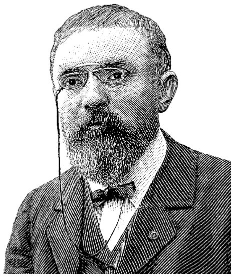

Volume 58, Number 4, 2022
ISSN 0246-0203

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

58/4

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Vol. 58, No. 4 (November, 2022) 1869–2381

ANNALES DE L'INSTITUT HENRI POINCARÉ
PROBABILITÉS ET STATISTIQUES

Rédacteurs en chef / *Chief Editors*

Giambattista GIACOMIN
Université Paris Cité
5 rue Thomas Mann, 75013 Paris, France
giambattista.giacomin@u-paris.fr

Yueyun HU
Université Sorbonne Paris Nord
99 Av. J-B Clément, 93430 Villetaneuse, France
hu@math.univ-paris13.fr

Comité de Rédaction / *Editorial Board*

E. AÏDÉKON *(Fudan University)*
S. ARLOT *(Université Paris-Sud)*
J. BERTOIN *(Universität Zürich)*
P. BOURGADE *(New York Univ.)*
F. CARAVENNA *(Univ. Milano-Bicocca)*
D. CHAFAI *(Ecole Normale Supérieure, Paris)*
I. CORWIN *(Columbia University)*
A. DEBUSSCHE *(École Normale Supérieure de Rennes)*
I. DUMITRIU *(UC San Diego)*
B. GESS *(Universität Bielefeld)*
S. GOUËZEL *(Université de Nantes)*
A. GUILLIN *(Clermont-Auvergne University)*
M. HAIRER *(Imperial College London)*
M. HOFFMANN *(Univ. Paris-Dauphine)*
N. HOLDEN *(ETH Zurich)*
T. HUTCHCROFT *(Cambridge University)*
A. NACHMIAS *(Tel Aviv University)*
J. NORRIS *(Cambridge University)*
R. RHODES *(Univ. Aix-Marseille)*
J. ROUSSEAU *(University of Oxford)*
M. SASADA *(University of Tokyo)*
P. SOUSI *(Cambridge University)*
B. DE TILIÈRE *(Univ. Paris-Dauphine)*
V. WACHTEL *(Universität München)*
H. WEBER *(Univ. of Warwick)*

Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques (ISSN 0246-0203), Volume 58, Number 4, November 2022. Published quarterly by Association des Publications de l'Institut Henri Poincaré.

POSTMASTER: Send address changes to Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques, Dues and Subscriptions Office, PO Box 729, Middletown, Maryland 21769, USA.

Copyright © 2022 Association des Publications de l'Institut Henri Poincaré
Printed in the United States of America

Président et directeur de la publication : Cédric Villani
Périodicité : 4 nos / an

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1869–1886
https://doi.org/10.1214/21-AIHP1232
© Association des Publications de l'Institut Henri Poincaré, 2022

Sharp threshold for two-dimensional majority dynamics percolation

Caio Alves[1,a] and Rangel Baldasso[2,b]

[1]Institute of Mathematics, University of Leipzig – Augustusplatz 10, 04109 Leipzig, Germany. [a]caiotmalves@gmail.com
[2]Bar-Ilan University, 5290002, Ramat Gan, Israel. [b]r.baldasso@math.leidenuniv.nl

Abstract. In this work we consider the two-dimensional percolation model arising from the majority dynamics process at a given time $t \in \mathbb{R}_+$. We show the emergence of a sharp threshold phenomenon for the box crossing event at the critical probability parameter $p_c(t)$ with polynomial size window. We then use this result in order to obtain stretched-exponential bounds on the one-arm event probability in the subcritical phase. Our results are based on differential inequalities derived from the OSSS inequality, inspired by the recent developments by Ahlberg, Broman, Griffiths, and Morris and by Duminil-Copin, Raoufi, and Tassion. We also provide analogous results for percolation in the voter model.

Résumé. Dans ce travail, nous considérons le modèle de percolation bidimensionnel résultant du processus de dynamique d'opinions avec règle de la majorité à un temps donné $t \in \mathbb{R}_+$. Nous montrons l'existence d'un phénomène de seuil pour l'événement de traversée d'une région au paramètre de probabilité critique $p_c(t)$, avec une fenêtre de taille polynomiale. Nous utilisons ensuite ce résultat pour obtenir des bornes exponentielles étirées sur la probabilité de la présence d'un bras dans la phase sous-critique. Nos résultats sont basés sur des inégalités différentielles dérivées de l'inégalité OSSS et inspirées des développements récents d'Ahlberg, Broman, Griffiths et Morris et de Duminil-Copin, Raoufi et Tassion. Nous fournissons également des résultats analogues pour la percolation dans le modèle du votant.

MSC2020 subject classifications: 60K35; 82C43; 82C27

Keywords: Percolation; Opinion dynamics; Sharp thresholds

References

[1] D. Ahlberg and R. Baldasso. Noise sensitivity and Voronoi percolation. *Electron. J. Probab.* **23** (2018). MR3878133 https://doi.org/10.1214/18-ejp233
[2] D. Ahlberg, E. Broman, S. Griffiths and R. Morris. Noise sensitivity in continuum percolation. *Israel J. Math.* **201** (2) (2014) 847–899. MR3265306 https://doi.org/10.1007/s11856-014-1038-y
[3] D. Ahlberg, V. Tassion and A. Teixeira. Sharpness of the phase transition for continuum percolation in $\mathbb{R}^2$. *Probab. Theory Related Fields* **172** (1–2) (2018) 525–581. MR3851838 https://doi.org/10.1007/s00440-017-0815-8
[4] G. Amir and R. Baldasso. Percolation in majority dynamics, 2019. ArXiv preprint. Available at arXiv:1902.03349. MR4059191 https://doi.org/10.1214/20-ejp414
[5] F. Camia, C. M. Newman and V. Sidoravicius. Approach to fixation for zero-temperature stochastic Ising models on the hexagonal lattice. In *In and Out of Equilibrium* 163–183. Springer, Berlin, 2002. MR1901952
[6] D. Dereudre and P. Houdebert. Sharp phase transition for the continuum Widom–Rowlinson model, 2018. ArXiv preprint. Available at arXiv:1807.04988. MR4255179 https://doi.org/10.1214/20-aihp1082
[7] H. Duminil-Copin, A. Raoufi and V. Tassion. Subcritical phase of d-dimensional poisson-boolean percolation and its vacant set, 2018. ArXiv preprint. Available at arXiv:1805.00695. MR4149823 https://doi.org/10.5802/ahl.43
[8] H. Duminil-Copin, A. Raoufi and V. Tassion. Exponential decay of connection probabilities for subcritical Voronoi percolation in $\mathbb{R}^d$. *Probab. Theory Related Fields* **173** (1–2) (2019) 479–490. MR3916112 https://doi.org/10.1007/s00440-018-0838-9
[9] H. Duminil-Copin, A. Raoufi and V. Tassion. Sharp phase transition for the random-cluster and Potts models via decision trees. *Ann. of Math.* **189** (1) (2019) 75–99. MR3898174 https://doi.org/10.4007/annals.2019.189.1.2
[10] A. Gandolfi, M. Keane and L. Russo. On the uniqueness of the infinite occupied cluster in dependent two-dimensional site percolation. *Ann. Probab.* **16** (3) (1988) 1147–1157. MR0942759
[11] S. Muirhead and H. Vanneuville. The sharp phase transition for level set percolation of smooth planar Gaussian fields, 2018. ArXiv preprint. Available at arXiv:1806.11545. MR4076787 https://doi.org/10.1214/19-AIHP1006

[12] R. O'Donnell, M. Saks, O. Schramm and R. A. Servedio. Every decision tree has an influential variable. In *46th Annual IEEE Symposium on Foundations of Computer Science (FOCS'05)* 31–39. IEEE, New York, 2005.
[13] T. Omer and R. J. Tessler. Majority dynamics and the retention of information. *Israel J. Math.* **206** (1) (2015) 483–507. MR3319649 https://doi.org/10.1007/s11856-014-1148-2
[14] S. Popov and A. Teixeira. Soft local times and decoupling of random interlacements. *J. Eur. Math. Soc. (JEMS)* **17** (10) (2015) 2545–2593. MR3420516 https://doi.org/10.4171/JEMS/565
[15] L. Russo. An approximate zero-one law. *Z. Wahrsch. Verw. Gebiete* **61** (1) (1982) 129–139. MR0671248 https://doi.org/10.1007/BF00537230
[16] M. Talagrand. On Russo's approximate zero-one law. *Ann. Probab.* **22** (3) (1994) 1576–1587. MR1303654
[17] V. Tassion. Crossing probabilities for Voronoi percolation. *Ann. Probab.* **44** (5) (2016) 3385–3398. MR3551200 https://doi.org/10.1214/15-AOP1052
[18] E. H. Theodore. A correlation inequality for Markov processes in partially ordered state spaces. *Ann. Probab.* (1977) 451–454. MR0433650 https://doi.org/10.1214/aop/1176995804
[19] J. van den Berg. Sharpness of the percolation transition in the two-dimensional contact process. *Ann. Appl. Probab.* (2011) 374–395. MR2778387 https://doi.org/10.1214/10-AAP702

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1887–1899
https://doi.org/10.1214/21-AIHP1231
© Association des Publications de l'Institut Henri Poincaré, 2022

Constrained-degree percolation in random environment

Rémy Sanchis[1,a], Diogo C. dos Santos[2,b] and Roger W. C. Silva[2,c]

[1]*Departamento de Matemática, Universidade Federal de Minas Gerais, Brazil.* [a]*rsanchis@mat.ufmg.br*
[2]*Departamento de Estatística, Universidade Federal de Minas Gerais, Brazil.* [b]*diogoc@ufmg.br,* [c]*rogerwcs@est.ufmg.br*

Abstract. We consider the Constrained-degree percolation model in random environment on the square lattice. In this model, each vertex v has an independent random constraint κ_v which takes the value $j \in \{0, 1, 2, 3\}$ with probability ρ_j. Each edge e attempts to open at a random uniform time U_e in $[0, 1]$, independently of all other edges. It succeeds if at time U_e both its end-vertices have degrees strictly smaller than their respectively attached constraints. We show that this model undergoes a non-trivial phase transition when ρ_3 is sufficiently large. The proof consists of a decoupling inequality, the continuity of the probability for local events, and a coarse-graining argument.

Résumé. Nous considérons le modèle de percolation à degré contraint dans un environnement aléatoire sur le réseau carré. Dans ce modèle, chaque sommet v a une contrainte aléatoire indépendante κ_v qui prend la valeur $j \in \{0, 1, 2, 3\}$ avec probabilité ρ_j. Chaque arête e tente de s'ouvrir à un temps uniforme aléatoire U_e dans $[0, 1]$, indépendamment de toutes les autres arêtes. Il réussit si à l'instant U_e ses deux sommets d'extrémité ont des degrés strictement inférieurs à leurs contraintes respectives attachées. Nous montrons que ce modèle subit une transition de phase non triviale lorsque ρ_3 est suffisamment grand. La preuve consiste en une inégalité de découplage, la continuité de la probabilité des événements locaux et un argument de renormalisation.

MSC2020 subject classifications: Primary 60K35; secondary 82B43

Keywords: Exponential decay of correlations; Constrained-degree percolation; Renormalization; Continuity

References

[1] D. Ahlberg, V. Tassion and A. Teixeira. Sharpness of the phase transition for continuum percolation in $\mathbb{R}^2$. *Probab. Theory Related Fields* **172** (2018) 525–581. MR3851838 https://doi.org/10.1007/s00440-017-0815-8
[2] G. Amir and R. Baldasso. Percolation in majority dynamics. *Electron. J. Prob.* **25** (2020). 18 pp.. MR4059191 https://doi.org/10.1214/20-ejp414
[3] R. Baldasso and A. Teixeira. How can a clairvoyant particle escape the exclusion process. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (2018) 2177–2202. MR3865670 https://doi.org/10.1214/17-AIHP868
[4] B. Bollobás and O. Riordan. The critical probability for random Voronoi percolation in the plane is $\frac{1}{2}$. *Probab. Theory Related Fields* **136** (2006) 417–468. MR2257131 https://doi.org/10.1007/s00440-005-0490-z
[5] M. Bramson, R. Durrett and R. Schonmann. The contact process in a random environment. *Ann. Probab.* **19** (1991) 960–983. MR1112403
[6] M. Campanino and A. Klein. Decay of two-point functions for $(d + 1)$-dimensional percolation, Ising and Potts models with d-dimensional disorder. *Comm. Math. Phys.* **135** (1991) 483–497. MR1091574
[7] M. Campanino, A. Klein and J. F. Perez. Localization in the ground state of the Ising model with a random transverse field. *Comm. Math. Phys.* **135** (1991) 499–515. MR1091575
[8] B. N. B. de Lima, R. Sanchis, D. C. dos Santos, V. Sidoravicius and R. Teodoro. The constrained-degree percolation model. *Stochastic Process. Appl.* **130** (2020) 5492–5509. MR4127336 https://doi.org/10.1016/j.spa.2020.03.014
[9] A. P. Furlan, D. C. dos Santos, R. M. Ziff and R. Dickman. Jamming and percolation of dimers in restricte-valence random sequential absortion. *Phys. Rev. Res.* **2** (2020), 043027.
[10] O. Garet, R. Marchand and I. Marcovici. Does Eulerian percolation on $\mathbb{Z}^2$ percolate? *ALEA Lat. Am. J. Probab. Math. Stat.* **15** (2018) 279–294. MR3795450 https://doi.org/10.30757/alea.v15-13
[11] G. Grimmett and S. Janson. Random graphs with forbidden vertex degrees. *Random Structures Algorithms* **37** (2010) 137–175. MR2676027 https://doi.org/10.1002/rsa.20307
[12] G. Grimmett and Z. Li. The 1-2 model. *Contemp. Math.* **969** (2017) 139–152. MR3715445 https://doi.org/10.1090/conm/696/14020
[13] C. Hoffman. Phase transition in dependent percolation. *Comm. Math. Phys.* **254** (2005) 1–22. MR2116736 https://doi.org/10.1007/s00220-004-1240-2
[14] A. E. Holroyd and Z. Li. Constrained percolation in two dimensions. *Ann. Inst. Henri Poincaré D* **8** (3) (2021) 323–375. MR4321220 https://doi.org/10.4171/aihpd/106

[15] J. Jonasson, E. Mossel and Y. Peres. Percolation in a dependent random environment. *Random Structures Algorithms* **16** (2000) 333–343. MR1761579 https://doi.org/10.1002/1098-2418(200007)16:4<333::AID-RSA3>3.3.CO;2-3

[16] H. Kesten, V. Sidoravicius and M. E. Vares. Oriented percolation in a random environment. Available at arXiv:1207.3168.

[17] A. Klein. Extinction of contact and percolation processes in random environment. *Ann. Probab.* **22** (1994) 1227–1251. MR1303643

[18] Z. Li. Constrained percolation, Ising model, and XOR Ising model on planar lattices. *Random Structures Algorithms* **57** (2020) 474–525. MR4129728 https://doi.org/10.1002/rsa.20924

[19] B. M. McCoy and T. T. Wu. Theory of a two-dimensional Ising model with random impurities. I. Thermodynamics. *Phys. Rev.* **176** (1968) 631–643. MR0286431

[20] R. Teodoro. Constrained-degree Percolation. PhD Thesis, IMPA, 2014.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1900–1941
https://doi.org/10.1214/21-AIHP1222
© Association des Publications de l'Institut Henri Poincaré, 2022

Continuity of the time constant in a continuous model of first passage percolation

Jean-Baptiste Gouéré[1,a] and Marie Théret[2,b]

[1] Institut Denis-Poisson - UMR CNRS 7013, Université de Tours, Parc de Grandmont, 37200 Tours, France. [a] jean-baptiste.gouere@lmpt.univ-tours.fr
[2] Modal'X - UMR CNRS 9023, UPL, Université Paris Nanterre, 92000 Nanterre, France, and FP2M, CNRS FR 2036. [b] marie.theret@parisnanterre.fr

Abstract. For a given dimension $d \geq 2$ and a finite measure ν on $(0, +\infty)$, we consider ξ a Poisson point process on $\mathbb{R}^d \times (0, +\infty)$ with intensity measure $dc \otimes \nu$ where dc denotes the Lebesgue measure on $\mathbb{R}^d$. We consider the Boolean model $\Sigma = \bigcup_{(c,r)\in\xi} B(c, r)$ where $B(c, r)$ denotes the open ball centered at c with radius r. For every $x, y \in \mathbb{R}^d$, we define $T(x, y)$ as the minimum time needed to travel from x to y by a traveler that walks at speed 1 outside Σ and at infinite speed inside Σ. By a standard application of Kingman's subadditive theorem, one easily shows that $T(0, x)$ behaves like $\mu\|x\|$ when $\|x\|$ goes to infinity, where μ is a constant named the time constant in classical first passage percolation. In this paper we investigate the regularity of μ as a function of the measure ν associated with the underlying Boolean model. One of the key results is a uniform control of the length of "nice" geodesics. In the course of the proof, we need a continuum analogue of the BK inequality for unions of disjoint occurrences of events.

