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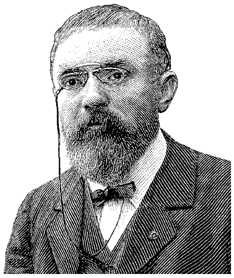

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A stochastic Gronwall inequality and applications to moments, strong completeness, strong local Lipschitz continuity, and perturbations

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Abstract. There are numerous applications of the classical (deterministic) Gronwall inequality. Recently, Michael Scheutzow discovered a stochastic Gronwall inequality which provides upper bounds for p-th moments, $p \in (0, 1)$, of the supremum of nonnegative scalar continuous processes which satisfy a linear integral inequality. In this article we complement this with upper bounds for p-th moments, $p \in [2, \infty)$, of the supremum of general Itô processes which satisfy a suitable one-sided affine-linear growth condition. As example applications, we improve known results on strong local Lipschitz continuity in the starting point of solutions of stochastic differential equations (SDEs), on (exponential) moment estimates for SDEs, on strong completeness of SDEs, and on perturbation estimates for SDEs.

Résumé. La version classique (déterministe) de l'inégalité de Gronwall possède de nombreuses applications. Récemment Michael Scheutzow a proposé une version stochastique de cette inégalité. Cette dernière permet de majorer le moment d'ordre $p \in (0, 1)$ du supremum des processus réels continus qui satisfont une inégalité intégrale linéaire. Dans cet article nous complétons ce résultat. Nous déterminons des majorants pour les moments d'ordre $p \in [2, \infty)$ du supremum des processus généraux de Itô qui vérifient une certaine condition de croissance affine. Comme application, nous affinons des résultats existants concernant les équations différentielles stochastiques : lipschitziannité locale uniforme en temps par rapport au point de départ, majorations des moments (exponentiels), existence d'une modification continue par rapport au couple temps-point de départ (i.e. strong completeness), théorie des perturbations.

MSC2020 subject classifications: Primary 60H10; secondary 60E15

Keywords: Stochastic Gronwall inequality; Stochastic Gronwall lemma; Martingale inequality; Exponential moments; Strong completeness; Strong local Lipschitz continuity; Perturbation theory

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New lower bounds for trace reconstruction

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Abstract. We improve the lower bound on worst case trace reconstruction from $\Omega(\frac{n^{5/4}}{\sqrt{\log n}})$ to $\Omega(\frac{n^{3/2}}{\log^7 n})$. As a consequence, we improve the lower bound on average case trace reconstruction from $\Omega(\frac{\log^{9/4} n}{\sqrt{\log\log n}})$ to $\Omega(\frac{\log^{5/2} n}{(\log\log n)^7})$.

Résumé. Nous améliorons la borne inférieure, dans le cas pire, de la reconstruction par trace de $\Omega(\frac{n^{5/4}}{\sqrt{\log n}})$ à $\Omega(\frac{n^{3/2}}{\log^7 n})$. Comme conséquence, nous améliorons la borne inférieure, dans le cas moyenné, de la reconstruction par trace de $\Omega(\frac{\log^{9/4} n}{\sqrt{\log\log n}})$ à $\Omega(\frac{\log^{5/2} n}{(\log\log n)^7})$.

MSC2020 subject classifications: 62B10

Keywords: Statistics

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The spectral gap of sparse random digraphs

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Abstract. The second largest eigenvalue of a transition matrix P has connections with many properties of the underlying Markov chain, and especially its convergence rate towards the stationary distribution. In this paper, we give an asymptotic upper bound for the second eigenvalue when P is the transition matrix of the simple random walk over a random directed graph with given degree sequence. This is the first result concerning the asymptotic behavior of the spectral gap for sparse non-reversible Markov chains with an unknown stationary distribution. An immediate consequence of our result is a proof of the Alon conjecture for directed regular graphs.

Résumé. La seconde plus grande valeur propre d'une matrice de transition P capture de nombreuses propriétés de la chaîne de Markov sous-jacente, comme par exemple la vitesse de convergence vers l'état stationnaire. Dans cet article, on démontre une borne supérieure asymptotique pour la seconde valeur propre de P, lorsque P est la matrice de transition de la marche aléatoire simple sur un graphe dirigé avec une suite de degrés fixée. Il s'agit du premier résultat sur le comportement asymptotique de chaînes de Markov parcimonieuses et non-réversibles, dans lesquelles la loi stationnaire n'est pas connue. Une conséquence immédiate de notre résultat est une démonstration de la conjecture d'Alon pour les graphes dirigés.

MSC2020 subject classifications: 60B20; 05C20; 05C80; 47A11

Keywords: Random graphs; Directed configuration graph; Degree sequence; Second eigenvalue; High trace method

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Two-temperatures overlap distribution for the 2D discrete Gaussian free field[1]

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Abstract. In this paper, we prove absence of temperature chaos for the two-dimensional discrete Gaussian free field using the convergence of the full extremal process, which has been obtained recently by Biskup and Louidor. This means that the overlap of two points chosen under Gibbs measures at different temperatures has a nontrivial distribution. Whereas this distribution is the same as for the random energy model when the two points are sampled at the same temperature, we point out here that they are different when temperatures are distinct: more precisely, we prove that the mean overlap of two points chosen under Gibbs measures at different temperatures for the DGFF is strictly smaller than the REM's one. Therefore, although neither of these models exhibits temperature chaos, one could say that the DGFF is more chaotic in temperature than the REM.

