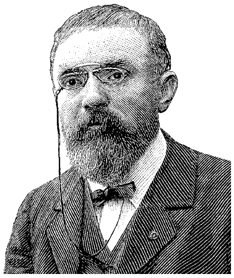

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Self-avoiding walk on $\mathbb{Z}^2$ with Yang–Baxter weights: Universality of critical fugacity and 2-point function

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Abstract. We consider a self-avoiding walk model (SAW) on the faces of the square lattice $\mathbb{Z}^2$. This walk can traverse the same face twice, but crosses any edge at most once. The weight of a walk is a product of local weights: each square visited by the walk yields a weight that depends on the way the walk passes through it. The local weights are parametrised by angles $\theta \in [\frac{\pi}{3}, \frac{2\pi}{3}]$ and satisfy the Yang–Baxter equation. The self-avoiding walk is embedded in the plane by replacing the square faces of the grid with rhombi with corresponding angles.

By means of the Yang–Baxter transformation, we show that the 2-point function of the walk in the half-plane does not depend on the rhombic tiling (*i.e.* on the angles chosen). In particular, this statistic concides with that of the self-avoiding walk on the hexagonal lattice. Indeed, the latter can be obtained by choosing all angles θ equal to $\frac{\pi}{3}$.

For the hexagonal lattice, the critical fugacity of SAW was recently proved to be equal to $1+\sqrt{2}$. We show that the same is true for any choice of angles. In doing so, we also give a new short proof to the fact that the partition function of self-avoiding bridges in a strip of the hexagonal lattice tends to 0 as the width of the strip tends to infinity. This proof also yields a quantitative bound on the convergence.

Résumé. On considère un modèle de marches auto-évitantes sur les faces du réseau carré $\mathbb{Z}^2$. Ce type de marche peut traverser la même face deux fois, mais traverse chaque arrête au plus une fois. Le poids d'une telle marche est le produit de poids locaux : chaque face visitée contribue par un poids qui dépend de la façon dont la marche la traverse. Les poids locaux associés à chaque face sont paramétrés par des angles $\theta \in [\frac{\pi}{3}, \frac{2\pi}{3}]$ et satisfont l'équation de Yang–Baxter. La marche est plongée dans le plan en remplaçant les faces carrées du réseau par des losanges d'angles correspondant à leur poids.

À l'aide de la transformation de Yang–Baxter, on montre que la fonction à deux points de la marche dans le demi-plan ne dépend pas des angles des losanges. En particulier, cette statistique coïncide avec celle de la marche aléatoire sur le réseau hexagonal – celle-ci est obtenue en choisissant tous les angles θ égaux à $\frac{\pi}{3}$.

La fugacité critique des marches auto-évitantes sur le réseau hexagonal a été calculée récemment : elle vaut $1+\sqrt{2}$. Nous montrons que la même chose est valable pour tout choix d'angles. A cette occasion, on donne une nouvelle preuve du fait que la fonction de partition des ponts auto-évitants dans une bande du réseau hexagonal tend vers 0 quand la largeur de la bande tend vers l'infini. De plus, on montre une borne quantitative sur le taux de convergence.

MSC2020 subject classifications: 60K35; 60D05; 82B23; 82B20

Keywords: Self-avoiding walk; Yang–Baxter; Universality; Isoradial graphs; Rhombic tiling; Critical fugacity

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Hausdorff dimension of the uniform measure of Galton–Watson trees without the XlogX condition

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Abstract. We consider a Galton–Watson tree with offspring distribution ν of finite mean. The uniform measure on the boundary of the tree is obtained by putting mass 1 on each vertex of the n-th generation and taking the limit $n \to \infty$. In the case $E[\nu \log(\nu)] < \infty$, this measure has been well studied, and it is known that the Hausdorff dimension of the measure is equal to $\log(m)$ (*J. Lond. Math. Soc. (2)* **24** (1981) 373–384; *Ergodic Theory Dynam. Systems* **15** (1995) 593–619). When $E[\nu \log(\nu)] = \infty$, we show that the dimension drops to 0. This answers a question of Lyons, Pemantle and Peres (In *Classical and Modern Branching Processes. Proceedings of the IMA Workshop* (1997) 223–237 Springer).

Résumé. Nous considérons un arbre de Galton–Watson dont le nombre d'enfants ν a une moyenne finie. La mesure uniforme sur la frontière de l'arbre s'obtient en chargeant chaque sommet de la n-ième génération avec une masse 1, puis en prenant la limite $n \to \infty$. Dans le cas $E[\nu \log(\nu)] < \infty$, cette mesure est bien étudiée, et l'on sait que la dimension de Hausdorff de la mesure est égale à $\log(m)$ (*J. Lond. Math. Soc. (2)* **24** (1981) 373–384; *Ergodic Theory Dynam. Systems* **15** (1995) 593–619). Lorsque $E[\nu \log(\nu)] = \infty$, nous montrons que la dimension est 0. Cela répond à une question posée par Lyons, Pemantle et Peres (In *Classical and Modern Branching Processes. Proceedings of the IMA Workshop* (1997) 223–237 Springer).

MSC2020 subject classifications: 60J80; 28A78

Keywords: Galton–Watson tree; Hausdorff dimension

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Sparse random matrices have simple spectrum

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Abstract. Let M_n be a class of symmetric sparse random matrices, with independent entries $M_{ij} = \delta_{ij}\xi_{ij}$ for $i \le j$. δ_{ij} are i.i.d. Bernoulli random variables taking the value 1 with probability $p \ge n^{-1+\delta}$ for any constant $\delta > 0$ and ξ_{ij} are i.i.d. centered, subgaussian random variables. We show that with high probability this class of random matrices has simple spectrum (i.e. the eigenvalues appear with multiplicity one). We can slightly modify our proof to show that the adjacency matrix of a sparse Erdős–Rényi graph has simple spectrum for $n^{-1+\delta} \le p \le 1 - n^{-1+\delta}$. These results are optimal in the exponent. The result for graphs has connections to the notorious graph isomorphism problem.