Résumé. Etant donnés une dimension $d \geq 2$ et une mesure finie ν sur $(0, +\infty)$, nous considérons ξ un processus ponctuel de Poisson sur $\mathbb{R}^d \times (0, +\infty)$ de mesure d'intensité $dc \otimes \nu$ où dc désigne la mesure de Lebesgue sur $\mathbb{R}^d$. Nous considérons le modèle booléen $\Sigma = \bigcup_{(c,r)\in\xi} B(c, r)$ où $B(c, r)$ désigne la boule ouverte centrée en c et de rayon r. Pour tous $x, y \in \mathbb{R}^d$, nous définissons $T(x, y)$ comme le temps minimal nécessaire pour voyager de x à y pour un voyageur qui se déplace à vitesse 1 en dehors de Σ et à vitesse infinie dans Σ. Par une application standard du théorème ergodique sous-additif de Kingman, on peut facilement prouver que $T(0, x)$ se comporte comme $\mu\|x\|$ quand $\|x\|$ tend vers l'infini, où μ est une constante appelée la constante de temps en percolation de premier passage classique. Dans cet article, nous étudions la régularité de μ comme fonction de la mesure ν associée au modèle booléen sous-jacent. Un des résultats clés est un contrôle uniforme de la longueur de "bonnes" géodésiques. Au cours de la preuve, nous avons recours à un analogue continu de l'inégalité BK pour des unions d'occurrences disjointes d'évènements.

MSC2020 subject classifications: Primary 60K35; secondary 82B43

Keywords: Boolean model; Continuum percolation; First passage percolation; Time constant; Continuity

References

[1] A. Auffinger, M. Damron and J. Hanson. 50 years of first passage percolation. In *University Lecture Series*, **68**. American Mathematical Society, 2017. MR3729447 https://doi.org/10.1090/ulect/068

[2] E. Bates Empirical distributions, geodesic lengths, and a variational formula in first-passage percolation, 2020.

[3] C. Bezuidenhout and G. Grimmett. Exponential decay for subcritical contact and percolation processes. *Ann. Probab.* **19** (3) (1991) 984–1009. MR1112404

[4] R. Cerf and B. Dembin The time constant for Bernoulli percolation is Lipschitz continuous strictly above p_c. Working paper or preprint, 2021. MR4234130 https://doi.org/10.1051/ps/2021005

[5] R. Cerf and M. Théret. Weak shape theorem in first passage percolation with infinite passage times. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (3) (2016) 1351–1381. MR3531712 https://doi.org/10.1214/15-AIHP686

[6] J. T. Cox. The time constant of first-passage percolation on the square lattice. *Adv. in Appl. Probab.* **12** (4) (1980) 864–879. MR588407 https://doi.org/10.2307/1426745

[7] J. T. Cox and R. Durrett. Some limit theorems for percolation processes with necessary and sufficient conditions. *Ann. Probab.* **9** (4) (1981) 583–603. MR624685 https://doi.org/10.1051/ps/2021005

[8] J. T. Cox and H. Kesten. On the continuity of the time constant of first-passage percolation. *J. Appl. Probab.* **18** (4) (1981) 809–819. MR633228 https://doi.org/10.1051/ps/2021005

[9] M. Damron and P. Tang. Superlinearity of geodesic length in 2D critical first-passage percolation. In *Sojourns in Probability Theory and Statistical Physics – II* 101–122. V. Sidoravicius (Ed.). Springer Singapore, Singapore, 2019. MR2209700 https://doi.org/10.1051/ps/2021005

[10] B. Dembin. Regularity of the time constant for a supercritical Bernoulli percolation. *ESAIM Probab. Stat.* **25** (2021) 109–132. MR4234130 https://doi.org/10.1051/ps/2021005

[11] O. Garet and R. Marchand. Asymptotic shape for the chemical distance and first-passage percolation on the infinite Bernoulli cluster. *ESAIM Probab. Stat.* **8** (2004) 169–199 (electronic). MR2085613 https://doi.org/10.1051/ps:2004009

[12] O. Garet, R. Marchand, E. B. Procaccia and M. Théret. Continuity of the time and isoperimetric constants in supercritical percolation. *Electron. J. Probab.* **22** (2017), 78. MR3710798 https://doi.org/10.1214/17-EJP90

[13] J.-B. Gouéré and R. Marchand. Continuous first-passage percolation and continuous greedy paths model: Linear growth. *Ann. Appl. Probab.* **18** (6) (2008) 2300–2319. MR2474537 https://doi.org/10.1214/08-AAP523

[14] J.-B. Gouéré and M. Théret. Positivity of the time constant in a continuous model of first passage percolation. *Electron. J. Probab.* **22** (2017), 21. MR4234130 https://doi.org/10.1214/17-EJP67

[15] G. Grimmett. *Percolation*, 2nd edition. *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]* **321**. Springer-Verlag, Berlin, 1999. MR1707339 https://doi.org/10.1007/978-3-662-03981-6

[16] H. Kesten. Aspects of first passage percolation. In *École d'été de probabilités de Saint-Flour, XIV-1984* 125–264. *Lecture Notes in Math.* **1180**. Springer, Berlin, 1986. MR0876084 https://doi.org/10.1007/BFb0074919

[17] A. Krishnan, F. Rassoul-Agha and T. Seppäläinen Geodesic length and shifted weights in first-passage percolation, 2021.

[18] G. Last and M. Penrose. *Lectures on the Poisson Process*. *Institute of Mathematical Statistics Textbooks* **7**, xx+293. Cambridge University Press, Cambridge, 2018. MR3791470 https://doi.org/10.1051/ps/2021005

[19] J. B. Martin. Linear growth for greedy lattice animals. *Stochastic Process. Appl.* **98** (1) (2002) 43–66. MRMR1884923 https://doi.org/10.1051/ps/2021005

[20] R. Meester and R. Roy. *Continuum Percolation*. *Cambridge Tracts in Mathematics* **119**, x+238. Cambridge University Press, Cambridge, 1996. MR1409145 https://doi.org/10.1017/CBO9780511895357

[21] J. van den Berg. A note on disjoint-occurrence inequalities for marked Poisson point processes. *J. Appl. Probab.* **33** (2) (1996) 420–426. MR1385351 https://doi.org/10.2307/3215065

[22] J. van den Berg and H. Kesten. Inequalities with applications to percolation and reliability. *J. Appl. Probab.* **22** (3) (1985) 556–569. MR0799280 https://doi.org/10.1017/s0021900200029326

[23] Y. Zhang. Supercritical behaviors in first-passage percolation. *Stochastic Process. Appl.* **59** (2) (1995) 251–266. MR1357654 https://doi.org/10.1016/0304-4149(95)00051-8

[24] Y. Zhang and Y. C. Zhang. A limit theorem for N_{0n}/n in first-passage percolation. *Ann. Probab.* **12** (4) (1984) 1068–1076. MRMR757770 https://doi.org/10.1051/ps/2021005

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1942–1958
https://doi.org/10.1214/21-AIHP1236
© Association des Publications de l'Institut Henri Poincaré, 2022

Negative correlation of adjacent Busemann increments

Ian Alevy[a,c] and Arjun Krishnan[b,d]

Mathematics Department, University of Rochester, Hylan 1008, Rochester, NY 14627, USA. [a]*ian.alevy@rochester.edu*, [b]*arjunkc@gmail.com;* [c]*https://people.math.rochester.edu/faculty/ialevy/*, [d]*https://people.math.rochester.edu/faculty/akrish11/*

Abstract. We consider i.i.d. last-passage percolation on $\mathbb{Z}^2$ with weights having distribution F and time-constant g_F. We provide an explicit condition on the large deviation rate function for independent sums of F that determines when some adjacent Busemann function increments are negatively correlated. As an example, we prove that Bernoulli(p) weights for $p > p^* \approx 0.6504$ satisfy this condition. We prove this condition by establishing a direct relationship between the negative correlations of adjacent Busemann increments and the dominance of g_F by the function describing the time-constant of last-passage percolation with exponential or geometric weights.

Résumé. Nous considérons la percolation de dernier passage i.i.d. sur $\mathbb{Z}^2$ avec des poids de loi F et de constante temporelle g_F. Nous donnons une condition explicite sur la fonction de taux de grande déviation de la somme de variables aléatoires indépendantes de loi F, qui détermine quand certains accroissements de Busemann adjacents sont négativement corrélés. À titre d'exemple nous montrons que les poids Bernoulli(p) avec $p > p^* \approx 0.6504$ vérifient cette condition. Nous obtenons cette condition en établissant un lien direct entre les corrélations négatives des accroissements de Busemann adjacents et la domination de la constante temporelle g_F par la fonction qui décrit la constante de temps de la percolation de dernier passage avec poids exponentiels ou géométriques.

MSC2020 subject classifications: 60K35; 60K37

Keywords: Busemann function; Negative correlation criterion; Time-constant domination; Large deviation rate-function

References

[1] D. Ahlberg and C. Hoffman. Random coalescing geodesics in first-passage percolation. 2016. Available at arXiv:1609.02447 [math.PR]. MR2462555 https://doi.org/10.1214/07-AAP510
[2] A. Auffinger and M. Damron. The scaling relation $\chi = 2\xi - 1$ for directed polymers in a random environment. *ALEA Lat. Am. J. Probab. Math. Stat.* **10** (2) (2013) 857–880. MR3141825
[3] A. Auffinger and M. Damron. Differentiability at the edge of the percolation cone and related results in first-passage percolation. *Probab. Theory Related Fields* **156** (1–2) (2013) 193–227. Available at http://www.ams.org/mathscinet-getitem?mr=3055257 (visited on 08/25/2014). MR3055257 https://doi.org/10.1007/s00440-012-0425-4
[4] M. Balázs, E. Cator and T. Seppäläinen. Cube root fluctuations for the corner growth model associated to the exclusion process. *Electron. J. Probab.* **11** (42) (2006) 1094–1132. MR2268539 https://doi.org/10.1214/EJP.v11-366
[5] R. Basu, S. Sarkar and A. Sly. Coalescence of geodesics in exactly solvable models of last passage percolation. *J. Math. Phys.* **60** (9) (2019), 093301, 22. Available at https://doi-org.ezp.lib.rochester.edu/10.1063/1.5093799.
[6] H. Busemann. *The Geometry of Geodesics* x+422. Academic Press, New York, N.Y., 1955. MR0075623
[7] S. Chatterjee. The universal relation between scaling exponents in first-passage percolation. *Ann. of Math. (2)* **177** (2) (2013) 663–697. MR3010809 https://doi.org/10.4007/annals.2013.177.2.7
[8] M. Damron and J. Hanson. Busemann functions and infinite geodesics in two-dimensional first-passage percolation. *Comm. Math. Phys.* **325** (3) (2014) 917–963. https://doi.org/10.1007/s00220-013-1875-y
[9] M. Damron and X. Wang. Entropy reduction in Euclidean first-passage percolation. *Electron. J. Probab.* **21** (2016) Paper No. 65, 23. MR3580031 https://doi.org/10.1214/16-EJP12
[10] R. Durrett and T. M. Liggett. The shape of the limit set in Richardson's growth model. *Ann. Probab.* **9** (2) (1981) 186–193. Available at http://www.ams.org/mathscinet-getitem?mr=606981 (visited on 10/31/2013). MR0606981
[11] A. Gandolfi and H. Kesten. Greedy lattice animals. II. Linear growth. *Ann. Appl. Probab.* **4** (1) (1994) 76–107. Available at http://links.jstor.org/sici?sici=1050-5164(199402)4:1%3C76:GLAILG%3E2.0.C0;2-5&origin=MSN. MR1258174
[12] U. Gangopadhyay. Fluctuations of Transverse Increments in Two-dimensional First Passage Percolation. Available at arXiv:2011.14686 [math] (Nov. 2020) (visited on 02/27/2021).

[13] N. Georgiou, F. Rassoul-Agha and T. Seppäläinen. Variational formulas and cocycle solutions for directed polymer and percolation models. *Comm. Math. Phys.* **346** (2) (2016) 741–779. MR3535900 https://doi.org/10.1007/s00220-016-2613-z

[14] N. Georgiou, F. Rassoul-Agha and T. Seppäläinen. Stationary cocycles and Busemann functions for the corner growth model. *Probab. Theory Related Fields* **169** (1–2) (2017) 177–222. MR3704768 https://doi.org/10.1007/s00440-016-0729-x

[15] N. Georgiou, F. Rassoul-Agha and T. Seppäläinen. Geodesics and the competition interface for the corner growth model. *Probab. Theory Related Fields* **169** (1–2) (2017) 223–255. https://doi.org/10.1007/s00440-016-0734-0

[16] G. R. Grimmett and D. R. Stirzaker. *Probability and Random Processes*, 3rd edition. xii+596. Oxford University Press, New York, 2001. MR2059709

[17] C. Janjigian and F. Rassoul-Agha. Busemann functions and Gibbs measures in directed polymer models on Z^2. *Ann. Probab.* **48** (2) (2020) 778–816. MR4089495 https://doi.org/10.1214/19-AOP1375

[18] K. Johansson. Shape fluctuations and random matrices. *Comm. Math. Phys.* **209** (2) (2000) 437–476. https://doi.org/10.1007/s002200050027

[19] K. Johansson. Transversal fluctuations for increasing subsequences on the plane. *Probab. Theory Related Fields* **116** (4) (2000) 445–456. https://doi.org/10.1007/s004400050258

[20] J. Krug and H. Spohn. Universality classes for deterministic surface growth. *Phys. Rev. A* **38** (8) (1988) 4271. MR0966369 https://doi.org/10.1103/PhysRevA.38.4271

[21] C. Licea and C. M. Newman. Geodesics in two-dimensional first-passage percolation. *Ann. Probab.* **24** (1) (1996) 399–410. https://doi.org/10.1214/aop/1042644722

[22] C. Licea, C. M. Newman and M. S. Piza. Superdiffusivity in first-passage percolation. *Probab. Theory Related Fields* **106** (4) (1996) 559–591. MR1421992 https://doi.org/10.1007/s004400050075

[23] R. Marchand. Strict inequalities for the time constant in first passage percolation. *Ann. Appl. Probab.* **12** (3) (2002), 1001–1038. Available at http://www.ams.org/mathscinet-getitem?mr=1925450 (visited on 11/01/2013). MR1925450 https://doi.org/10.1214/aoap/1031863179

[24] J. B. Martin. Linear growth for greedy lattice animals. *Stochastic Process. Appl.* **98** (1) (2002) 43–66. MR1884923 https://doi.org/10.1016/S0304-4149(01)00142-9

[25] J. B. Martin. Limiting shape for directed percolation models. *Ann. Probab.* **32** (4) (2004) 2908–2937. https://doi.org/10.1214/009117904000000838

[26] C. M. Newman. A surface view of first-passage percolation. In *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Zürich, 1994)*. 1017–1023. Birkhäuser, Basel, 1995. Available at http://www.ams.org/mathscinet-getitem?mr=1404001 (visited on 11/01/2013). MR1404001

[27] H. Rost. Non-equilibrium behaviour of a many particle process: Density profile and local equilibria. *Z. Wahrscheinlichkeitstheor. Verw. Geb.* **58** (1) (1981) 41–53. MR0635270 https://doi.org/10.1007/BF00536194

[28] T. Seppäläinen. Zero correlation and the exponential limit shape. *Personal Communication.* (2018).

[29] J. T. Cox et al. Greedy lattice animals. I. Upper bounds. *Ann. Appl. Probab.* **3** (4) (1993) 1151–1169. Available at http://links.jstor.org/sici?sici=1050-5164(199311)3:4%3C1151:GLAIUB%3E2.0.C0;2-H&origin=MSN. MR1241039

[30] C. A. Tracy and H. Widom. Level-spacing distributions and the Airy kernel. *Comm. Math. Phys.* **159** (1) (1994) 151–174. Available at http://projecteuclid.org/euclid.cmp/1104254495. MR1257246

[31] J. van den Berg and H. Kesten. Inequalities for the time constant in first-passage percolation. *Ann. Appl. Probab.* **3** (1) (1993) 56–80. Available at http://www.ams.org/mathscinet-getitem?mr=1202515 (visited on 11/01/2013). MR1202515

[32] W. Whitt. The effect of variability in the $GI/G/s$ queue. *J. Appl. Probab.* **17** (4) (1980) 1062–1071. Available at https://doi-org.ezp.lib.rochester.edu/10.2307/3213215.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1959–1980
https://doi.org/10.1214/21-AIHP1233
© Association des Publications de l'Institut Henri Poincaré, 2022

Typicality and entropy of processes on infinite trees

Ágnes Backhausz[1,2,a], Charles Bordenave[3,c] and Balázs Szegedy[2,b]

[1]*Faculty of Science, ELTE Eötvös Loránd University and Alfréd Rényi Institute of Mathematics, Budapest, Hungary.* [a]*agnes.backhausz@ttk.elte.hu*
[2]*Alfréd Rényi Institute of Mathematics, Budapest, Hungary.* [b]*szegedy.balazs@renyi.mta.hu*
[3]*Institut de Mathématiques de Marseille, CNRS & Aix-Marseille University, Marseille, France.* [c]*charles.bordenave@univ-amu.fr*

Abstract. Consider a uniformly sampled random d-regular graph on n vertices. If d is fixed and n goes to ∞ then we can relate typical (large probability) properties of such random graph to a family of invariant random processes (called "typical" processes) on the infinite d-regular tree T_d. This correspondence between ergodic theory on T_d and random regular graphs is already proven to be fruitful in both directions. This paper continues the investigation of typical processes with a special emphasis on entropy. We study a natural notion of micro-state entropy for invariant processes on T_d. It serves as a quantitative refinement of the notion of typicality and is tightly connected to the asymptotic free energy in statistical physics. Using entropy inequalities, we provide new sufficient conditions for typicality for edge Markov processes. We also extend these notions and results to processes on unimodular Galton–Watson random trees.

