Bordes, S. Mottelet and P. Vandekerkhove. Semiparametric estimation of a two-component mixture model. *Ann. Statist.* **34** (3) (2006) 1204–1232. MR2278356 https://doi.org/10.1214/009053606000000353
[5] F. Bunea, A. B. Tsybakov, M. H. Wegkamp and A. Barbu. Spades and mixture models. *Ann. Statist.* **38** (4) (2010) 2525–2558. MR2676897 https://doi.org/10.1214/09-AOS790
[6] C. Butucea and P. Vandekerkhove. Semiparametric mixtures of symmetric distributions. *Scand. J. Stat.* **41** (1) (2014) 227–239. MR3181141 https://doi.org/10.1111/sjos.12015
[7] T. T. Cai, X. J. Jeng and J. Jin. Optimal detection of heterogeneous and heteroscedastic mixtures. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **73** (5) (2011) 629–662. https://doi.org/10.1111/j.1467-9868.2011.00778.x. MR2867452 https://doi.org/10.1111/j.1467-9868.2011.00778.x
[8] T. T. Cai, J. Jin and M. G. Low. Estimation and confidence sets for sparse normal mixtures. *Ann. Statist.* **35** (6) (2007) 2421–2449. MR2382653 https://doi.org/10.1214/009053607000000334
[9] J. H. Chen. Optimal rate of convergence for finite mixture models. *Ann. Statist.* **23** (1) (1995) 221–233. MR1331665 https://doi.org/10.1214/aos/1176324464
[10] A. P. Dempster, N. M. Laird and D. B. Rubin. Maximum likelihood from incomplete data via the EM algorithm. *J. Roy. Statist. Soc. Ser. B* **39** (1) (1977) 1–38. With discussion. MR0501537
[11] S. Frühwirth-Schnatter. *Finite Mixture and Markov Switching Models*. *Springer Series in Statistics*, xx+492. Springer, New York, 2006. MR2265601
[12] S. Gadat, J. Kahn, C. Marteau and C. Maugis-Rabusseau Parameter recovery in two-component contamination mixtures: The L^2 strategy. Preprint, hal-01713035, 2018. Available at https://hal.archives-ouvertes.fr/hal-01713035.

[13] C. R. Genovese and L. Wasserman. Rates of convergence for the Gaussian mixture sieve. *Ann. Statist.* **28** (4) (2000) 1105–1127. MR1810921
[14] S. Ghosal and A. W. van der Vaart. Entropies and rates of convergence for maximum likelihood and Bayes estimation for mixtures of normal densities. *Ann. Statist.* **29** (5) (2001) 1233–1263. MR1873329
[15] P. Heinrich and J. Kahn. Strong identifiability and optimal minimax rates for finite mixture estimation. *Ann. Statist.* **46** (6A) (2018) 2844–2870. MR3851757
[16] N. Ho and X. Nguyen. Convergence rates of parameter estimation for some weakly identifiable finite mixtures. *Ann. Statist.* **44** (6) (2016) 2726–2755. MR3576559 https://doi.org/10.1214/16-AOS1444
[17] N. Ho and X. Nguyen. On strong identifiability and convergence rates of parameter estimation in finite mixtures. *Electron. J. Stat.* **10** (1) (2016) 271–307. https://doi.org/10.1214/16-EJS1105. MR3466183 https://doi.org/10.1214/16-EJS1105
[18] P. J. Huber et al. Robust estimation of a location parameter. *Ann. Math. Stat.* **35** (1) (1964) 73–101. MR161415
[19] D. R. Hunter, S. Wang and T. P. Hettmansperger. Inference for mixtures of symmetric distributions. *Ann. Statist.* **35** (1) (2007) 224–251. MR2332275 https://doi.org/10.1214/009053606000001118
[20] W. Kruijer, J. Rousseau and A. W. van der Vaart. Adaptive Bayesian density estimation with location-scale mixtures. *Electron. J. Stat.* **4** (2010) 1225–1257. MR2735885 https://doi.org/10.1214/10-EJS584
[21] B. Laurent, C. Marteau and C. Maugis-Rabusseau. Non asymptotic detection of two component mixtures with unknown means. *Bernoulli* **22** (2016) 242–274. MR3449782 https://doi.org/10.3150/14-BEJ657
[22] L. Le Cam and G. Yang. *Asymptotics in Statistics: Some Basic Concepts*. *Springer Series in Statistics*. Springer Verlag, New-York, 2000. MR1784901 https://doi.org/10.1007/978-1-4612-1166-2
[23] C. Maugis and B. Michel. A non asymptotic penalized criterion for Gaussian mixture model selection. *ESAIM Probab. Stat.* **15** (2011) 41–68. MR2870505 https://doi.org/10.1051/ps/2009004
[24] C. Maugis-Rabusseau and B. Michel. Adaptive density estimation for clustering with Gaussian mixtures. *ESAIM Probab. Stat.* **17** (2013) 698–724. https://doi.org/10.1051/ps/2012018. MR3126158 https://doi.org/10.1051/ps/2012018
[25] G. McLachlan and D. Peel. *Finite Mixture Models*. Wiley series in Probability and Statistics, 2000. MR1789474 https://doi.org/10.1002/0471721182
[26] X. Nguyen. Convergence of latent mixing measures in finite and infinite mixture models. *Ann. Statist.* **41** (1) (2013) 370–400. MR3059422 https://doi.org/10.1214/12-AOS1065
[27] R. K. Patra and B. Sen. Estimation of a two-component mixture model with applications to multiple testing. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **78** (4) (2016) 869–893. MR3534354 https://doi.org/10.1111/rssb.12148
[28] C. Stein. Estimation of the mean of a multivariate normal distribution. *Ann. Statist.* **9** (1981) 1135–1151. MR0630098
[29] C. F. J. Wu. On the convergence properties of the EM algorithm. *Ann. Statist.* **11** (1983) 95–103. MR684867
[30] B. Yu Assouad, Fano, and Le Cam. *Festschrift for Lucien Le Cam*. Springer Verlag, 1997. MR1462931

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1419–1439
https://doi.org/10.1214/19-AIHP1008

Size of a minimal cutset in supercritical first passage percolation[1]

Barbara Dembin[a] and Marie Théret[b]

[a]*LPSM UMR 8001, Université Paris Diderot, Sorbonne Paris Cité, CNRS, F-75013 Paris, France. E-mail:* bdembin@lpsm.paris
[b]*Modal'X, UPL, Univ. Paris Nanterre, F92000 Nanterre, France. E-mail:* marie.theret@parisnanterre.fr

Abstract. We consider the standard model of i.i.d. first passage percolation on $\mathbb{Z}^d$ given a distribution G on $[0,+\infty]$ (including $+\infty$). We suppose that $G(\{0\}) > 1 - p_c(d)$, *i.e.*, the edges of positive passage time are in the subcritical regime of percolation on $\mathbb{Z}^d$. We consider a cylinder of basis an hyperrectangle of dimension $d-1$ whose sides have length n and of height $h(n)$ with $h(n)$ negligible compared to n (*i.e.*, $h(n)/n \to 0$ when n goes to infinity). We study the maximal flow from the top to the bottom of this cylinder. We already know that the maximal flow renormalized by n^{d-1} converges towards the flow constant which is null in the case $G(\{0\}) > 1 - p_c(d)$. The study of maximal flow is associated with the study of sets of edges of minimal capacity that cut the top from the bottom of the cylinder. If we denote by ψ_n the minimal cardinality of such a set of edges, we prove here that ψ_n/n^{d-1} converges almost surely towards a constant.