Résumé. On définit une classe M_n de matrices symétriques clairsemées, à coefficients indépendants, en posant $M_{ij} = \delta_{ij}\xi_{ij}$ pour $i \le j$, où les δ_{ij} sont des variables aléatoires de Bernoulli i.i.d. prenant la valeur 1 avec probabilité $p \ge n^{-1+\delta}$ pour une constante $\delta > 0$ arbitraire, et les ξ_{ij} sont des variables aléatoires sous-gaussiennes i.i.d. centrées. Nous montrons qu'avec une grande probabilité, cette classe de matrices aléatoires a un spectre simple, c'est-à-dire que les valeurs propres sont de multiplicité 1. Une légère modification de la démonstration de ce résultat permet de montrer montrer que la matrice d'adjacence d'un graphe d'Erdős–Rényi clairsemé a un spectre simple pour $n^{-1+\delta} \le p \le 1 - n^{-1+\delta}$. Ces résultats sont optimaux en les exposants. Le résultat pour les graphes a des liens avec le célèbre problème de l'isomorphisme de graphe.

MSC2020 subject classifications: Primary 60B20; 15B52; secondary 05C80

Keywords: Random matrices; Sparse matrices; Eigenvalue degeneracy; Random graphs; Graph isomorphism

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Hard-edge asymptotics of the Jacobi growth process

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Abstract. We introduce a two parameter ($\alpha, \beta > -1$) family of interacting particle systems with determinantal correlation kernels expressible in terms of Jacobi polynomials $\{P_k^{(\alpha,\beta)}\}_{k\geq 0}$. The family includes previously discovered Plancherel measures for the infinite-dimensional orthogonal and symplectic groups. The construction uses certain multivariate BC-type orthogonal polynomials that generalize the characters of these groups.

The local asymptotics near the hard edge where one expects distinguishing behavior yields the multi-time (α, β)-dependent discrete Jacobi kernel and the multi-time β-dependent hard-edge Pearcey kernel. The hard-edge Pearcey kernel has previously appeared in the asymptotics of non-intersecting squared Bessel paths at the hard edge.

Résumé. Nous introduisons une famille à deux paramètres ($\alpha, \beta > -1$) de systèmes de particules en interaction, avec des noyaux de corrélation déterminantaux qui s'expriment en termes des polynômes de Jacobi $\{P_k^{(\alpha,\beta)}\}_{k\geq 0}$. Cette famille comprend les mesures de Plancherel pour les groupes orthogonal et symplectique de dimension finie, qui avaient été découvertes auparavant. La construction utilise certains polynômes orthogonaux multivariés de type BC qui généralisent les caractères de ces derniers groupes.

Les asymptotiques locales près du bord dur, où l'on attend un comportement caractéristique, font intervenir le noyau de Jacobi discret multi-temps dépendant de (α, β), et le noyau de Pearcey du bord dur multi-temps dépendant de (α, β). Le noyau de Pearcey du bord dur était apparu précédemment dans les asymptotiques au bord dur de processus de carrés de Bessel s'évitant mutuellement.

MSC2020 subject classifications: 60J25

Keywords: Determinantal point process; Hard-edge Pearcey; Jacobi

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Hanson–Wright inequality in Banach spaces

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Abstract. We discuss two-sided bounds for moments and tails of quadratic forms in Gaussian random variables with values in Banach spaces. We state a natural conjecture and show that it holds up to additional logarithmic factors. Moreover in a certain class of Banach spaces (including L_r-spaces) these logarithmic factors may be eliminated. As a corollary we derive upper bounds for tails and moments of quadratic forms in subgaussian random variables, which extend the Hanson–Wright inequality.

Résumé. Nous étudions des bornes bilatères pour les moments et queues de distribution de formes quadratiques de variables aléatoires gaussiennes à valeurs dans des espaces de Banach. Nous formulons une conjecture naturelle, et en proposons une preuve à des facteurs logarithmiques près. De plus nous montrons que ces facteurs logarithmiques sont éliminables pour une certaine classe d'espaces de Banach incluant les espaces L_r. Comme corollaire, nous obtenons une majoration pour les moments et la queue de distribution de formes quadratiques de variables aléatoires sous-gaussiennes, qui étend l'inégalité de Hanson–Wright.

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Keywords: Tail and moment inequalities; Quadratic forms; Hanson–Wright inequality; Gaussian chaoses; Gaussian processes; Metric entropy

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Sparse space–time models: Concentration inequalities and Lasso

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Abstract. Inspired by Kalikow-type decompositions, we introduce a new stochastic model of infinite neuronal networks, for which we establish sharp oracle inequalities for Lasso methods and restricted eigenvalue properties for the associated Gram matrix with high probability. These results hold even if the network is only partially observed. The main argument rely on the fact that concentration inequalities can easily be derived whenever the transition probabilities of the underlying process admit a *sparse space–time representation*.

Résumé. En s'inspirant des décompositions de Kalikow, nous introduisons un nouveau modèle de réseaux neuronaux infinis, pour lesquels nous établissons des inégalités d'oracle précises pour des méthodes Lasso et des propriétés de valeur propre restreinte pour la matrice de Gram associée avec grande probabilité. Ces résultats sont vrais même si le réseau n'est que partiellement observé. L'argument principal est d'établir des inégalités de concentration quand les probabilités de transition sous-jacentes ont une représentation parcimonieuse en temps et espace.

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Keywords: Restricted eigenvalue; Chains of infinite order; Perfect simulation; Concentration inequalities; Oracle inequalities; Lasso estimator; Stochastic neuronal networks

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Fluctuation lower bounds in planar random growth models

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Abstract. We prove $\sqrt{\log n}$ lower bounds on the order of growth fluctuations in three planar growth models (first-passage percolation, last-passage percolation, and directed polymers) under no assumptions on the distribution of vertex or edge weights other than the minimum conditions required for avoiding pathologies. Such bounds were previously known only for certain restrictive classes of distributions. In addition, the first-passage shape fluctuation exponent is shown to be at least $1/8$, extending previous results to more general distributions.

Résumé. Nous montrons des bornes inférieures de $\sqrt{\log n}$ pour l'ordre des fluctuations de trois modèles planaires de croissance (percolation de premier passage, percolation de dernier passage et polymères dirigés) sans autre hypothèse sur la loi des poids des sommets ou des arêtes que les conditions minimales permettant d'éviter les cas pathologiques. De telles bornes étaient connues auparavant seulement pour certaines classes restreintes de lois. De surcroît, nous montrons que l'exposant des fluctuations autour de la forme limite pour la percolation de premier passage est au moins $1/8$, ce qui étend des résultats précédents à des lois plus générales.

MSC2020 subject classifications: 60E15; 60K35; 82D60; 60K37

Keywords: First-passage percolation; Corner growth model; Directed polymers

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Entropy and expansion

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Abstract. Shearer's inequality bounds the sum of joint entropies of random variables in terms of the total joint entropy. We give another lower bound for the same sum in terms of the individual entropies when the variables are functions of independent random seeds. The inequality involves a constant characterizing the expansion properties of the system.