Résumé. On considère un graphe d-régulier aléatoire avec n sommets uniformément distribué. Si d est fixé et n tend vers l'infini, nous pouvons alors relier les propriétés typiques (de grande probabilité) d'un tel graphe aléatoire avec une famille de processus aléatoires invariants (dénommés processus "typiques") sur l'arbre d-régulier infini T_d. Cette correspondance entre théorie ergodique sur T_d et graphes réguliers aléatoires s'est déjà révélée fructueuse dans les deux directions. Ce papier poursuit l'investigation des processus typiques avec un accent mis sur l'entropie. Nous y étudions une notion naturelle d'entropie micro-état pour les processus invariants sur T_d. Elle sert de raffinement quantitatif à la notion de typicalité et elle est intimement reliée à l'énergie libre asymptotique en physique statistique. Au moyen d'inégalités entropiques, nous démontrons des nouvelles conditions suffisantes de typicalité pour des processus markoviens sur les arêtes de l'arbre. Nous étendons aussi ces notions et résultats à des processus sur des arbres de Galton–Watson unimodulaires.

MSC2020 subject classifications: Primary 05C80; secondary 37A35

Keywords: Invariant process; Sofic entropy; Infinite trees

References

[1] D. Aldous and R. Lyons. Processes on unimodular random networks. *Electron. J. Probab.* **12** (2007) 1454–1508. MR2354165 https://doi.org/10.1214/EJP.v12-463
[2] A. Backhausz and B. Szegedy. On large-girth regular graphs and random processes on trees. *Random Structures Algorithms* **53** (3) (2018) 389–416. MR3854040 https://doi.org/10.1002/rsa.20769
[3] A. Backhausz and B. Szegedy. On the almost eigenvectors of random regular graphs. *Ann. Probab.* **47** (3) (2019) 1677–1725. MR3945757 https://doi.org/10.1214/18-AOP1294
[4] A. Backhausz, B. Szegedy and B. Virág. Ramanujan graphings and correlation decay in local algorithms. *Random Structures Algorithms* **47** (3) (2015) 424–435. MR3385741 https://doi.org/10.1002/rsa.20562
[5] A. Backhausz and B. Virág. Spectral measures of factor of i.i.d. processes on vertex-transitive graphs. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (4) (2017) 2260–2278. MR3729654 https://doi.org/10.1214/16-AIHP790
[6] M. Bayati, D. Gamarnik and P. Tetali. Combinatorial approach to the interpolation method and scaling limits in sparse random graphs. *Ann. Probab.* **41** (6) (2013) 4080–4115. MR3161470 https://doi.org/10.1214/12-AOP816
[7] E. A. Bender and E. Canfield. The asymptotic number of labeled graphs with given degree sequences. *J. Combin. Theory Ser. A* **24** (3) (1978) 296–307. MR0505796 https://doi.org/10.1016/0097-3165(78)90059-6
[8] I. Benjamini and O. Schramm. Recurrence of distributional limits of finite planar graphs. *Electron. J. Probab.* **6** (23) (2001) 13. MR1873300 https://doi.org/10.1214/EJP.v6-96
[9] B. Bollobás. The independence ratio of regular graphs. *Proc. Amer. Math. Soc.* **83** (2) (1981) 433–436. MR0624948 https://doi.org/10.2307/2043545

[10] B. Bollobás. *Random Graphs*, 2nd edition. *Cambridge Studies in Advanced Mathematics* **73**. Cambridge University Press, Cambridge, 2001. MR1864966 https://doi.org/10.1017/CBO9780511814068

[11] C. Bordenave. Normalité asymptotique des vecteurs propres d'un graphe régulier aléatoire (d'après Backhausz et Szegedy). In *Séminaire Bourbaki, 71e année, 2018–2019*, **1151**, 2018. MR4224633 https://doi.org/10.24033/ast

[12] C. Bordenave and P. Caputo. Large deviations of empirical neighborhood distribution in sparse random graphs. *Probab. Theory Related Fields* **163** (1–2) (2015) 149–222. MR3405616 https://doi.org/10.1007/s00440-014-0590-8

[13] L. P. Bowen. A brief introduction of sofic entropy theory. In *Proceedings of the International Congress of Mathematicians – Rio de Janeiro 2018. Vol. III. Invited Lectures* 1847–1866. World Sci. Publ., Hackensack, NJ, 2018. MR3966832

[14] A. Coja-Oghlan and W. Perkins. Spin systems on Bethe lattices. *Comm. Math. Phys.* **372** (2) (2019) 441–523. MR4032871 https://doi.org/10.1007/s00220-019-03544-y

[15] E. Csóka, V. Harangi and B. Virág. Entropy and expansion. *Ann. Inst. Henri Poincaré Probab. Stat.* **56** (4) (2020) 2428–2444. MR4164843 https://doi.org/10.1214/19-AIHP1044

[16] P. Delgosha and V. Anantharam. A notion of entropy for stochastic processes on marked rooted graphs, 2019. Available at arXiv:1908.00964.

[17] J. Ding, A. Sly and N. Sun. Maximum independent sets on random regular graphs. *Acta Math.* **217** (2) (2016) 263–340. MR3689942 https://doi.org/10.1007/s11511-017-0145-9

[18] W. Duckworth and N. C. Wormald. On the independent domination number of random regular graphs. *Combin. Probab. Comput.* **15** (4) (2006) 513–522. MR2238043 https://doi.org/10.1017/S0963548305007431

[19] P. Erdős and T. Gallai. Graphs with prescribed degrees of vertices (Hungarian). *Mat. Lapok* **11** (1960) 264–274. MR0123538

[20] S. Franz and M. Leone. Replica bounds for optimization problems and diluted spin systems. *J. Stat. Phys.* **111** (3–4) (2003) 535–564. MR1972121 https://doi.org/10.1023/A:1022885828956

[21] J. Friedman. A proof of Alon's second eigenvalue conjecture and related problems. *Mem. Amer. Math. Soc.* **195** (910) (2008) viii+100. MR2437174 https://doi.org/10.1090/memo/0910

[22] D. Gamarnik. Right-convergence of sparse random graphs. *Probab. Theory Related Fields* **160** (1–2) (2014) 253–278. MR3256814 https://doi.org/10.1007/s00440-013-0528-6

[23] F. Guerra. Broken replica symmetry bounds in the mean field spin glass model. *Comm. Math. Phys.* **233** (1) (2003) 1–12. MR1957729 https://doi.org/10.1007/s00220-002-0773-5

[24] H. Hatami, L. Lovász and B. Szegedy. Limits of locally-globally convergent graph sequences. *Geom. Funct. Anal.* **24** (1) (2014) 269–296. MR3177383 https://doi.org/10.1007/s00039-014-0258-7

[25] M. Lelarge and M. Oulamara. Replica bounds by combinatorial interpolation for diluted spin systems. *J. Stat. Phys.* **173** (3–4) (2018) 917–940. MR3876913 https://doi.org/10.1007/s10955-018-1964-6

[26] L. Lovász. *Large Networks and Graph Limits*. *American Mathematical Society Colloquium Publications.* **60**. American Mathematical Society, Providence, RI, 2012. MR3012035 https://doi.org/10.1090/coll/060

[27] M. Mezard and A. Montanari. *Information, Physics, and Computation*. Oxford University Press, Inc., USA, 2009. MR2518205 https://doi.org/10.1093/acprof:oso/9780198570837.001.0001

[28] D. Puder. Expansion of random graphs: New proofs, new results. *Invent. Math.* **201** (3) (2015) 845–908. MR3385636 https://doi.org/10.1007/s00222-014-0560-x

[29] J. Salez. The interpolation method for random graphs with prescribed degrees. *Combin. Probab. Comput.* **25** (3) (2016) 436–447. MR3482664 https://doi.org/10.1017/S0963548315000139

[30] R. van der Hofstad Random Graphs and Complex Networks Cambridge Series in Statistical and Probabilistic Mathematics 2. Cambridge University Press. To appear. MR3617364 https://doi.org/10.1017/9781316779422

[31] N. C. Wormald. Differential equations for random processes and random graphs. *Ann. Appl. Probab.* **5** (4) (1995) 1217–1235. MR1384372

[32] N. C. Wormald. Models of random regular graphs. In *Surveys in Combinatorics, 1999 (Canterbury)* 239–298. *London Math. Soc. Lecture Note Ser.* **267**. Cambridge Univ. Press, Cambridge, 1999. MR1725006

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1981–1997
https://doi.org/10.1214/21-AIHP1234
This research was funded, in whole or in part, by [IPM, Grant 98490118]. A CC BY 4.0 license is applied to this article arising from this submission, in accordance with the grant's open access conditions

Counter examples to invariant circle packing

Ali Khezeli[1,2,a,b]

[1]*Department of Applied Mathematics, Faculty of Mathematical Sciences, Tarbiat Modares University, P.O. Box 4115-134, Tehran, Iran.* [a]*khezeli@modares.ac.ir*
[2]*School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box: 19395-5746, Tehran, Iran.* [b]*alikhezeli@ipm.ir*

Abstract. In this work, a one-ended unimodular random planar triangulation is constructed that has no invariant circle packing; i.e., a circle packing such that its distribution is invariant under translations. This gives a negative answer to a problem posed by I. Benjamini and A. Timar (2019). A natural weaker problem is the existence, for unimodular graphs, of point-stationary circle packings, which are random circle packings that satisfy a certain mass transport principle. It is shown that this problem is related to the large scale properties of the circle packings and the answer is again negative. Two examples are provided with two different approaches: Using indistinguishability and approximation by finite graphs.

Résumé. Dans ce travail, on construit une triangulation planaire aléatoire unimodulaire avec une extrémité qui n'a pas d'empilement de cercle invariant par translation. Ceci répond de façon négative à un problème posé par I. Benjamini et A. Timar (2019). Un autre problème naturel et moins exigeant est l'existence, pour les graphes unimodulaires, d'empilements de cercles ponctuellement-stationnaire, c'est à dire, d'empilements aléatoires de cercles vérifiant un certain principe de transport de masse. Il est démontré que ce problème est lié aux propriétés de grande échelle des empilements de cercles et que la réponse est encore négative. On donne deux exemples fondés sur deux approches différentes, l'un sur l'indistinguabilité, et l'autre sur l'approximation par des graphes finis.

MSC2020 subject classifications: Primary 60D05; secondary 60K99; 55C26

Keywords: Circle packing; Unimodular random graphs; Stationary point process; Point-stationary point process

References

[1] M. Aizenman and S. Warzel. The canopy graph and level statistics for random operators on trees. *Math. Phys. Anal. Geom.* **9** (4) (2006) 291–333. MR2329431 https://doi.org/10.1007/s11040-007-9018-3
[2] D. Aldous. Asymptotic fringe distributions for general families of random trees. *Ann. Appl. Probab.* **1** (2) (1991) 228–266. MR1102319
[3] D. Aldous. Triangulating the circle, at random. *Amer. Math. Monthly* **101** (3) (1994) 223–233. MR1264002 https://doi.org/10.2307/2975599
[4] D. Aldous and R. Lyons. Processes on unimodular random networks. *Electron. J. Probab.* **12** (2007) 1454–1508. MR2354165 https://doi.org/10.1214/EJP.v12-463
[5] O. Angel, T. Hutchcroft, A. Nachmias and G. Ray. Unimodular hyperbolic triangulations: Circle packing and random walk. *Invent. Math.* **206** (1) (2016) 229–268. MR3556528 https://doi.org/10.1007/s00222-016-0653-9
[6] O. Angel, T. Hutchcroft, A. Nachmias and G. Ray. Hyperbolic and parabolic unimodular random maps. *Geom. Funct. Anal.* **28** (4) (2018) 879–942. MR3820434 https://doi.org/10.1007/s00039-018-0446-y
[7] O. Angel and O. Schramm. Uniform infinite planar triangulations. *Comm. Math. Phys.* **241** (2–3) (2003) 191–213. MR2013797 https://doi.org/10.1007/978-1-4419-9675-6_16
[8] F. Baccelli, M.-O. Haji-Mirsadeghi and A. Khezeli. Eternal family trees and dynamics on unimodular random graphs. In *Unimodularity in Randomly Generated Graphs* 85–127. *Contemp. Math.* **719**. Amer. Math. Soc., Providence, RI, 2018. MR3880014 https://doi.org/10.1090/conm/719/14471
[9] F. Baccelli, M.-O. Haji-Mirsadeghi and A. Khezeli. Unimodular Hausdorff and Minkowski Dimensions. Preprint. MR4350981 https://doi.org/10.1214/21-ejp692
[10] F. Baccelli, M.-O. Haji-Mirsadeghi and A. Khezeli. On the Dimension of Unimodular Discrete Spaces, Part II: Relations with Growth Rate. Preprint.
[11] I. Benjamini and A. Timar. Invariant embeddings of unimodular random planar graphs. *Electron. J. Probab.* **26** (2021) 1–18. MR4283370 https://doi.org/10.1214/21-ejp665
[12] N. Curien and I. Kortchemski. Random non-crossing plane configurations: A conditioned Galton-Watson tree approach. *Random Structures Algorithms* **45** (2) (2014) 236–260. MR3245291 https://doi.org/10.1002/rsa.20481
[13] O. Gurel-Gurevich and A. Nachmias. Recurrence of planar graph limits. *Ann. of Math. (2)* **177** (2) (2013) 761–781. MR3010812 https://doi.org/10.4007/annals.2013.177.2.10

[14] Z. He and O. Schramm. Fixed points, Koebe uniformization and circle packings. *Ann. of Math. (2)* **137** (2) (1993) 369–406. MR1207210 https://doi.org/10.2307/2946541
[15] Z. He and O. Schramm. Hyperbolic and parabolic packings. *Discrete Comput. Geom.* **14** (2) (1995) 123–149. MR1331923 https://doi.org/10.1007/BF02570699
[16] H. Hennion. Limit theorems for products of positive random matrices. *Ann. Probab.* **25** (4) (1997) 1545–1587. MR1487428 https://doi.org/10.1214/aop/1023481103
[17] A. Khezeli. Shift-coupling of random rooted graphs and networks. In *Unimodularity in Randomly Generated Graphs* 175–211. *Contemp. Math.* **719**. Amer. Math. Soc., Providence, RI, 2018. MRMR3880017 https://doi.org/10.1090/conm/719/14474
[18] P. Koebe. *Kontaktprobleme der konformen Abbildung*. Hirzel, Leipzig, 1936.
[19] G. Last and H. Thorisson. Invariant transports of stationary random measures and mass-stationarity. *Ann. Probab.* **37** (2) (2009) 790–813. MR2510024 https://doi.org/10.1214/08-AOP420
[20] W. P. Thurston. *The Geometry and Topology of Three-Manifolds*. Princeton University, Princeton, NJ, 1979.
[21] A. Timar. Unimodular random planar graphs are sofic. ArXiv preprint, 2019. Available at arXiv:1910.01307.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 1998–2028
https://doi.org/10.1214/21-AIHP1237
© Association des Publications de l'Institut Henri Poincaré, 2022

Rate of estimation for the stationary distribution of stochastic damping Hamiltonian systems with continuous observations

Sylvain Delattre[1,a], Arnaud Gloter[2,b] and Nakahiro Yoshida[3,c]

[1]Laboratoire de Probabilités Statistique et Modélisation UMR 7599, Université Denis Diderot, 77251 Paris Cedex 05, France. [a]delattre@math.jussieu.fr
[2]Laboratoire de Mathématiques et Modélisation d'Evry, Université Paris-Saclay, CNRS, Univ Evry, 91037, Evry, France. [b]arnaud.gloter@univ-evry.fr
[3]Graduate School of Mathematical Sciences, University of Tokyo: 3-8-1 Komaba, Meguro-ku, Tokyo 153-8914, Japan. [c]nakahiro@ms.u-tokyo.ac.jp

Abstract. We study the problem of the non-parametric estimation for the density π of the stationary distribution of a stochastic two-dimensional damping Hamiltonian system $(Z_t)_{t\in[0,T]} = (X_t, Y_t)_{t\in[0,T]}$. From the continuous observation of the sampling path on $[0, T]$, we study the rate of estimation for $\pi(x_\star, y_\star)$ as $T \to \infty$ where $(x_\star, y_\star) \in \mathbb{R}^2$. We show that kernel based estimators can achieve the rate T^{v} for some explicit exponent $v \in (0, 1/2)$. One finding is that the rate of estimation depends on the smoothness of π and is completely different with the rate appearing in the standard i.i.d. setting or in the case of two-dimensional non–degenerate diffusion processes. Especially, this rate depends also on $y_\star$. Moreover, we obtain a minimax lower bound on the L^2-risk for pointwise estimation, with the same rate T^{v}, up to $\ln(T)$ terms.