Résumé. Considérons le modèle de percolation de premier passage i.i.d. dans $\mathbb{Z}^d$ associé à la distribution G sur $[0,+\infty]$ (en incluant $+\infty$). Supposons que $G(\{0\}) > 1 - p_c(d)$, *i.e.*, les arêtes ayant un temps de passage strictement positif sont dans un régime sous-critique de percolation dans $\mathbb{Z}^d$. Considérons un cylindre ayant pour base un hyperrectangle de dimension $d-1$ de côté de longueur n et de hauteur $h(n)$ avec $h(n)$ négligeable devant n (*i.e.*, $h(n)/n \to 0$ quand n tend vers l'infini). Nous nous intéressons à la quantité maximale de flux pouvant circuler de haut en bas dans le cylindre. Le flux maximal renormalisé par n^{d-1} converge vers la constante de flux qui est nulle dans le cas où $G(\{0\}) > 1 - p_c(d)$. L'étude du flux maximal est équivalente à l'étude des ensembles d'arêtes de capacité minimale séparant le haut du bas du cylindre. Notons ψ_n le cardinal minimal de tels ensembles d'arêtes, nous prouvons ici que ψ_n/n^{d-1} converge presque sûrement vers une constante.

MSC: Primary 60K35; secondary 82B20

Keywords: First passage percolation; Maximal flow; Minimal cutset; Size of a cutset

References

[1] B. Bollobás. *Graph Theory: An Introductory Course*. *Graduate Texts in Mathematics* **63**. Springer-Verlag, New York, 1979. MR0536131
[2] S. Boucheron, G. Lugosi and P. Massart. *A Nonasymptotic Theory of Independence*. *Concentration Inequalities*. Oxford University Press, Oxford, 2013. MR3185193 https://doi.org/10.1093/acprof:oso/9780199535255.001.0001
[3] R. Cerf. The Wulff crystal in Ising and percolation models. In *École d'Été de Probabilités de Saint Flour*. *Lecture Notes in Mathematics* **1878**. Springer-Verlag, New York, 2006. MR2241754
[4] R. Cerf and M. Théret. Law of large numbers for the maximal flow through a domain of $\mathbb{R}^d$ in first passage percolation. *Trans. Amer. Math. Soc.* **363** (7) (2011) 3665–3702. MR2775823 https://doi.org/10.1090/S0002-9947-2011-05341-9
[5] R. Cerf and M. Théret. Lower large deviations for the maximal flow through a domain of $\mathbb{R}^d$ in first passage percolation. *Probab. Theory Related Fields* **150** (3–4) (2011) 635–661. MR2824869 https://doi.org/10.1007/s00440-010-0287-6
[6] R. Cerf and M. Théret. Upper large deviations for the maximal flow through a domain of $\mathbb{R}^d$ in first passage percolation. *Ann. Appl. Probab.* **21** (6) (2011) 2075–2108. MR2895410 https://doi.org/10.1214/10-AAP732
[7] R. Cerf and M. Théret. Maximal stream and minimal cutset for first passage percolation through a domain of $\mathbb{R}^d$. *Ann. Probab.* **42** (3) (2014) 1054–1120. MR3189066 https://doi.org/10.1214/13-AOP851
[8] R. Cerf and M. Théret. Weak shape theorem in first passage percolation with infinite passage times. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (3) (2016) 1351–1381. MR3531712 https://doi.org/10.1214/15-AIHP686
[9] M. Damron and P. Tang. Superlinearity of geodesic length in $2D$ critical first-passage percolation. (2016). Available at arXiv:1610.02593. To appear in *Festschrift for Chuck Newman*.
[10] G. Grimmett. *Percolation*. Springer-Verlag, New York, 1989. MR0995460
[11] G. Grimmett and H. Kesten. First-passage percolation, network flows and electrical resistances. *Z. Wahrsch. Verw. Gebiete* **66** (3) (1984) 335–366. MR0751574 https://doi.org/10.1007/BF00533701

[12] J. M. Hammersley and D. J. A. Welsh. First-passage percolation, subadditive processes, stochastic networks, and generalized renewal theory. In *In Proc. Internat. Res. Semin., Statist. Lab., Univ. California, Berkeley, California* 61–110. Springer-Verlag, New York, 1965. MR0198576

[13] H. Kesten. Aspects of first passage percolation. In *École d'été de probabilités de Saint-Flour, XIV—1984* 125–264. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876084 https://doi.org/10.1007/BFb0074919

[14] H. Kesten. Surfaces with minimal random weights and maximal flows: A higher dimensional version of first-passage percolation. *Illinois J. Math.* **31** (1) (1987) 99–166. MR0869483

[15] U. Krengel and R. Pyke. Uniform pointwise ergodic theorems for classes of averaging sets and multiparameter subadditive processes. *Stochastic Process. Appl.* **26** (2) (1987) 289–296. MR0923110 https://doi.org/10.1016/0304-4149(87)90182-7

[16] R. Rossignol and M. Théret. Lower large deviations and laws of large numbers for maximal flows through a box in first passage percolation. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** (4) (2010) 1093–1131. MR2744888 https://doi.org/10.1214/09-AIHP346

[17] R. Rossignol and M. Théret. Existence and continuity of the flow constant in first passage percolation. *Electron. J. Probab.* **23** (2018) 99. MR3862614 https://doi.org/10.1214/18-EJP214

[18] R. T. Smythe. Multiparameter subadditive processes. *Ann. Probab.* **4** (5) (1976) 772–782. MR0423495 https://doi.org/10.1214/aop/1176995983

[19] Y. Zhang. Supercritical behaviors in first-passage percolation. *Stochastic Process. Appl.* **59** (2) (1995) 251–266. MR1357654 https://doi.org/10.1016/0304-4149(95)00051-8

[20] Y. Zhang. Critical behavior for maximal flows on the cubic lattice. *J. Stat. Phys.* **98** (3–4) (2000) 799–811. MR1749233 https://doi.org/10.1023/A:1018631726709