Résumé. Dans cet article, nous montrons l'absence de chaos en température pour le champ libre gaussien discret 2-dimensionnel (DGFF) en utilisant la convergence du processus extrémal obtenue récemment par Biskup et Louidor. Cela signifie que le chevauchement entre deux configurations choisies selon les mesures de Gibbs à différentes températures a une distribution non triviale à la limite. Alors que cette distribution est identique à celle du modèle d'énergies aléatoires (REM) dans le cas de configurations choisies à la même température, nous montrons ici que ces distributions sont distinctes dans le cas de températures différentes. Plus précisément, nous démontrons que le chevauchement moyen entre deux configurations choisies selon les mesures de Gibbs à différentes températures pour le DGFF est strictement plus petit que celui obtenu dans le cas du REM. Ainsi, bien qu'aucun de ces modèles ne présente de chaos en température, on peut dire que le DGFF est moins chaotique en température que le REM.

MSC2020 subject classifications: 60G15; 82D30; 60G70

Keywords: 2D discrete Gaussian free field; Log-correlated fields; Spin glasses; Overlap; Chaos in temperature; Random energy model

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Stochastic optimal transport with free end time

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Abstract. We consider a stochastic transportation problem between two prescribed probability distributions (a source and a target) over processes with general drift dependence and with free end times. First, and in order to establish a dual principle, we associate two equivalent formulations of the primal problem in order to guarantee its convexity and lower semi-continuity with respect to the source and target distributions. We exhibit an equivalent Eulerian formulation, whose dual variational principle is given by Hamilton–Jacobi–Bellman type variational inequalities. In the case where the drift is bounded, regularity results on the minimizers of the Eulerian problem then enable us to prove attainment in the corresponding dual problem. We also address attainment when the drift component of the cost defining Lagrangian L is superlinear $L \approx |u|^p$ with $1 < p < 2$, in which case the setting is reminiscent of our approach – in a previous work – on deterministic controlled transport problems with free end time. We finally address criteria under which the optimal drift and stopping time are unique, namely strict convexity in the drift component and monotonicity in time of the Lagrangian.

Résumé. Nous considérons un problème de transport stochastique entre deux distributions de probabilité prescrites (une source et une cible) sur des processus avec une dépendance générale à la dérivé et avec des temps de fin libres. Premièrement, et afin d'établir un principe dual, nous associons deux formulations équivalentes du problème primal afin de garantir sa convexité et sa semi-continuité inférieure par rapport aux distributions source et cible. Nous présentons une formulation Eulérienne équivalente, dont le principe variationnel dual est donné par des inégalités variationnelles de type Hamilton–Jacobi–Bellman. Dans le cas où la dérivé est bornée, les résultats de régularité sur les minimiseurs du problème Eulérien nous permettent alors de prouver l'atteinte du problème dual correspondant. Nous abordons également la réalisation lorsque la composante de derivé du coût définissant le lagrangien L est $L \approx |u|^p$ superlinéaire avec $1 < p < 2$, auquel cas le réglage rappelle notre approche – dans un travail précédent – sur les problèmes de transport contrôlé déterministe avec un temps de fin libre. Nous abordons enfin les critères selon lesquels la dérivé optimale et le temps d'arrêt sont uniques, à savoir la convexité stricte dans la composante de dérivé et la monotonie dans le temps du lagrangien.

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Keywords: Stochastic optimal transport; Kantorovich duality; Viscosity solutions

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Stochastic approximation of quasi-stationary distributions for diffusion processes in a bounded domain

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Abstract. We study a random process with reinforcement, which evolves following the dynamics of a given diffusion process in a bounded domain and is resampled according to its occupation measure when it reaches the boundary. We show that its occupation measure converges to the unique quasi-stationary distribution of the diffusion process absorbed at the boundary of the domain. Our proofs use recent results in the theory of quasi-stationary distributions and stochastic approximation techniques.

Résumé. Nous étudions un processus stochastique avec renforcement, qui évolue suivant une diffusion dans un domaine borné, avec ré-échantillonnage suivant sa mesure d'occupation lorsqu'il atteint la frontière. Nous montrons que sa mesure d'occupation converge vers l'unique distribution quasi-stationnaire de la diffusion absorbée au bord du domaine. Nos preuves s'appuient sur des résultats récents en théorie des distributions quasi-stationnaires et sur des techniques d'approximation stochastique.

MSC2020 subject classifications: Primary 60B12; 60J60; 60B10; 60F99; secondary 60J70

Keywords: Random processes with reinforcement; Stochastic approximation; Pseudo-asymptotic trajectories; Quasi-stationary distributions

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Total variation estimates in the Breuer–Major theorem

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Abstract. This paper provides estimates for the convergence rate of the total variation distance in the framework of the Breuer–Major theorem, assuming some smoothness properties of the underlying function. The results are proved by applying new bounds for the total variation distance between a random variable expressed as a divergence and a standard Gaussian random variable, which are derived by a combination of techniques of Malliavin calculus and Stein's method. The representation of a functional of a Gaussian sequence as a divergence is established by introducing a shift operator on the expansion in Hermite polynomials. Some applications to the asymptotic behavior of power variations of the fractional Brownian motions and to the estimation of the Hurst parameter using power variations are presented.