Résumé. Nous étudions le problème de l'estimation non paramétrique pour la densité π de la distribution stationnaire d'une équation différentielle stochastique bi-dimensionnelle correspondant à un système Hamiltonien amorti $(Z_t)_{t\in[0,T]} = (X_t, Y_t)_{t\in[0,T]}$. Depuis l'observation continue d'une trajectoire sur $[0, T]$, nous étudions la vitesse d'estimation pour $\pi(x_\star, y_\star)$ quand $T \to \infty$ et $(x_\star, y_\star) \in \mathbb{R}^2$. Nous montrons que des estimateurs à noyau atteignent la vitesse T^{v} pour un exposant explicite $v \in (0, 1/2)$. Nous trouvons que la vitesse d'estimation dépend de la régularité de π et est complètement différente des vitesses apparaissant dans le cadre classique des variables i.i.d. ou dans celui d'une diffusion bi-dimensionnelle non dégénérée. En particulier, cette vitesse dépend aussi de $y_\star$. De plus, nous obtenons une minoration du risque minimax L^2 pour l'estimation ponctuelle, avec la même vitesse T^{v}, à des facteurs $\ln(T)$ près.

MSC2020 subject classifications: Primary 62G07; 62G20; secondary 60J60

Keywords: Hypo-elliptic diffusion; Non-parametric estimation; Stationary measure; Minimax lower bound

References

[1] C. Amorino. Rate of estimation for the stationary distribution of jump-processes over anisotropic Holder classes. *Electron. J. Stat.* **15** (2021) 5067–5116. MR4347384 https://doi.org/10.1214/21-ejs1913
[2] C. Amorino and A. Gloter. Invariant density adaptive estimation for ergodic jump–diffusion processes over anisotropic classes. *J. Statist. Plann. Inference* **213** (2021) 106–129. MR4184676 https://doi.org/10.1016/j.jspi.2020.11.006
[3] P. Cattiaux, J. León and C. Prieur. Estimation for stochastic damping Hamiltonian systems under partial observation I. Invariant density. *Stochastic Process. Appl.* **124** (3) (2014) 1236–1260. MR3148012 https://doi.org/10.1016/j.spa.2013.10.008
[4] F. Comte and C. Lacour. Anisotropic adaptive kernel deconvolution. *Ann. Inst. Henri Poincaré Probab. Stat.* **49** (2) (2013) 569–609. MR3088382 https://doi.org/10.1214/11-aihp470
[5] F. Comte and F. Merlevède. Adaptive estimation of the stationary density of discrete and continuous time mixing processes. *ESAIM Probab. Stat.* **6** (2002) 211–238. MR1943148 https://doi.org/10.1051/ps:2002012
[6] F. Comte and F. Merlevède. Super optimal rates for nonparametric density estimation via projection estimators. *Stochastic Process. Appl.* **115** (5) (2005) 797–826. MR2132599 https://doi.org/10.1016/j.spa.2004.12.004
[7] F. Comte, C. Prieur and A. Samson. Adaptive estimation for stochastic damping Hamiltonian systems under partial observation. *Stochastic Process. Appl.* **127** (11) (2017) 3689–3718. MR3707242 https://doi.org/10.1016/j.spa.2017.03.011
[8] A. Dalalyan and M. Reiß. Asymptotic statistical equivalence for ergodic diffusions: The multidimensional case. *Probab. Theory Related Fields* **137** (1) (2007) 25–47. MR2278451 https://doi.org/10.1007/s00440-006-0502-7
[9] S. Ditlevsen and A. Samson. Hypoelliptic diffusions: Discretization, filtering and inference from complete and partial observations. *J. Roy. Statist. Soc. Ser. B* **81** (2019) 361–384. MR3928146

[10] S. Ditlevsen and M. Sørensen. Inference for observations of integrated diffusion processes. *Scand. J. Stat.* **31** (2004) 417–429. MR2087834 https://doi.org/10.1111/j.1467-9469.2004.02_023.x

[11] A. Goldenshluger and O. Lepski. Bandwidth selection in kernel density estimation: Oracle inequalities and adaptive minimax optimality. *Ann. Statist.* **39** (3) (2011) 1608–1632. MR2850214 https://doi.org/10.1214/11-AOS883

[12] V. Konakov, S. Menozzi and S. Molchanov. Explicit parametrix and local limit theorems for some degenerate diffusion processes. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** (4) (2010) 908–923. MR2744877 https://doi.org/10.1214/09-AIHP207

[13] Y. Kutoyants. *Statistical Inference for Ergodic Diffusion Processes*. *Springer Series in Statistics*, xiv+481 pp. Springer, London, 2004. MR2144185 https://doi.org/10.1007/978-1-4471-3866-2

[14] J. R. Leòn and A. Samson. Hypoelliptic stochastic FitzHugh-Nagumo neuronal model: Mixing, up-crossing and estimation of the spike rate. *Ann. Appl. Probab.* **28** (2018) 2243–2274. MR3843828 https://doi.org/10.1214/17-AAP1355

[15] H.-T. Nguyen. Density estimation in a continuous-time stationary Markov process. *Ann. Statist.* **7** (2) (1979) 341–348. MR0520244

[16] Y. Nishiyama. Estimation for the invariant law of an ergodic diffusion process based on high-frequency data. *J. Nonparametr. Stat.* **23** (4) (2011) 909–915. MR2854246 https://doi.org/10.1080/10485252.2011.591397

[17] E. Schmisser. Non parametric estimation of the derivatives of the stationary density for stationary processes. *ESAIM Probab. Stat.* **17** (2013) 33–69. MR3002995 https://doi.org/10.1051/ps/2011102

[18] C. Strauch. Adaptive invariant density estimation for ergodic diffusions over anisotropic classes. *Ann. Statist.* **46** (6B) (2018) 3451–3480. MR3852658 https://doi.org/10.1214/17-AOS1664

[19] D. Talay. Stochastic Hamiltonian systems: Exponential convergence to the invariant measure, and discretization by the implicit Euler scheme. *Markov Process. Related Fields* **8** (2002) 1–36. MR1924934

[20] A. Tsybakov. *Introduction to Nonparametric Estimation*. *Springer Series in Statistics*, xii+214 pp. Springer, New York, 2009. MR2724359 https://doi.org/10.1007/b13794

[21] L. Wu. Large and moderate deviations and exponential convergence for stochastic damping Hamiltonian systems. *Stochastic Process. Appl.* **91** (2001) 205–238. MR1807683 https://doi.org/10.1016/S0304-4149(00)00061-2

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2029–2064
https://doi.org/10.1214/21-AIHP1235
© Association des Publications de l'Institut Henri Poincaré, 2022

Adaptive invariant density estimation for continuous-time mixing Markov processes under sup-norm risk

Niklas Dexheimer[1,a], Claudia Strauch[1,b] and Lukas Trottner[2,c]

[1]*Department of Mathematics, Aarhus University, Aarhus, Denmark.* [a]*dexheimer@math.au.dk,* [b]*strauch@math.au.dk*
[2]*Institute of Mathematics, University of Mannheim, Mannheim, Germany.* [c]*trottner@math.au.dk*

Abstract. Up to now, the nonparametric analysis of multidimensional continuous-time Markov processes has focussed strongly on specific model choices, mostly related to symmetry of the semigroup. While this approach allows to study the performance of estimators for the characteristics of the process in the minimax sense, it restricts the applicability of results to a rather constrained set of stochastic processes and in particular hardly allows incorporating jump structures. As a consequence, for many models of applied and theoretical interest, no statement can be made about the robustness of typical statistical procedures beyond the beautiful, but limited framework available in the literature. To contribute to the statistical understanding in more general situations, we demonstrate how combining β-mixing assumptions on the process and heat kernel bounds on the transition density representing controls on the long- and short-time transitional behaviour, allow to obtain sup-norm and L^2 kernel invariant density estimation rates that match the well-understood case of reversible multidimensional diffusion processes and are faster than in a sampled discrete data scenario. Moreover, we demonstrate how, up to log-terms, optimal sup-norm *adaptive* invariant density estimation can be achieved within our framework, based on tight uniform moment bounds and deviation inequalities for empirical processes associated to additive functionals of Markov processes. The underlying assumptions are verifiable with classical tools from stability theory of continuous-time Markov processes and PDE techniques, which opens the door to evaluate statistical performance for a vast amount of popular Markov models. We highlight this point by showing how multidimensional jump SDEs with Lévy-driven jump part under different coefficient assumptions can be seamlessly integrated into our framework, thus establishing novel adaptive sup-norm estimation rates for this class of processes.

Résumé. La statistique non-paramétrique des processus de Markov reste le plus souvent confinée à des modèles spécifiques, essentiellement en lien avec des propriétés de symétrie du semigroupe de Markov associé. Bien que cette approche permette une étude minimax des estimateurs des paramètres caractérisant la dynamique du processus, elle restreint les résultats à un ensemble relativement contraint de modèles envisageables, ne permettant par exemple l'incorporation de sauts que difficilement. En conséquence, pour de nombreux modèles intéressants tant en théorie qu'en pratique, peu de propriétés sur la robustesse de procédures habituelles sont disponibles au delà d'un cadre élégant mais limité. Afin de contribuer à une meilleure compréhension dans des situations plus générales, nous démontrons comment la combinaison d'hypothèses de β-mélange avec des bornes de type noyau de la chaleur sur les probabilités de transitions qui contrôlent le comportement temps long et temps court du processus permettent d'obtenir des vitesses d'estimation en norme uniforme et dans L^2 de la densité invariante compatibles avec le cas bien compris des diffusions multidimensionelles réversibles, et qui sont plus rapides que pour des observations discrètes. De plus, nous montrons comment, à des termes logarithmiques près, l'estimation adaptative optimale en norme uniforme est atteignable dans notre cadre, en nous reposant sur des bornes fines de déviation du processus empirique associé à des fonctionnelles additives. Les hypothèses sous-jacentes sont facilement vérifiables avec des outils classiques de stabilité des processus de Markov ainsi qu'avec des techniques d'EDP. Ceci permet l'étude des performance statistiques d'estimateurs pour une classe plus large de modèles markoviens populaires. Nous montrons en particulier comment des EDS avec sauts conduites par des processus de Lévy sont incorporées dans notre cadre, ce qui nous permet d'établir de nouvelles vitesses d'estimation en perte uniforme pour cette classe de processus.

MSC2020 subject classifications: Primary 62M05; secondary 62G05; 62G20; 60G10; 60J25

Keywords: Nonparametric statistics; sup-norm risk; Adaptive estimation; Statistics for stochastic processes; Jump SDEs

References

[1] R. Adamczak. A tail inequality for suprema of unbounded empirical processes with applications to Markov chains. *Electron. J. Probab.* **13** (2008) 1000–1034. MR2424985 https://doi.org/10.1214/EJP.v13-521

[2] R. Adamczak and W. Bednorz. Exponential concentration inequalities for additive functionals of Markov chains. *ESAIM Probab. Stat.* **19** (2015) 440–481. MR3423302 https://doi.org/10.1051/ps/2014032
[3] L. H. R. Alvarez. Stochastic forest stand value and optimal timber harvesting. *SIAM J. Control Optim.* **42** (6) (2004) 1972–1993. MR2080924 https://doi.org/10.1137/S0363012901393456
[4] C. Amorino and A. Gloter. Invariant density adaptive estimation for ergodic jump-diffusion processes over anisotropic classes. *J. Statist. Plann. Inference* **213** (2021) 106–129. MR4184676 https://doi.org/10.1016/j.jspi.2020.11.006
[5] C. Amorino and A. Gloter Minimax rate of estimation for invariant densities associated to continuous stochastic differential equations over anisotropic Holder classes. Preprint, 2021. Available at arXiv:2110.02774.
[6] C. Amorino and E. Nualart Optimal convergence rates for the invariant density estimation of jump-diffusion processes. Preprint, 2021. Available at arXiv:2101.08548.
[7] D. Applebaum. *Lévy Processes and Stochastic Calculus*, 2nd edition. *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2009. MR3087758 https://doi.org/10.1017/CBO9780511809781
[8] D. Bakry, P. Cattiaux and A. Guillin. Rate of convergence for ergodic continuous Markov processes: Lyapunov versus Poincaré. *J. Funct. Anal.* **254** (3) (2008) 727–759. MR2381160 https://doi.org/10.1016/j.jfa.2007.11.002
[9] Y. Baraud. A Bernstein-type inequality for suprema of random processes with applications to model selection in non-Gaussian regression. *Bernoulli* **16** (4) (2010) 1064–1085. MR2759169 https://doi.org/10.3150/09-BEJ245
[10] P. Bertail and G. Ciołek. New Bernstein and Hoeffding type inequalities for regenerative Markov chains. *ALEA Lat. Am. J. Probab. Math. Stat.* **16** (1) (2019) 259–277. MR3914351 https://doi.org/10.30757/alea.v16-09
[11] J. Bierkens, G. O. Roberts and P.-A. Zitt. Ergodicity of the zigzag process. *Ann. Appl. Probab.* **29** (4) (2019) 2266–2301. MR3983339 https://doi.org/10.1214/18-AAP1453
[12] R. M. Blumenthal and R. K. Getoor. *Markov Processes and Potential Theory*. *Pure and Applied Mathematics* **29**. Academic Press, New York–London, 1968. MR0264757
[13] D. Bosq. Parametric rates of nonparametric estimators and predictors for continuous time processes. *Ann. Statist.* **25** (3) (1997) 982–1000. MR1447737 https://doi.org/10.1214/aos/1069362734
[14] D. Bosq. *Nonparametric Statistics for Stochastic Processes: Estimation and Prediction*, 2nd edition. *Lecture Notes in Statistics* **110**, Springer-Verlag, New York, 1998. MR1640691 https://doi.org/10.1007/978-1-4612-1718-3
[15] S. Boucheron, G. Lugosi and P. Massart. *Concentration Inequalities: A Nonasymptotic Theory of Independence*. Oxford University Press, Oxford, 2013. MR3185193 https://doi.org/10.1093/acprof:oso/9780199535255.001.0001
[16] R. C. Bradley. Basic properties of strong mixing conditions. A survey and some open questions. *Probab. Surv.* **2** (2005) 107–144. Update of, and a supplement to, the 1986 original. MR2178042 https://doi.org/10.1214/154957805100000104
[17] N. Brosse, A. Durmus and E. Moulines. The promises and pitfalls of stochastic gradient Langevin dynamics. In *Advances in Neural Information Processing Systems* **31** 8268–8278. S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi and R. Garnett (Eds). Curran Associates, Inc., Red Hook, 2018.
[18] J. V. Castellana and M. R. Leadbetter. On smoothed probability density estimation for stationary processes. *Stochastic Process. Appl.* **21** (2) (1986) 179–193. MR833950 https://doi.org/10.1016/0304-4149(86)90095-5
[19] P. Cattiaux and A. Guillin. Deviation bounds for additive functionals of Markov processes. *ESAIM Probab. Stat.* **12** (2008) 12–29. MR2367991 https://doi.org/10.1051/ps:2007032
[20] M.-F. Chen. *Eigenvalues, Inequalities, and Ergodic Theory*. *Probability and Its Applications (New York)*. Springer-Verlag London, Ltd., London, 2005. MR2105651
[21] Z.-Q. Chen, E. Hu, L. Xie and X. Zhang. Heat kernels for non-symmetric diffusion operators with jumps. *J. Differential Equations* **263** (10) (2017) 6576–6634. https://doi.org/10.1016/j.jde.2017.07.023
[22] S. Christensen, C. Strauch and L. Trottner Learning to reflect: A unifying approach for data-driven stochastic control strategies. Preprint, 2021. Available at arXiv:2104.11496.
[23] S. J. M. Clémencon. Moment and probability inequalities for sums of bounded additive functionals of regular Markov chains via the Nummelin splitting technique. *Statist. Probab. Lett.* **55** (3) (2001) 227–238. MR1867526 https://doi.org/10.1016/S0167-7152(00)00236-4
[24] F. Comte and F. Merlevède. Adaptive estimation of the stationary density of discrete and continuous time mixing processes. *ESAIM Probab. Stat.* **6** (2002) 211–238. New directions in time series analysis (Luminy, 2001). MR1943148 https://doi.org/10.1051/ps:2002012
[25] G. Da Prato, K. D. Elworthy and J. Zabczyk. Strong Feller property for stochastic semilinear equations. *Stoch. Anal. Appl.* **13** (1) (1995) 35–45. MR1313205 https://doi.org/10.1080/07362999508809381
[26] G. Da Prato and J. Zabczyk. Smoothing properties of transition semigroups in Hilbert spaces. *Stoch. Stoch. Rep.* **35** (2) (1991) 63–77. MR1110991 https://doi.org/10.1080/17442509108833690
[27] A. Dalalyan. Sharp adaptive estimation of the drift function for ergodic diffusions. *Ann. Statist.* **33** (6) (2005) 2507–2528. MR2253093 https://doi.org/10.1214/009053605000000615
[28] A. Dalalyan and M. Reiß. Asymptotic statistical equivalence for ergodic diffusions: The multidimensional case. *Probab. Theory Related Fields* **137** (1–2) (2007) 25–47. MR2278451 https://doi.org/10.1007/s00440-006-0502-7
[29] J. A. Davydov. Mixing conditions for Markov chains. *Teor. Veroyatn. Primen.* **18** (1973) 321–338. MR0321183
[30] J. Dedecker, P. Doukhan, G. Lang, J. R. León R., S. Louhichi and C. Prieur. *Weak Dependence: With Examples and Applications*. *Lecture Notes in Statistics* **190**. Springer, New York, 2007. MR2338725
[31] J. Dedecker and S. Gouëzel. Subgaussian concentration inequalities for geometrically ergodic Markov chains. *Electron. Commun. Probab.* **20** (2015) no. 64, 12. MR3407208 https://doi.org/10.1214/ECP.v20-3966
[32] N. Dexheimer and C. Strauch Estimating the characteristics of stochastic damping Hamiltonian systems from continuous observations. Preprint, 2021. Available at arXiv:2109.13190.
[33] S. Dirksen. Tail bounds via generic chaining. *Electron. J. Probab.* **20** (2015) no. 53, 29. MR3354613 https://doi.org/10.1214/EJP.v20-3760
[34] D. Down and S. P. Meyn. Piecewise linear test functions for stability and instability of queueing networks. *Queueing Syst. Theory Appl.* **27** (3–4) (1997) 205–226 (1998). MR1625069 https://doi.org/10.1023/A:1019166115653
[35] D. Down, S. P. Meyn and R. L. Tweedie. Exponential and uniform ergodicity of Markov processes. *Ann. Probab.* **23** (4) (1995) 1671–1691. MR1379163