[21] Y. Zhang. Limit theorems for maximum flows on a lattice. *Probab. Theory Related Fields*, 2018. MR3800832 https://doi.org/10.1007/s00440-017-0775-z

[22] Y. Zhang and Y. C. Zhang. A limit theorem for N_{0n}/n in first-passage percolation. *Ann. Probab.* **12** (4) (1984) 1068–1076. MR0757770

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1440–1464
https://doi.org/10.1214/19-AIHP1009

Local densities for a class of degenerate diffusions

Alberto Lanconelli[a,1], Stefano Pagliarani[b] and Andrea Pascucci[c]

[a]*Dipartimento di Matematica, Università di Bari Aldo Moro, Bari, Italy. E-mail: alberto.lanconelli@uniba.it*
[b]*DIES, Università di Udine, Udine, Italy. E-mail: stefano.pagliarani@uniud.it*
[c]*Dipartimento di Matematica, Università di Bologna, Bologna, Italy. E-mail: andrea.pascucci@unibo.it*

Abstract. We study a class of $\mathbb{R}^d$-valued continuous strong Markov processes that are generated, only locally, by an ultra-parabolic operator with coefficients that are regular w.r.t. the intrinsic geometry induced by the operator itself and not w.r.t. the Euclidean one. The first main result is a local Itô formula for functions that are not twice-differentiable in the classical sense, but only intrinsically w.r.t. to a set of vector fields, related to the generator, satisfying the Hörmander condition. The second main contribution, which builds upon the first one, is an existence and regularity result for the local transition density.

Résumé. Dans cet article on étudie une classe de processus de type Markov fort continus à valeurs dans $\mathbb{R}^d$ qui sont engendrés, seulement localement, par un opérateur ultra-parabolique avec des coefficients réguliers par rapport à la géométrie intrinsèque induite par l'opérateur lui-même et non par rapport à la géométrie euclidienne. On obtient deux résultats importants. Le premier est une formule locale d'Itô valable pour des fonctions qui ne sont pas deux fois différentiables dans le sens classique, mais seulement intrinsèquement par rapport à un ensemble de champs de vecteurs, liés au générateur, satisfaisant à la condition d'Hörmander. La deuxième contribution, qui s'appuie sur la première, est un résultat d'existence et de régularité pour la densité de transition locale.

MSC: 60J60; 60J35; 35H20

Keywords: Hörmander condition; Intrinsic geometry; Intrinsic Hölder spaces; Kolmogorov equations; Local densities; Strong Feller property

References

[1] L. Ambrosio, G. Da Prato and A. Mennucci. *Introduction to Measure Theory and Integration*. *Appunti. Scuola Normale Superiore di Pisa (Nuova Serie) [Lecture Notes. Scuola Normale Superiore di Pisa (New Series)]* **10**. Edizioni della Normale, Pisa, 2011. MR3379909 https://doi.org/10.1007/978-88-7642-386-4
[2] P. Baldi. *An Introduction Through Theory and Exercises*. *Stochastic Calculus. Universitext*. Springer, Cham, 2017. MR3726894 https://doi.org/10.1007/978-3-319-62226-2
[3] V. Bally, L. Caramellino and P. Pigato. Tube estimates for diffusions under a local strong Hörmander condition. *Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 2320–2369. https://doi.org/10.1214/18-AIHP950
[4] G. Cibelli, S. Polidoro and F. Rossi. Sharp Estimates for Geman–Yor Processes and applications to Arithmetic Average Asian options. *J. Math. Pures Appl. (9)* **129** (2019) 87–130. MR3998791 https://doi.org/10.1016/j.matpur.2018.12.009
[5] F. Delarue and S. Menozzi. Density estimates for a random noise propagating through a chain of differential equations. *J. Funct. Anal.* **259** (6) (2010) 1577–1630. MR2659772 https://doi.org/10.1016/j.jfa.2010.05.002
[6] M. Di Francesco and A. Pascucci. On a class of degenerate parabolic equations of Kolmogorov type. *Appl. Math. Res. Express. AMRX* **3** (2005) 77–116. MR2180050
[7] M. Di Francesco and S. Polidoro. Schauder estimates, Harnack inequality and Gaussian lower bound for Kolmogorov-type operators in non-divergence form. *Adv. Differential Equations* **11** (11) (2006) 1261–1320. MR2277064
[8] A. Friedman. *Stochastic Differential Equations and Applications. Vol. 1. Probability and Mathematical Statistics* **28**. Academic Press [Harcourt Brace Jovanovich, Publishers], New York–London, 1975. MR0494490
[9] S. Kusuoka and D. Stroock. Applications of the Malliavin calculus. II. *J. Fac. Sci., Univ. Tokyo, Sect. 1A, Math.* **32** (1) (1985) 1–76. MR0783181
[10] E. Lanconelli and S. Polidoro. On a class of hypoelliptic evolution operators. *Rend. Semin. Mat. Univ. Politec. Torino* **1** (1994) 29–63. Partial differential equations, II (Turin, 1993). MR1289901
[11] M. Manfredini. The Dirichlet problem for a class of ultraparabolic equations. *Adv. Differential Equations* **2** (5) (1997) 831–866. MR1751429
[12] S. Pagliarani and A. Pascucci. The exact Taylor formula of the implied volatility. *Finance Stoch.* **21** (3) (2017) 661–718. MR3663640 https://doi.org/10.1007/s00780-017-0330-x
[13] S. Pagliarani, A. Pascucci and M. Pignotti. Intrinsic Taylor formula for Kolmogorov-type homogeneous groups. *J. Math. Anal. Appl.* **435** (2) (2016) 1054–1087. MR3429628 https://doi.org/10.1016/j.jmaa.2015.10.080

[14] S. Pagliarani and M. Pignotti Intrinsic Taylor formula for non-homogeneous Kolmogorov-type Lie groups. Preprint, 2017. Available at arXiv:1707.01422v2.

