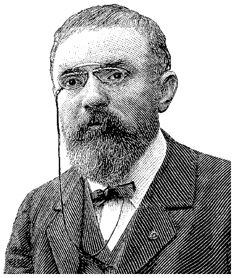

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Heavy-tailed configuration models at criticality

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Abstract. We study the critical behavior of the component sizes for the configuration model when the tail of the degree distribution of a randomly chosen vertex is a regularly-varying function with exponent $\tau - 1$, where $\tau \in (3,4)$. The component sizes are shown to be of the order $n^{(\tau-2)/(\tau-1)}L(n)^{-1}$ for some slowly-varying function $L(\cdot)$. We show that the re-scaled ordered component sizes converge in distribution to the ordered excursions of a thinned Lévy process. This proves that the scaling limits for the component sizes for these heavy-tailed configuration models are in a different universality class compared to the Erdős–Rényi random graphs. Also the joint re-scaled vector of ordered component sizes and their surplus edges is shown to have a distributional limit under a strong topology. Our proof resolves a conjecture by Joseph (*Ann. Appl. Probab.* **24** (2014) 2560–2594) about the scaling limits of uniform simple graphs with i.i.d. degrees in the critical window, and sheds light on the relation between the scaling limits obtained by Joseph and in this paper, which appear to be quite different. Further, we use percolation to study the evolution of the component sizes and the surplus edges within the critical scaling window, which is shown to converge in finite dimension to the augmented multiplicative coalescent process introduced by Bhamidi et al. (*Probab. Theory Related Fields* **160** (2014) 733–796). The main results of this paper are proved under rather general assumptions on the vertex degrees. We also discuss how these assumptions are satisfied by some of the frameworks that have been studied previously.

Résumé. Nous étudions le comportement critique des tailles des composantes du modèle de configuration lorsque la queue de la loi du degré d'un sommet choisi uniformément au hasard est une fonction à variation régulière d'exposant $\tau - 1$, où $\tau \in (3,4)$. Nous montrons que les tailles des composantes sont d'ordre $n^{(\tau-2)/(\tau-1)}L(n)^{-1}$ où $L(\cdot)$ est une fonction à variation lente. Nous montrons également que les tailles des composantes, une fois ordonnées et remises à l'échelle, convergent en loi vers les longueurs ordonnées des excursions d'un processus de Lévy raréfié. Ceci montre que les limites d'échelle des tailles des composantes pour ces modèles de configuration à queue lourde sont dans une classe d'universalité différente des graphes aléatoires d'Erdős–Rényi. De plus nous montrons que le vecteur de ces tailles de composantes remises à l'échelle et de leurs excès respectifs convergent en loi pour une topologie forte. Notre approche résout une conjecture de Joseph (*Ann. Appl. Probab.* **24** (2014) 2560–2594) sur les limites d'échelle de graphes simples uniformes à degrés i.i.d. dans la fenêtre critique, et met en lumière les relations entre les limites d'échelle obtenues par Joseph et celles considérées dans cet article, qui se révèlent très différentes. Par ailleurs, nous utilisons un modèle de percolation pour étudier l'évolution des tailles des composantes et des arêtes en excès à l'intérieur de la fenêtre critique, dont nous montrons qu'elle converge au sens des marginales de dimension finie vers le coalescent multiplicatif augmenté introduit par Bhamidi et al. (*Probab. Theory Related Fields* **160** (2014) 733–796). Les résultats principaux de cet article sont montrés sous des hypothèses assez générales sur les degrés des sommets, et nous discutons des situations déjà considérées où ces hypothèses sont vérifiées.

MSC: 60C05; 05C80

Keywords: Critical configuration model; Heavy-tailed degree; Thinned Lévy process; Augmented multiplicative coalescent; Universality; Critical percolation

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Non-asymptotic Gaussian estimates for the recursive approximation of the invariant distribution of a diffusion

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Abstract. We obtain non-asymptotic Gaussian concentration bounds for the difference between the invariant distribution ν of an ergodic Brownian diffusion process and the empirical distribution of an approximating scheme with decreasing time step along a suitable class of (smooth enough) test functions f such that $f - \nu(f)$ is a coboundary of the infinitesimal generator. We show that these bounds can still be improved when some suitable squared-norms of the diffusion coefficient also belong to this class. We apply these estimates to design computable non-asymptotic confidence intervals for the approximating scheme. As a theoretical application, we finally derive non-asymptotic deviation bounds for the almost sure Central Limit Theorem.

Résumé. Nous obtenons des estimées de concentration gaussienne non asymptotiques pour la différence entre la mesure invariante ν d'une diffusion brownienne ergodique et la mesure empirique d'un schéma d'approximation à pas décroissants évaluée le long d'une classe admissible de fonctions tests f telles que $f - \nu(f)$ soit un co-bord du générateur infinitésimal. Nous montrons que ces bornes peuvent être améliorées lorsque le carré de certaines normes du coefficient de diffusion appartient également à cette classe. Nous déduisons de ces estimées des intervalles de confiance non asymptotiques explicitement calculables pour le schéma d'approximation. Nous obtenons également, en terme d'application théorique, des estimées de déviations non asymptotiques pour le théorème de la limite centrale presque sûr.

MSC: Primary 60H35; 60E15; secondary 35J15

Keywords: Invariant distribution; Diffusion processes; Inhomogeneous Markov chains; Non-asymptotic Gaussian concentration; Almost sure Central Limit Theorem

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The Skorokhod embedding problem for inhomogeneous diffusions

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Abstract. We solve the Skorokhod embedding problem for a class of stochastic processes satisfying an inhomogeneous stochastic differential equation (SDE) of the form $\mathrm{d}A_t = \mu(t, A_t)\,\mathrm{d}t + \sigma(t, A_t)\,\mathrm{d}W_t$. We provide sufficient conditions guaranteeing that for a given probability measure ν on $\mathbb{R}$ there exists a bounded stopping time τ and a real a such that the solution (A_t) of the SDE with initial value a satisfies $A_\tau \sim \nu$. We hereby distinguish the cases where (A_t) is a solution of the SDE in a weak or strong sense. Our construction of embedding stopping times is based on a solution of a fully coupled forward-backward SDE. We use the so-called method of decoupling fields for verifying that the FBSDE has a unique solution. Finally, we sketch an algorithm for putting our theoretical construction into practice and illustrate it with a numerical experiment.

Résumé. Nous résolvons le problème de plongement de Skorokhod pour une classe de processus stochastiques satisfaisant une équation différentielle stochastique (EDS) non homogène de la forme $\mathrm{d}A_t = \mu(t, A_t)\,\mathrm{d}t + \sigma(t, A_t)\,\mathrm{d}W_t$. Nous fournissons des conditions suffisantes garantissant que, pour une mesure de probabilité ν sur $\mathbb{R}$, il existe un temps d'arrêt borné τ et un réel a tels que la solution (A_t) de l'EDS avec condition initiale a satisfait $A_\tau \sim \nu$. Nous distinguons ici les cas où (A_t) est une solution de l'EDS dans un sens faible ou fort. Notre construction des temps d'arrêt de plongement est basée sur une solution d'une EDS progressive rétrograde totalement couplée. Nous utilisons la méthode des decoupling fields pour vérifier que l'EDSPR a une solution unique. Enfin, nous esquissons un algorithme pour mettre en pratique notre construction théorique et l'illustrons par une simulation numérique.