Our results generalize to entropy inequalities used in recent work in invariant settings, including the edge-vertex inequality for factor-of-IID processes, Bowen's entropy inequalities, and Bollobás's entropy bounds in random regular graphs.

The proof method yields inequalities for other measures of randomness, including covariance.

As an application, we give upper bounds for independent sets in both finite and infinite graphs.

Résumé. L'inégalité de Shearer limite la somme des entropies conjointes de variables aléatoires en termes d'entropie conjointe totale. Nous donnons une autre borne inférieure pour la même somme en termes d'entropies individuelles lorsque les variables sont des fonctions de nombres aléatoires indépendantes. Le coefficient de l'inégalité caractérise les propriétés d'expansion du système.

Nos résultats se généralisent aux inégalités d'entropie utilisées dans des travaux récents dans des contextes invariants, y compris l'inégalité arête-sommet pour les processus de facteur d'IID, les inégalités d'entropie de Bowen et les limites d'entropie de Bollobás dans des graphes réguliers aléatoires.

La méthode de la preuve produit des inégalités pour d'autres mesures de l'aléatoire, y compris la covariance.

Comme application, nous donnons des bornes supérieures pour des ensembles indépendants dans des graphes finis et infinis.

MSC2020 subject classifications: 94A17; 60K35; 37A50; 05E18; 05C69

Keywords: Entropy inequality; Expansion; Cheeger constant; Graph isoperimetry; Factor-of-IID; Local algorithm; Independent set

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Fluctuations of Biggins' martingales at complex parameters

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Abstract. The long-term behavior of a supercritical branching random walk can be described and analyzed with the help of Biggins' martingales, parametrized by real or complex numbers. The study of these martingales with complex parameters is a rather recent topic. Assuming that certain sufficient conditions for the convergence of the martingales to non-degenerate limits hold, we investigate the fluctuations of the martingales around their limits. We discover three different regimes. First, we show that for parameters with small absolute values, the fluctuations are Gaussian and the limit laws are scale mixtures of the real or complex standard normal laws. We also cover the boundary of this phase. Second, we find a region in the parameter space in which the martingale fluctuations are determined by the extremal positions in the branching random walk. Finally, there is a critical region (typically on the boundary of the set of parameters for which the martingales converge to a non-degenerate limit) where the fluctuations are stable-like and the limit laws are the laws of randomly stopped Lévy processes satisfying invariance properties similar to stability.

Résumé. Le comportement en temps long d'une marche aléatoire branchante surcritique peut être décrit et analysé en utilisant les martingales de Biggins, à paramètres réels ou complexes. L'étude de ces martingales prises en des paramètres complexes est un sujet d'étude assez récent. En supposant que certaines conditions pour leur convergence vers une limite non-dégénérée sont vérifiées, nous étudions les fluctuations de ces martingales autour de leurs limites. Nous observons trois régimes différents. D'abord, nous montrons que dans une région dans laquelle les paramètres sont de petite norme, les fluctuations sont gaussiennes, et les lois limites sont des mélanges de variables aléatoires gaussiennes réelles ou complexes. Nous obtenons également le comportement au bord de cette région. Dans un second temps, nous trouvons une région dans l'espace des paramètres dans laquelle les fluctuations des martingales sont déterminées par les valeurs extrêmes dans la marche aléatoire branchante. Finalement, il existe une région critique (typiquement sur le bord de l'ensemble des paramètres pour lesquels les martingales convergent vers une limite non-dégénérée) où les fluctuations sont de type stable, et les lois limites sont les lois de valeurs en un temps aléatoire de processus de Lévy satisfaisant des propriétés d'invariance similaires à la stabilité.

MSC2020 subject classifications: 60J80; 60F15

Keywords: Branching random walk; Central limit theorem; Complex martingales; Minimal position; Point processes; Rate of convergence; Stable processes

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A natural extension of Markov processes and applications to singular SDEs

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Abstract. We develop a general method for extending Markov processes to a larger state space such that the added points form a polar set. The so obtained extension is an improvement on the standard trivial extension in which case the process is made stuck in the added points, and it renders a new technique of constructing extended solutions to S(P)DEs from all starting points, in such a way that they are solutions at least after any strictly positive time. Concretely, we adopt this strategy to study SDEs with singular coefficients on an infinite dimensional state space (e.g. SPDEs of evolutionary type), for which one often encounters the situation where not every point in the space is allowed as an initial condition. The same can happen when constructing solutions of martingale problems or Markov processes from (generalized) Dirichlet forms, to which our new technique also applies.

Résumé. On établit une méthode générale pour élargir l'espace d'états d'un processus de Markov, de telle façon que l'ensemble des points ajoutés est polaire. Cette extension est une amélioration de l'extension triviale, dans quel cas le processus est bloqué dans les points ajoutés, et elle produit une technique nouvelle de construction des solutions étendues pour des ED(P) stochastiques, à partir de tous les points de départ, telle qu'elles soient des solutions au moins après tout moment de temps strictement positif. Concrètement, on adopte cette stratégie pour étudier des ED stochastiques avec des coefficients singuliers, sur un espace d'états de dimension infinie (par example des EDP stochastiques de type evolution), pour lesquelles on rencontre des situations où tous les points de l'espace ne sont pas des conditions initiales autorisées. La même chose peut se passer dans la construction des solutions pour des problèmes de martingales ou pour des processus de Markov à partir de formes de Dirichlet (généralisées), pour lesquelles notre nouvelle technique s'applique aussi.

MSC2020 subject classifications: 60H15; 60H10; 60J45; 60J35; 60J40; 60J57; 31C25; 47D07; 35R60; 60J25

Keywords: Stochastic differential equation on Hilbert spaces; Stochastic PDE; Martingale problem; Not allowed starting point; Girsanov transform; Nonregular drift; Dirichlet form; Right process; Fine topology

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Lower deviation and moderate deviation probabilities for maximum of a branching random walk

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Abstract. Given a supercritical branching random walk on $\mathbb{R}$ started from the origin, let M_n be the maximal position of individuals at the nth generation. Under some mild conditions, it is proved in (*Ann. Probab.* **41** (2013) 1362–1426) that as $n \to \infty$, $M_n - x^* n + \frac{3}{2\theta^*} \log n$ converges in law for some suitable constants x^* and θ^*. In this work, we investigate its moderate deviation, in other words, the convergence rates of

$$\mathbb{P}\left(M_n \le x^* n - \frac{3}{2\theta^*} \log n - \ell_n\right),$$

for any positive sequence (ℓ_n) such that $\ell_n = O(n)$ and $\ell_n \uparrow \infty$. As a by-product, we obtain lower deviation of M_n; i.e., the convergence rate of $\mathbb{P}(M_n \le xn)$ for $x < x^*$ in Böttcher case where the offspring number is at least two. We also apply our techniques to study the small ball probability of the limit of the so-called derivative martingale. Our results complete those in (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 233–260) and (*Electron. Commun. Probab.* **23** (2018) 1–12).