[15] A. Pascucci. *PDE and Martingale Methods in Option Pricing*. *Bocconi & Springer Series* **2**. Springer, Milan; Bocconi University Press, Milan, 2011. MR2791231 https://doi.org/10.1007/978-88-470-1781-8

[16] P. Pigato. Tube estimates for diffusion processes under a weak Hörmander condition. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2018) 1. MR3765891 https://doi.org/10.1214/16-AIHP805

[17] S. Polidoro. On a class of ultraparabolic operators of Kolmogorov–Fokker–Planck type. *Matematiche (Catania)* **49** (1) (1994) 53–105. MR1386366

[18] R. L. Schilling and J. Wang. Strong Feller continuity of Feller processes and semigroups. *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **15** (2) (2012), 1250010, 28. MR2957137 https://doi.org/10.1142/S0219025712500105

[19] C. Soize. *The Fokker–Planck Equation for Stochastic Dynamical Systems and Its Explicit Steady State Solutions*. *Series on Advances in Mathematics for Applied Sciences.* **17**. World Scientific Publishing Co., Inc., River Edge, NJ, 1994. MR1287386 https://doi.org/10.1142/9789814354110

[20] D. W. Stroock and S. R. S. Varadhan. *Multidimensional Diffusion Processes*. *Classics in Mathematics*. Springer-Verlag, Berlin, 2006. Reprint of the 1997 edition. MR2190038

[21] D. Talay. Stochastic Hamiltonian systems: Exponential convergence to the invariant measure, and discretization by the implicit Euler scheme. *Markov Process. Related Fields* **8** (2) (2002) 163–198. Inhomogeneous random systems (Cergy-Pontoise, 2001). MR1924934

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1465–1483
https://doi.org/10.1214/19-AIHP1010

Exponential weights in multivariate regression and a low-rankness favoring prior

Arnak S. Dalalyan

ENSAE/CREST, 5 Av. Le Chatelier, 91120 Palaiseau, France. E-mail: arnak.dalalyan@ensae.fr

Abstract. We establish theoretical guarantees for the expected prediction error of the exponentially weighted aggregate in the case of multivariate regression that is when the label vector is multidimensional. We consider the regression model with fixed design and bounded noise. The first new feature uncovered by our guarantees is that it is not necessary to require independence of the observations: a symmetry condition on the noise distribution alone suffices to get a sharp risk bound. This result needs the regression vectors to be bounded. A second curious finding concerns the case of unbounded regression vectors but independent noise. It turns out that applying exponential weights to the label vectors perturbed by a uniform noise leads to an estimator satisfying a sharp oracle inequality. The last contribution is the instantiation of the proposed oracle inequalities to problems in which the unknown parameter is a matrix. We propose a low-rankness favoring prior and show that it leads to an estimator that is optimal under weak assumptions.

Résumé. Nous établissons des garanties théoriques pour l'erreur de prédiction de l'agrégat par poids exponentiels dans le cadre de la régression multivariée, c'est-à-dire lorsqu'on souhaite prédire une étiquette multidimensionnelle. Nous considérons le modèle de régression à design fixe et à bruit borné. La première conséquence de nos garanties est qu'il n'est pas nécessaire d'exiger l'indépendance des observations : une condition de symétrie sur la seule distribution du bruit suffit pour obtenir des bornes précises. Ce résultat nécessite que les vecteurs de régression soient bornées. Une seconde conséquence curieuse concerne le cas des vecteurs de régression non bornés mais indépendant de bruit. Il s'avère que l'application des poids exponentiels aux vecteurs d'étiquette perturbés par un bruit uniforme conduit à un estimateur satisfaisant une inégalité d'oracle exacte. La dernière contribution est l'instanciation des inégalités d'oracle proposées à des problèmes dans lesquels le paramètre inconnu est une matrice. Nous proposons une loi a priori favorisant les matrices à faible rang et montrons qu'elle conduit à un estimateur optimal sous des hypothèses faibles.

MSC: Primary 62J05; secondary 62H12

Keywords: Oracle inequality; Exponential weights; Trace regression; Low-rank matrices

References

[1] P. Alquier. Bayesian methods for low-rank matrix estimation: Short survey and theoretical study. In *Algorithmic Learning Theory* 309–323. *Berlin, Heidelberg*, 2013. MR3133074 https://doi.org/10.1007/978-3-642-40935-6_22

[2] P. Alquier and B. Guedj. An oracle inequality for quasi-Bayesian nonnegative matrix factorization. *Math. Methods Statist.* **26** (1) (2017) 55–67. MR3656072 https://doi.org/10.3103/S1066530717010045

[3] P. Alquier and K. Lounici. PAC-Bayesian bounds for sparse regression estimation with exponential weights. *Electron. J. Stat.* **5** (2011) 127–145. MR2786484 https://doi.org/10.1214/11-EJS601

[4] J.-Y. Audibert. Fast learning rates in statistical inference through aggregation. *Ann. Statist.* **37** (4) (2009) 1591–1646. MR2533466 https://doi.org/10.1214/08-AOS623

[5] P. C. Bellec. Optimal bounds for aggregation of affine estimators. *Ann. Statist.* **46** (1) (2018) 30–59. MR3766945 https://doi.org/10.1214/17-AOS1540

[6] F. Bunea, Y. She and M. H. Wegkamp. Optimal selection of reduced rank estimators of high-dimensional matrices. *Ann. Statist.* **39** (2) (2011) 1282–1309. MR2816355 https://doi.org/10.1214/11-AOS876

[7] F. Bunea, Y. She and M. H. Wegkamp. Joint variable and rank selection for parsimonious estimation of high-dimensional matrices. *Ann. Statist.* **40** (5) (2012) 2359–2388. MR3097606 https://doi.org/10.1214/12-AOS1039

[8] E. J. Candès and T. Tao. The power of convex relaxation: Near-optimal matrix completion. *IEEE Trans. Inform. Theory* **56** (5) (2010) 2053–2080. MR2723472 https://doi.org/10.1109/TIT.2010.2044061

[9] O. Catoni. *Pac-Bayesian Supervised Classification: The Thermodynamics of Statistical Learning*. *Lecture Notes – Monograph Series* **56**. Institute of Mathematical Statistics, Beachwood, OH, 2007. MR2483528