MSC: Primary 60G40; secondary 60H10; 60J25

Keywords: Skorokhod embedding; Decoupling fields; FBSDE

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Global universality of Macdonald plane partitions

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Abstract. We study scaling limits of periodically weighted skew plane partitions with semilocal interactions and general boundary conditions. The semilocal interactions correspond to the Macdonald symmetric functions which are (q, t)-deformations of the Schur symmetric functions. We show that the height functions converge to a deterministic limit shape and that the global fluctuations are given by the 2-dimensional Gaussian free field as $q, t \to 1$ and the mesh size goes to 0. Specializing to the noninteracting case, this verifies the Kenyon–Okounkov conjecture for the case of the r^{volume} measure under general boundary conditions. Our approach uses difference operators on Macdonald processes.

Résumé. Nous étudions les limites d'échelle de partitions planes tordues (skew) pondérées périodiquement, avec des interactions semi-locales et des conditions au bord générales. Ces interactions correspondent aux fonctions symétriques de Macdonald, qui sont des (q, t)-déformations des fonctions de Schur symétriques. Nous montrons que les fonctions de hauteur convergent vers une forme limite déterministe et que les fluctuations globales sont données par le champ libre gaussien 2-dimensionnel, lorsque $q, t \to 1$ et que la maille du réseau tend vers 0. En se restreignant au cas sans interactions, ceci confirme la conjecture de Kenyon–Okounkov pour le cas de la mesure r^{volume} pour des conditions au bord générales. Notre approche utilise des opérateurs aux différences agissant sur les processus de Macdonald.

MSC: 33D52; 82B23

Keywords: Macdonald symmetric function; Macdonald process; Plane partition; Gaussian free field

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Sampling of probability measures in the convex order by Wasserstein projection

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Abstract. In this paper, for μ and ν two probability measures on $\mathbb{R}^d$ with finite moments of order $\varrho \geq 1$, we define the respective projections for the W_ϱ-Wasserstein distance of μ and ν on the sets of probability measures dominated by ν and of probability measures larger than μ in the convex order. The W_2-projection of μ can be easily computed when μ and ν have finite support by solving a quadratic optimization problem with linear constraints. In dimension $d = 1$, Gozlan et al. (*Ann. Inst. Henri Poincaré Probab. Stat.* **54** (3) (2018) 1667–1693) have shown that the projection of μ does not depend on ϱ. We explicit their quantile functions in terms of those of μ and ν. The motivation is the design of sampling techniques preserving the convex order in order to approximate Martingale Optimal Transport problems by using linear programming solvers. We prove convergence of the Wasserstein projection based sampling methods as the sample sizes tend to infinity and illustrate them by numerical experiments.

Résumé. Soient μ et ν deux mesures de probabilité sur $\mathbb{R}^d$ ayant un moment d'ordre $\varrho \geq 1$ fini. Dans ce papier, nous définissons respectivement les projections de μ et ν pour la distance de Wasserstein W_ϱ sur l'ensemble des probabilités dominées par ν et sur l'ensemble des probabilités dominant μ pour l'ordre convexe. Pour $\varrho = 2$, la projection de μ peut facilement être calculée lorsque μ et ν ont un support fini en résolvant un problème de minimisation quadratique avec des contraintes linéaires. En dimension $d = 1$, Gozlan et al. (*Ann. Inst. Henri Poincaré Probab. Stat.* **54** (3) (2018) 1667–1693) ont montré que la projection de μ ne dépend pas de ϱ. Nous donnons ici l'expression de la fonction quantile de cette projection à l'aide des fonctions quantiles de μ et ν. La motivation de cette étude est de fournir une méthode d'échantillonage permettant de préserver l'ordre convexe. Cela permet ensuite d'approcher les problèmes de transport optimal martingale en utilisant un solveur de programmation linéaire. Nous prouvons la convergence des méthodes d'échantillonage basées sur la projection Wasserstein lorsque la taille des échantillons tend vers l'infini, et illustrons cette convergence par des exemples numériques.

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Keywords: Convex order; Martingale optimal transport; Wasserstein distance; Sampling techniques; Linear programming

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Discrete rough paths and limit theorems

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Abstract. In this article, we consider limit theorems for some weighted type random sums (or discrete rough integrals). We introduce a general transfer principle from limit theorems for unweighted sums to limit theorems for weighted sums via rough path techniques. As a by-product, we provide a natural explanation of the various new asymptotic behaviors in contrast with the classical unweighted random sum case. We apply our principle to derive some weighted type Breuer–Major theorems, which generalize previous results to random sums that do not have to be in a finite sum of chaos. In this context, a Breuer–Major type criterion in notion of Hermite rank is obtained. We also consider some applications to realized power variations and to Itô's formulas in law. In the end, we study the asymptotic behavior of weighted quadratic variations for some multi-dimensional Gaussian processes.

Résumé. Dans cet article, nous étudions les théorèmes limite pour des sommes aléatoires pondérées (ou intégrales discrètes rugueuses). Nous introduisons un principe de transfert général entre les théorèmes limite pour les sommes non pondérées et pour les sommes pondérées, en utilisant des techniques de chemins rugueux. Comme conséquence, nous proposons une explication naturelle pour la diversité des nouveaux comportements asymptotiques par rapport aux cas des sommes aléatoires non pondérées. Nous appliquons notre principe pour obtenir des théorèmes de type Breuer–Major pondérés, qui généralisent des résultats précédents aux cas de sommes aléatoires qui ne sont pas dans une somme finie de chaos. Dans ce contexte, un critère de type Breuer–Major en termes de rang d'Hermite est obtenu. Nous considérons aussi des applications pour réaliser des variations de puissance et pour les formules d'Itô en loi. A cette fin, nous étudions le comportement asymptotique de variations quadratiques pondérées pour des processus Gaussiens multidimensionnels.

MSC: 60B10; 60G15; 60G22; 60H07

Keywords: Discrete rough paths; Discrete rough integrals; Weighted random sums; Limit theorems; Breuer–Major theorem

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Existence, uniqueness and coalescence of directed planar geodesics: Proof via the increment-stationary growth process

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Abstract. We present a proof of the almost sure existence, uniqueness and coalescence of directed semi-infinite geodesics in planar growth models that is based on properties of an increment-stationary version of the growth process. The argument is developed in the context of the exponential corner growth model. It uses coupling, planar monotonicity, and properties of the stationary growth process to derive the existence of Busemann functions, which in turn control geodesics. This soft approach is in some situations an alternative to the much-applied 20-year-old arguments of C. Newman and co-authors. Along the way we derive some related results such as the distributional equality of the directed geodesic tree and its dual, originally due to L. Pimentel.

Résumé. Nous présentons une preuve d'existence, d'unicité, et de coalescence presque sûre de géodésiques semi-infinies dirigées dans des modèles de croissance planaires. La preuve est basée sur des propriétés d'une version stationnaire du processus de croissance. L'argument est développé dans le contexte du modèle de la percolation dirigée de dernier passage. Il utilise un couplage, une monotonicité planaire, et des propriétés du processus de croissance stationnaire pour déduire l'existence de fonctions de Busemann, qui elles-mêmes contrôlent les géodésiques. Cette approche élémentaire est dans certains cas une alternative aux arguments de C. Newman et coauteurs, très utilisés depuis une vingtaine d'années. En cours de route, nous obtenons des résultats connexes tels que l'égalité distributionnelle de l'arbre de géodésiques dirigées et de son dual, initialement due à L. Pimentel.

MSC: 60K35; 65K37

Keywords: Busemann function; Coalescence; Cocycle; Competition interface; Corner growth model; Directed percolation; Geodesic; Last-passage percolation

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On temporal regularity of stochastic convolutions in 2-smooth Banach spaces[1]

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Abstract. We show that paths of solutions to parabolic stochastic differential equations have the same regularity in time as the Wiener process (as of the current state of art). The temporal regularity is considered in the Besov–Orlicz space $B^{1/2}_{\Phi_2,\infty}(0,T;X)$ where $\Phi_2(x)=\exp(x^2)-1$ and X is a 2-smooth Banach space.