Résumé. Étant donnée une marche aléatoire branchante surcritique sur $\mathbb{R}$ issue de l'origine, on note M_n la position maximale des individus à la n-ème génération. Sous des conditions raisonnables, il a été prouvé dans (*Ann. Probab.* **41** (2013) 1362–1426) que lorsque $n \to \infty$, $M_n - x^* n + \frac{3}{2\theta^*} \log n$ converge en loi pour certaines constantes appropriées x^* et θ^*. Dans cet article, nous envisageons la déviation modérée, autrement dit, les taux de convergence de

$$\mathbb{P}\left(M_n \le x^* n - \frac{3}{2\theta^*} \log n - \ell_n\right),$$

pour toute positive suite (ℓ_n) telle que $\ell_n = O(n)$ et $\ell_n \uparrow \infty$. En particulier, nous obtenons la déviation inférieure de M_n; c'est-à-dire, le taux de convergence de $\mathbb{P}(M_n \le xn)$ avec $x < x^*$ dans le cas Böttcher où le nombre d'enfants est au moins deux. Nous appliquons également ces techniques à l'étude de la petite déviation de la limite de la martingale dérivée. Notre résultats complètent ceux dans (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 233–260) et (*Electron. Commun. Probab.* **23** (2018) 1–12).

MSC2020 subject classifications: 60F10; 60J80; 60G50

Keywords: Branching random walk; Maximal position; Moderate deviation; Lower deviation; Schröder case; Böttcher case; Small ball probability; Derivative martingale

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Functional approximations via Stein's method of exchangeable pairs

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Abstract. We combine the method of exchangeable pairs with Stein's method for functional approximation. As a result, we give a general linearity condition under which an abstract Gaussian approximation theorem for stochastic processes holds. We apply this approach to estimate the distance of a sum of random variables, chosen from an array according to a random permutation, from a Gaussian mixture process. This result lets us prove a functional combinatorial central limit theorem. We also consider a graph-valued process and bound the speed of convergence of the distribution of its rescaled edge counts to a continuous Gaussian process.

Résumé. Nous combinons la méthode des paires échangeables avec la méthode d'approximation fonctionnelle de Stein. De cette façon, nous obtenons une condition générale de linéarité sous laquelle un résultat abstrait d'approximation Gaussienne est valide. Nous appliquons cette approche à l'estimation de la distance entre une somme de variables aléatoires, choisies dans un tableau par le biais d'une permutation aléatoire, et un mélange de processus Gaussiens. À partir de ce résultat, nous prouvons un théorème central limite fonctionnel combinatoire. Nous considérons également un graphe aléatoire et fournissons des bornes pour la vitesse de convergence de la loi de son nombre d'arêtes (aprés un changement d'échelle) vers un processus Gaussien continu.

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Keywords: Stein's method; Functional convergence; Exchangeable pairs; Stochastic processes

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Subordination methods for free deconvolution

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Abstract. We derive subordination functions for free additive and free multiplicative deconvolutions under mild moment conditions. Our results include an algorithm to calculate these subordination functions, and thus the associated Cauchy transforms, for complex numbers with imaginary part greater than a parameter depending on the measure to deconvolve. The existence of these subordination functions on such domains reduces the problem of free deconvolutions to the problem of the classical additive deconvolution with a Cauchy distribution. Thus, our results, combined with known methods for the deconvolution with a Cauchy distribution, allow us to solve the free deconvolution problem. We also present extensions of these results to the case of operator-valued deconvolutions.

Résumé. Nous dérivons des fonctions de subordination pour la déconvolution libre additive et multiplicative sous des conditions de moment faibles. Nos résultats incluent un algorithme pour calculer ces fonctions de subordination, et donc les transformées de Cauchy associées, pour les nombres complexes ayant une partie imaginaire supérieure à un paramètre dépendant de la mesure à déconvoler. L'existence des fonctions de subordination sur de tels domaines réduit le problème de la déconvolution libre au problème de la déconvolution additive classique par une distribution de Cauchy. Ainsi, nos résultats, combinés à des méthodes connues de déconvolution classique par une distribution de Cauchy, nous permettent de résoudre le problème de déconvolution libre. Nous présentons également des extensions de ces résultats au cas des déconvolutions à valeur opérateur.

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Keywords: Deconvolution; Free probability; Random matrices; Subodination

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Long-time limits and occupation times for stable Fleming–Viot processes with decaying sampling rates

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Abstract. A class of Fleming–Viot processes with decaying sampling rates and α-stable motions that correspond to distributions with growing populations are introduced and analyzed. Almost sure long-time scaling limits for these processes are developed, addressing the question of long-time population distribution for growing populations. Asymptotics in higher orders are investigated. Convergence of particle location occupation and inhabitation time processes are also addressed and related by way of the historical process. The basic results and techniques allow general Feller motion/mutation and may apply to other measure-valued Markov processes.

Résumé. Dans cet article, nous introduisons et analysons une classe de processus de Fleming–Viot, avec taux d'échantillonnage décroissant et déplacement α-stable, correspondant à des distributions de populations croissantes. Les théorèmes limites en temps long presque-sûr pour ces processus sont obtenus, répondant ainsi à la question de la distribution en temps long de la population dans le cas de populations croissantes. Les asymptotiques d'ordres supérieurs sont aussi obtenues. Les convergences des processus de temps d'occupation et d'habitation des particules sont considérées et reliées au moyen du processus historique. Les résultats et techniques autorisent des processus de Feller de déplacement/mutation généraux et peuvent s'appliquer à d'autres processus de Markov à valeurs mesures.

MSC2020 subject classifications: Primary 60J80; 60F15; secondary 60B10; 60G57

Keywords: Fleming–Viot process; α-Stable process; Historical process; Occupation times

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Cutoff for the Bernoulli–Laplace urn model with $o(n)$ swaps

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Abstract. We study the mixing time of the (n,k) Bernoulli–Laplace urn model, where $k \in \{0,1,\dots,n\}$. Consider two urns, each containing n balls, so that when combined they have precisely n red balls and n white balls. At each step of the process choose uniformly at random k balls from the left urn and k balls from the right urn and switch them simultaneously. We show that if $k = o(n)$, this Markov chain exhibits mixing time cutoff at $\frac{n}{4k}\log n$ and window of the order $\frac{n}{k}\log\log n$. This is an extension of a classical theorem of Diaconis and Shahshahani who treated the case $k=1$.