[36] N. H. Du, N. T. Dieu and N. N. Nhu. Conditions for permanence and ergodicity of certain SIR epidemic models. *Acta Appl. Math.* **160** (2019) 81–99. MR3934659 https://doi.org/10.1007/s10440-018-0196-8

[37] A. Durmus and É. Moulines. High-dimensional Bayesian inference via the unadjusted Langevin algorithm. *Bernoulli* **25** (4A) (2019) 2854–2882. MR4003567 https://doi.org/10.3150/18-BEJ1073

[38] J.-P. Eckmann and M. Hairer. Uniqueness of the invariant measure for a stochastic PDE driven by degenerate noise. *Comm. Math. Phys.* **219** (3) (2001) 523–565. MR1838749 https://doi.org/10.1007/s002200100424

[39] C. Fuchs. *Inference for Diffusion Processes – With Applications in Life Sciences*. Springer, Heidelberg, 2013. MR3015023 https://doi.org/10.1007/978-3-642-25969-2

[40] F. Gao, A. Guillin and L. Wu. Bernstein type's concentration inequalities for symmetric Markov processes. *Teor. Veroyatn. Primen.* **58** (3) (2013) 521–549.

[41] E. Giné and R. Nickl. An exponential inequality for the distribution function of the kernel density estimator, with applications to adaptive estimation. *Probab. Theory Related Fields* **143** (3–4) (2009) 569–596. MR2475673 https://doi.org/10.1007/s00440-008-0137-y

[42] M. Giordano and K. Ray Nonparametric Bayesian inference for reversible multi-dimensional diffusions, 2020.

[43] U. Grenander and M. I. Miller. Representations of knowledge in complex systems. *J. Roy. Statist. Soc. Ser. B* **56** (4) (1994) 549–603. With discussion and a reply by the authors. MR1293234

[44] M. Hairer and J. Mattingly. The strong Feller property for singular stochastic PDEs. *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (3) (2018) 1314–1340. MR3825883 https://doi.org/10.1214/17-AIHP840

[45] J. Hawkes. Potential theory of Lévy processes. *Proc. Lond. Math. Soc. (3)* **38** (2) (1979) 335–352. MR531166 https://doi.org/10.1112/plms/s3-38.2.335

[46] K. Ichihara and H. Kunita. A classification of the second order degenerate elliptic operators and its probabilistic characterization. *Z. Wahrsch. Verw. Gebiete* **30** (1974) 235–254. MR381007 https://doi.org/10.1007/BF00533476

[47] G. Jongbloed, F. H. van der Meulen and A. W. van der Vaart. Nonparametric inference for Lévy-driven Ornstein–Uhlenbeck processes. *Bernoulli* **11** (5) (2005) 759–791. MR2172840 https://doi.org/10.3150/bj/1130077593

[48] P. Kevei. Ergodic properties of generalized Ornstein–Uhlenbeck processes. *Stochastic Process. Appl.* **128** (1) (2018) 156–181. MR3729534 https://doi.org/10.1016/j.spa.2017.04.010

[49] Y. A. Kutoyants. *Statistical Inference for Ergodic Diffusion Processes*. *Springer Series in Statistics*. Springer, New York, 2004. MR2144185 https://doi.org/10.1007/978-1-4471-3866-2

[50] F. Leblanc. Density estimation for a class of continuous time processes. *Math. Methods Statist.* **6** (2) (1997) 171–199. MR1466626

[51] M. Lemanczyk. General Bernstein-like inequality for additive functionals of Markov chains. *J. Theor. Probab.* (2020). MR4289890 https://doi.org/10.1007/s10959-020-01006-z

[52] O. Lepski. Multivariate density estimation under sup-norm loss: Oracle approach, adaptation and independence structure. *Ann. Statist.* **41** (2) (2013) 1005–1034. MR3099129 https://doi.org/10.1214/13-AOS1109

[53] P. Lezaud. Chernoff and Berry–Esséen inequalities for Markov processes. *ESAIM Probab. Stat.* **5** (2001) 183–201. MR1875670 https://doi.org/10.1051/ps:2001108

[54] B. Maslowski. Strong Feller property for semilinear stochastic evolution equations and applications. In *Stochastic Systems and Optimization (Warsaw, 1988)* 210–224. *Lect. Notes Control Inf. Sci.* **136**. Springer, Berlin, 1989. MR1180781 https://doi.org/10.1007/BFb0002683

[55] H. Masuda. On multidimensional Ornstein–Uhlenbeck processes driven by a general Lévy process. *Bernoulli* **10** (1) (2004) 97–120. MR2044595 https://doi.org/10.3150/bj/1077544605

[56] H. Masuda. Ergodicity and exponential β-mixing bounds for multidimensional diffusions with jumps. *Stochastic Process. Appl.* **117** (1) (2007) 35–56. MR4289890 https://doi.org/10.1016/j.spa.2006.04.010

[57] H. Masuda. Erratum to: "Ergodicity and exponential β -mixing bound for multidimensional diffusions with jumps" [Stochastic Process. Appl. 117 (2007) 35–56]. *Stochastic Process. Appl.* **119** (2) (2009) 676–678. MR2552307 https://doi.org/10.1016/j.spa.2009.09.004

[58] F. Merlevède, M. Peligrad and E. Rio. Bernstein inequality and moderate deviations under strong mixing conditions. In *High Dimensional Probability V: The Luminy Volume* 273–292. *Inst. Math. Stat. (IMS) Collect.* **5**. Inst. Math. Statist., Beachwood, OH, 2009. MR2797953 https://doi.org/10.1214/09-IMSCOLL518

[59] S. Meyn and R. L. Tweedie. *Markov Chains and Stochastic Stability*, 2nd edition. Cambridge University Press, Cambridge, 2009. With a prologue by Peter W. Glynn. MR2509253 https://doi.org/10.1017/CBO9780511626630

[60] S. P. Meyn and R. L. Tweedie. Stability of Markovian processes. II. Continuous-time processes and sampled chains. *Adv. in Appl. Probab.* **25** (3) (1993) 487–517. MR1234294 https://doi.org/10.2307/1427521

[61] S. P. Meyn and R. L. Tweedie. Generalized resolvents and Harris recurrence of Markov processes. In *Doeblin and Modern Probability (Blaubeuren, 1991)* 227–250. *Contemp. Math.* **149**. Amer. Math. Soc., Providence, RI, 1993. MR1229967 https://doi.org/10.1090/conm/149/01273

[62] S. P. Meyn and R. L. Tweedie. Stability of Markovian processes. III. Foster–Lyapunov criteria for continuous-time processes. *Adv. in Appl. Probab.* **25** (3) (1993) 518–548. MR1234295 https://doi.org/10.2307/1427522

[63] R. Nickl and K. Ray. Nonparametric statistical inference for drift vector fields of multi-dimensional diffusions. *Ann. Statist.* **48** (3) (2020) 1383–1408. MR4124327 https://doi.org/10.1214/19-AOS1851

[64] E. Nummelin. *General Irreducible Markov Chains and Nonnegative Operators*. *Cambridge Tracts in Mathematics* **83**. Cambridge University Press, Cambridge, 1984. MR776608 https://doi.org/10.1017/CBO9780511526237

[65] E. Nummelin and R. L. Tweedie. Geometric ergodicity and R-positivity for general Markov chains. *Ann. Probab.* **6** (3) (1978) 404–420. MR474504 https://doi.org/10.1214/aop/1176995527

[66] D. Paulin. Concentration inequalities for Markov chains by Marton couplings and spectral methods. *Electron. J. Probab.* **20** (2015) no. 79, 32. MR3383563 https://doi.org/10.1214/EJP.v20-4039

[67] J. Paulsen. Sharp conditions for certain ruin in a risk process with stochastic return on investments. *Stochastic Process. Appl.* **75** (1) (1998) 135–148. MR1629034 https://doi.org/10.1016/S0304-4149(98)00012-X

[68] S. Peszat and J. Zabczyk. Strong Feller property and irreducibility for diffusions on Hilbert spaces. *Ann. Probab.* **23** (1) (1995) 157–172. MR1330765 https://doi.org/10.1007/s10959-020-01006-z

[69] E. Rio. *Théorie Asymptotique des Processus Aléatoires Faiblement Dépendants*. *Mathématiques & Applications* **31**. Springer, Berlin, 2000. MR2117923 https://doi.org/10.1007/s10959-020-01006-z

[70] E. Rio. *Asymptotic Theory of Weakly Dependent Random Processes*. *Probability Theory and Stochastic Modelling* **80**. Springer, Berlin, 2017. Translated from the 2000 French edition [MR2117923]. MR3642873 https://doi.org/10.1007/978-3-662-54323-8

[71] P.-M. Samson. Concentration of measure inequalities for Markov chains and Φ-mixing processes. *Ann. Probab.* **28** (1) (2000) 416–461. MR1756011 https://doi.org/10.1214/aop/1019160125

[72] K. Sato and M. Yamazato. Operator-self-decomposable distributions as limit distributions of processes of Ornstein–Uhlenbeck type. *Stochastic Process. Appl.* **17** (1) (1984) 73–100. MR738769 https://doi.org/10.1016/0304-4149(84)90312-0

[73] R. L. Schilling and J. Wang. Strong Feller continuity of Feller processes and semigroups. *Infin. Dimens. Anal. Quantum Probab. Relat. Top.* **15** (2) (2012) 1250010, 28. MR2957137 https://doi.org/10.1142/S0219025712500105

[74] M. Sharpe. *General Theory of Markov Processes*. *Pure and Applied Mathematics* **133**. Academic Press Inc., Boston, MA, 1988. MR958914 https://doi.org/10.1007/s10959-020-01006-z

[75] S. J. Sheu. Some estimates of the transition density of a nondegenerate diffusion Markov process. *Ann. Probab.* **19** (2) (1991) 538–561. MR1106275 https://doi.org/10.1007/s10959-020-01006-z

[76] C. Strauch. Sharp adaptive drift estimation for ergodic diffusions: The multivariate case. *Stochastic Process. Appl.* **125** (7) (2015) 2562–2602.

[77] C. Strauch. Exact adaptive pointwise drift estimation for multidimensional ergodic diffusions. *Probab. Theory Related Fields* **164** (1–2) (2016) 361–400. MR3449393 https://doi.org/10.1007/s00440-014-0614-4

[78] C. Strauch. Adaptive invariant density estimation for ergodic diffusions over anisotropic classes. *Ann. Statist.* **46** (6B) (2018) 3451–3480. MR3852658 https://doi.org/10.1214/17-AOS1664

[79] Y. Tamura and S. Yamada. Reliability analysis based on a jump diffusion model with two Wiener processes for cloud computing with big data. *Entropy* **17** (7) (2015) 4533–4546. MR4289890 https://doi.org/10.3390/e17074533

[80] A. B. Tsybakov. *Introduction to Nonparametric Estimation*. *Springer Series in Statistics*. Springer, New York, 2009. Revised and extended from the 2004 French original, Translated by Vladimir Zaiats. MR2724359 https://doi.org/10.1007/b13794

[81] R. L. Tweedie. Topological conditions enabling use of Harris methods in discrete and continuous time. *Acta Appl. Math.* **34** (1994) 175–188. MR1273853 https://doi.org/10.1007/BF00994264

[82] B. Tzen and M. Raginsky Neural stochastic differential equations: Deep latent Gaussian models in the diffusion limit. Preprint, 2019. Available at arXiv:1905.09883.

[83] A. Y. Veretennikov. On Castellana–Leadbetter's condition for diffusion density estimation. *Stat. Inference Stoch. Process.* **2** (1) (1999) 1–9 (2000). MR1918625 https://doi.org/10.1023/A:1009996608986

[84] G. Viennet. Inequalities for absolutely regular sequences: Application to density estimation. *Probab. Theory Related Fields* **107** (4) (1997) 467–492. MR1440142 https://doi.org/10.1007/s004400050094

[85] V. A. Volkonskiĭ and J. A. Rozanov. Some limit theorems for random functions. II. *Teor. Veroyatn. Primen.* **6** (1961) 202–215. MR0137141 https://doi.org/10.1007/s10959-020-01006-z

[86] F. Xi and G. Yin. The strong Feller property of switching jump-diffusion processes. *Statist. Probab. Lett.* **83** (3) (2013) 761–767. MR3040301 https://doi.org/10.1016/j.spl.2012.11.021

[87] F. Xi and C. Zhu. Jump type stochastic differential equations with non-Lipschitz coefficients: Non-confluence, Feller and strong Feller properties, and exponential ergodicity. *J. Differential Equations* **266** (8) (2019) 4668–4711. MR3912730 https://doi.org/10.1016/j.jde.2018.10.006

[88] C. Zhu and G. Yin. On strong Feller, recurrence, and weak stabilization of regime-switching diffusions. *SIAM J. Control Optim.* **48** (3) (2009) 2003–2031. MR2516197 https://doi.org/10.1137/080712532

[89] J. D. Zund. George David Birkhoff and John von Neumann: A question of priority and the ergodic theorems, 1931–1932. *Historia Math.* **29** (2) (2002) 138–156. MR1896971 https://doi.org/10.1006/hmat.2001.2338

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2065–2093
https://doi.org/10.1214/21-AIHP1219
© Association des Publications de l'Institut Henri Poincaré, 2022

Maximum of branching Brownian motion in a periodic environment

Eyal Lubetzky[1,a], Chris Thornett[1,b] and Ofer Zeitouni[2,1,c]

[1] Courant Institute, New York University, 251 Mercer Street, New York, NY 10012, USA. [a]eyal@courant.nyu.edu, [b]thornett@cims.nyu.edu
[2] Department of Mathematics, Weizmann Institute of Science, POB 26, Rehovot 76100, Israel. [c]ofer.zeitouni@weizmann.ac.il

Abstract. We study the maximum of Branching Brownian motion (BBM) with branching rates that vary in space, via a periodic function of a particle's location. This corresponds to a variant of the F-KPP equation in a periodic medium, extensively studied in the last 15 years, admitting pulsating fronts as solutions. Recent progress on this PDE due to Hamel, Nolen, Roquejoffre and Ryzhik ('16) implies tightness for the centered maximum of BBM in a periodic environment. Here we establish the convergence in distribution of specific subsequences of this centered maximum, and identify the limiting distribution. Consequently, we find the asymptotic shift between the solution to the corresponding F-KPP equation with Heavyside initial data and the pulsating wave, thereby answering a question of Hamel et al. Analogous results are given for the cases where the Brownian motion is replaced by an Ito diffusion with periodic coefficients, as well as for nearest-neighbor branching random walks.