Résumé. Nous montrons que les trajectoires des solutions des équations aux deriveés partielles stochastiques paraboliques ont la même régularité en temps que le processus de Wiener (aussi loin que vont les connaissances actuelles en la matière). La régularité temporelle est considérée dans l'espace de Besov–Orlicz $B^{1/2}_{\Phi_2,\infty}(0,T;X)$ où $\Phi_2(x)=\exp(x^2)-1$ et X est un espace de Banach 2-lisse.

MSC: 60G17; 46E35; 60J65; 60H15

Keywords: Temporal regularity; Stochastic convolution; 2-smooth Banach space; Besov–Orlicz space

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Superdiffusions with super-exponential growth: Construction, mass and spread

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Abstract. Superdiffusions corresponding to differential operators of the form $Lu + \beta u - \alpha u^2$ with mass creation (potential) terms $\beta(\cdot)$ that are 'large functions' are studied. Our construction for superdiffusions with large mass creations works for the branching mechanism $\beta u - \alpha u^{1+\gamma}$, $0 < \gamma < 1$, as well.

Let $D \subseteq \mathbb{R}^d$ be a domain in $\mathbb{R}^d$. When β is large, the generalized principal eigenvalue λ_c of $L + \beta$ in D is typically infinite. Let $\{T_t, t \geq 0\}$ denote the Schrödinger semigroup of $L + \beta$ in D with zero Dirichlet boundary condition. Under the mild assumption that there exists an $0 < h \in C^2(D)$ so that $T_t h$ is finite-valued for all $t \geq 0$, we show that there is a unique $\mathcal{M}_{\text{loc}}(D)$-valued Markov process that satisfies a log-Laplace equation in terms of the minimal nonnegative solution to a semilinear initial value problem. Although for super-Brownian motion (SBM) this assumption requires β to be less than quadratic, the quadratic case will be treated as well.

When $\lambda_c = \infty$, the usual machinery, including martingale methods and PDE as well as other similar techniques cease to work effectively, both for the construction and for the investigation of the large time behavior of superdiffusions. In this paper, we develop the following two new techniques for the study of the local/global growth of mass and for the spread of superdiffusions:

- a generalization of the Fleischmann–Swart 'Poisson-coupling,' linking superprocesses with branching diffusions;
- the introduction of a new concept: the '*p-generalized principal eigenvalue.*'

The precise growth rate for the total population of SBM with $\alpha(x) = \beta(x) = 1 + |x|^p$ for $p \in [0, 2]$ is given in this paper.

Résumé. Nous étudions des superdiffusions qui correspondent à des opérateurs différentiels de la forme $Lu + \beta u - \alpha u^2$ tels que le terme de création de masse (potentiel) $\beta(\cdot)$ est une « grande fonction ». Notre construction pour les superdiffusions avec un grand terme de création de masse fonctionne aussi pour le mécanisme de branchement $\beta u - \alpha u^{1+\gamma}$, $0 < \gamma < 1$.

Soit $D \subseteq \mathbb{R}^d$ un domaine dans $\mathbb{R}^d$. Lorsque β est grand, la valeur propre principale généralisée λ_c de $L + \beta$ dans D est typiquement infinie. Soit $\{T_t, t \geq 0\}$ le semigroupe de Schrödinger de $L + \beta$ dans D avec condition aux limites de Dirichlet égale à zéro. Sous l'hypothèse légère qu'il existe un $0 < h \in C^2(D)$ tel que $T_t h$ a une valeur finie pour tout $t \geq 0$, nous montrons qu'il existe un unique processus de Markov à valeurs dans $\mathcal{M}_{\text{loc}}(D)$ satisfaisant une équation log-Laplace en fonction de la solution positive minimale d'un problèmeaux valeurs initiales semi-linéaire. Bien que pour le super-mouvement brownien (SMB), cette hypothèse demande que la fonction β soit dominée par une fonction quadratique, nous traitons aussi le cas quadratique.

Quand $\lambda_c = \infty$, les techniques habituelles, y compris les méthodes de martingale et les équations différentielles partielles, ainsi que d'autres techniques similaires, cessent d'être efficaces pour la construction des superdiffusions et pour l'étude de leur comportement en temps grand.

Dans cet article, nous développons les deux nouvelles techniques suivantes pour l'étude de la croissance locale/globale de la masse et pour l'étude de la propagation des superdiffusions :

- une généralisation du « Poisson-coupling » de Fleischmann–Swart, liant les super-processus aux diffusions branchantes ;
- l'introduction d'un nouveau concept : la « *valeur propre principale p-généralisée* ».

Nous identifions aussi le taux de croissance précis de la population totale du SMB pour $\alpha(x) = \beta(x) = 1 + |x|^p$ et $p \in [0, 2]$.

MSC: Primary 60J60; secondary 60J80

Keywords: Super-Brownian motion; Spatial branching processes; Superdiffusion; Super-exponential growth; Generalized principal eigenvalue; p-generalized principal eigenvalue; Poisson-coupling; Semi-orbit; Nonlinear h-transform; Weighted superprocess

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The Green's function on the double cover of the grid and application to the uniform spanning tree trunk

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Abstract. We compute the Green's function on the double cover of $\mathbb{Z}^2$, branched over a vertex or a face. We use this result to compute the local statistics of the "trunk" of the uniform spanning tree on the square lattice, i.e., the limiting probabilities of cylinder events conditional on the path connecting far away points passing through a specified edge. We also show how to compute the local statistics of large-scale triple points of the uniform spanning tree, where the trunk branches. The method reduces the problem to a dimer system with isolated monomers, and we compute the inverse Kasteleyn matrix using the Green's function on the double cover of the square lattice. For the trunk, the probabilities of cylinder events are in $\mathbb{Q}[\sqrt{2}]$, while for the triple points the probabilities are in $\mathbb{Q}[1/\pi]$.

Résumé. Nous calculons la fonction de Green sur le revêtement à deux feuillets de $\mathbb{Z}^2$, ramifié au-dessus d'un sommet ou d'une face. Nous utilisons ce résultat pour calculer les statistiques locales du « tronc » de l'arbre couvrant minimal sur le réseau carré, c'est-à-dire les probabilités limites des événements cylindriques conditionnées à ce que le chemin connectant deux sommets éloignés passe par une arête donnée. Nous montrons également comment calculer les statistiques locales des points triples à grande échelle de l'arbre couvrant minimal, où le tronc se sépare. La méthode consiste à ramener le problème à un système de dimères avec des monomères isolés, et nous calculons l'inverse de la matrice de Kasteleyn à l'aide de la fonction de Green sur le revêtement deux feuillets du réseau carré. Pour le tronc, les probabilités des événements cylindriques sont dans $\mathbb{Q}[\sqrt{2}]$, tandis que pour les points triples, les probabilités sont dans $\mathbb{Q}[1/\pi]$.