Résumé. Nous étudions le temps de mélange de l'urne de Bernoulli–Laplace de paramètres (n,k), où $k \in \{0,1,\dots,n\}$. On considère deux urnes, chacune contenant n boules, telles que combinées elles ont exactement n boules rouges et n boules blanches. A chaque étape du processus, on choisit au hasard k boules dans chaque urne et on les échange. Nous montrons que si $k = o(n)$, le temps de mélange de cette chaîne de Markov exhibe un phénomène de coupure à l'instant $\frac{n}{4k}\log n$ avec une fenêtre d'ordre $\frac{n}{k}\log\log n$. Ceci donne une extension du théorème classique de Diaconis et Shahshahani qui traitait le cas $k=1$.

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Keywords: Markov chain; Mixing time; Bernoulli–Laplace urn model; Cutoff phenomenon

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Semi-Markov processes, integro-differential equations and anomalous diffusion-aggregation

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Abstract. In this article integro-differential Volterra equations whose convolution kernel depends on the vector variable are considered and a connection of these equations with a class of semi-Markov processes is established. The variable order $\alpha(x)$-fractional diffusion equation is a particular case of our analysis and it turns out that it is associated with a suitable (non-independent) time-change of the Brownian motion. The resulting process is semi-Markovian and its paths have intervals of constancy, as it happens for the delayed Brownian motion, suitable to model trapping effects induced by the medium. However in our scenario the interval of constancy may be position dependent and this means traps of space-varying depth as it happens in a disordered medium. The strength of the trapping is investigated by means of the asymptotic behaviour of the process: it is proved that, under some technical assumptions on $\alpha(x)$, traps make the process non-diffusive in the sense that it spends a negligible amount of time out of a neighborhood of the region $\operatorname{argmin}(\alpha(x))$ to which it converges in probability under some more restrictive hypotheses on $\alpha(x)$.

Résumé. Dans cet article, les équations de Volterra intégro-différentielles dont le noyau de convolution dépend de la variable vectorielle sont considérées et une relation entre ces équations et une classe de processus semi-Markoviens est établie. L'équation de diffusion fractionnelle d'ordre variable $\alpha(x)$ est un cas particulier de notre analyse et elle se révèle être associée à un changement de temps approprié (non indépendant) du mouvement Brownien. Le processus résultant est semi-markovien et ses trajectoires ont des intervalles de constance, comme cela arrive pour le mouvement Brownien retardé, adapté pour modéliser les effets de piégeage induits par le milieu. Cependant, dans notre scénario, l'intervalle de constance peut dépendre de la position et cela signifie des pièges de profondeur variant dans l'espace comme cela se produit dans un milieu désordonné. La force du piégeage est étudiée au moyen du comportement asymptotique du processus: il est démontré que, sous certaines hypothèses techniques sur $\alpha(x)$, les pièges rendent le processus non diffusif en ce sens qu'il passe un temps négligeable hors d'un voisinage de la région $\operatorname{argmin}(\alpha(x))$ vers laquelle il converge en probabilité sous quelques hypothèses plus restrictives sur $\alpha(x)$.

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Comparing with octopi

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Abstract. Operator inequalities with a geometric flavour have been successful in studying mixing of random walks and quantum mechanics. We suggest a new way to extract such inequalities using the octopus inequality of Caputo, Liggett and Richthammer.

Résumé. Les inégalités d'opérateurs de nature géométrique ont été très utiles pour étudier le mélange des marches aléatoires et la mécanique quantique. Nous suggérons une nouvelle approche pour exhiber des inégalités de ce type en utilisant l'inégalité « pieuvre » (octopus inequality) de Caputo, Liggett et Richthammer.

MSC2020 subject classifications: 60B15; 20C30

Keywords: The interchange process; The stirring process; Random walk on the symmetric group; The quantum Heisenberg ferromagnet; Mixing times; The octopus inequality

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On the convergence of random tridiagonal matrices to stochastic semigroups

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Abstract. We develop an improved version of the stochastic semigroup approach to study the edge of β-ensembles pioneered by Gorin and Shkolnikov (*Ann. Probab.* **46** (2018) 2287–2344), and later extended to rank-one additive perturbations by the author and Shkolnikov (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 1402–1438). Our method is applicable to a significantly more general class of random tridiagonal matrices than that considered in (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 1402–1438; *Ann. Probab.* **46** (2018) 2287–2344), including some non-symmetric cases that are not covered by the stochastic operator formalism of Bloemendal, Ramírez, Rider, and Virág (*Probab. Theory Related Fields* **156** (2013) 795–825; *J. Amer. Math. Soc.* **24** (2011) 919–944).

We present two applications of our main results: Firstly, we prove the convergence of β-Laguerre-type (i.e., sample covariance) random tridiagonal matrices to the stochastic Airy semigroup and its rank-one spiked version. Secondly, we prove the convergence of the eigenvalues of a certain class of non-symmetric random tridiagonal matrices to the spectrum of a continuum Schrödinger operator with Gaussian white noise potential.

Résumé. Nous développons une version améliorée de l'approche de *stochastic semigroup* pour étudier l'extrémité des ensembles bêta introduits par Gorin et Shkolnikov (*Ann. Probab.* **46** (2018) 2287–2344), ensuite étendue aux ensembles bêta gaussiens avec perturbation de rang un par l'auteur et Shkolnikov (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 1402–1438). Notre méthode est applicable à une classe nettement plus générale de matrices tridiagonales aléatoires que celles dans (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 1402–1438; *Ann. Probab.* **46** (2018) 2287–2344), y compris certains cas non symétriques qui ne sont pas couverts par la méthode de *stochastic operators* introduite par Bloemendal, Ramírez, Rider et Virág (*Probab. Theory Related Fields* **156** (2013) 795–825; *J. Amer. Math. Soc.* **24** (2011) 919–944).

Nous présentons deux applications de nos principaux résultats : Premièrement, nous prouvons la convergence de matrices tridiagonales aléatoires de type β-Laguerre (c.-à-d., matrices de covariances empiriques) vers le semi-groupe du *stochastic Airy operator* et sa perturbation de rang un. Deuxièmement, nous prouvons la convergence des valeurs propres d'une certaine classe de matrices tridiagonales aléatoires non symétriques vers le spectre d'opérateurs de Schrödinger avec bruit blanc gaussien.