[10] D. Dai, P. Rigollet, L. Xia and T. Zhang. Aggregation of affine estimators. *Electron. J. Stat.* **8** (1) (2014) 302–327. MR3192554 https://doi.org/10.1214/14-EJS886
[11] A. S. Dalalyan. Theoretical guarantees for approximate sampling from smooth and log-concave densities. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **79** (3) (2017) 651–676. MR3641401 https://doi.org/10.1111/rssb.12183
[12] A. S. Dalalyan and J. Salmon. Sharp oracle inequalities for aggregation of affine estimators. *Ann. Statist.* **40** (4) (2012) 2327–2355. MR3059085 https://doi.org/10.1214/12-AOS1038
[13] A. S. Dalalyan and A. B. Tsybakov. Aggregation by exponential weighting and sharp oracle inequalities. In *Learning Theory* 97–111. *LNCS* **4539**, 2007. MR2397581 https://doi.org/10.1007/978-3-540-72927-3_9
[14] A. S. Dalalyan and A. B. Tsybakov. Aggregation by exponential weighting, sharp pac-Bayesian bounds and sparsity. *Mach. Learn.* **72** (1–2) (2008) 39–61.
[15] A. S. Dalalyan and A. B. Tsybakov. Mirror averaging with sparsity priors. *Bernoulli* **18** (3) (2012) 914–944. MR2948907 https://doi.org/10.3150/11-BEJ361
[16] A. S. Dalalyan and A. B. Tsybakov. Sparse regression learning by aggregation and Langevin Monte-Carlo. *J. Comput. System Sci.* **78** (5) (2012) 1423–1443. MR2926142 https://doi.org/10.1016/j.jcss.2011.12.023
[17] A. Durmus and E. Moulines. Sampling from strongly log-concave distributions with the Unadjusted Langevin Algorithm. Technical Report, 2016. Available at arXiv:1605.01559.
[18] E. I. George. Combining minimax shrinkage estimators. *J. Amer. Statist. Assoc.* **81** (394) (1986) 437–445. MR0845881
[19] E. I. George. Minimax multiple shrinkage estimation. *Ann. Statist.* **14** (1) (1986) 188–205. MR0829562 https://doi.org/10.1214/aos/1176349849
[20] S. Gerchinovitz. Sparsity regret bounds for individual sequences in online linear regression. *J. Mach. Learn. Res.* **14** (1) (2013) 729–769. MR3049487
[21] A. J. Izenman. Reduced-rank regression for the multivariate linear model. *J. Multivariate Anal.* **5** (2) (1975) 248–264. MR0373179 https://doi.org/10.1016/0047-259X(75)90042-1
[22] A. Juditsky, P. Rigollet and A. B. Tsybakov. Learning by mirror averaging. *Ann. Statist.* **36** (5) (2008) 2183–2206. MR2458184 https://doi.org/10.1214/07-AOS546
[23] G. Leung. Information Theory and Mixing Least Squares Regression. PhD thesis, Yale University, 2004.
[24] G. Leung and A. R. Barron. Information theory and mixing least-squares regressions. *IEEE Trans. Inform. Theory* **52** (8) (2006) 3396–3410. MR2242356 https://doi.org/10.1109/TIT.2006.878172
[25] S. Negahban and M. J. Wainwright. Estimation of (near) low-rank matrices with noise and high-dimensional scaling. *Ann. Statist.* **39** (2) (2011) 1069–1097. MR2816348 https://doi.org/10.1214/10-AOS850
[26] P. Rigollet and A. Tsybakov. Exponential screening and optimal rates of sparse estimation. *Ann. Statist.* **39** (2) (2011) 731–771. MR2816337 https://doi.org/10.1214/10-AOS854
[27] P. Rigollet and A. B. Tsybakov. Sparse estimation by exponential weighting. *Statist. Sci.* **27** (4) (2012) 558–575. MR3025134 https://doi.org/10.1214/12-STS393
[28] A. Rohde and A. B. Tsybakov. Estimation of high-dimensional low-rank matrices. *Ann. Statist.* **39** (2) (2011) 887–930. MR2816342 https://doi.org/10.1214/10-AOS860
[29] O. Wintenberger. Optimal learning with Bernstein online aggregation. *Mach. Learn.* **106** (1) (2017) 119–141. MR3592152 https://doi.org/10.1007/s10994-016-5592-6
[30] L. Yang, J. Fang, H. Duan, H. Li and B. Zeng. Fast low-rank Bayesian matrix completion with hierarchical Gaussian prior models. *IEEE Trans. Signal Process.* **66** (11) (2018) 2804–2817. MR3816906 https://doi.org/10.1109/tsp.2018.2816575
[31] Y. Yang. Adaptive estimation in pattern recognition by combining different procedures. *Statist. Sinica* **10** (4) (2000) 1069–1089. MR1804557
[32] Y. Yang. Combining different procedures for adaptive regression. *J. Multivariate Anal.* **74** (1) (2000) 135–161. MR1790617 https://doi.org/10.1006/jmva.1999.1884

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2020, Vol. 56, No. 2, 1484–1513
https://doi.org/10.1214/19-AIHP1011

Stein's method for functions of multivariate normal random variables

Robert E. Gaunt

School of Mathematics, The University of Manchester, Manchester, M13 9PL, United Kingdom. E-mail: robert.gaunt@manchester.ac.uk

Abstract. By the continuous mapping theorem, if a sequence of d-dimensional random vectors $(\mathbf{W}_n)_{n\geq 1}$ converges in distribution to a multivariate normal random variable $\Sigma^{1/2}\mathbf{Z}$, then the sequence of random variables $(g(\mathbf{W}_n))_{n\geq 1}$ converges in distribution to $g(\Sigma^{1/2}\mathbf{Z})$ if $g:\mathbb{R}^d\rightarrow\mathbb{R}$ is continuous. In this paper, we develop Stein's method for the problem of deriving explicit bounds on the distance between $g(\mathbf{W}_n)$ and $g(\Sigma^{1/2}\mathbf{Z})$ with respect to smooth probability metrics. We obtain several bounds for the case that the j-component of $\mathbf{W}_n$ is given by $W_{n,j}=\frac{1}{\sqrt{n}}\sum_{i=1}^n X_{ij}$, where the X_{ij} are independent. In particular, provided g satisfies certain differentiability and growth rate conditions, we obtain an order $n^{-(p-1)/2}$ bound, for smooth test functions, if the first p moments of the X_{ij} agree with those of the normal distribution. If p is an even integer and g is an even function, this convergence rate can be improved further to order $n^{-p/2}$. These convergence rates are shown to be of optimal order. We apply our general bounds to some examples, which include the distributional approximation of asymptotically chi-square distributed statistics; the approximation of expectations of smooth functions of binomial and Poisson random variables; rates of convergence in the delta method; and a quantitative variance-gamma approximation of the D_2^* statistic for alignment-free sequence comparison in the case of binary sequences.

Résumé. Par le théorème de l'application continue, si une suite $(\mathbf{W}_n)_{n\geq 1}$ de vecteurs aléatoires de dimension d converge en loi vers une loi normale multivariée $\Sigma^{1/2}\mathbf{Z}$, alors la suite des variables aléatoires $(g(\mathbf{W}_n))_{n\geq 1}$ converge en loi vers $g(\Sigma^{1/2}\mathbf{Z})$ si $g:\mathbb{R}^d\rightarrow\mathbb{R}$ est continue. Dans cet article, nous développons la méthode de Stein pour obtenir des bornes explicites sur la distance entre $g(\mathbf{W}_n)$ et $g(\Sigma^{1/2}\mathbf{Z})$, pour des métriques lisses sur l'espaces des probabilités. Nous obtenons plusieurs bornes dans le cas où la j-ème coordonnée de $\mathbf{W}_n$ est donnée par $W_{n,j}=\frac{1}{\sqrt{n}}\sum_{i=1}^n X_{ij}$, où les X_{ij} sont indépendants. En particulier, si g vérifie certains conditions de dérivabilité et de croissance, nous obtenons une borne d'ordre $n^{-(p-1)/2}$, pour des fonctions-test lisses, si les p premiers moments des X_{ij} coïncident avec ceux de la loi normale. Si p est un entier pair et g est une fonction paire, ce taux de convergence peut être encore amélioré en $n^{-p/2}$. Nous montrons que ces taux de convergence sont d'ordre optimal. Nous appliquons nos bornes générales à quelques exemples, incluant l'approximation en loi de statistiques suivant asymptotiquement une loi du chi-deux; l'approximation d'espérances de fonctions lisses de variables aléatoires de loi binomiales ou de Poisson; des taux de convergence pour la méthode δ; et une approximation variance-gamma quantitative de la statistique D_2^* pour la comparaison sans alignement, dans le cas de suites binaires.