MSC: 82B41

Keywords: Loop-erased random walk; Laplacian; Green's function

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Thresholds for vanishing of 'Isolated' faces in random Čech and Vietoris–Rips complexes

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Abstract. We study combinatorial connectivity for two models of random geometric complexes. These two models – Čech and Vietoris–Rips complexes – are built on a homogeneous Poisson point process of intensity n on a d-dimensional torus, $d > 1$, using balls of radius r_n. In the former, the k-simplices/faces are formed by subsets of $(k+1)$ Poisson points such that the balls of radius r_n centred at these points have a mutual interesection and in the latter, we require only a pairwise intersection of the balls. Given a (simplicial) complex (i.e., a collection of k-simplices for all $k \geq 1$), we can connect k-simplices via $(k+1)$-simplices ('up-connectivity') or via $(k-1)$-simplices ('down-connectivity). Our interest is to understand these two combinatorial notions of connectivity for the random Čech and Vietoris–Rips complexes asymptotically as $n \to \infty$. In particular, we analyse in detail the threshold radius for vanishing of isolated k-faces for up and down connectivity of both types of random geometric complexes. Though it is expected that the threshold radius $r_n = \Theta((\frac{\log n}{n})^{1/d})$ in coarse scale, our results give tighter bounds on the constants in the logarithmic scale as well as shed light on the possible second-order correction factors. Further, they also reveal interesting differences between the phase transition in the Čech and Vietoris–Rips cases. The analysis is interesting due to non-monotonicity of the number of isolated k-faces (as a function of the radius) and leads one to consider 'monotonic' vanishing of isolated k-faces. The latter coincides with the vanishing threshold mentioned above at a coarse scale (i.e., $\log n$ scale) but differs in the $\log\log n$ scale for the Čech complex with $k = 1$ in the up-connected case. For the case of up-connectivity in the Vietoris–Rips complex and for r_n in the critical window, we also show a Poisson convergence for the number of isolated k-faces when $k \leq d$.

Résumé. Nous étudions la connectivité combinatoire pour deux modèles de complexes géométriques aléatoires. Ces deux modèles – les complexes de Čech et de Vietoris–Rips – sont construits sur la base d'un processus de Poisson homogène d'intensité n sur un tore de dimension d, $d > 1$, en utilisant des boules de rayon r_n. Dans le premier, les k-simplexes/faces sont formés par les sous-ensembles de $k+1$ points du processus de Poisson tels que l'intersection des boules de rayon r_n centrées en ces points est non vide, et dans le second, nous demandons seulement que les intersections deux-à-deux des boules soient non vides. Étant donné un complexe simplicial (c'est-à-dire une collection de k-simplexes pour tous $k \geq 1$), nous pouvons connecter les k-simplexes via les $(k+1)$-simplexes (connectivité par le haut) ou via les $(k-1)$-simplexes (connectivité par le bas).

Notre objectif est de comprendre ces deux notions combinatoires de connectivité pour les complexes de Čech et Vietoris–Rips asymptotiquement lorsque $n \to \infty$.

En particulier, nous analysons en détail le rayon critique pour la disparition des k-faces isolées pour la connectivité par le haut et par le bas dans les deux types de complexes géométriques aléatoires. Bien qu'il soit attendu que le rayon critique soit $r_n = \Theta((\frac{\log n}{n})^{1/d})$ dans une échelle grossière, nos résultats donnent des bornes plus fines sur les constantes dans l'échelle logarithmique et suggère les possibles facteurs correctifs de second ordre. De plus, ils révèlent aussi des différences intéressantes entre les transitions de phase entre les cas de Čech et Vietoris–Rips.

L'analyse est intéressante du fait de la non monotonie du nombre de k-faces isolées (comme fonction du rayon) ce qui conduit à considérer une version monotone de la disparition des k-faces. Cette dernière coïncide avec le seuil de disparition mentionné précédemment à une échelle grossière (c'est-à-dire à une échelle $\log n$) mais diffère à l'échelle $\log\log n$ pour le complexe de Čech avec $k = 1$ pour la connectivité par le haut.

Dans le cas de la connectivité par le haut dans le cas du complexe de Vietoris–Rips et pour r_n dans la fenêtre critique, nous montrons aussi une convergence vers un processus de Poisson pour le nombre de k-faces isolées quand $k \leq d$.

MSC: Primary 60D05; 05E45; secondary 60B99; 05C80

Keywords: Random geometric complexes; Random hypergraphs; Connectivity; Maximal faces; Phase transition; Poisson convergence

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Spread of an infection on the zero range process

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Abstract. We study the spread of an infection on top of a moving population. The environment evolves as a zero range process on the integer lattice starting in equilibrium. At time zero, the set of infected particles is composed by those which are on the negative axis, while particles at the right of the origin are considered healthy. A healthy particle immediately becomes infected if it shares a site with an infected particle. We prove that the front of the infection wave travels to the right with positive and finite velocity. As a central step in the proof of these results, we prove a space-time decoupling for the zero range process which is interesting on its own. Using a sprinkling technique, we derive an estimate on the correlation of functions of the space of trajectories whose supports are sufficiently far away.

Résumé. Nous étudions la propagation d'une infection dans une population en déplacement. L'environnement évolue comme un processus « zero range » sur le réseau entier partant de l'équilibre. Au temps zéro, l'ensemble des particules infectées est composé des particules sur l'axe négatif, alors que les particules à droite de l'origine sont considérées comme saines. Une particule saine devient immédiatement malade si elle partage un site avec une particule malade. Nous prouvons que le front de l'infection se déplace vers la droite à vitesse positive finie. Une étape-clef dans la preuve de ces résultats consiste à prouver un découplage en espace-temps pour le processus « zero range » qui est intéressant en soi. Par un argument de « sprinkling » nous déduisons une estimée sur la corrélation de fonctions de l'espace des trajectoires dont les supports sont suffisamment éloignés.

MSC: 60K37; 60K35; 82C22

Keywords: Zero range process; Decoupling; Infection process

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Hypothesis testing via Euclidean separation

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Abstract. We discuss an "operational" approach to testing convex composite hypotheses when the underlying distributions are heavy-tailed. It relies upon Euclidean separation of convex sets and can be seen as an extension of the approach to testing by convex optimization developed in (*Electron. J. Stat.* **9** (2015) 1645–1712; *Electron. J. Stat.* **10** (2016) 2204–2242). In particular, we show how one can construct quasi-optimal testing procedures for families of distributions which are majorated, in a certain precise sense, by a sub-spherical symmetric one and study the relationship between tests based on Euclidean separation and "potential-based tests." We apply the promoted methodology in the problem of sequential detection and illustrate its practical implementation in an application to sequential detection of changes in the input of a dynamic system.

Résumé. Nous proposons une méthode « opérationnelle » pour le problème de tests d'hypothèses composites convexes lorsque les distributions sous-jacentes possèdent des queues lourdes. Elle s'appuie sur la séparation euclidienne des ensembles convexes et peut être vue comme une extension de la méthode développée dans (*Electron. J. Stat.* **9** (2015) 1645–1712 ; *Electron. J. Stat.* **10** (2016) 2204–2242) pour l'étude des tests d'hypothèses reposant sur des techniques d'optimisation convexe. En particulier, nous montrons comment construire des tests quasi-optimaux pour des familles de distributions qui sont majorées, dans un sens précis, par une distribution symétrique quasi-sphérique et étudions la relation entre les tests basés sur la séparation euclidienne et les tests utilisant des potentiels. Nous appliquons la méthodologie proposée au problème de la détection séquentielle et décrivons sa mise en oeuvre pour la détection séquentielle de ruptures dans l'entrée d'un système dynamique.

MSC: Primary 62G10; 90C25; 62F03; secondary 62M10

Keywords: Hypothesis testing; Nonparametric testing; Composite hypothesis testing; Statistical applications of convex optimization

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Stability and mean-field limits of age dependent Hawkes processes

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Abstract. In the last decade, Hawkes processes have received a lot of attention as good models for functional connectivity in neural spiking networks. In this paper we consider a variant of this process; the age dependent Hawkes process, which incorporates individual post-jump behavior into the framework of the usual Hawkes model. This allows to model recovery properties such as refractory periods, where the effects of the network are momentarily being suppressed or altered. We show how classical stability results for Hawkes processes can be improved by introducing age into the system. In particular, we neither need to a priori bound the intensities nor to impose any conditions on the Lipschitz constants. When the interactions between neurons are of mean-field type, we study large network limits and establish the propagation of chaos property of the system.