Résumé. Nous étudions le maximum du mouvement Brownian branchant (BBM) avec des taux de branchement qui varient en l'espace, via une fonction périodique. Ceci correspond à une variante de l'équation de F-KPP en milieu périodique, largement étudiée au cours des 15 dernière années, qui admet des fronts pulsés comme solutions. Les progrès récents sur cette EDP de Hamel, Nolen, Roquejoffre and Ryzhik ('16) impliquent la tension du maximum centré du BBM en milieu périodique. Ici, nous établissons la convergence en distribution de sous-suites spécifiques de ce maximum centré, et identifions la loi limite. Par conséquent, nous trouvons le décalage asymptotique entre la solution de l'équation F-KPP correspondante avec les données initiales Heavyside et l'onde pulsatoire, répondant ainsi à une question de Hamel et al. Des résultats analogues sont donnés pour les cas où le mouvement Brownien est remplacé par une diffusion de Ito à coefficients périodiques, ainsi que pour les marches aléatoires branchantes au plus proche voisin.

MSC2020 subject classifications: 60J65; 60J80

Keywords: Branching Brownian motion; F-KPP in periodic medium

References

[1] E. Aïdékon. Convergence in law of the minimum of a branching random walk. *Ann. Probab.* **41** (2013) 1362–1426. MR3098680 https://doi.org/10.1214/12-AOP750
[2] H. Berestycki and F. Hamel. Front propagation in periodic excitable media. *Comm. Pure Appl. Math.* **55** (8) (2002) 949–1032. MR1900178 https://doi.org/10.1002/cpa.3022
[3] H. Berestycki, F. Hamel and N. Nadirashvili. The speed of propagation for KPP type problems. I. Periodic framework. *J. Eur. Math. Soc. (JEMS)* **7** (2) (2005) 173–213. MR2127993 https://doi.org/10.4171/JEMS/26
[4] H. Berestycki, F. Hamel and L. Roques. Équations de réaction-diffusion et modèles d'invasions biologiques dans les milieux périodiques. *C. R. Math. Acad. Sci. Paris* **339** (8) (2004) 549–554. MR2111029 https://doi.org/10.1016/j.crma.2004.07.025
[5] A. Bovier. *Gaussian Processes on Trees: From Spin Glasses to Branching Brownian Motion*. *Cambridge Studies in Advanced Mathematics* **163**. Cambridge University Press, Cambridge, 2017. MR3618123 https://doi.org/10.1017/9781316675779
[6] M. Bramson. Convergence of solutions of the Kolmogorov equation to travelling waves. *Mem. Amer. Math. Soc.* **44** (285) (1983) iv–190. MR0705746 https://doi.org/10.1090/memo/0285
[7] M. Bramson, J. Ding and O. Zeitouni. Convergence in law of the maximum of nonlattice branching random walk. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (4) (2016) 1897–1924. MR3573300 https://doi.org/10.1214/15-AIHP703
[8] M. D. Bramson. Maximal displacement of branching Brownian motion. *Comm. Pure Appl. Math.* **31** (5) (1978) 531–581. MR0494541 https://doi.org/10.1002/cpa.3160310502
[9] A. Dembo and O. Zeitouni. *Large Deviations Techniques and Applications*. *Stochastic Modelling and Applied Probability* **38**. Springer-Verlag, Berlin, 2010. MR2571413 https://doi.org/10.1007/978-3-642-03311-7
[10] R. A. Fisher. The wave of advance of advantageous genes. *Annu. Eugen.* **7** (4) (1937) 355–369.

[11] I. Grama, R. Lauvergnat and E. Le Page. Conditioned local limit theorems for random walks on finite Markov chains. *Probab. Theory Related Fields* **176** (2020) 669–735. MR4055198 https://doi.org/10.1007/s00440-019-00948-8

[12] F. Hamel, J. Nolen, J.-M. Roquejoffre and L. Ryzhik. The logarithmic delay of KPP fronts in a periodic medium. *J. Eur. Math. Soc. (JEMS)* **18** (3) (2016) 465–505. MR3463416 https://doi.org/10.4171/JEMS/595

[13] F. Hamel and L. Roques. Uniqueness and stability properties of monostable pulsating fronts. *J. Eur. Math. Soc. (JEMS)* **13** (2) (2011) 345–390. MR2746770 https://doi.org/10.4171/JEMS/256

[14] S. C. Harris and M. I. Roberts. The many-to-few lemma and multiple spines. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (1) (2017) 226–242. MR3606740 https://doi.org/10.1214/15-AIHP714

[15] J.-P. Kahane and J. Peyrière. Sur certaines martingales de Benoit Mandelbrot. *Adv. Math.* **22** (2) (1976) 131–145. MR0431355 https://doi.org/10.1016/0001-8708(76)90151-1

[16] T. Kato. *Perturbation Theory for Linear Operators*. *Classics in Mathematics*. Springer-Verlag, Berlin, 1995. Reprint of the 1980 edition. MR1335452

[17] A. Kolmogorov, I. Petrovskii and N. Piskunov. A study of the diffusion equation with increase in the amount of substance, and its application to a biological problem. *Bull. Moscow State Univ. Ser. A: Math. Mech.* **1** (6) (1937) 1–25.

[18] S. P. Lalley and T. Sellke. A conditional limit theorem for the frontier of a branching Brownian motion. *Ann. Probab.* **15** (3) (1987) 1052–1061. MR0893913

[19] P. Maillard. Branching Brownian motion with selection. PhD thesis, University of Paris, 2012.

[20] H. P. McKean. Application of Brownian motion to the equation of Kolmogorov–Petrovskii–Piskunov. *Comm. Pure Appl. Math.* **28** (3) (1975) 323–331. MR0400428 https://doi.org/10.1002/cpa.3160280302

[21] J. Peyrière. Turbulence et dimension de Hausdorff. *C. R. Acad. Sci. Paris Sér. A* **278** (1974) 567–569. MR0431354

[22] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Grundlehren der mathematischen Wissenschaften.* Springer-Verlag, Berlin, 1999. MR1725357 https://doi.org/10.1007/978-3-662-06400-9

[23] Z. Shi. *Branching Random Walks*. *Lecture Notes in Mathematics* **2151**. Springer-Verlag, Berlin, 2015. MR3444654 https://doi.org/10.1007/978-3-319-25372-5

[24] O. Zeitouni. Branching random walks and Gaussian fields. In *Probability and Statistical Physics in St. Petersburg* 437–471. *Proc. Sympos. Pure Math.* **91**. Amer. Math. Soc., Providence, RI, 2016. MR3526836

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2094–2113
https://doi.org/10.1214/21-AIHP1212
© Association des Publications de l'Institut Henri Poincaré, 2022

A simple backward construction of branching Brownian motion with large displacement and applications

Julien Berestycki[1,a], Éric Brunet[2,b], Aser Cortines[3,c] and Bastien Mallein[4,d]

[1]University of Oxford, United Kingdom, [a]julien.berestycki@stats.ox.ac.uk
[2]Sorbonne Université, Laboratoire de Physique de l'École Normale Supérieure, PSL Research University; Université Paris-Cité, CNRS, France, [b]Eric.Brunet@lps.ens.fr
[3]Institute für Mathematik, Universität Zürich, Switzerland, [c]aser.cortinespeixoto@math.uzh.ch
[4]Université Sorbonne Paris Nord, LAGA, UMR 7539, F- 93430, Villetaneuse, France and DMA, École normale supérieure, Université PSL, 75005 Paris, France, [d]mallein@math.univ-paris13.fr

Abstract. In this article, we study the extremal processes of branching Brownian motions conditioned on having an unusually large maximum. The limiting point measures form a one-parameter family and are the decoration point measures in the extremal processes of several branching processes, including branching Brownian motions with variable speed and multitype branching Brownian motions. We give a new, alternative representation of these point measures and we show that they form a continuous family. This also yields a simple probabilistic expression for the constant that appears in the large deviation probability of having a large displacement. As an application, we show that Bovier and Hartung's (*ALEA Lat. Am. J. Probab. Math. Stat.* **12** (2015) 261–291) results about variable speed branching Brownian motion also describe the extremal point process of branching Ornstein–Uhlenbeck processes.

Résumé. Le présent article a pour objet l'étude du processus extrémal du mouvement Brownien branchant conditionné à avoir une particule anormalement loin à droite. Ces mesures ponctuelles limites forment une famille à un paramètre et apparaissent dans les processus extrémaux de plusieurs processus de branchement tels que le mouvement Brownien branchant avec vitesse variable ou certains mouvement Brownien branchants multitype. Nous donnons une nouvelle représentation de ces mesures ponctuelles et nous montrons qu'elles forment une famille continue de lois. Nous obtenons ainsi une expression probabiliste simple de la constante qui apparaît dans le principe de grande déviation pour un déplacement anormal de la particule la plus à droite d'un mouvement Brownien branchant. Finalement, nous appliquons ces résultats pour montrer que les travaux de Bovier et Hartung (*ALEA Lat. Am. J. Probab. Math. Stat.* **12** (2015) 261–291) sur le mouvement Brownien branchant avec vitesse variable décrivent également le processus extrémal du processus d'Ornstein–Uhlenbeck branchant.

MSC2020 subject classifications: 60J80; 60J65; 60F10

Keywords: Branching Brownian motion; Large deviations; Extremal process; Branching Ornstein–Uhlenbeck process

References

[1] E. Aïdékon. Convergence in law of the minimum of a branching random walk. *Ann. Probab.* **41** (3A) (2013) 1362–1426. MR3098680 https://doi.org/10.1214/12-AOP750
[2] E. Aïdékon, J. Berestycki, É. Brunet and Z. Shi. Branching Brownian motion seen from its tip. *Probab. Theory Related Fields* **157** (1–2) (2013) 405–451. MR3101852 https://doi.org/10.1007/s00440-012-0461-0
[3] L.-P. Arguin, D. Belius and P. Bourgade. Maximum of the characteristic polynomial of random unitary matrices. *Comm. Math. Phys.* **349** (2) (2017) 703–751. MR3594368 https://doi.org/10.1007/s00220-016-2740-6
[4] L.-P. Arguin, A. Bovier and N. Kistler. The extremal process of branching Brownian motion. *Probab. Theory Related Fields* **157** (3–4) (2013) 535–574. MR3129797 https://doi.org/10.1007/s00440-012-0464-x
[5] M. A. Belloum and B. Mallein. Anomalous spreading of reducible multitype branching Brownian motion. *Electron. J. Probab.* **26** (61) (2021) 39. MR4254803 https://doi.org/10.1214/21-ejp629
[6] J. Berestycki Topics on Branching Brownian motion. Oxford Probability, 2014. Lecture notes. Available at http://www.stats.ox.ac.uk/~berestyc/Articles/EBP18_v2.pdf.
[7] J. D. Biggins. Uniform convergence of martingales in the branching random walk. *Ann. Probab.* **20** (1) (1992) 137–151. Available at https://projecteuclid.org/euclid.aop/1176989921. MR1143415

[8] J. D. Biggins and A. E. Kyprianou. Measure change in multitype branching. *Adv. in Appl. Probab.* **36** (2) (2004) 544–581. MR2058149 https://doi.org/10.1239/aap/1086957585

[9] M. Biskup and O. Louidor. Full extremal process, cluster law and freezing for the two-dimensional discrete Gaussian free field. *Adv. Math.* **330** (2018) 589–687. MR3787554 https://doi.org/10.1016/j.aim.2018.02.018

[10] A. Bovier. *Gaussian Processes on Trees: From Spin Glasses to Branching Brownian Motion*. *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 2016. MR3618123 https://doi.org/10.1017/9781316675779

[11] A. Bovier and L. Hartung. The extremal process of two-speed branching Brownian motion. *Electron. J. Probab.* **19** (18) (2014) 28. MR3164771 https://doi.org/10.1214/EJP.v19-2982

[12] A. Bovier and L. Hartung. Variable speed branching Brownian motion 1. Extremal processes in the weak correlation regime. *ALEA Lat. Am. J. Probab. Math. Stat.* **12** (1) (2015) 261–291. Available at http://alea.impa.br/articles/v12/12-11.pdf. MR3351476

[13] A. Bovier and L. Hartung. From 1 to 6: A finer analysis of perturbed branching Brownian motion. *Comm. Pure Appl. Math.* **73** (7) (2020) 1490–1525. MR4156608 https://doi.org/10.1002/cpa.21893

[14] M. Bramson, J. Ding and O. Zeitouni. Convergence in law of the maximum of the two-dimensional discrete Gaussian free field. *Comm. Pure Appl. Math.* **69** (1) (2016) 62–123. MR3433630 https://doi.org/10.1002/cpa.21621

[15] É. Brunet and B. Derrida. Statistics at the tip of a branching random walk and the delay of traveling waves. *Europhys. Lett.* **87** (6) (2009) 60010. Available at http://stacks.iop.org/0295-5075/87/i=6/a=60010.

[16] É. Brunet, B. Derrida, A. H. Mueller and S. Munier. Phenomenological theory giving the full statistics of the position of fluctuating pulled fronts. *Phys. Rev. E* **73** (5) (2006). https://doi.org/10.1103/physreve.73.056126

[17] D. Buraczewski and M. Maślanka. Large deviation estimates for branching random walks. *ESAIM Probab. Stat.* **23** (2019) 823–840. Available at https://arxiv.org/abs/1708.06540. MR4045541 https://doi.org/10.1051/ps/2019006

[18] B. Chauvin and A. Rouault. KPP equation and supercritical branching Brownian motion in the subcritical speed area. Application to spatial trees. *Probab. Theory Related Fields* **80** (2) (1988) 299–314. MR0968823 https://doi.org/10.1007/BF00356108

[19] A. Cortines and B. Mallein. The genealogy of an exactly solvable Ornstein–Uhlenbeck type branching process with selection. *Electron. Commun. Probab.* **23** (13) (2018) Id/No 98. MR3896836 https://doi.org/10.1214/18-ECP197

[20] B. Derrida, B. Meerson and P. V. Sasorov. Large-displacement statistics of the rightmost particle of the one-dimensional branching Brownian motion. *Phys. Rev. E* **93** (2016) 042139. https://doi.org/10.1103/PhysRevE.93.042139

[21] B. Derrida and Z. Shi. Large deviations for the rightmost position in a branching Brownian motion. In *Springer Proceedings in Mathematics & Statistics* 303–312. Springer International Publishing, 2017. MR3747671 https://doi.org/10.1007/978-3-319-65313-6_12

[22] M. Fang. Tightness for maxima of generalized branching random walks. *J. Appl. Probab.* **49** (3) (2012) 652–670. MR3012090 https://doi.org/10.1239/jap/1346955324

[23] N. Gantert and T. Höfelsauer. Large deviations for the maximum of a branching random walk. *Electron. Commun. Probab.* **23** (2018) 12. MR3812066 https://doi.org/10.1214/18-ECP135

[24] J.-P. Kahane. Une inégalité du type de Slepian et Gordon sur les processus gaussiens. *Israel J. Math.* **55** (1986) 109–110. MR0858463 https://doi.org/10.1007/BF02772698

[25] N. Kistler and M. A. Schmidt. From Derrida's random energy model to branching random walks: From 1 to 3. *Electron. Commun. Probab.* **20** (47) (2015) 12. Available at https://projecteuclid.org/euclid.ecp/1465320974. MR3358969 https://doi.org/10.1214/ecp.v20-4189

[26] S. P. Lalley and T. Sellke. A conditional limit theorem for the frontier of a branching Brownian motion. *Ann. Probab.* **15** (3) (1987) 1052–1061. MR0893913 https://doi.org/10.1214/aop/1176992080

[27] R. Lyons. A simple path to Biggins' martingale convergence for branching random walk. In *Classical and Modern Branching Processes* 217–221. *Minneapolis, MN, 1994*. *IMA Vol. Math. Appl.* **84**. Springer, New York, 1997. MR1601749 https://doi.org/10.1007/978-1-4612-1862-3_17

[28] R. Lyons, R. Pemantle and Y. Peres. Conceptual proofs of $L \log L$ criteria for mean behavior of branching processes. *Ann. Probab.* **23** (3) (1995) 1125–1138. Available at http://links.jstor.org/sici?sici=0091-1798(199507)23:3<1125:CPOLCF>2.0.CO;2-Y&origin=MSN. MR1349164

[29] T. Madaule. First order transition for the branching random walk at the critical parameter. *Stochastic Process. Appl.* **126** (2) (2016) 470–502. MR3434991 https://doi.org/10.1016/j.spa.2015.09.008

[30] P. Maillard and O. Zeitouni. Slowdown in branching Brownian motion with inhomogeneous variance. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (3) (2016) 1144–1160. MR3531703 https://doi.org/10.1214/15-AIHP675

[31] R. Rhodes and V. Vargas. Gaussian multiplicative chaos and applications: A review. *Probab. Surv.* **11** (2014) 315–392. MR3274356 https://doi.org/10.1214/13-PS218

[32] E. Subag and O. Zeitouni. Freezing and decorated Poisson point processes. *Comm. Math. Phys.* **337** (1) (2015) 55–92. MR3324155 https://doi.org/10.1007/s00220-015-2303-2

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2114–2128
https://doi.org/10.1214/21-AIHP1228
© Association des Publications de l'Institut Henri Poincaré, 2022

Variance linearity for real Gaussian zeros

Raphaël Lachièze-Rey[a]

Université de Paris, Laboratoire MAP5, UMR CNRS 8145, 45 Rue des Saints-Pères, F-75006 Paris, France. [a]*raphael.lachiezerey@parisdescartes.fr*

Abstract. We investigate the zero set of a stationary Gaussian process on the real line, and in particular give lower bounds for the variance of the number of points and of linear statistics on a large interval, in all generality. We prove that this point process is never hyperuniform, i.e. the variance is at least linear, and give a necessary condition to have linear variance, which is close to be sufficient. We study the class of symmetric Bernoulli convolutions and give an example where the zero set is maximally rigid, weakly mixing, and not hyperuniform.