MSC: 60F05

Keywords: Stein's method; Functions of multivariate normal random variables; Multivariate normal approximation; Rate of convergence; Delta method; Sequence comparison

References

[1] B. Arras, E. Azmoodeh, G. Poly and Y. Swan. A bound on the 2-Wasserstein distance between linear combinations of independent random variables. *Stochastic Process. Appl.* **129** (2019) 2341–2375. MR3958435 https://doi.org/10.1016/j.spa.2018.07.009

[2] B. Arras, E. Azmoodeh, G. Poly and Y. Swan. Stein characterizations for linear combinations of gamma random variables. *Braz. J. Probab. Stat.* To appear, 2019.

[3] B. Arras, G. Mijoule, G. Poly and Y. Swan. A new approach to the Stein–Tikhomirov method: With applications to the second Wiener chaos and Dickman convergence, 2017. Available at arXiv:1605.06819v2.

[4] E. Azmooden, G. Peccati and G. Poly. Convergence towards linear combinations of chi-squared random variables: A Malliavin-based approach. In *Séminaire de Probabilités XLVII (Special Volume in Memory of Marc Yor)* 339–367, 2015. MR3444306 https://doi.org/10.1007/978-3-319-18585-9_16

[5] A. D. Barbour. Stein's method for diffusion approximations. *Probab. Theory Related Fields* **84** (1990) 297–322. MR1035659 https://doi.org/10.1007/BF01197887

[6] B. Blaisdell. A measure of the similarity of sets of sequences not requiring sequence alignment. *Proc. Natl. Acad. Sci. USA* **83** (1986) 5155–5159.
[7] T. Bonis. Rates in the Central Limit Theorem and diffusion approximation via Stein's Method, 2018. Available at arXiv:1506.06966v6.
[8] A. Braverman and J. G. Dai. High order steady-state diffusion approximation of the Erlang-C system, 2016. Available at arXiv:1602.02866. MR1490535
[9] S. Chatterjee and E. Meckes. Multivariate normal approximation using exchangeable pairs. *ALEA Lat. Am. J. Probab. Math. Stat.* **4** (2008) 257–283. MR2453473
[10] L. H. Y. Chen, L. Goldstein and Q.-M. Shao. *Normal Approximation by Stein's Method*. Springer, Berlin, 2011. MR2732624 https://doi.org/10.1007/978-3-642-15007-4
[11] M. Clausel, F. Roueff, M. S. Taqqu and C. Tudor. High order chaotic limits for wavelet scalogram under long-range dependence. *ALEA Lat. Am. J. Probab. Math. Stat.* **10** (2013) 979–1011. MR3151747
[12] T. A. Courtade, M. Fathi and A. Pananjady. Existence of Stein kernels under a spectral gap, and discrepancy bound. *Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 777–790. MR3949953 https://doi.org/10.1214/18-aihp898
[13] F. Daly. Upper bounds for Stein-type operators. *Electron. J. Probab.* **13** (2008) 566–587. MR2399291 https://doi.org/10.1214/EJP.v13-479
[14] C. Döbler. New developments in Stein's method with applications. Ph.D. thesis, Ruhr-Universität Bochum, 2012.
[15] C. Döbler. Stein's method for the half-normal distribution with applications to limit theorems related to the simple random walk. *ALEA Lat. Am. J. Probab. Math. Stat.* **12** (2015) 171–191. MR3343481
[16] C. Döbler. Stein's method of exchangeable pairs for the beta distribution and generalizations. *Electron. J. Probab.* **20** (2015) paper no. 109, 34. MR3418541 https://doi.org/10.1214/EJP.v20-3933
[17] C. Döbler, R. E. Gaunt and S. J. Vollmer. An iterative technique for bounding derivatives of solutions of Stein equations. *Electron. J. Probab.* **22** (2017) paper no. 96, 39. MR3724564 https://doi.org/10.1214/17-EJP118
[18] R. Eden and J. Viquez. Nourdin–Peccati analysis on Wiener and Wiener–Poisson space for general distributions. *Stochastic Process. Appl.* **125** (2015) 182–216. MR3274696 https://doi.org/10.1016/j.spa.2014.09.001
[19] P. Eichelsbacher and C. Thäle. Malliavin–Stein method for variance-gamma approximation on Wiener space. *Electron. J. Probab.* **20** (2015) paper no. 123, 28. MR3425543 https://doi.org/10.1214/EJP.v20-4136
[20] X. Fang, Q.-M. Shao and L. Xu. Multivariate approximations in Wasserstein distance by Stein's method and Bismut's formula. *Probab. Theory Related Fields* **174** (2019) 945–979. MR3980309 https://doi.org/10.1007/s00440-018-0874-5
[21] M. Fathi. Higher-order Stein kernels for Gaussian approximation, 2018. Available at arXiv:1812.02703.
[22] M. Fathi. Stein kernels and moment maps. *Ann. Probab.* **47** (2019) 2172–2185. MR3980918 https://doi.org/10.1214/18-AOP1305
[23] T. Gallouët, G. Mijoule and Y. Swan. Regularity of solutions of the Stein equation and rates in the multivariate central limit theorem, 2018. Available at arXiv:1805.01720.
[24] R. E. Gaunt. Variance-gamma approximation via Stein's method. *Electron. J. Probab.* **19** (2014) paper no. 38, 33. MR3194737 https://doi.org/10.1214/EJP.v19-3020
[25] R. E. Gaunt. Rates of convergence in normal approximation under moment conditions via new bounds on solutions of the Stein equation. *J. Theoret. Probab.* **29** (2016) 231–247. MR3463084 https://doi.org/10.1007/s10959-014-0562-z
[26] R. E. Gaunt. On Stein's method for products of normal random variables and zero bias couplings. *Bernoulli* **23** (2017) 3311–3345. MR3654808 https://doi.org/10.3150/16-BEJ848
[27] R. E. Gaunt. Products of normal, beta and gamma random variables: Stein operators and distributional theory. *Braz. J. Probab. Stat.* **32** (2018) 437–466. MR3787761 https://doi.org/10.1214/16-BJPS349
[28] R. E. Gaunt. Stein operators for variables from the third and fourth Wiener chaoses. *Statist. Probab. Lett.* **145** (2019) 118–126. MR3873897 https://doi.org/10.1016/j.spl.2018.09.001
[29] R. E. Gaunt. Wasserstein and Kolmogorov error bounds for variance-gamma approximation via Stein's method I. *J. Theoret. Probab.* To appear, 2019.
[30] R. E. Gaunt, S. Iyengar, A. B. Olde Daalhuis and B. Simsek. An asymptotic expansion for the normalizing constant of the Conway–Maxwell–Poisson distribution. *Ann. Inst. Statist. Math.* **71** (2019) 163–180. MR3898430 https://doi.org/10.1007/s10463-017-0629-6
[31] R. E. Gaunt, G. Mijoule and Y. Swan. An algebra of Stein operators. *J. Math. Anal. Appl.* **469** (2019) 260–279. MR3857522 https://doi.org/10.1016/j.jmaa.2018.09.015
[32] R. E. Gaunt, A. M. Pickett and G. Reinert. Chi-square approximation by Stein's method with application to Pearson's statistic. *Ann. Appl. Probab.* **27** (2017) 720–756. MR3655852 https://doi.org/10.1214/16-AAP1213
[33] R. E. Gaunt and G. Reinert. The rate of convergence of some asymptotically chi-square distributed statistics by Stein's method, 2016. Available at arXiv:1603.01889.
[34] L. Goldstein and G. Reinert. Stein's method and the zero bias transformation with application to simple random sampling. *Ann. Appl. Probab.* **7** (1997) 935–952. MR1484792 https://doi.org/10.1214/aoap/1043862419
[35] L. Goldstein and G. Reinert. Zero biasing in one and higher dimensions, and applications. In *Stein's Method and Applications* 1–18. *Lect. Notes Ser. Inst. Math. Sci. Natl. Univ. Singap.* **5**. Singapore Univ. Press, Singapore, 2005. MR2201883 https://doi.org/10.1142/9789812567673_0001
[36] L. Goldstein and Y. Rinott. Multivariate normal approximations by Stein's method and size bias couplings. *J. Appl. Probab.* **33** (1996) 1–17. MR1371949 https://doi.org/10.1017/s0021900200103675
[37] F. Götze. On the rate of convergence in the multivariate CLT. *Ann. Probab.* **19** (1991) 724–739. MR1106283
[38] P. Hall. *The Bootstrap and Edgeworth Expansion*. Springer–Verlag, Berlin, 1991. MR1145237 https://doi.org/10.1007/978-1-4612-4384-7
[39] H. Huang. Error bounds on multivariate normal approximations for word count statistics. *Adv. Appl. Probab.* **34** (2002) 559–586. MR1929598 https://doi.org/10.1239/aap/1033662166
[40] D. R. Hunter. Notes for a graduate-level course in asymptotics for statisticians. Penn State University, 2014.
[41] M. R. Kantorovitz, H. S. Booth, C. J. Burden and S. R. Wilson. Asymptotic behavior of k-word matches between two uniformly distributed sequences. *J. Appl. Probab.* **44** (2007) 788–805. MR2355592 https://doi.org/10.1239/jap/1189717545
[42] M. J. Kasprzak, A. B. Duncan and S. J. Vollmer. Note on A. Barbour's paper on Stein's method for diffusion approximations. *Electron. Commun. Probab.* **22** (2017) 1–8. MR3645505 https://doi.org/10.1214/17-ECP54
[43] C. Lefèvre and S. Utev. Exact norms of a Stein-type operator and associated stochastic orderings. *Probab. Theory Related Fields* **127** (2003) 353–366. MR2018920 https://doi.org/10.1007/s00440-003-0288-9