Résumé. Depuis la dernière décennie il s'est avéré que la classe des processus de Hawkes fournit un bon modèle pour décrire la connectivité fonctionnelle dans un réseau de neurones. Dans cet article nous étudions une variante de ce processus, le processus de Hawkes structuré en âge. Cette structure en âge rajoute un comportement individuel après les sauts à la dynamique de chaque composante, ce qui permet en particulier de décrire une période refractaire durant laquelle l'influence du réseau est supprimée ou au moins modifiée. Nous améliorons les résultats de stabilité classiques pour les processus de Hawkes dans ce cadre. En particulier, nous n'avons ni besoin de supposer que les intensités sont bornées, ni d'imposer une condition aux normes Lipschitz des fonctions taux de saut. Lorsque les interactions entre les neurones sont du type champ moyen, nous étudions les limites en grande population et nous démontrons la propriété de propagation du chaos du système.

MSC: 60G55; 60G57; 60K05

Keywords: Multivariate nonlinear Hawkes processes; Multivariate point processes; Mean-field approximations; Age dependency; Stability; Coupling; Piecewise deterministic Markov processes

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Random permutations with logarithmic cycle weights

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Abstract. We consider random permutations on $\mathfrak{S}_n$ with logarithmic growing cycles weights and study the asymptotic behavior as the length n tends to infinity. We show that the cycle count process converges to a vector of independent Poisson variables and also compute the total variation distance between both processes. Next, we prove a central limit theorem for the total number of cycles. Furthermore we establish a shape theorem and a functional central limit theorem for the Young diagrams associated to random permutations under this measure. We prove these results using tools from complex analysis and combinatorics. In particular we have to apply the method of singularity analysis to generating functions of the form $\exp((-\log(1-z))^{k+1})$ with $k \geq 1$, which have not yet been studied in the literature.

Résumé. Nous considérons les permutations aléatoires sur $\mathfrak{S}_n$ dont les poids des cycles sont à croissance logarithmique et nous étudions le comportement asymptotique quand la longueur n tend vers l'infini. Nous montrons que le processus de comptage des cycles converge vers un vecteur de variables de Poisson indépendantes et nous calculons également la distance en variation totale entre les deux processus. Ensuite, nous prouvons un théorème central limite pour le nombre total de cycles. En outre, nous établissons un théorème de forme et un théorème central limite fonctionnel pour les diagrammes de Young associés à des permutations aléatoires sous cette mesure.

Nous prouvons ces résultats à l'aide d'outils d'analyse complexe et de combinatoire. En particulier nous devons appliquer la méthode d'analyse de singularité aux fonctions génératrices de la forme $\exp((-\log(1-z))k+1)$ avec $k \geq 1$, qui n'ont pas encore été étudiées dans la littérature.

MSC: 60F17; 60F05; 60C05; 40E05

Keywords: Random permutations; Cycle counts; Total variation distance; Total number of cycles; Singularity analysis; Limit shape; Functional central limit theorem; Tauberian theorem

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Exceptional graphs for the random walk

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Abstract. If $\mathcal{W}$ is the simple random walk on the square lattice $\mathbb{Z}^2$, then $\mathcal{W}$ induces a random walk $\mathcal{W}_G$ on any spanning subgraph $G \subset \mathbb{Z}^2$ of the lattice as follows: viewing $\mathcal{W}$ as a uniformly random infinite word on the alphabet $\{\mathbf{x}, -\mathbf{x}, \mathbf{y}, -\mathbf{y}\}$, the walk $\mathcal{W}_G$ starts at the origin and follows the directions specified by $\mathcal{W}$, only accepting steps of $\mathcal{W}$ along which the walk $\mathcal{W}_G$ does not exit G. For any fixed $G \subset \mathbb{Z}^2$, the walk $\mathcal{W}_G$ is distributed as the simple random walk on G, and hence $\mathcal{W}_G$ is almost surely recurrent in the sense that $\mathcal{W}_G$ visits every site reachable from the origin in G infinitely often. This fact naturally leads us to ask the following: does $\mathcal{W}$ almost surely have the property that $\mathcal{W}_G$ is recurrent for *every* $G \subset \mathbb{Z}^2$? We answer this question negatively, demonstrating that exceptional subgraphs exist almost surely. In fact, we show more to be true: exceptional subgraphs continue to exist almost surely for a countable collection of independent simple random walks, but on the other hand, there are almost surely no exceptional subgraphs for a branching random walk.

Résumé. Une marche aléatoire simple $\mathcal{W}$ sur $\mathbb{Z}^2$ induit pour chaque sous-graphe couvrant $G \subset \mathbb{Z}^2$ une marche $\mathcal{W}_G$: notamment, si on regarde $\mathcal{W}$ comme un mot infini uniforme sur l'alphabet $\{\mathbf{x}, -\mathbf{x}, \mathbf{y}, -\mathbf{y}\}$, alors la marche $\mathcal{W}_G$ commence à l'origine et suit les directions definis par $\mathcal{W}$, en acceptant seulement les pas de $\mathcal{W}$ le long desquels $\mathcal{W}_G$ reste sur G. Pour chaque $G \subset \mathbb{Z}^2$ fixé, la marche $\mathcal{W}_G$ a la loi d'une marche aléatoire simple sur G et alors $\mathcal{W}_G$ est presque sûrement récurrent dans le sens que $\mathcal{W}_G$ visite chaque sommet de G connexe à l'origine un nombre infini de fois. Alors, une question naturelle surgit : est-ce que presque sûrement pour $\mathcal{W}$, les marches $\mathcal{W}_G$ sont récurrents pour *tous* $G \subset \mathbb{Z}^2$ à la fois? Dans cet article, on répond à cette question d'une manière négative, en montrent l'existence des graphes exceptionnels sur lesquels la marche est transitoire. En fait, on montre que même si on considère un nombre dénombrable des marches aléatoires simples indépendantes, alors des graphes exceptionnels existent. D'autre coté, on montre que il n'existe pas des graphes exceptionnels pour la marche aléatoire branchante.

MSC: Primary 05C81; secondary 60G50

Keywords: Random walks; Recurrence and transience; Exceptional graphs; Traversal sequences

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On boundary detection

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Abstract. Given a sample of a random variable supported by a smooth compact manifold $M \subset \mathbb{R}^d$, we propose a test to decide whether the boundary of M is empty or not with no preliminary support estimation. The test statistic is based on the maximal distance between a sample point and the average of its k_n-nearest neighbors. We prove that the level of the test can be estimated, that, with probability one, its power is one for n large enough, and that there exists a consistent decision rule. Heuristics for choosing a convenient value for the k_n parameter and identifying observations close to the boundary are also given. We provide a simulation study of the test.

Résumé. Soit un n-échantillon issus d'une loi supportée par M, une variété compacte suffisament régulière. On propose un test de l'hypothèse nulle $\partial M = \varnothing$ contre l'hypothèse alternative $\partial M \neq \varnothing$ qui ne nécessite pas d'estimation de M préliminaire. La statistique de test est la distance maximale (adéquatement renormalisée) entre une observation et la moyenne de ses k_n-plus proches voisins. On montre que le niveau du test peut être estimé, que sa puissance est 1 lorsque n est suffisament grand et, enfin, qu'il existe une règle de décision consistente. De manière pratique, on propose aussi une heuristique pour le choix de k_n et pour l'indentification des observations proches du bord. Ces résultats sont illustrés par des simulations.