Résumé. On étudie l'ensemble formé par les zéros d'un processus Gaussien stationnaire sur la droite réelle, et on donne en particulier des bornes inférieures sur la variance du nombre de zéros et des statistiques linéaires sur un grand intervalle, en toute généralité. On montre que ce processus de points n'est jamais hyperuniforme, i.e. la variance est toujours au moins linéaire, et on donne une condition nécessaire pour avoir une variance linéaire, cette condition est proche d'être suffisante. On étudie la classe des convolutions symmétriques de Bernoulli et donnons un exemple où l'ensemble des zéros est maximalement rigide, faiblement mélangeant, et pas hyperuniforme.

MSC2020 subject classifications: 60G10; 60G15; 60G55

Keywords: Gaussian fields; Point processes; Crossings; Nodal set; Excursion; Hyperuniformity; Rigidity; Chaos decomposition; Linear statistics

References

[1] M. Ancona and T. Letendre. Zeros of smooth stationary Gaussian processes. *Electron. J. Probab.* **26** (2021) 1–81. MR4262341 https://doi.org/10.1214/21-ejp637
[2] E. Assaf, J. Buckley and N. Feldheim. An asymptotic formula for the variance of the number of zeroes of a stationary Gaussian process. Available at arXiv:2101.04052. MR3851839 https://doi.org/10.1007/s00440-017-0816-7
[3] J. Azaïs, F. Dalmao and J. R. Leon. Studying the winding number of a Gaussian process: The real method, 2020. Available at https://arxiv.org/pdf/2004.10333.pdf.
[4] J. Azaïs and M. Wschebor. *Level Sets and Extrema of Random Processes and Fields*. Wiley, New York, 2009. MR2478201 https://doi.org/10.1002/9780470434642
[5] J. Buckley and N. Feldheim. The winding of stationary Gaussian processes. *Probab. Theory Related Fields* **172** (2018) 583–614. MR3851839 https://doi.org/10.1007/s00440-017-0816-7
[6] S. Chatterjee. Rigidity of the three-dimensional hierarchical Coulomb gas. *Probab. Theory Related Fields* **175** (2019) 1123–1176. MR4026615 https://doi.org/10.1007/s00440-019-00912-6
[7] C. Corduneanu, N. Gheorghiu and V. Barbu. *Almost Periodic Functions*. Chelsea Publishing Company, New York, 1989. MR0481915
[8] H. Cramér and M. R. Leadbetter. *Stationary and Related Stochastic Processes*. Wiley, New York, 1967. MR2108670
[9] J. Cuzick. A central limit theorem for the number of zeros of a stationary Gaussian process. *Ann. Probab.* **4** (4) (1976) 547–556. MR0420809 https://doi.org/10.1214/aop/1176996026
[10] D. Dereudre, A. Hardy, T. Leblé and M. Maïda. DLR equations and rigidity for the sine-beta process. *Comm. Pure Appl. Math.* **74** (1) (2020) 172–222. MR4178183 https://doi.org/10.1002/cpa.21963
[11] H. Dym and P. McKean. *Gaussian Processes, Function Theory, and the Inverse Spectral Problem*. Academic Press, New York, 1976. MR0448523
[12] N. D. Feldheim. Variance of the number of zeroes of shift-invariant Gaussian analytic functions. *Israel J. Math.* **227** (2018) 753–792. MR3846340 https://doi.org/10.1007/s11856-018-1737-6
[13] D. Geman. On the variance of the number of zeros of a stationary Gaussian process. *Ann. Math. Stat.* **43** (1972) 977–982. MR0301791 https://doi.org/10.1214/aoms/1177692560
[14] S. Ghosh, M. Krishnapur and Y. Peres. Continuum percolation for Gaussian zeroes and Ginibre eigenvalues. *Ann. Probab.* **44** (5) (2016) 3357–3384. MR3551199 https://doi.org/10.1214/15-AOP1051
[15] S. Ghosh and J. L. Lebowitz. Fluctuations, large deviations and rigidity in hyperuniform systems: A brief survey. *Indian J. Pure Appl. Math.* **48** (2017) 609–631. MR3741696 https://doi.org/10.1007/s13226-017-0248-1

[16] S. Ghosh and Y. Peres. Rigidity and tolerance in point processes: Gaussian zeros and Ginibre eigenvalues. *Duke Math. J.* **166** (10) (2017) 1789–1858. MR3679882 https://doi.org/10.1215/00127094-2017-0002

[17] J. B. Hough M. Khrishnapur Y. Peres and B. Viràg. *Zeros of Gaussian Analytic Functions and Determinantal Point Processes*. *University Lecture Series*. Institute of Mathematical Statistics 2009. MR2552864 https://doi.org/10.1090/ulect/051

[18] M. Kac. On the average number of real roots of a random algebraic equation. *Bull. Amer. Math. Soc.* **49** (1943) 314–320. MR0007812 https://doi.org/10.1090/S0002-9904-1943-07912-8

[19] M. Kratz. Level crossings and other level functionals of stationary Gaussian processes. *Probab. Surv.* **3** (2006) 230–288. MR2264709 https://doi.org/10.1214/154957806000000087

[20] M. Kratz and J. R. Leon. Central limit theorems for level functionals of stationary Gaussian processes and fields. *J. Theor. Probab.* **14** (3) (2001) 639–672. MR1860517 https://doi.org/10.1023/A:1017588905727

[21] R. Lachièze-Rey. Diophantine Gaussian excursions and random walks. Available at https://arxiv.org/abs/2104.07290.

[22] E. Lukacs. *Characteristic Functions*. Charles Griffin & Co 1970. MR0346874

[23] G. Maruyama. The harmonic analysis of stationary stochastic processes. *Mem. Fac. Sci. Kyushu Univ., Ser. A* **4** (1949) 45–106. MR0032127

[24] F. Nazarov and M. Sodin. Fluctuations in random complex zeroes: Asymptotic normality revisited. *Int. Math. Res. Not.* **24** (2011) 5720–5759. MR2863379 https://doi.org/10.1093/imrn/rnr007

[25] Y. Peres and A. Sly. Rigidity and tolerance for perturbed lattices, 2014. Available at arXiv:1409.4490.

[26] J. Rosinski and T. Zak. The equivalence of ergodicity and weak mixing for infinitely divisible processes. *J. Theor. Probab.* **10** (1) (1997) 73–86. MR1432616 https://doi.org/10.1023/A:1022690230759

[27] E. Slud. Multiple Wiener–Ito integral expansions for level-crossing-count functionals. *Probab. Theory Related Fields* **87** (1991) 349–364. MR1084335 https://doi.org/10.1007/BF01312215

[28] P. P. Varjú. On the dimension of Bernoulli convolutions for all transcendental parameters. *Ann. of Math.* **189** (2019) 1001–1011. MR3961088 https://doi.org/10.4007/annals.2019.189.3.9

[29] I. Wigman. Fluctuations of the nodal length of random spherical harmonics. *Comm. Math. Phys.* **298** (2010) 787–831. MR2670928 https://doi.org/10.1007/s00220-010-1078-8

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2129–2164
https://doi.org/10.1214/21-AIHP1227
© Association des Publications de l'Institut Henri Poincaré, 2022

Wilson loops in finite Abelian lattice gauge theories

Malin P. Forsström[a], Jonatan Lenells[b] and Fredrik Viklund[c]

Department of Mathematics, KTH Royal Institute of Technology, 100 44 Stockholm, Sweden. [a]*malinpf@kth.se,* [b]*jlenells@kth.se,* [c]*fredrik.viklund@math.kth.se*

Abstract. We consider lattice gauge theories on $\mathbb{Z}^4$ with Wilson action and structure group $\mathbb{Z}_n$. We compute the expectation of Wilson loop observables to leading order in the weak coupling regime, extending and refining a recent result of Chatterjee. Our proofs use neither duality relations nor cluster expansion techniques.

Résumé. Nous considérons les théories de jauge sur réseau sur $\mathbb{Z}^4$ avec l'action de Wilson et le groupe de structure $\mathbb{Z}_n$. Nous calculons l'espérance des observables de la boucle de Wilson à l'ordre principal dans le régime de couplage faible, prolongeant et affinant un résultat récent de Chatterjee. Nos preuves n'utilisent ni relations de dualité ni techniques d'expansion d'amas.

MSC2020 subject classifications: 70S15; 81T13; 81T25; 82B20

Keywords: Lattice gauge theory; Wilson loops

References

[1] C. Borgs. Translation symmetry breaking in four dimensional lattice gauge theories. *Comm. Math. Phys.* **96** (1984) 251–284. MR0768257
[2] S. Cao. Wilson loop expectations in lattice gauge theories with finite gauge groups. *Comm. Math. Phys.* **380** (2020) 1439–1505. MR4179732 https://doi.org/10.1007/s00220-020-03912-z
[3] S. Chatterjee. Wilson loops in Ising lattice gauge theory. *Comm. Math. Phys.* **377** (2020) 307–340. MR4107931 https://doi.org/10.1007/s00220-020-03738-9
[4] J. Fröhlich and T. Spencer. Massless phases and symmetry restoration in Abelian gauge theories and spin systems. *Comm. Math. Phys.* **83** (1982) 411–454. MR0649811
[5] C. Gattringer and C. Lang. *Quantum Chromodynamics on the Lattice: An Introductory Presentation*. *Lect. Notes Phys.* **788**. Springer, Berlin, 2010. MR2574000 https://doi.org/10.1007/978-3-642-01850-3
[6] J. Ginibre. General formulation of Griffiths' inequalities. *Comm. Math. Phys.* **16** (1970) 310–328. MR0269252
[7] K. Wilson. Confinement of quarks. *Phys. Rev. D* **10** (1974) 2445–2459.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2165–2200
https://doi.org/10.1214/21-AIHP1230
© Association des Publications de l'Institut Henri Poincaré, 2022

Lévy area without approximation

Isao Sauzedde[a]

Mathematical institute, University of Oxford, Oxford, England. [a]*isao.sauzedde@maths.ox.ac.uk*

Abstract. We give asymptotic estimations on the area of the set of points with large Brownian winding, and study the average winding between a planar Brownian motion and a Poisson point process of large intensity on the plane. This allows us to give a new definition of the Lévy area which does not rely on piecewise linear approximations of the Brownian path.

Résumé. On donne une estimation asymptotique de l'aire de l'ensemble des points autour desquels le mouvement brownian s'enlace un grand nombre de fois, et on étudie l'enlacement moyen entre un mouvement brownian plan et un processus de Poisson de grande intensité dans le plan. Cela nous permet de donner une nouvelle définition de l'aire de Lévy, qui ne repose pas sur des approximations linéaires par morceaux du chemin brownien.

MSC2020 subject classifications: Primary 60J65; secondary 60B20; 60E07

Keywords: Stokes' formula; Planar Brownian motion; Lévy's area

References

[1] T. F. Banchoff and W. F. Pohl. A generalization of the isoperimetric inequality. *J. Differential Geom.* **6** (1971/72) 175–192. Available at http://projecteuclid.org/euclid.jdg/1214430403. MR0305319

[2] J. Bertoin and W. Werner. Asymptotic windings of planar Brownian motion revisited via the Ornstein–Uhlenbeck process. In *Séminaire de Probabilités, XXVIII* 138–152. *Lecture Notes in Math.* **1583**. Springer, Berlin, 1994. MR1329109 https://doi.org/10.1007/BFb0073842

[3] H. Boedihardjo and X. Geng. Simple piecewise geodesic interpolation of simple and Jordan curves with applications. *Constr. Approx.* **42** (1) (2015) 161–180. MR3361454 https://doi.org/10.1007/s00365-014-9257-z

[4] R. Chen and L. A. Shepp. On the sum of symmetric random variables. *Amer. Statist.* **37** (3) (1983) 237. MR713833 https://doi.org/10.2307/2683381

[5] W. Feller. *An Introduction to Probability Theory and Its Applications. Vol. II. Second Edition*, xxiv+669. John Wiley & Sons, Inc., New York-London-Sydney, 1971. MR0270403

[6] J. Franchi. Enroulements asymptotiques du mouvement brownien autour de lacets dans une variété riemannienne compacte de dimension 3. *Stochastic Process. Appl.* **52** (2) (1994) 251–272. MR1290698 https://doi.org/10.1016/0304-4149(94)90028-0

[7] P. K. Friz and M. Hairer. *A Course on Rough Paths: With an Introduction to Regularity Structures*. *Universitext*, xiv+251. Springer, Cham, 2014. MR3289027 https://doi.org/10.1007/978-3-319-08332-2

[8] P. K. Friz and N. B. Victoir. *Multidimensional Stochastic Processes as Rough Paths: Theory and Applications*. *Cambridge Studies in Advanced Mathematics* **120**, xiv+656. Cambridge University Press, Cambridge, 2010. MR2604669 https://doi.org/10.1017/CBO9780511845079

[9] C. Garban and J. A. Trujillo Ferreras. The expected area of the filled planar Brownian loop is $\pi/5$. *Comm. Math. Phys.* **264** (3) (2006) 797–810. MR2217292 https://doi.org/10.1007/s00220-006-1555-2

[10] O. Johnson and R. Samworth. Central limit theorem and convergence to stable laws in Mallows distance. *Bernoulli* **11** (5) (2005) 829–845. MR2172843 https://doi.org/10.3150/bj/1130077596

[11] J.-F. Le Gall and M. Yor. Étude asymptotique des enlacements du mouvement brownien autour des droites de l'espace. *Probab. Theory Related Fields* **74** (4) (1987) 617–635. MR0876259 https://doi.org/10.1007/BF00363519

[12] J.-F. Le Gall and M. Yor. Enlacements du mouvement brownien autour des courbes de l'espace. *Trans. Amer. Math. Soc.* **317** (2) (1990) 687–722. MR946219 https://doi.org/10.2307/2001484

[13] T. J. Lyons, M. Caruana and T. Lévy. *Differential Equations Driven by Rough Paths*. *Lecture Notes in Mathematics* **1908**, xviii+109. Springer,, Berlin, 2007. Lectures from the 34th Summer School on Probability Theory held in Saint-Flour, July 6–24, 2004, With an introduction concerning the Summer School by Jean Picard. MR2314753

[14] R. Mansuy and M. Yor. *Aspects of Brownian Motion*. *Universitext*, xiv+195. Springer-Verlag, Berlin, 2008. MR2454984 https://doi.org/10.1007/978-3-540-49966-4

[15] J. Pitman and M. Yor. Asymptotic laws of planar Brownian motion. *Ann. Probab.* **14** (3) (1986) 733–779. Available at http://links.jstor.org/sici?sici=0091-1798(198607)14:3<733:ALOPBM>2.0.CO;2-Q&origin=MSN. MR841582

[16] J. Pitman and M. Yor. Further asymptotic laws of planar Brownian motion. *Ann. Probab.* **17** (3) (1989) 965–1011. Available at http://links.jstor.org/sici?sici=0091-1798(198907)17:3<965:FALOPB>2.0.CO;2-H&origin=MSN. MR1009441

[17] Z. Shi. Windings of Brownian motion and random walks in the plane. *Ann. Probab.* **26** (1) (1998) 112–131. MR1617043 https://doi.org/10.1214/aop/1022855413

[18] F. Spitzer. Some theorems concerning 2-dimensional Brownian motion. *Trans. Amer. Math. Soc.* **87** (1958) 187–197. MR104296 https://doi.org/10.2307/1993096

[19] W. Werner. Sur les points autour desquels le mouvement brownien plan tourne beaucoup. *Probab. Theory Related Fields* **99** (1) (1994) 111–144. MR1273744 https://doi.org/10.1007/BF01199592

[20] W. Werner. Formule de Green, lacet brownien plan et aire de Lévy. *Stochastic Process. Appl.* **57** (2) (1995) 225–245. MR1334970 https://doi.org/10.1016/0304-4149(94)00052-U

[21] M. Yor. Loi de l’indice du lacet brownien, et distribution de Hartman–Watson. *Z. Wahrsch. Verw. Gebiete* **53** (1) (1980) 71–95. MR0576898 https://doi.org/10.1007/BF00531612

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2201–2215
https://doi.org/10.1214/21-AIHP1226
© Association des Publications de l'Institut Henri Poincaré, 2022

Eigenvalues for the minors of Wigner matrices

Jiaoyang Huang[a]

Courant Institute, NYU, 251 Mercer St # 801, New York, NY 10012, USA. [a]*jh4427@nyu.edu*

Abstract. The eigenvalues for the minors of real symmetric ($\beta = 1$) and complex Hermitian ($\beta = 2$) Wigner matrices form the Wigner corner process, which is a multilevel interlacing particle system. In this paper, we study the microscopic scaling limit of the Wigner corner process both near the spectral edge and in the bulk, and prove they are universal. We show: (i) Near the spectral edge, the corner process exhibit a decoupling phenomenon, as first observed in (*Adv. Math.* **304** (2017) 90–130). Individual extreme particles have Tracy-Widom$_\beta$ distribution; the spacings between the extremal particles on adjacent levels converge to independent Gamma distributions in a much smaller scale. (ii) In the bulk, the microscopic scaling limit of the Wigner corner process is given by the bead process for general Sine$_\beta$ process, as constructed recently in (*Probab. Theory Related Fields* **179** (2021) 589–647).