Résumé. Cet article fournit des estimations pour la vitesse de convergence de la variation totale dans le cadre du théorème de Breuer–Major, en supposant quelques propriétés de régularité de la fonction sous-jacente. Les résultats se démontrent en appliquant des nouvelles bornes pour la distance en variation totale entre une variable aléatoire qui s'exprime comme une divergence et une variable aléatoire gaussienne, qu'on obtient en combinant des techniques du calcul de Malliavin et la méthode de Stein. On établit la représentation d'une fonctionnelle d'une suite gaussienne comme une divergence en introduisant un opérateur de décalage sur le développement en polynômes d'Hermite. Quelques applications au comportement asymptotique des variations puissance pour le mouvement Brownien fractionnaire et à l'estimation du paramètre de Hurst sont aussi présentées.

MSC2020 subject classifications: 60F05; 60H07; 60G10; 60G22

Keywords: Breuer–Major theorem; Total variation; Stein's method; Malliavin calculus; Hermite rank

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Fractional moments of the stochastic heat equation

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Abstract. Consider the solution $\mathcal{Z}(t,x)$ of the one-dimensional stochastic heat equation, with a multiplicative spacetime white noise, and with the delta initial data $Z(0,x)=\delta(x)$. For any real $p>0$, we obtained detailed estimates of the pth moment of $e^{t/12}\mathcal{Z}(2t,0)$, as $t\to\infty$, and from these estimates establish the one-point upper-tail large deviation principle of the Kardar–Parisi–Zhang equation. The deviations have speed t and rate function $\Phi_+(y)=\frac{4}{3}y^{3/2}$. Our result confirms the existing physics predictions [*Europhys. Lett.* **113** (2016) 60004] and also [*Phys. Rev. E* **94** (2016) 032108].

Résumé. Nous considérons la solution $\mathcal{Z}(t,x)$ de l'équation de la chaleur stochastique unidimensionnelle, avec un bruit blanc multiplicatif en espace et en temps, et pour toute condition initiale de Dirac $Z(0,x)=\delta(x)$. Pour tout réel $p>0$, nous obtenons une estimée précise du p-ième moment de $e^{t/12}\mathcal{Z}(2t,0)$, lorsque $t\to\infty$, et à partir de ces estimées, nous établissons une borne supérieure de grandes déviations pour l'équation de Kardar–Parisi–Zhang. Les déviations ont pour vitesse t et fonction de taux $\Phi_+(y)=\frac{4}{3}y^{3/2}$. Nos résultats confirment les prédictions des physiciens [*Europhys. Lett.* **113** (2016) 60004] et aussi [*Phys. Rev. E* **94** (2016) 032108].

MSC2020 subject classifications: Primary 60F10; secondary 60H15

Keywords: Kardar–Parisi–Zhang equation; Stochastic heat equation; Large deviations; Fredholm determinants

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The Schelling model on $\mathbb{Z}$

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Abstract. A version of the Schelling model on $\mathbb{Z}$ is defined, where two types of agents are allocated on the sites. An agent prefers to be surrounded by other agents of its own type, and may choose to move if this is not the case. It then sends a request to an agent of opposite type chosen according to some given moving distribution and, if the move is beneficial for both agents, they swap location. We show that certain choices in the dynamics are crucial for the properties of the model. In particular, the model exhibits different asymptotic behavior depending on whether the moving distribution has bounded or unbounded support. Furthermore, the behavior changes if the agents are lazy in the sense that they only swap location if this strictly improves their situation. Generalizations to a version that includes multiple types are discussed. The work provides a rigorous analysis of so called Kawasaki dynamics on an infinite structure with local interactions.

Résumé. Une version du modèle de Schelling sur $\mathbb{Z}$ est définie, où deux types d'agents sont distribués sur les sites. Un agent préfère être entouré d'agents du même type que lui, et peut choisir de se déplacer si ce n'est pas le cas. Il envoie alors une demande à un agent de type opposé suivant une certain loi de déplacement, et si ce déplacement est bénéfique pour les deux alors ils échangent leurs positions respectives. Nous montrons que certains choix de dynamiques sont cruciaux pour les propriétés du modèle. En particulier, le modèle montre différents comportements asymptotiques suivant si la distribution des déplacements a un support borné ou non borné. De plus, le comportement change si les agents sont paresseux, dans le sens où ils n'échangent leurs positions que si cela améliore strictement leurs situations. Les généralisations aux cas impliquant plus de types sont aussi discutées. Ce travail apporte une analyse rigoureuse de la dynamique de Kawasaki sur une structure infinie avec interactions locales.

MSC2020 subject classifications: 60K35; 60J75

Keywords: Schelling segregation model; Voter model; Interacting particle systems; Kawasaki dynamics; Asymptotic behavior

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Asymptotics of maximum likelihood estimators based on Markov chain Monte Carlo methods

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Abstract. In complex statistical models, in which exact computation of the likelihood is intractable, Monte Carlo methods can be applied to approximate maximum likelihood estimates. In this paper we consider approximation obtained via Markov chain Monte Carlo. We prove consistency and asymptotic normality of the resulting estimator, when both sample sizes (the initial and Monte Carlo one) tend to infinity. Our results can be applied to models with intractable normalizing constants and missing data models. We also investigate properties of estimators in numerical experiments.