MSC2020 subject classifications: Primary 60B20; 60H25; 47D08; secondary 60J55

Keywords: Random tridiagonal matrices; Feynman-Kac formulas; Stochastic Airy operator; Stochastic Airy semigroup; Random walk occupation measures; Brownian local time; Strong invariance principles

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Induced graphs of uniform spanning forests

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Abstract. Given a subgraph H of a graph G, the induced graph of H is the largest subgraph of G whose vertex set is the same as that of H. Our paper concerns the induced graphs of the components of $\mathsf{WSF}(G)$, the wired uniform spanning forest on G, and, to a lesser extent, $\mathsf{FSF}(G)$, the free uniform spanning forest. We show that the induced graph of each component of $\mathsf{WSF}(\mathbb{Z}^d)$ is almost surely recurrent when $d \geq 8$. Moreover, the effective resistance between two points on the ray of the tree to infinity within a component grows linearly when $d \geq 9$. For any vertex-transitive graph G, we establish the following resampling property: Given a vertex o in G, let $\mathcal{T}_o$ be the component of $\mathsf{WSF}(G)$ containing o and $\overline{\mathcal{T}_o}$ be its induced graph. Conditioned on $\overline{\mathcal{T}_o}$, the tree $\mathcal{T}_o$ is distributed as $\mathsf{WSF}(\overline{\mathcal{T}_o})$. For any graph G, we also show that if $\mathcal{T}_o$ is the component of $\mathsf{FSF}(G)$ containing o and $\overline{\mathcal{T}_o}$ is its induced graph, then conditioned on $\overline{\mathcal{T}_o}$, the tree $\mathcal{T}_o$ is distributed as $\mathsf{FSF}(\overline{\mathcal{T}_o})$.

Résumé. Étant donné un sous-graphe H d'un graphe G, le graphe induit de H est le plus grand sous-graphe de G dont l'ensemble de sommets est le même que celui de H. Notre article concerne les graphes induits des composants connexes de $\mathsf{WSF}(G)$, la forêt recouvrante uniforme câblée sur G, et, dans une moindre mesure, $\mathsf{FSF}(G)$, la forêt recouvrante uniforme libre. Nous montrons que le graphe induit de chaque composant de $\mathsf{WSF}(\mathbb{Z}^d)$ est presque sûrement récurrent lorsque $d \geq 8$. De plus, la résistance effective entre deux points du rayon de l'arbre à l'infini au sein d'un composant croît linéairement lorsque $d \geq 9$. Pour tout graphe transitif à sommets G, nous établissons la propriété de rééchantillonnage suivante: Étant donné un sommet o dans G, soit $\mathcal{T}_o$ le composant de $\mathsf{WSF}(G)$ qui contient o et $\overline{\mathcal{T}_o}$ son graphe induit. Conditionné sur $\overline{\mathcal{T}_o}$, l'arbre $\mathcal{T}_o$ est distribué comme $\mathsf{WSF}(\overline{\mathcal{T}_o})$. Pour tout graphe G, nous montrons également que si $\mathcal{T}_o$ est le composant de $\mathsf{FSF}(G)$ qui contient o et $\overline{\mathcal{T}_o}$ est son graphe induit, alors conditionné sur $\overline{\mathcal{T}_o}$, l'arbre $\mathcal{T}_o$ est distribué comme $\mathsf{FSF}(\overline{\mathcal{T}_o})$.

MSC2020 subject classifications: Primary 60K35; 60J10; secondary 60G50; 60K37

Keywords: Resampling; Recurrence; Effective resistance; Loop-erased random walk

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Cutoff for random walk on dynamical Erdős–Rényi graph

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Abstract. We consider dynamical percolation on the complete graph K_n, where each edge refreshes its state at rate $\mu \ll 1/n$, and is then declared open with probability $p = \lambda/n$ where $\lambda > 1$. We study a random walk on this dynamical environment which jumps at rate $1/n$ along every open edge. We show that the mixing time of the full system exhibits cutoff at $\frac{3}{2}\log n/\mu$. We do this by showing that the random walk component mixes faster than the environment process; along the way, we control the time it takes for the walk to become isolated.

Résumé. Nous considérons le modèle de percolation dynamique sur le graphe complet K_n, où chaque arête réactualise son état au taux $\mu \ll 1/n$, et est ensuite déclarée ouverte avec probabilité $p = \lambda/n$, où $\lambda > 1$. Nous étudions une marche aléatoire sur cet environnement dynamique qui saute à taux $1/n$ à travers chaque arête ouverte. Nous montrons que le temps de mélange de tout ce processus a un cutoff au temps $\frac{3}{2}\log n/\mu$. Nous l'obtenons en montrant que la composante marche aléatoire mélange plus vite que le processus d'environnement; au passage nous contrôlons le temps que met la marche avant d'être isolée.

MSC2020 subject classifications: 05C81; 60J27; 60K35; 60K37

Keywords: Dynamical percolation; Erdős–Rényi; Random walk; Mixing times; Coupling

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Convergence of local supermartingales

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Abstract. We characterize the event of convergence of a local supermartingale. Conditions are given in terms of its predictable characteristics and quadratic variation. The notion of stationarily local integrability plays a key role.

Résumé. Nous caractérisons l'événement de convergence d'une surmartingale locale. Les conditions sont exprimées en termes de ses caractéristiques prévisibles et de sa variation quadratique. La notion d'intégrabilité stationnairement locale joue un rôle clé.

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Keywords: Supermartingale convergence; Stationary localization

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On the Poisson boundary of the relativistic Brownian motion

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Abstract. In this paper, we determine the Poisson boundary of the relativistic Brownian motion in two classes of Lorentzian manifolds, namely model manifolds of constant scalar curvature and Robertson–Walker space–times, the latter constituting a large family of curved manifolds. Our objective is two fold: on the one hand, to understand the interplay between the geometry at infinity of these manifolds and the asymptotics of random sample paths, in particular to compare the stochastic compactification given by the set of exit points of the process to classical purely geometric compactifications such as the conformal or causal boundaries. On the other hand, we want to illustrate the power of the dévissage method introduced by the authors (in *Séminaire de Probabilités XLVIII* (2016) 199–229 Springer), method which we show to be particularly well suited in the geometric contexts under consideration here.