MSC: 62G10; 62H15

Keywords: Geometric inference; Boundary; Test; Nearest-neighbors

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On stiff problems via Dirichlet forms

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Abstract. The stiff problem is concerned with a thermal conduction model with a singular barrier of zero volume. In this paper, we shall build the phase transitions for the stiff problems in one-dimensional space. It turns out that every phase transition definitely depends on the total thermal resistance of the barrier, and the three phases correspond to the so-called impermeable pattern, semi-permeable pattern and permeable pattern of thermal conduction respectively. For each pattern, the related boundary condition of the flux at the barrier is also derived. Mathematically, we shall introduce and explore the so-called snapping out Markov process, which is the probabilistic counterpart of semi-permeable pattern in the stiff problem.

Résumé. Le problème raide (the « Stiff problem ») concerne un modèle de diffusion de la chaleur avec une barrière singulière de volume zéro. Dans ce papier, nous établissons le diagramme de phase pour le problème raide en dimension 1 : il y a trois phases (la phase imperméable, la phase demi-perméable et la phase perméable) et chaque phase dépend de la résistance thermique totale de la barrière. De plus, pour chaque phase, nous identifions la condition au bord pour le flux sur la barrière. Du point de vue mathématique, nous introduisons et étudions le processus de Markov à transition brusque (the snapping out Markov process), qui donne une interprétation probabiliste de la phase demi-perméable dans le problème raide.

MSC: 31C25; 60J25; 60J45; 60J50

Keywords: Stiff problems; Phase transitions; Dirichlet forms; Mosco convergences; Snapping out Markov processes

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Bayesian linear inverse problems in regularity scales

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Abstract. We obtain rates of contraction of posterior distributions in inverse problems defined by scales of smoothness classes. We derive abstract results for general priors, with contraction rates determined by Galerkin approximation. The rate depends on the amount of prior concentration near the true function and the prior mass of functions with inferior Galerkin approximation. We apply the general result to non-conjugate series priors, showing that these priors give near optimal and adaptive recovery in some generality, Gaussian priors, and mixtures of Gaussian priors, where the latter are also shown to be near optimal and adaptive. The proofs are based on general testing and approximation arguments, without explicit calculations on the posterior distribution. We are thus not restricted to priors based on the singular value decomposition of the operator. We illustrate the results with examples of inverse problems resulting from differential equations.

Résumé. Nous obtenons le taux de contraction des distributions a posteriori dans les problèmes inverses définis par des classes d'échelles de régularité. Nous obtenons des résultats abstraits pour des lois a posteriori générales déterminées par des approximations de type Galerkin. Le taux dépend du niveau de concentration de la loi a priori au voisinage des vrais paramètres et de la probabilité a priori de l'ensemble des paramètres avec approximation Galerkin inférieure. Nous appliquons le résultat abstrait a trois types de lois a priori : au cas des séries aléatoires non conjuguées, montrant ainsi que ces mesures a priori donnent une récupération presque optimale sous des hypothèses assez générales ; au cas des mesures gaussiennes ; et au cas des mélanges de gaussiennes, où il est également démontré que ces derniers sont presque optimaux et adaptatifs. Les preuves sont basées sur des tests statistiques et arguments d'approximation, sans calculs explicites sur la loi a posteriori. Nous ne sommes donc pas limités aux lois a priori basées sur la décomposition en valeurs singulières de l'opérateur. Nous illustrons les résultats par des exemples de problèmes inverses résultant d'équations différentielles.

MSC: Primary 62G20; secondary 35R30

Keywords: Adaptive estimation; Gaussian prior; Hilbert scale; Linear inverse problem; Nonparametric Bayesian estimation; Posterior contraction rate; Random series prior; Regularity scale; White noise

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On principal curves with a length constraint

Sur les courbes principales avec contrainte de longueur

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Abstract. In this paper, we are interested in the following problem: find a curve f minimizing the quantity $\mathbb{E}[\min_{t\in[0,1]} \|X - f(t)\|^2]$, where X is a random variable, under a length constraint. This question is known in the probability and statistical learning context as length-constrained principal curves optimization, as introduced in (*IEEE Trans. Pattern Anal. Mach. Intell.* **22** (2000) 281–297), and it also corresponds to a version of the "average-distance problem" studied in the calculus of variation and shape optimization community (*Ann. Sc. Norm. Super. Pisa Cl. Sci.* **II** (4) (2003) 631–678; *Progr. Nonlinear Differential Equations Appl.* **51** (2002) 41–65).

We investigate the theoretical properties satisfied by a principal curve $f : [0, 1] \to \mathbb{R}^d$ with length at most L, associated to a probability distribution with second-order moment. We suppose that the probability distribution is not supported on the image of a curve with length L. Studying open as well as closed optimal curves, we show that they have finite curvature. We also derive an Euler–Lagrange equation. This equation is then used to show that a length-constrained principal curve in two dimensions has no multiple point. Finally, some examples of optimal curves are presented.

Résumé. Dans cet article, nous nous intéressons au problème suivant : étant donné une variable aléatoire X, trouver une courbe f minimisant la quantité $\mathbb{E}[\min_{t\in[0,1]} \|X - f(t)\|^2]$, sous contrainte de longueur. Dans le contexte des probabilités et de l'apprentissage statistique, cette question est connue sous le nom d'optimisation de courbes principales avec contrainte de longueur, selon la définition introduite dans (*IEEE Trans. Pattern Anal. Mach. Intell.* **22** (2000) 281–297) ; elle correspond également à une version du « problème de distance moyenne » étudié dans la communauté de calcul des variations et d'optimisation de formes (*Ann. Sc. Norm. Super. Pisa Cl. Sci.* **II** (4) (2003) 631–678 ; *Progr. Nonlinear Differential Equations Appl.* **51** (2002) 41–65).

Nous étudions les propriétés théoriques vérifiées par une courbe principale $f : [0, 1] \to \mathbb{R}^d$ de longueur au plus L, associée à une loi de probabilité ayant un moment d'ordre deux. Nous faisons l'hypothèse que la loi de probabilité n'est pas à support dans l'image d'une courbe de longueur L. Etudiant des courbes optimales ouvertes ou fermées, nous montrons qu'elles ont une courbure finie. Nous obtenons également une équation d'Euler–Lagrange. Cette équation est ensuite utilisée pour montrer qu'une courbe principale de longueur contrainte en dimension deux n'a pas de point multiple. Enfin, nous présentons quelques exemples de courbes optimales.

MSC: Primary 60E99; secondary 35B38; 49Q10; 49Q20

Keywords: Principal curves; Average-distance problem; Quantization of probability measures; Length constraint; Finite curvature

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Spectral radii of sparse random matrices

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Abstract. We establish bounds on the spectral radii for a large class of sparse random matrices, which includes the adjacency matrices of inhomogeneous Erdős–Rényi graphs. Our error bounds are sharp for a large class of sparse random matrices. In particular, for the Erdős–Rényi graph $G(n, d/n)$, our results imply that the smallest and second-largest eigenvalues of the adjacency matrix converge to the edges of the support of the asymptotic eigenvalue distribution provided that $d/\log n \to \infty$. Together with the companion paper (Benaych-Georges, Bordenave and Knowles (2017)), where we analyse the extreme eigenvalues in the complementary regime $d/\log n \to 0$, this establishes a crossover in the behaviour of the extreme eigenvalues at $d \asymp \log n$. Our results also apply to non-Hermitian sparse random matrices, corresponding to adjacency matrices of directed graphs. The proof combines (i) a new inequality between the spectral radius of a matrix and the spectral radius of its nonbacktracking version together with (ii) a new application of the method of moments for nonbacktracking matrices.