Résumé. Afin d'inférer de nombreux modèles statistiques, pour lesquels le calcul exact de la vraisemblance n'est pas possible, de nombreuses méthodes peuvent être cependant mises en place pour obtenir des estimées du maximum de vraisemblance. Ces méthodes pour la plupart sont basées sur des approches de type Monte Carlo. Dans cet article, nous considérons une approche à partir de méthodes de Monte Carlo par chaînes de Markov pour lesquelles nous montrons la consistance et la normalité asymptotique de l'estimateur ainsi défini. Nos résultats peuvent s'appliquer à de nombreux modèles pour lesquels la constante de normalisation n'est pas calculable ou est à données manquantes. Nous illustrons enfin cela à partir des nombreuses expériences numériques.

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Keywords: Intractable normalizing constant; Markov chain; Maximum likelihood estimation; Missing data model; Monte Carlo method

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Limit law for the cover time of a random walk on a binary tree

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Abstract. Let $\mathcal{T}_n$ denote the binary tree of depth n augmented by an extra edge connected to its root. Let $\mathcal{C}_n$ denote the cover time of $\mathcal{T}_n$ by simple random walk. We prove that the sequence of random variables $\sqrt{\mathcal{C}_n 2^{-(n+1)}} - m_n$, where m_n is an explicit constant, converges in distribution as $n \to \infty$, and identify the limit.

Résumé. Soit $\mathcal{T}_n$ l'arbre binaire de profondeur n, augmenté par une arête attachée à la racine. Soit $\mathcal{C}_n$ le temps de recouvrement de $\mathcal{T}_n$ par une marche aléatoire simple. Nous montrons que la suite de variables aléatoires $\sqrt{\mathcal{C}_n 2^{-(n+1)}} - m_n$, avec m_n une constante explicite, converge quand $n \to \infty$. Nous identifions la limite.

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Keywords: Cover time; Binary tree; Barrier estimates

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Conditional measures for Pfaffian point processes: Conditioning on a bounded domain

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Abstract. For a Pfaffian point process we show that its Palm measures, its normalised compositions with multiplicative functionals, and its conditional measures with respect to fixing the configuration in a bounded subset are Pfaffian point processes whose kernels we find explicitly.

Résumé. Pour un processus Pfaffien, nous démontrons que ses mesures de Palm, ses compositions avec les fonctionnelles multiplicatives normalisdées, ainsi que ses mesures conditionnelles obtenues en fixant la configuration dans un sous-ensemble borndé, sont encore des processus Pfaffiens dont les noyaux de corrdélation sont explicitement trouvdés.

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Gaussian fluctuations for the directed polymer partition function in dimension $d \geq 3$ and in the whole L^2-region

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Abstract. We consider the discrete directed polymer model with i.i.d. environment and we study the fluctuations of the tail $n^{(d-2)/4}(W_\infty - W_n)$ of the normalized partition function. It was proven by Comets and Liu (*J. Math. Anal. Appl.* **455** (2017) 312–335), that for sufficiently high temperature, the fluctuations converge in distribution towards the product of the limiting partition function and an independent Gaussian random variable. We extend the result to the whole L^2-region of temperature, which is predicted to be the maximal high-temperature region where the Gaussian fluctuations should occur under the considered scaling. To do so, we manage to avoid the heavy 4th-moment computation and instead rely on the local limit theorem for polymers (*Fund. Math.* **147** (1995) 173–180; *Ann. Inst. Henri Poincaré Probab. Stat.* **42** (2006) 521–534) and homogenization.

Résumé. Nous considérons le polymère dirigé discret en environnement i.i.d. et nous étudions les fluctuations de la queue $n^{(d-2)/4}(W_\infty - W_n)$ de la fonction de partition normalisée. Comets et Liu (*J. Math. Anal. Appl.* **455** (2017) 312–335) ont montré que pour une température suffisamment haute, les fluctuations convergent en loi vers le produit de la fonction de partition limite et d'une variable aléatoire gaussienne indépendante. Dans cet article, nous étendons le résultat à toute la région de température L^2. Il est attendu que ce soit la région optimale où les fluctuations gaussiennes apparaissent sous le changement d'échelle considéré. Pour y parvenir, nous évitons le calcul de moments d'ordre 4 et nous utilisons à la place le théorème local limite pour les polymères (*Fund. Math.* **147** (1995) 173–180 ; *Ann. Inst. Henri Poincaré Probab. Stat.* **42** (2006) 521–534) ainsi qu'un argument d'homogénéisation.

MSC2020 subject classifications: Primary 60K37; 60K37; secondary 60F05

Keywords: Directed polymers; Random environment; Rate of convergence; Martingale central limit theorem

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A reverse Aldous–Broder algorithm

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Abstract. The Aldous–Broder algorithm provides a way of sampling a uniformly random spanning tree for finite connected graphs using simple random walk. Namely, start a simple random walk on a connected graph and stop at the cover time. The tree formed by all the first-entrance edges has the law of a uniform spanning tree. Here we show that the tree formed by all the last-exit edges also has the law of a uniform spanning tree. This answers a question of Tom Hayes and Cris Moore from 2010. The proof relies on a bijection that is related to the BEST theorem in graph theory. We also give other applications of our results, including new proofs of the reversibility of loop-erased random walk, of the Aldous–Broder algorithm itself, and of Wilson's algorithm.