Résumé. Dans cet article, nous déterminons la frontière de Poisson du mouvement brownien relativiste dans deux classes de variétés lorentziennes, les espaces modèles de courbure constante et les espaces de Robertson–Walker qui constituent une vaste famille d'espace-temps courbes. Notre objectif est double : d'une part, il s'agit de comprendre les relations entre la géométrie à l'infini de ces variétés et le comportement asymptotique des trajectoires browniennes, avec comme objectif de comparer la compactification stochastique formée par les points de sortie du processus aux frontières purement géométriques, conformes ou causales. D'autre part, nous souhaitons illustrer la pertinence de la méthode de dévissage introduite par les auteurs (*Séminaire de Probabilités XLVIII* (2016) 199–229 Springer), méthode qui s'avère particulièrement bien adaptée aux différents contextes géométriques considérés.

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Keywords: Relativistic Brownian motion; Poisson boundary; Dévissage method; Causal boundary; Conformal boundary

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Eigenvectors distribution and quantum unique ergodicity for deformed Wigner matrices

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Abstract. We analyze the distribution of eigenvectors for mesoscopic, mean-field perturbations of diagonal matrices, in the bulk of the spectrum. Our results apply to a generalized $N \times N$ Rosenzweig–Porter model. We prove that the eigenvector entries are asymptotically Gaussian with a specific variance. For a well spread initial spectrum, this variance profile universally follows a heavy-tailed Cauchy distribution. In the case of smooth entries, we also obtain a strong form of quantum unique ergodicity in the form of a strong concentration inequality for the mass of eigenvectors on a given set of coordinates. The proof relies on a priori local laws for this model as given in (*Ann. Probab.* **44** (2016) 2349–2425; *Comm. Math. Phys.* **355** (2017) 949–1000; *Comm. Math. Phys.* **350** (2017) 231–278), and the eigenvector moment flow from (*Comm. Math. Phys.* **350** (2017) 231–278; Bourgade et al. 2018).

Résumé. Nous analysons la distribution des vecteurs propres de perturbations mésoscopiques de matrices diagonales à l'intérieur du spectre. Nos résultats s'appliquent a un modèle généralisé de Rosenzweig–Porter. Nous prouvons que les entrées des vecteurs propres sont asymptotiquement gaussiennes avec une variance explicite. Lorsque le spectre initial est bien étalé, ce profil de variance suit de manière universelle une distribution de Cauchy à queue lourde. Lorsque les entrées sont lisses, nous obtenons aussi une forme forte d'unique ergodicité quantique sous la forme d'une inégalité de concentration sur la masse des vecteurs propres sur un domaine fixé de coordonnées. La preuve se base sur des lois locales a priori données dans (*Ann. Probab.* **44** (2016) 2349–2425; *Comm. Math. Phys.* **355** (2017) 949–1000; *Comm. Math. Phys.* **350** (2017) 231–278) et le flot des moments des vecteurs propres de (*Comm. Math. Phys.* **350** (2017) 231–278; Bourgade et al. 2018).

MSC2020 subject classifications: Primary 60B20; secondary 58J51

Keywords: Deformed random matrices; Eigenvector distribution; Quantum unique ergodicity; Dyson Brownian motion

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Obliquely reflected backward stochastic differential equations

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Abstract. In this paper, we study existence and uniqueness to multidimensional Reflected Backward Stochastic Differential Equations in a non-empty open convex domain, allowing for oblique directions of reflection. In a Markovian framework, combining *a priori* estimates for penalised equations and compactness arguments, we obtain existence results under quite weak assumptions on the driver of the BSDEs and the direction of reflection, which is allowed to depend on both Y and Z. In a non Markovian framework, we obtain existence and uniqueness result for direction of reflection depending on time and Y in smooth convex domain. We make use in this case of stability estimates that require some regularity conditions on the direction of reflection only.

Résumé. Nous étudions dans cet article l'existence et l'unicité des solutions d'équations différentielles stochastiques rétrogrades multidimensionnelles réfléchies dans un domaine ouvert convexe non vide avec une possible obliquité de la direction de réflexion. Dans le cadre markovien, en utilisant des estimées *a priori* pour les équations pénalisées et des arguments de compacité, nous obtenons un résultat d'existence sous des hypothèses faibles sur le générateur de l'EDSR et la direction de réflexion qui peut dépendre de Y et Z. Dans un cadre non markovien, nous obtenons un résultat d'existence et d'unicité lorsque la direction de réflexion dépend uniquement du temps et de Y et que le domaine de réflexion est régulier. Pour ce faire, nous utilisons des estimées de stabilité qui nécessitent des conditions de régularité portant uniquement sur la direction de réflexion.

MSC2020 subject classifications: 93E20; 65C99; 60H30

Keywords: BSDE; Reflected BSDE; Oblique reflection

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Free energy of bipartite spherical Sherrington–Kirkpatrick model

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Abstract. We consider the free energy of the bipartite spherical Sherrington–Kirkpatrick model and determine the limiting free energy at every temperature. We also prove the convergence of the law of the fluctuations of the free energy at non-critical temperature. The limit is given by the Gaussian distribution for all high temperatures and by the GOE Tracy–Widom distribution for all low temperatures. The result is universal and the analysis is applicable to a more general setting including the case where the disorders are non-identically distributed.

Résumé. Nous considérons l'énergie libre du modèle sphérique bipartite de Sherrington–Kirkpatrick et déterminons l'énergie libre limite à chaque température. Nous prouvons également la convergence de la loi des fluctuations de l'énergie libre à température non critique. La limite est donnée par la distribution Gaussienne pour toutes les températures élevées et par la distribution de Tracy–Widom GOE pour toutes les températures basses. Le résultat est universel et l'analyse est applicable à un cadre plus général, y compris le cas où le désordre est distribué de manière non identique.

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Keywords: Free energy; Bipartite spherical SK model; Phase transition

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The random transposition dynamics on random regular graphs and the Gaussian free field

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Abstract. A single permutation, seen as union of disjoint cycles, represents a regular graph of degree two. Consider d many independent random permutations and superimpose their graph structures. This is the well known permutation model of a random regular (multi-) graph of degree $2d$. We consider a two dimensional field of d permutations indexed by size and time. The size of each permutation grows by coupled Chinese Restaurant Processes, while in time, each permutation evolves according to the random transposition chain. Via the permutation model, this projects to give a two parameter family of graphs growing in size ("dimension") and evolving over time. Asymptotically in this random graph ensemble one observes a remarkable evolution of short cycles and linear eigenvalue statistics in dimension and time. In dimension, it was shown by Johnson and Pal (*Ann. Probab.* **42** (2014) 1396–1437) that cycle counts are described by a Poisson field of Yule processes. Here, we give a Poisson random surface description in dimension and time of the cycle process, for every d. As d grows to infinity, the fluctuation of the limiting cycle counts, converges to a Gaussian process indexed by dimension and time. The marginal along dimension turns out to be the Gaussian Free Field and the process is stationary in time. Similar covariance structure appears in eigenvalue fluctuations of the minor process of a real symmetric Wigner matrix whose coordinates evolve as i.i.d. stationary stochastic processes. Thus this article describes a Poisson analogue of a natural Markovian dynamics on the Gaussian free field and its path properties.