Résumé. Nous établissons des bornes sur le rayon spectral pour une grande classe de matrices aléatoires creuses, qui inclut les matrices d'adjacence des graphes Erdős–Rényi inhomogènes. Nos bornes d'erreur sont optimales pour une grande classe de matrices aléatoires. En particulier, pour le graphe Erdős–Rényi $G(n, d/n)$, nos résultats impliquent que la plus petite et la deuxième plus grande valeurs propres de la matrice d'adjacence convergent vers les bords du support de la distribution asymptotique des valeurs propres sous la condition $d/\log n \to \infty$. Avec le papier (Benaych-Georges, Bordenave and Knowles (2017)), où nous analysons les valeurs propres extrêmes dans le régime complémentaire $d/\log n \to 0$, ceci établit une transition dans le comportement des valeurs propres dans le régime $d \asymp \log n$. Nos résultats s'appliquent aussi aux matrices non-hermitiennes, correspondant à des matrices d'adjacence de graphes dirigés. La démonstration combine (i) une nouvelle inégalité reliant le rayon spectral d'une matrice et le rayon spectral de sa version nonbacktracking avec (ii) une nouvelle application de la méthode des moments pour les matrices nonbacktracking.

MSC: 60B20; 15B52; 05C80

Keywords: Random matrices; Erdős–Rényi graphs; Sparse matrices nonbacktracking matrices; Spectral radius

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Performance guarantees for policy learning

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Abstract. This article gives performance guarantees for the regret decay in optimal policy estimation. We give a margin-free result showing that the regret decay for estimating a within-class optimal policy is second-order for empirical risk minimizers over Donsker classes when the data are generated from a fixed data distribution that does not change with sample size, with regret decaying at a faster rate than the standard error of an efficient estimator of the value of an optimal policy. We also present a result giving guarantees on the regret decay of policy estimators for the case that the policy falls within a restricted class and the data are generated from local perturbations of a fixed distribution, where this guarantee is uniform in the direction of the local perturbation. Finally, we give a result from the classification literature that shows that faster regret decay is possible via plug-in estimation provided a margin condition holds. Three examples are considered. In these examples, the regret is expressed in terms of either the mean value or the median value, and the number of possible actions is either two or finitely many.

Résumé. Cet article présente des garanties de performance concernant la vitesse à laquelle le regret s'amenuise dans le cadre de l'estimation d'une politique d'action optimale. Si la politique optimale est définie comme optimale relativement à un ensemble de politiques formant une classe de Donsker, et si elle est estimée par minimisation sur cet ensemble d'une estimation du regret vu comme une fonction sur celui-ci, alors un premier résultat révèle que la vitesse est de second ordre dès lors que les observations sont générées sous une loi qui ne change pas à mesure que leur nombre augmente. Plus spécifiquement, le regret de l'estimateur de la politique optimale s'amenuise plus rapidement que l'écart type d'un estimateur efficace de la valeur d'une politique optimale. Ce résultat ne nécessite pas le recours à une hypothèse de marge. Un second résultat porte sur la vitesse à laquelle le regret de l'estimateur de la politique optimale s'amenuise lorsque les observations sont générées sous des lois définies comme des perturbations locales d'une loi de référence fixe, la garantie de performance étant alors uniforme relativement aux directions de perturbation. Finalement, un troisième résultat montre qu'il est possible d'atteindre des vitesses plus rapides en mettant en œuvre une procédure d'estimation par substitution à la condition qu'une hypothèse de marge soit satisfaite. Ce résultat s'inspire de la littérature consacrée à la classification. Trois exemples illustrent nos trouvailles. Dans ceux-ci, le regret s'exprime en termes de valeur moyenne ou de valeur médiane, et les actions envisageables sont au nombre de deux ou bien en nombre fini.

MSC: 62F12; 62G20; 62H30; 62P10

Keywords: Individualized treatment rules; Personalized medicine; Policy learning; Precision medicine

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Diffusive scaling of the Kob–Andersen model in $\mathbb{Z}^d$

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Abstract. We consider the Kob–Andersen model, a cooperative lattice gas with kinetic constraints which has been widely analysed in the physics literature in connection with the study of the liquid/glass transition. We consider the model in a finite box of linear size L with sources at the boundary. Our result, which holds in any dimension and significantly improves upon previous ones, establishes for any positive vacancy density q a purely diffusive scaling of the relaxation time $T_{\mathrm{rel}}(q, L)$ of the system. Furthermore, as $q \downarrow 0$ we prove upper and lower bounds on $L^{-2}T_{\mathrm{rel}}(q, L)$ which agree with the physicists belief that the dominant equilibration mechanism is a cooperative motion of rare large droplets of vacancies. The main tools combine a recent set of ideas and techniques developed to establish universality results for kinetically constrained spin models, with methods from bootstrap percolation, oriented percolation and canonical flows for Markov chains.

Résumé. On considère le modèle de Kob–Andersen (KA), un modèle de particules sur réseau avec dynamique conservative et contraintes cinétiques. KA a été étudié en profondeur dans la literature physique en relation avec l'étude de la transition liquide/verre. On étudie le modèle dans une boite finie de taille L avec un réservoir de particules aux bords. Notre résultat, qui est valable en toute dimension et qui améliore d'une façon très significative les résultats précédents, établit que pour toute densité de sites vides q telle que $q > 0$, le temps de relaxation du système, $T_{\mathrm{rel}}(q; L)$, est purement diffusif. De plus, on établit des bornes supérieures et inférieures pour $L^{-2}T_{\mathrm{rel}}(q; L)$ qui sont en accord avec l'heuristique des physiciens selon laquelle, quand $q \downarrow 0$, le mécanisme dominant de relaxation est un mouvement coopératif de grandes et rares gouttelettes formées par des sites vides. Nos outils principaux mélangent : un ensemble de techniques et idées developpées pour établir les résultats d'universalité pour les modèles de spin avec contraintes cinétiques ; des méthodes developpeés en percolation bootstrap ; des méthodes developpées en percolation dirigée ; et la technique de chemins canoniques pour les chaînes de Markov.

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Keywords: Kawasaki dynamics; Spectral gap; Kinetically constrained models

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An entropic interpolation problem for incompressible viscous fluids

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Abstract. In view of studying incompressible inviscid fluids, Brenier introduced in the late 80's a relaxation of a geodesic problem addressed by Arnold in 1966. Instead of *inviscid* fluids, the present paper is devoted to incompressible *viscous* fluids. A natural analogue of Brenier's problem is introduced, where generalized flows are no more supported by absolutely continuous paths, but by Brownian sample paths. It turns out that this new variational problem is an entropy minimization problem with marginal constraints entering the class of convex minimization problems.

This paper explores the connection between this variational problem and Brenier's original problem. Its dual problem is derived and the general form of its solution is described. Under the restrictive assumption that the pressure is a nice function, the kinematics of its solution is made explicit and its relation with viscous fluid dynamics is discussed.