Résumé. L'algorithme d'Aldous–Broder fournit un moyen d'échantillonner un arbre couvrant aléatoire uniforme pour des graphes connexes finis en utilisant une marche aléatoire simple. Il procède de la façon suivante : commençons une marche aléatoire simple sur un graphe connexe et arrêtons-nous au temps de couverture. L'arbre formé par toutes les arêtes de première entrée a la loi d'un arbre couvrant uniforme. Nous montrons ici que l'arbre formé par toutes les arêtes de dernière sortie a également la loi d'un arbre couvrant uniforme. Cela répond à une question de Tom Hayes et Cris Moore de 2010. La preuve repose sur une bijection qui est liée au théorème BEST de la théorie des graphes. Nous donnons également d'autres applications de nos résultats, comprenant de nouvelles preuves de la réversibilité de la marche aléatoire à boucles effacées, de l'algorithme d'Aldous–Broder lui-même, et de l'algorithme de Wilson.

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Keywords: Spanning trees; Loop-erased random walk; BEST theorem

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Total variation distance for discretely observed Lévy processes: A Gaussian approximation of the small jumps

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Abstract. It is common practice to treat small jumps of Lévy processes as Wiener noise and to approximate its marginals by a Gaussian distribution. However, results that allow to quantify the goodness of this approximation according to a given metric are rare. In this paper, we clarify what happens when the chosen metric is the total variation distance. Such a choice is motivated by its statistical interpretation; if the total variation distance between two statistical models converges to zero, then no test can be constructed to distinguish the two models and they are therefore asymptotically equally informative. We elaborate a fine analysis of a Gaussian approximation for the small jumps of Lévy processes in total variation distance. Non-asymptotic bounds for the total variation distance between n discrete observations of small jumps of a Lévy process and the corresponding Gaussian distribution are presented and extensively discussed. As a byproduct, new upper bounds for the total variation distance between discrete observations of Lévy processes are provided. The theory is illustrated by concrete examples.

Résumé. Il est habituel d'assimiler les petits sauts d'un processus de Lévy à un mouvement brownien et d'approcher leurs marginales par des distributions gaussiennes. Cependant, les résultats permettant de quantifier cette approximation selon une métrique donnée sont rares. Dans cet article, nous la quantifions pour la distance en variation totale. Un tel choix s'explique par son interprétation statistique : si la distance en variation totale entre deux modèles statistiques tend vers 0, alors il n'existe aucun test permettant de distinguer les deux modèles, qui sont alors asymptotiquement équivalents. Nous contrôlons ici finement la distance en variation totale entre n incréments des petits sauts d'un processus de Lévy et n variables aléatoires gaussiennes : des bornes non asymptotiques pour la distance en variation totale sont données et discutées. Une conséquence de ces résultats est l'obtention de nouvelles bornes supérieures pour le contrôle en variation totale entre n incréments de deux processus de Lévy. Plusieurs exemples viennent illustrer ces résultats.

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Keywords: Lévy processes; Total variation distance; Small jumps; Gaussian approximation; Statistical test

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Brownian motion on stable looptrees

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Abstract. In this article, we introduce Brownian motion on α-stable looptrees using resistance techniques, where $\alpha \in (1, 2)$. We prove an invariance principle characterising it as the scaling limit of random walks on discrete looptrees, and prove precise local and global bounds on its heat kernel. We also conduct a detailed investigation of the volume growth properties of stable looptrees, and show that the random volume and heat kernel fluctuations are locally log-logarithmic, and globally logarithmic around leading terms of r^α and $t^{\frac{-\alpha}{\alpha+1}}$ respectively. These volume fluctuations are the same order as for the Brownian continuum random tree, but the upper volume fluctuations (and corresponding lower heat kernel fluctuations) are different to those of stable trees.

Résumé. Dans cet article, nous introduisons le mouvement brownien sur les arbres à boucles α-stables en utilisant des techniques de réseaux électriques, pour $\alpha \in (1, 2)$. Nous démontrons un principe d'invariance énonçant que ce processus est la limite de marches aléatoires sur des arbres à boucles discrets, et nous montrons des bornes précises locales et globales sur son noyau de la chaleur. Nous menons également une étude approfondie des propriétés de croissance de volume des arbres à boucles stables, et montrons que les fluctuations aléatoires de volume et du noyau de la chaleur sont localement en logarithme itéré, et globalement logarithmiques, autour d'un terme dominant de r^α et $t^{\frac{-\alpha}{\alpha+1}}$ respectivement. Ces fluctuations de volume sont du même ordre que pour l'arbre continu aléatoire brownien, mais les bornes supérieures des fluctuations de volume (et les bornes inférieures correspondantes des fluctuations du noyau de la chaleur) sont différentes de celles des arbres stables.

MSC2020 subject classifications: Primary 60K37; 60F17; 60G57; 60G52; 54E70

Keywords: Random stable looptree; Volume fluctuations; Heat kernel estimates; Stable Lévy process

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Scaling limits and fluctuations for random growth under capacity rescaling

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Abstract. We evaluate a strongly regularised version of the Hastings–Levitov model HL(α) for $0 \le \alpha < 2$. Previous results have concentrated on the small-particle limit where the size of the attaching particle approaches zero in the limit. However, we consider the case where we rescale the whole cluster by its capacity before taking limits, whilst keeping the particle size fixed. We first consider the case where $\alpha = 0$ and show that under capacity rescaling, the limiting structure of the cluster is not a disk, unlike in the small-particle limit. Then we consider the case where $0 < \alpha < 2$ and show that under the same rescaling the cluster approaches a disk. We also evaluate the fluctuations and show that, when represented as a holomorphic function, they behave like a Gaussian field dependent on α. Furthermore, this field becomes degenerate as α approaches 0 and 2, suggesting the existence of phase transitions at these values.