[44] C. Ley, G. Reinert and Y. Swan. Stein's method for comparison of univariate distributions. *Probab. Surv.* **14** (2017) 1–52. MR3595350 https://doi.org/10.1214/16-PS278

[45] R. A. Lippert, H. Huang and M. S. Waterman. Distributional regimes for the number of k-word matches between two random sequences. *Proc. Natl. Acad. Sci. USA* **99** (2002) 13980–13989. MR1944413 https://doi.org/10.1073/pnas.202468099

[46] H. Luk. Stein's method for the gamma distribution and related statistical applications. Ph.D. thesis, University of Southern California, 1994. MR2693204

[47] R. Lundstrom. Stochastic models and statistical methods for DNA sequence data. Ph.D. thesis, University of Utah, Salt Lake City, UT, 1990. MR2685411

[48] T. W. Ma. Higher chain formula proved by combinatorics. *Electron. J. Combin.* **16** (1) (2009) N21. MR2515761

[49] E. S. Meckes. On Stein's method for multivariate normal approximation. *Inst. Math. Stat. Collect.* **5** (2009) 153–178. MR2797946 https://doi.org/10.1214/09-IMSCOLL511

[50] I. Nourdin and G. Peccati. Stein's method on Wiener chaos. *Probab. Theory Related Fields* **145** (2009) 75–118. MR2520122 https://doi.org/10.1007/s00440-008-0162-x

[51] I. Nourdin and G. Peccati. *Normal Approximations with Malliavin Calculus: From Stein's Method to Universality*. *Cambridge Tracts in Mathematics*. Cambridge University Press, Cambridge, 2012. MR2962301 https://doi.org/10.1017/CBO9781139084659

[52] I. Nourdin, G. Peccati and Y. Swan. Integration by parts and representation of information functionals. In *Information Theory (ISIT), 2014 IEEE International Symposium*, 2014.

[53] F. W. J. Olver, D. W. Lozier, R. F. Boisvert and C. W. Clark. *NIST Handbook of Mathematical Functions*. Cambridge University Press, Cambridge, 2010. MR2655349

[54] G. Peccati. Quantitative CLTs on a Gaussian space: A survey of recent developments. *ESAIM Proc.* **44** (2014) 61–78. MR3178608 https://doi.org/10.1051/proc/201444003

[55] E. Peköz, A. Röllin and N. Ross. Generalized gamma approximation with rates for urns, walks and trees. *Ann. Probab.* **44** (2016) 1776–1816. MR3502594 https://doi.org/10.1214/15-AOP1010

[56] A. Pickett. Rates of convergence of χ^2 approximations via Stein's method. Ph.D. thesis, University of Oxford, 2004.