Résumé. Une permutation donnée, vue comme réunion de cycles disjoints, représente un graphe régulier de degré 2. Considérons d permutations aléatoires indépendantes, en superposons leurs structures de graphes. Ceci est le modèle de permutation bien connu donnant un (multi-)graphe régulier aléatoire de degré $2d$. Nous considérons un champ 2-dimensionnel de d permutations indexé par la taille et le temps. La taille de chaque permutation croît selon des processus couplés de restaurants chinois, tandis que chaque permutation évolue dans le temps selon une chaîne de transpositions aléatoires. À travers le modèle de permutation, ceci se projette en une famille à deux paramètres de graphes qui croissent en taille (« dimension ») et qui évoluent en temps. Dans cet ensemble de graphes aléatoires, on observe asymptotiquement une évolution remarquable des petits cycles et des statistiques linéaires des valeurs propres en dimension et en temps. En dimension, il avait été montré par Johnson et Pal (*Ann. Probab.* **42** (2014) 1396–1437) que les nombres de cycles sont décrits par un champ poissonnien de processus de Yule. Ici, nous donnons une description en dimension et en temps du processus des cycles en termes en termes d'un surface aléatoire poissonnienne, pour tout d. Lorsque d tend vers l'infini, les fluctuations des nombres de cycles convergent vers un processus gaussien indexé par la dimension et le temps. Les marginales en dimension se trouvent être le champ libre gaussien, et le processus est stationnaire en temps. Une structure similaire de covariance apparaît dans les fluctuations des valeurs propres du processus des mineurs d'une matrice de Wigner réelle symétrique dont les coordonnées évoluent selon des processus stochastiques stationnaires i.i.d.. Ainsi, cet article décrit un analogue poissonnien d'une dynamique markovienne naturelle sur le champ libre gaussien, et étudie ses propriétés trajectorielles.

MSC2020 subject classifications: 60B20; 60C05

Keywords: Random regular graphs; Chinese restaurant process; Random transpositions; Virtual permutations; Gaussian free field; Minor process; Dyson Brownian motion

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Spectral gap of sparse bistochastic matrices with exchangeable rows

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Abstract. We consider a random bistochastic matrix of size n of the form MQ where M is a uniformly distributed permutation matrix and Q is a given bistochastic matrix. Under sparsity and regularity assumptions on Q, we prove that the second largest eigenvalue of MQ is essentially bounded by the normalized Hilbert–Schmidt norm of Q when n grows large. We apply this result to random walks on random regular digraphs.

Résumé. Considérons une matrice bi-stochastique aléatoire de taille n et de la forme MQ avec M une matrice de permutation uniformément distribuée et Q une matrice bi-stochastique fixée. Sous des conditions de parcimonie et de régularité sur Q, on démontre que la deuxième plus grande valeur propre de MQ est essentiellement bornée par la norme de Hilbert–Schmidt normalisée de Q lorsque n est très grand. Ce résultat s'applique aux marches au hasard sur les graphes aléatoires dirigés réguliers.

MSC2020 subject classifications: 60B20; 60C05; 05C80

Keywords: Spectral gap; Random bistochastic matrices; High trace method; Tangled-free paths

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A functional limit theorem for coin tossing Markov chains

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Abstract. We prove a functional limit theorem for Markov chains that, in each step, move up or down by a possibly state dependent constant with probability $1/2$, respectively. The theorem entails that the law of every one-dimensional regular continuous strong Markov process in natural scale can be approximated with such Markov chains arbitrarily well. The functional limit theorem applies, in particular, to Markov processes that cannot be characterized as solutions to stochastic differential equations. Our results allow to practically approximate such processes with irregular behavior; we illustrate this with Markov processes exhibiting sticky features, e.g., sticky Brownian motion and a Brownian motion slowed down on the Cantor set.

Résumé. Nous prouvons un théorème limite fonctionnelle pour les chaînes de Markov qui, à chaque étape, montent ou descendent avec probabilité $1/2$ d'une constante dépendante de l'état. Le théorème implique que la loi de chaque processus de Markov uni-dimensionel, fort, continu, régulier et à l'échelle naturelle peut être approximée par de telles chaînes de Markov avec précision quelconque. Le théorème limite fonctionnelle s'applique en particulier aux processus de Markov qui ne peuvent pas être caractérisés comme solutions d'une équation différentielle stochastique. Notamment nos résultats permettent d'approximer de tels processus avec un comportement irrégulier; nous illustrons cela avec des processus de Markov «collants», par exemple, le mouvement brownien «collant» et un mouvement brownien ralenti sur l'ensemble de Cantor.

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Keywords: One-dimensional Markov process; Speed measure; Markov chain approximation; Functional limit theorem; Sticky Brownian motion; Sticky reflection; Slow reflection; Brownian motion slowed down on the Cantor set

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A Central Limit Theorem for the stochastic wave equation with fractional noise

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Abstract. We study the one-dimensional stochastic wave equation driven by a Gaussian multiplicative noise, which is white in time and has the covariance of a fractional Brownian motion with Hurst parameter $H \in [1/2, 1)$ in the spatial variable. We show that the normalized spatial average of the solution over $[-R, R]$ converges in total variation distance to a normal distribution, as R tends to infinity. We also provide a functional Central Limit Theorem.

Résumé. Nous étudions l'équation des ondes en une dimension, perturbée par un bruit gaussien multiplicatif, qui est blanc en temps et qui a la covariance d'un mouvement brownien fractionnaire avec paramètre de Hurst $H \in [1/2, 1)$ dans la variable d'espace. Nous démontrons que la moyenne spatiale normalisée de la solution sur un intervalle $[-R, R]$ converge, en la distance de la variation totale, vers une loi normale, quand R tend vers l'infini. Nous prouvons aussi un théorème central limite fonctionnel.

MSC2020 subject classifications: 60H15; 60H07; 60G15; 60F05

Keywords: Stochastic wave equation; Central Limit Theorem; Malliavin calculus; Stein's method

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