Résumé. Nous étudions une version fortement régularisée du modèle de Hastings–Levitov HL(α) pour $0 \le \alpha < 2$. Des résultats antérieurs se concentraient sur la limite des petites particules, où la taille de chaque particule rattachée se rapproche asymptotiquement de zéro. Par contraste, nous étudions le cas où l'on renormalise l'amas tout entier par sa capacité avant de passer à la limite, tout en laissant fixe la taille des particules. Nous considérons tout d'abord le cas où $\alpha = 0$ et montrons que sous cette renormalisation par la capacité, et contrairement au cas de la limite des petites particules, la structure limite de l'amas n'est pas un disque. Ensuite, nous considérons le cas où $0 < \alpha < 2$, et démontrons que sous cette même renormalisation, l'amas tend vers un disque. Nous estimons également les fluctuations en montrons que, lorsqu'on les représente par une fonction holomorphe, ces dernières se comportent comme un champ gaussien dépendant de α. De plus, ce champ devient dégénéré lorsque α tend vers 0 ou 2, ce qui suggère l'existence de transitions de phase en ces valeurs.

MSC2020 subject classifications: Primary 60Fxx; secondary 30C35; 60D05; 82C24

Keywords: Planar random growth; Scaling limits; Fluctuations; Phase transitions

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Conformal covariance of the Liouville quantum gravity metric for $\gamma \in (0, 2)$

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Abstract. For $\gamma \in (0, 2)$, $U \subset \mathbb{C}$, and an instance h of the Gaussian free field (GFF) on U, the γ-Liouville quantum gravity (LQG) surface associated with (U, h) is formally described by the Riemannian metric tensor $e^{\gamma h}(dx^2 + dy^2)$ on U. Previous work by the authors showed that one can define a canonical metric (distance function) D_h on U associated with a γ-LQG surface. We show that this metric is conformally covariant in the sense that it respects the coordinate change formula for γ-LQG surfaces. That is, if U, $\widetilde{U}$ are domains, $\phi\colon U \to \widetilde{U}$ is a conformal transformation, $Q = 2/\gamma + \gamma/2$, and $\widetilde{h} = h \circ \phi^{-1} + Q \log|(\phi^{-1})'|$, then $D_h(z, w) = D_{\widetilde{h}}(\phi(z), \phi(w))$ for all $z, w \in U$. This proves that D_h is intrinsic to the quantum surface structure of (U, h), i.e., it does not depend on the particular choice of parameterization.

Résumé. Pour $\gamma \in (0, 2)$, $U \subset \mathbb{C}$, et une réalisation h du champ libre gaussien (GFF) sur U, la γ-surface de gravité quantique de Liouville (LQG) associée à (U, h) est décrite formellement par le tenseur métrique riemannien $e^{\gamma h}(dx^2 + dy^2)$ sur U. De précédents travaux des auteurs ont montré que l'on peut définir une métrique (fonction de distance) canonique D_h sur U associée à une γ-surface LQG. Nous montrons que cette métrique est conformément covariante au sens où elle respecte les formules de changement de variables pour les γ-surfaces LQG. Précisément, si U, $\widetilde{U}$ sont des domaines, et $\phi\colon U \to \widetilde{U}$ est une transformation conforme, en notant $Q = 2/\gamma + \gamma/2$ et $\widetilde{h} = h \circ \phi^{-1} + Q \log|(\phi^{-1})'|$, alors $D_h(z, w) = D_{\widetilde{h}}(\phi(z), \phi(w))$ pour tout $z, w \in U$. Ceci montre que D_h est une quantité intrinsèque de la structure de surface quantique de (U, h), au sens où elle ne dépend pas d'un choix particulier de paramétrisation.

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Keywords: Liouville quantum gravity; Gaussian free field; Coordinate change; Conformal covariance; LQG metric

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McKean–Vlasov SDEs under measure dependent Lyapunov conditions

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Abstract. We prove the existence of weak solutions to McKean–Vlasov SDEs defined on a domain $D \subseteq \mathbb{R}^d$ with continuous and unbounded coefficients and degenerate diffusion coefficient. Using differential calculus for the flow of probability measures due to Lions, we introduce a novel integrated condition for Lyapunov functions in an infinite dimensional space $D \times \mathcal{P}(D)$, where $\mathcal{P}(D)$ is a space of probability measures on D. Consequently we show existence of solutions to the McKean–Vlasov SDEs on $[0, \infty)$. This leads to a probabilistic proof of the existence of a stationary solution to the nonlinear Fokker–Planck–Kolmogorov equation under very general conditions. Finally, we prove uniqueness under an integrated condition based on a Lyapunov function. This extends the standard monotone-type condition for uniqueness.