Résumé. Afin d'étudier les fluides incompressibles non-visqueux, Brenier a introduit à la fin des années 80 une relaxation du problème géodésique posé par Arnold en 1966. Dans le présent article, nous nous intéressons aux fluides visqueux incompressibles. Nous définissons un analogue naturel du problème de Brenier, où les flots généralisés ne sont plus supportés par des trajectoires absolument continues, mais plutôt par des trajectoires browniennes. Ce nouveau problème variationnel devient un problème de minimisation d'entropie avec des contraintes de lois marginales, et il entre dans le cadre des problèmes de minimisation convexe. Nous étudions le lien entre ce problème variationnel et le problème originel de Brenier. Nous déterminons le problème dual et nous décrivons la forme générale de sa solution. Sous l'hypothèse additionnelle que la pression est une fonction régulière, nous déterminons la cinématique de la solution et sa relation avec la dynamique des fluides visqueux.

MSC: 76D03; 28D20; 49Q20; 49S05; 60G99

Keywords: Incompressible viscous fluids; Entropy minimization; Diffusion processes; Convex duality; Stochastic velocities

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Noise reinforcement for Lévy processes

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Abstract. In a step reinforced random walk, at each integer time and with a fixed probability $p \in (0, 1)$, the walker repeats one of his previous steps chosen uniformly at random, and with complementary probability $1 - p$, the walker makes an independent new step with a given distribution. Examples in the literature include the so-called elephant random walk and the shark random swim. We consider here a continuous time analog, when the random walk is replaced by a Lévy process. For sub-critical (or admissible) memory parameters $p < p_c$, where p_c is related to the Blumenthal–Getoor index of the Lévy process, we construct a noise reinforced Lévy process. Our main result shows that the step-reinforced random walks corresponding to discrete time skeletons of the Lévy process, converge weakly to the noise reinforced Lévy process as the time-mesh goes to 0.

Résumé. Dans une marche aléatoire à pas renforcés, à chaque instant entier et avec une probabilité fixée $p \in (0, 1)$, le marcheur répète un de ses précédents pas tiré uniformément au hasard, et avec probabilité $1 - p$ effectue un nouveau pas indépendant de loi donnée. Comme exemples dans la littérature figurent l'elephant random walk et le shark random swim. Nous nous intéressons ici à un analogue en temps continu, c'est-à-dire lorsque la marche aléatoire est remplacée par un processus de Lévy. Pour des paramètres de mémoire sous-critiques (ou encore admissibles) $p < p_c$, où p_c est lié à l'indice de Blumenthal–Getoor du processus de Lévy, nous construisons un processus de Lévy à bruit renforcé. Notre résultat principal établit la convergence en loi des marches aléatoires à pas renforcés associées aux squelettes discrets d'un processus de Lévy, vers le processus de Lévy à bruit renforcé, lorsque que le pas de la subdivision du temps tend vers 0.

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Keywords: Reinforcement; Preferential attachment; Lévy process; Yule–Simon distribution; Blumenthal–Getoor index

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Stability of the logarithmic Sobolev inequality via the Föllmer process

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Abstract. We study the stability and instability of the Gaussian logarithmic Sobolev inequality, in terms of covariance, Wasserstein distance and Fisher information, addressing several open questions in the literature. We first establish an improved logarithmic Sobolev inequality which is at the same time scale invariant and dimension free. As a corollary, we show that if the covariance of the measure is bounded by the identity, one may obtain a sharp and dimension-free stability bound in terms of the Fisher information matrix. We then investigate under what conditions stability estimates control the covariance, and when such control is impossible. For the class of measures whose covariance matrix is dominated by the identity, we obtain optimal dimension-free stability bounds which show that the deficit in the logarithmic Sobolev inequality is minimized by Gaussian measures, under a fixed covariance constraint. On the other hand, we construct examples showing that without the boundedness of the covariance, the inequality is not stable. Finally, we study stability in terms of the Wasserstein distance, and show that even for the class of measures with a bounded covariance matrix, it is hopeless to obtain a dimension-free stability result. The counterexamples provided motivate us to put forth a new notion of stability, in terms of proximity to mixtures of the Gaussian distribution. We prove new estimates (some dimension-free) based on this notion. These estimates are strictly stronger than some of the existing stability results in terms of the Wasserstein metric. Our proof techniques rely heavily on stochastic methods.

Résumé. On étudie les propriétés de stabilité et d'instabilité de l'inégalité de Sobolev logarithmique gaussienne, en termes de covariance, de distance de Wasserstein et d'information de Fisher, répondant à plusieurs questions ouvertes dans la littérature. On établit d'abord une forme améliorée de l'inégalité de Sobolev logarithmique qui est à la fois invariante par transformation linéaire et indépendante de la dimension. Comme corollaire, on obtient une inégalité de stabilité optimale et indépendante de la dimension pour les mesures dont la covariance est majorée par l'identité. On se penche ensuite sur la question de savoir dans quelle mesure le déficit dans l'inégalité de Sobolev logarithmique contrôle la covariance de la mesure. On montre notamment que si la covariance est majorée par l'identité, alors à covariance fixée, la mesure gaussienne minimise ce déficit. D'un autre côté on présente un contrexemple montrant que sans hypothèse de covariance bornée, l'inégalité est instable. Enfin, on étudie la question de la stabilité en termes de distance de Wasserstein, et on montre que même en se restreignant aux mesures dont la covariance est bornée, il n'est pas possible d'obtenir un résultat de stabilité qui soit indépendant de la dimension. Les contrexemples que nous exhibons suggèrent une nouvelle notion de stabilité, en terme de proximité de la mesure à un mélange de gaussiennes. On démontre plusieurs résultats dans cette direction, certains étant indépendants de la dimension. Ces résultats sont par ailleurs plus forts que certains résultats de stabilité qu'on trouve dans la littérature. Nos techniques de preuve reposent fortement sur des méthodes stochastiques.

MSC: 39B62; 60J60

Keywords: Quantitative functional inequalities; Stochastic methods

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Concentration of Markov chains with bounded moments

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Abstract. Let $\{W_t\}_{t=1}^{\infty}$ be a finite state stationary Markov chain, and suppose that f is a real-valued function on the state space. If f is bounded, then Gillman's *expander Chernoff bound* (1993) provides concentration estimates for the random variable $f(W_1) + \cdots + f(W_n)$ that depend on the spectral gap of the Markov chain and the assumed bound on f. Here we obtain analogous inequalities assuming only that the q'th moment of f is bounded for some $q \geq 2$. Our proof relies on reasoning that differs substantially from the proofs of Gillman's theorem that are available in the literature, and it generalizes to yield dimension-independent bounds for mappings f that take values in an $L_p(\mu)$ for some $p \geq 2$, thus answering (even in the Hilbertian special case $p = 2$) a question of Kargin (*Ann. Appl. Probab.* **17** (4) (2007) 1202–1221).

Résumé. Soit $\{W_t\}_{t=1}^{\infty}$ une chaîne de Markov stationnaire à états finis, et supposons que f soit une fonction réelle définie sur l'espace d'états. Si f est bornée, *l'inégalité de Chernoff pour les graphes expanseurs* (1993) de Gillman fournit des estimations de concentration pour la variable aléatoire $f(W_1) + \cdots + f(W_n)$ qui dépendent du trou spectral de la chaîne de Markov et la borne sur f. Nous obtenons ici des inégalités analogues en supposant seulement que le q-ème moment de f est borné pour un certain $q \geq 2$. Notre démonstration repose sur un raisonnement qui diffère substantiellement des démonstrations du théorème de Gillman disponibles dans la littérature, et elle se généralise de façon à générer des bornes indépendantes de la dimension pour les applications f qui prennent des valeurs dans un $L_p(\mu)$ pour $p \geq 2$, répondant ainsi (même dans le cas spécial hilbertien $p = 2$) à une question de Kargin (*Ann. Appl. Probab.* **17** (4) (2007) 1202–1221).

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Keywords: Markov chains; Concentration bounds; Expander graphs; Gillman's theorem

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