[57] I. Pinelis and R. Molzon. Optimal-order bounds on the rate of convergence to normality in the multivariate delta method. *Electron. J. Stat.* **10** (2016) 1001–1063. MR3486424 https://doi.org/10.1214/16-EJS1133

[58] M. Raič. A multivariate CLT for decomposable random vectors with finite second moments. *J. Theoret. Probab.* **17** (2004) 573–603. MR2091552 https://doi.org/10.1023/B:JOTP.0000040290.44087.68

[59] G. Reinert. Three general approaches to Stein's method. In *An Introduction to Stein's Method* 183–221. A. D. Barbour and L. H. Y. Chen (Eds) *Lect. Notes Ser. Inst. Math. Sci. Natl. Univ. Singap.* **4**. Singapore Univ. Press, Singapore, 2005. MR2235451 https://doi.org/10.1142/9789812567680_0004

[60] G. Reinert, D. Chew, F. Sun and M. S. Waterman. Alignment free sequence comparison (I): Statistics and power. *J. Comput. Biol.* **16** (2009) 1615–1634. MR2578699 https://doi.org/10.1089/cmb.2009.0198

[61] G. Reinert and A. Röllin. Multivariate normal approximations with Stein's method of exchangeable pairs under a general linearity condition. *Ann. Probab.* **37** (2009) 2150–2173. MR2573554 https://doi.org/10.1214/09-AOP467

[62] G. Reinert, S. Schbath and M. S. Waterman. In *Applied Combinatorics on Words*. M. Lothaire (Ed.). Cambridge University Press, Cambridge, 2005. MR2165687 https://doi.org/10.1017/CBO9781107341005

[63] N. Ross. Fundamentals of Stein's method. *Probab. Surv.* **8** (2011) 210–293. MR2861132 https://doi.org/10.1214/11-PS182

[64] C. Stein. A bound for the error in the normal approximation to the distribution of a sum of dependent random variables. In *Proc. Sixth Berkeley Symp. Math. Statis. Prob.* 583–602, **2**. Univ. California Press, Berkeley, 1972. MR0402873

[65] C. Stein. *Approximate Computation of Expectations*. IMS, Hayward, CA, 1986. MR0882007

[66] A. Stuart and J. K. Ord. *Kendall's Advanced Theory of Statistics: Volume I*, 5th edition. Charles Griffin, 1987. MR0019869

[67] A. Zhai. A high-dimensional CLT in $\mathcal{W}_2$ distance with near optimal convergence rate. *Probab. Theory Related Fields* **170** (2018) 821–845. MR3773801 https://doi.org/10.1007/s00440-017-0771-3

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Recommandations aux auteurs Instructions to authors

Les *Annales de l'I.H.P., Probabilités et Statistiques*, sont une revue internationale publiant des articles originaux en français et en anglais. Le journal publie des articles de qualité reflétant les différents aspects des processus stochastiques, de la statistique mathématique et des domaines contigus.

A compter du 14 avril 2008, les Annales ont adopté le système EJMS pour soumettre et traiter les articles. Les auteurs sont encouragés à utiliser ce système et peuvent y accéder à l'adresse http://www.e-publications.org/ims/submission/. Des informations complémentaires se trouvent à http://www.imstat.org/aihp/mansub.html. Les soumissions par courriel à l'Editeur sont toujours possible en envoyant l'article sous forme de fichiers PDF ou TeX à l'adresse Ann.IHP.PS@math.univ-lyon1.fr. Les articles peuvent être écrits en français ou en anglais. Sous le titre, les auteurs indiqueront leurs prénoms et noms ainsi qu'une désignation succincte de leur laboratoire – notamment l'adresse. Afin de faciliter la communication, il est aussi souhaitable que les auteurs fournissent un numéro de fax et une adresse de courrier électronique.

Les articles doivent être accompagnés d'un résumé précisant clairement les points essentiels développés dans l'article. Pour les articles en français, l'auteur est invité à fournir la traduction en anglais de ce résumé. Pour les articles en anglais, l'auteur est invité à fournir un résumé en français. Le comité éditorial pourra effectuer ces traductions le cas échéant.

Les références seront numérotées continûment, renvoyant à la liste bibliographique indiquant l'initiale du prénom + le nom de l'auteur, le titre de la publication, le titre de la Revue, l'année, la tomaison ou le cas échéant le numéro, les pages de début et de fin d'article et, dans le cas d'un livre, l'éditeur, le lieu et l'année d'édition.

Les articles acceptés pour publication sont considérés comme ne varietur. Les auteurs recevront une seule épreuve de leur article : celle-ci devra être retournée à l'éditeur dans le délai maximal d'une semaine. Toutes modifications ou corrections excessives autres que celles provenant d'erreurs typographiques peuvent être facturées aux auteurs. La publication des articles ou mémoires est gratuite, la facturation des pages est optionnelle. L'auteur correspondant recevra un fichier pdf de leur article final par courrier electronique.

Les auteurs sont encouragés à préparer leurs manuscrits au moyen de l'un des logiciels Plain TeX, LaTeX ou AMS TeX. Un support LATEX se trouve à l'adresse http://www.e-publications.org/ims/support/

The *Annales de l'I.H.P., Probabilites et Statistiques* is an international Journal publishing original articles in French or in English. The Journal publishes papers of high quality representing different aspects of the theory of stochastic processes, mathematical statistics and related fields.

Effective April 14, 2008, the Annales has adopted an Electronic Journal Management System (EJMS) for submission and handling of papers by the editorial board and reviewers for its journal. Authors are encouraged to use this system and should access EJMS at http://www.e-publications.org/ims/submission/. Please see http://www.imstat.org/aihp/mansub.html for additional details. Submissions will still be accepted via email in the form of a TeX or PDF to the Editor at Ann.IHP.PS@math.univ-lyon1.fr. Papers may be written in French or in English. The authors give their name below the title, together with their current affiliation and address. In order to facilitate communication, the authors should also provide a fax number and an e-mail address.

Articles should begin with a summary explaining the basic points developed in the article. For papers in English, the authors should provide the translation in French of this summary. The Editorial committee will supply the translation if necessary. The authors of articles in French should provide the translation in English of this summary.

References should be numbered, referring to the bibliography giving for each reference the initial and name of the author followed by the title of the publication, the name of the Journal, the volume, the year, the pages of the article, and in the case of a book, the editor, the place and year of edition.

Papers accepted for publication are considered as ne variatur. The corresponding author will receive email regarding proofs, which should be sent back to the publisher within one week.

A charge may be made for any excessive corrections other than those due to typographical errors.

The publication of articles is free, page charges are optional. Every corresponding author will receive a pdf file via email of the final article. Paper offprints may be purchased by using the IMS Offprint Purchase Order Forms below.

The authors are encouraged to prepare their manuscripts using Plain TeX, LaTeX or AMS TeX. A LaTeX support page is available at http://www.e-publications.org/ims/support/

Indexations: *Current Contents (PC&ES), Zentralblatt für Mathematik, Inspec, Current Index to statistics, Pascal (INIST), Science Citation Index, SciSearch®, Research Alert®, Compu Math Citation Index®. Also covered in the abstract and citation database SCOPUS®.*