Résumé. Nous démontrons l'existence de solutions faibles à des EDS de McKean–Vlasov définies sur un domaine $D \subseteq \mathbb{R}^d$ avec des coefficients continus et non bornés et un coefficient de diffusion dégénéré. En utilisant le calcul différentiel dû à Lions pour la loi de probabilité marginale, nous introduisons une nouvelle condition de Lyapunov dans un espace de dimension infinie $D \times \mathcal{P}(D)$, où $\mathcal{P}(D)$ est l'espace de mesures de probabilité sur D. Ainsi nous démontrons l'existence, globale en temps sur $[0, \infty)$, de solutions à des EDS de McKean–Vlasov. Cela conduit également à une preuve probabiliste de l'existence d'une solution stationnaire à l'équation non linéaire de Fokker–Planck–Kolmogorov associée, sous des conditions très générales. Finalement, nous démontrons l'unicité sous nos conditions de Lyapunov. Nos conditions permettent notamment d'étendre la condition de monotonie sous laquelle est habituellement établie l'unicité.

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Keywords: Mckean–Vlasov equations; Mean-field equations; Wasserstein calculus

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Random interlacements for vertex-reinforced jump processes

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Abstract. We introduce random interlacements for transient vertex-reinforced jump processes on a general graph G. Using increasing finite subgraphs G_n of G with wired boundary conditions, we show convergence of the vertex-reinforced jump process on G_n observed in a finite window to the random interlacement observed in the same window.

Résumé. Nous introduisons les entrelacs aléatoires pour le processus de sauts renforcé par sommets sur un graphe général G. A l'aide d'une suite croissante de sous-graphes G_n de G avec condition aux limites de Dirichlet, nous montrons la convergence du processus de sauts renforcé par sommets sur G_n observé sur une fenêtre finie vers l'entrelacs aléatoire observé sur la même fenêtre.

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Keywords: Random interlacement; Vertex-reinforced jump process

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Directed polymer in γ-stable random environments

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Abstract. The transition from a weak-disorder (diffusive) to a strong-disorder (localized) phase for directed polymers in a random environment is a well studied phenomenon. In the most common setup, it is established that the phase transition is trivial when the transversal dimension d equals 1 or 2 (the diffusive phase is reduced to $\beta = 0$) while when $d \geq 3$, there is a critical temperature $\beta_c \in (0, \infty)$ which delimits the two phases. The proof of the existence of a diffusive regime for $d \geq 3$ is based on a second moment method (*Comm. Math. Phys.* **123** (1989) 529–534, *Ann. Probab.* **34** (2006) 1746–1770, *J. Stat. Phys.* **52** (1988) 609–626), and thus relies heavily on the assumption that the variable which encodes the disorder intensity (which in most of the mathematics literature assumes the form $e^{\beta\eta_x}$), has finite second moment. The aim of this work is to investigate how the presence/absence of phase transition may depend on the dimension d in the case when the disorder variable displays a heavier tail. To this end we replace $e^{\beta\eta_x}$ by $(1+\beta\omega_x)$ where ω_x is in the domain of attraction of a stable law with parameter $\gamma \in (1, 2)$. In this setup we show that a non-trivial phase transition occurs if and only if $\gamma > 1 + 2/d$. More precisely, when $\gamma \leq 1 + 2/d$, the free energy of the system is smaller than its annealed counterpart at every temperature whereas when $\gamma > 1 + 2/d$ the martingale sequence of renormalized partition functions converges to an almost surely positive random variable for all β sufficiently small.

Résumé. Le passage d'un désordre faible (phase diffusive) à un désordre fort (phase localisée) pour des polymères dirigés dans un environnement aléatoire est un phénomène bien étudié. Dans la configuration la plus courante, il est établi que la transition de phase est triviale lorsque la dimension transversale d vaut 1 ou 2 (la phase diffusive est réduite à $\beta = 0$) tandis que lorsque $d \geq 3$, il existe une température critique $\beta_c \in (0, \infty)$ qui délimite les deux phases. La preuve de l'existence d'un régime diffusif pour $d \geq 3$ est basée sur une méthode du second moment (*Comm. Math. Phys.* **123** (1989) 529–534, *Ann. Probab.* **34** (2006) 1746–1770, *J. Stat. Phys.* **52** (1988) 609–626), et repose donc fortement sur l'hypothèse que la variable qui encode l'intensité du désordre (qui dans la plupart de la littérature mathématique prend la forme $e^{\beta\eta_x}$), a un second moment fini. Le but de ce travail est d'étudier comment la présence/absence de transition de phase peut dépendre de la dimension d dans le cas où la variable désordre affiche une queue plus lourde. Pour cela on remplace $e^{\beta\eta_x}$ par $(1 + \beta\omega_x)$ où ω_x est dans le domaine d'attraction d'une loi stable avec le paramètre $\gamma \in (1, 2)$. Dans cette configuration, nous montrons qu'une transition de phase non triviale se produit si et seulement si $\gamma > 1 + 2/d$. Plus précisément, lorsque $\gamma \leq 1 + 2/d$, l'énergie libre du système est plus petite que celle de son homologue recuit à chaque température alors que lorsque $\gamma > 1 + 2/d$ la suite martingale des fonctions de partition renormalisées converge vers une variable aléatoire presque sûrement positive pour tout β suffisamment petit.

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Rare event process and entry times distribution for arbitrary null sets on compact manifolds

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Abstract. We establish the general equivalence between rare event process for arbitrary continuous functions whose maximal values are achieved on non-trivial sets, and the entry times distribution for arbitrary measure zero sets. We then use it to show that for differentiable maps on a compact Riemannian manifold that can be modeled by Young's towers, the rare event process and the limiting entry times distribution both converge to compound Poisson distributions. A similar result is also obtained on Gibbs–Markov systems, for both cylinders and open sets. We also give explicit expressions for the parameters of the limiting distribution, and a simple criterion for the limiting distribution to be Poisson. This can be applied to a large family of continuous observables that achieve their maximum on a non-trivial set with zero measure.

Résumé. Nous établissons l'équivalence générale entre les processus d'événements rares pour des fonctions continues arbitraires dont les valeurs maximales sont atteintes sur des ensembles non-triviaux, et la distribution des temps d'entrée pour des ensembles de mesure nulle arbitraires. Nous utilisons ensuite cette équivalence afin de montrer que, pour des applications différentiables sur une variété riemannienne compacte qui peuvent être réalisées par des tours de Young, le processus d'événements rares et la distribution limite des temps d'entrée convergent tous deux vers des lois de Poisson composées. Un résultat similaire est également obtenu pour des systèmes de Gibbs–Markov, à la fois pour des ensembles cylindriques et ouverts. Nous donnons également des expressions explicites pour les paramètres de la loi limite, et un critère simple garantissant que cette dernière est une loi de Poisson. Tout ceci peut être appliqué à une grande famille d'observables continues qui atteignent leur maximum sur un ensemble non-trivial de mesure nulle.

MSC2020 subject classifications: Primary 37A50; secondary 37D25; 60B10; 60G70

Keywords: Rare event process; Compound Poisson distribution; Young's towers; Mixing dynamical systems

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On conditioning a self-similar growth-fragmentation by its intrinsic area

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Abstract. The genealogical structure of self-similar growth-fragmentations can be described in terms of a branching random walk. The so-called intrinsic area $\mathtt{A}$ arises in this setting as the terminal value of a remarkable additive martingale. Motivated by connections with some models of random planar geometry, the purpose of this work is to investigate the effect of conditioning a self-similar growth-fragmentation on its intrinsic area. The distribution of $\mathtt{A}$ is a fixed point of a useful smoothing transform which enables us to establish the existence of a regular density a and to determine the asymptotic behavior of $a(r)$ as $r \to \infty$ (this can be seen as a local version of Kesten–Grincevičius–Goldie theorem's for random affine fixed point equations in a particular setting). In turn, this yields a family of martingales from which the formal conditioning on $\mathtt{A} = r$ can be realized by probability tilting. We point at a limit theorem for the conditional distribution given $\mathtt{A} = r$ as $r \to \infty$, and also observe that such conditioning still makes sense under the so-called canonical measure for which the growth-fragmentation starts from 0.

Résumé. La structure généalogique d'un processus de croissance-fragmentation auto-similaire peut être décrite en termes d'une marche aléatoire branchante. Son aire intrinsèque $\mathtt{A}$ apparait dans ce cadre comme la valeur terminale d'une martingale additive remarquable. L'objet de ce travail est l'étude du conditionnement du processus par l'aire intrinsèque ; il est motivé par certains modèles de géométrie aléatoire plane. La loi de $\mathtt{A}$ est le point fixe d'une transformation de lissage qui permet d'établir l'existence d'une densité régulière a et de déterminer le comportement asymptotique de $a(r)$ lorsque $r \to \infty$ (ce qui peut être vu comme une version locale du théorème de Kesten–Grincevičius–Goldie pour les points fixes d' une équation aléatoire affine dans un cadre particulier). Cela conduit à une famille de martingales à partir desquelles le conditionnement par l'évènement $\mathtt{A} = r$ peut être réalisé au moyen d'un changement de probabilités. Nous obtenons un théorème limite pour la loi conditionnelle sachant $\mathtt{A} = r$ lorsque $r \to \infty$, et observons également qu'un tel conditionnement garde un sens sous la mesure dite canonique pour laquelle le processus de croissance-fragmentation part de 0.

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Keywords: Growth-fragmentation; Branching process; Self-similarity; Smoothing transform; Intrinsic martingale

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Quantitative homogenization of differential forms

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Abstract. We develop a quantitative theory of stochastic homogenization in a general framework involving differential forms. Inspired by recent progress in the uniformly elliptic setting, the analysis relies on the study of certain sub- and superadditive quantities. We establish an algebraic rate of convergence for these quantities and an algebraic error estimate for the homogenization of the Dirichlet problem. Most of the ideas needed in this article come from two distinct theories, the theory of quantitative stochastic homogenization, and the generalization of the main results of functional analysis and of the regularity theory of second-order elliptic equations to the setting of differential forms.

Résumé. Dans cet article, nous étendons la théorie de l'homogénéisation stochastique quantitative au cadre des formes différentielles. L'analyse repose sur l'étude de certaines quantités sous et sur-additives qui ont été utilisées pour développer la théorie dans le cadre des équations uniformément elliptiques. Dans un premier temps, nous démontrons que ces quantités convergent et quantifions leur vitesse de convergence. Une fois ce résultat établi, nous démontrons un théorème d'homogénéisation pour le problème de Dirichlet et obtenons que l'erreur tend vers 0 avec une vitesse de convergence algébrique. Les outils utilisés proviennent de deux théories distinctes, la théorie de l'homogénéisation stochastique quantitative et la généralisation des principaux résultats de l'analyse fonctionnelle et de la théorie de la régularité pour les équations elliptiques au cadre des formes différentielles.

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