Résumé. Les valeurs propres des mineurs des matrices de Wigner symétriques réelles ($\beta = 1$) et hermitiennes complexes ($\beta = 2$) forment le processus des coins de matrices de Wigner, qui est un système multi-niveaux des particules entrelacées. Dans cet article, nous étudions la limite d'échelle microscopique du processus des coins de matrices de Wigner à la fois près du bord spectral et loin du bord, et prouvons qu'elles sont universelles. Nous montrons : (i) Près du bord spectral, le processus des coins présente un phénomène de découplage, comme observé pour la première fois dans (*Adv. Math.* **304** (2017) 90–130). Les particules extrémales individuelles ont une distribution Tracy-Widom$_\beta$; les espacements entre les particules extrémales sur les niveaux adjacents convergent vers des distributions gamma indépendantes à une échelle beaucoup plus petite. (ii) Loin du bord, la limite d'échelle microscopique du processus des coins de matrices de Wigner est donnée par le processus des perles pour le processus Sine$_\beta$ général, tel qu'il a été construit récemment dans (*Probab. Theory Related Fields* **179** (2021) 589–647).

MSC2020 subject classifications: 60B20; 60G55

Keywords: Random matrix; Eigenvalues; Wigner corner process; Bead process

References

[1] A. Adhikari and Z. Che. Edge universality of correlated Gaussians. *Electron. J. Probab.* **24** (2019) Paper No. 44. MR3949269 https://doi.org/10.1214/19-EJP273
[2] M. Adler, P. van Moerbeke and D. Wang. Random matrix minor processes related to percolation theory. *Random Matrices Theory Appl.* **2** (4) (2013), 1350008. MR3149438 https://doi.org/10.1142/S2010326313500081
[3] A. Aggarwal. Bulk universality for generalized Wigner matrices with few moments. *Probab. Theory Related Fields* **173** (1–2) (2019) 375–432. MR3916110 https://doi.org/10.1007/s00440-018-0836-y
[4] A. Aggarwal, P. Lopatto and H.-T. Yau. Goe statistics for lévy matrices. *Journal of the European Mathematical Society* (2021). MR4310816 https://doi.org/10.4171/jems/1089
[5] J. Alt, L. Erdős, T. Krüger and D. Schröder. Correlated random matrices: Band rigidity and edge universality. *Ann. Probab.* **48** (2) (2020) 963–1001. MR4089499 https://doi.org/10.1214/19-AOP1379
[6] Y. Baryshnikov. GUEs and queues. *Probab. Theory Related Fields* **119** (2) (2001) 256–274. MR1818248 https://doi.org/10.1007/PL00008760
[7] R. Bauerschmidt, J. Huang, A. Knowles and H.-T. Yau. Bulk eigenvalue statistics for random regular graphs. *Ann. Probab.* **45** (6A) (2017) 3626–3663. MR3729611 https://doi.org/10.1214/16-AOP1145
[8] P. Bourgade, L. Erdős, H.-T. Yau and J. Yin. Fixed energy universality for generalized Wigner matrices. *Comm. Pure Appl. Math.* **69** (10) (2016) 1815–1881. MR3541852 https://doi.org/10.1002/cpa.21624
[9] C. Boutillier. The bead model and limit behaviors of dimer models. *Ann. Probab.* **37** (1) (2009) 107–142. MR2489161 https://doi.org/10.1214/08-AOP398
[10] Z. Che. Universality of random matrices with correlated entries. *Electron. J. Probab.* **22** (2017) Paper No. 30. MR3629874 https://doi.org/10.1214/17-EJP46
[11] L. Erdős, A. Knowles, H.-T. Yau and J. Yin. Spectral statistics of Erdős–Rényi graphs II: Eigenvalue spacing and the extreme eigenvalues. *Comm. Math. Phys.* **314** (3) (2012) 587–640. MR2964770 https://doi.org/10.1007/s00220-012-1527-7

[12] L. Erdős, T. Krüger and D. Schröder. Random matrices with slow correlation decay. *Forum Math. Sigma* **7** (2019) e8. MR3941370 https://doi.org/10.1017/fms.2019.2

[13] L. Erdős, S. Péché, J. A. Ramirez, B. Schlein and H.-T. Yau. Bulk universality for Wigner matrices. *Comm. Pure Appl. Math.* **63** (7) (2010) 895–925. MR2662426 https://doi.org/10.1002/cpa.20317

[14] L. Erdős, J. Ramirez, B. Schlein, T. Tao, V. Vu and H.-T. Yau. Bulk universality for Wigner Hermitian matrices with subexponential decay. *Math. Res. Lett.* **17** (4) (2010) 667–674. MR2661171 https://doi.org/10.4310/MRL.2010.v17.n4.a7

[15] L. Erdős, B. Schlein and H.-T. Yau. Local semicircle law and complete delocalization for Wigner random matrices. *Comm. Math. Phys.* **287** (2) (2009) 641–655. MR2481753 https://doi.org/10.1007/s00220-008-0636-9

[16] L. Erdős, B. Schlein and H.-T. Yau. Universality of random matrices and local relaxation flow. *Invent. Math.* **185** (1) (2011) 75–119. MR2810797 https://doi.org/10.1007/s00222-010-0302-7

[17] L. Erdős, B. Schlein, H.-T. Yau and J. Yin. The local relaxation flow approach to universality of the local statistics for random matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **48** (1) (2012) 1–46. MR2919197 https://doi.org/10.1214/10-AIHP388

[18] L. Erdős and H.-T. Yau. Gap universality of generalized Wigner and β-ensembles. *J. Eur. Math. Soc. (JEMS)* **17** (8) (2015) 1927–2036. MR3372074 https://doi.org/10.4171/JEMS/548

[19] L. Erdős, H.-T. Yau and J. Yin. Universality for generalized Wigner matrices with Bernoulli distribution. *J. Comb.* **2** (1) (2011) 15–81. MR2847916 https://doi.org/10.4310/JOC.2011.v2.n1.a2

[20] L. Erdős, H.-T. Yau and J. Yin. Bulk universality for generalized Wigner matrices. *Probab. Theory Related Fields* **154** (1–2) (2012) 341–407. MR2981427 https://doi.org/10.1007/s00440-011-0390-3

[21] L. Erdős, H.-T. Yau and J. Yin. Rigidity of eigenvalues of generalized Wigner matrices. *Adv. Math.* **229** (3) (2012) 1435–1515. MR2871147 https://doi.org/10.1016/j.aim.2011.12.010

[22] V. Gorin and M. Shkolnikov. Multilevel Dyson Brownian motions via Jack polynomials. *Probab. Theory Related Fields* **163** (3–4) (2015) 413–463. MR3418747 https://doi.org/10.1007/s00440-014-0596-2

[23] V. Gorin and M. Shkolnikov. Interacting particle systems at the edge of multilevel Dyson Brownian motions. *Adv. Math.* **304** (2017) 90–130. MR3558206 https://doi.org/10.1016/j.aim.2016.08.034

[24] F. Götze, A. Naumov and A. Tikhomirov. Local semicircle law under fourth moment condition. *J. Theor. Probab.* **33** (3) (2020) 1327–1362. MR4125959 https://doi.org/10.1007/s10959-019-00907-y

[25] J. Huang, B. Landon and H.-T. Yau. Bulk universality of sparse random matrices. *J. Math. Phys.* **56** (12) (2015), 123301. MR3429490 https://doi.org/10.1063/1.4936139

[26] K. Johansson. Universality of the local spacing distribution in certain ensembles of Hermitian Wigner matrices. *Comm. Math. Phys.* **215** (3) (2001) 683–705. MR1810949 https://doi.org/10.1007/s002200000328

[27] K. Johansson and E. Nordenstam. Eigenvalues of GUE minors. *Electron. J. Probab.* **11** (50) (2006) 1342–1371. MR2268547 https://doi.org/10.1214/EJP.v11-370

[28] B. Landon, P. Sosoe and H.-T. Yau. Fixed energy universality of Dyson Brownian motion. *Adv. Math.* **346** (2019) 1137–1332. MR3914908 https://doi.org/10.1016/j.aim.2019.02.010

[29] J. O. Lee and K. Schnelli. Edge universality for deformed Wigner matrices. *Rev. Math. Phys.* **27** (8) (2015), 1550018. MR3405746 https://doi.org/10.1142/S0129055X1550018X

[30] J. O. Lee, K. Schnelli, B. Stetler and H.-T. Yau. Bulk universality for deformed Wigner matrices. *Ann. Probab.* **44** (3) (2016) 2349–2425. MR3502606 https://doi.org/10.1214/15-AOP1023

[31] J. O. Lee and J. Yin. A necessary and sufficient condition for edge universality of Wigner matrices. *Duke Math. J.* **163** (1) (2014) 117–173. MR3161313 https://doi.org/10.1215/00127094-2414767

[32] M. L. Mehta. *Random Matrices*, 3rd edition. *Pure and Applied Mathematics (Amsterdam)* **142**. Elsevier/Academic Press, Amsterdam, 2004. MR2129906

[33] J. Najnudel and B. Virág. Uniform point variance bounds in classical beta ensembles. *Random Matrices: Theory and Applications* (2020), 2150033.

[34] J. Najnudel and B. Virág. The bead process for beta ensembles. *Probab. Theory Related Fields* **179** (3) (2021) 589–647. MR4242623 https://doi.org/10.1007/s00440-021-01034-8

[35] Open problems: AIM workshop on random matrices(dec 2010). https://aimath.org/WWN/randommatrices/randommatrices.pdf. Accessed: 2019-7-17.

[36] Y. G. Sinai and A. B. Soshnikov. A refinement of Wigner's semicircle law in a neighborhood of the spectrum edge for random symmetric matrices. *Funktsional. Anal. i Prilozhen.* **32** (2) (1998) 56–79. MR1647832 https://doi.org/10.1007/BF02482597

[37] S. Sodin. Several applications of the moment method in random matrix theory. In *Proceedings of the International Congress of Mathematicians—Seoul 2014. Vol. III* 451–475. Kyung Moon Sa, Seoul, 2014. MR3729037

[38] S. Sodin. A limit theorem at the spectral edge for corners of time-dependent Wigner matrices. *Int. Math. Res. Not. IMRN* **17** (2015) 7575–7607. MR3403994 https://doi.org/10.1093/imrn/rnu180

[39] A. Soshnikov. Universality at the edge of the spectrum in Wigner random matrices. *Comm. Math. Phys.* **207** (3) (1999) 697–733. MR1727234 https://doi.org/10.1007/s002200050743

[40] T. Tao and V. Vu. Random matrices: Universality of local eigenvalue statistics up to the edge. *Comm. Math. Phys.* **298** (2) (2010) 549–572. MR2669449 https://doi.org/10.1007/s00220-010-1044-5

[41] T. Tao and V. Vu. Random matrices: Universality of local spectral statistics of non-Hermitian matrices. *Ann. Probab.* **43** (2) (2015) 782–874. MR3306005 https://doi.org/10.1214/13-AOP876

[42] C. A. Tracy and H. Widom. Level-spacing distributions and the Airy kernel. *Comm. Math. Phys.* **159** (1) (1994) 151–174. MR1257246

[43] C. A. Tracy and H. Widom. On orthogonal and symplectic matrix ensembles. *Comm. Math. Phys.* **177** (3) (1996) 727–754. MR1385083

[44] E. P. Wigner. Characteristic vectors of bordered matrices with infinite dimensions. *Ann. of Math. (2)* **62** (1955) 548–564. MR0077805 https://doi.org/10.2307/1970079

[45] E. P. Wigner. Characteristic vectors of bordered matrices with infinite dimensions. II. *Ann. of Math. (2)* **65** (1957) 203–207. MR0083848 https://doi.org/10.2307/1969956

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2216–2249
https://doi.org/10.1214/21-AIHP1238
© Association des Publications de l'Institut Henri Poincaré, 2022

Fluctuations for matrix-valued Gaussian processes

Mario Diaz[1,a], Arturo Jaramillo[2,b] and Juan Carlos Pardo[2,c]

[1] *Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, Mexico City, Mexico.* [a] *mario.diaz@sigma.iimas.unam.mx*
[2] *Centro de Investigación en Matemáticas A.C., Calle Jalisco s/n, CP 36240, Guanajuato, Mexico.* [b] *jagil@cimat.mx*, [c] *jcpardo@cimat.mx*

Abstract. We consider a symmetric matrix-valued Gaussian process $Y^{(n)} = (Y^{(n)}(t); t \geq 0)$ and its empirical spectral measure process $\mu^{(n)} = (\mu_t^{(n)}; t \geq 0)$. Under some mild conditions on the covariance function of $Y^{(n)}$, we find an explicit expression for the limit distribution of

$$Z_F^{(n)} := \big(\big(Z_{f_1}^{(n)}(t), \dots, Z_{f_r}^{(n)}(t)\big); t \geq 0\big),$$

where $F = (f_1, \dots, f_r)$, for $r \geq 1$, with each component belonging to a large class of test functions, and

$$Z_f^{(n)}(t) := n \int_{\mathbb{R}} f(x)\mu_t^{(n)}(\mathrm{d}x) - n\mathbb{E}\bigg[\int_{\mathbb{R}} f(x)\mu_t^{(n)}(\mathrm{d}x)\bigg].$$

More precisely, we establish the stable convergence of $Z_F^{(n)}$ and determine its limiting distribution. An upper bound for the total variation distance of the law of $Z_f^{(n)}(t)$ to its limiting distribution, for a test function f and $t \geq 0$ fixed, is also given.

Résumé. Nous considérons un processus gaussien symétrique à valeurs matricielles $Y^{(n)} = (Y^{(n)}(t); t \geq 0)$ et son processus des mesures spectrales empiriques $\mu^{(n)} = (\mu_t^{(n)}; t \geq 0)$. Sous des conditions assez faibles sur la fonction de covariance de $Y^{(n)}$ nous trouvons une expression explicite pour la loi limite de

$$Z_F^{(n)} := \big(\big(Z_{f_1}^{(n)}(t), \dots, Z_{f_r}^{(n)}(t)\big); t \geq 0\big),$$

où $F = (f_1, \dots, f_r)$, pour $r \geq 1$, avec chaque composant appartenant à une grande classe des fonctions test, et

$$Z_f^{(n)}(t) := n \int_{\mathbb{R}} f(x)\mu_t^{(n)}(\mathrm{d}x) - n\mathbb{E}\bigg[\int_{\mathbb{R}} f(x)\mu_t^{(n)}(\mathrm{d}x)\bigg].$$

Plus précisḿent, nous établissons la convergence stable de $Z_F^{(n)}$ et nous déterminons sa loi limite. Nous donnons également une borne supérieure pour la distance en variation totale entre la loi de $Z_f^{(n)}(t)$ et sa loi limite, pour une fonction test f et $t \geq 0$ fixés.

MSC2020 subject classifications: 60G15; 60B20; 60F05; 60H07; 60H05

Keywords: Malliavin calculus; Matrix-valued Gaussian processes; Central limit theorem; Skorokhod integration; Gaussian orthogonal ensemble

References

[1] G. W. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, New York, 2010. MR2760897
[2] G. W. Anderson and O. Zeitouni. A CLT for a band matrix model. *Probab. Theory Related Fields* **134** (2) (2006) 283–338. MR2222385 https://doi.org/10.1007/s00440-004-0422-3
[3] Z. Bai and J. W. Silverstein. CLT for linear spectral statistics of large-dimensional sample covariance matrices. *Ann. Probab.* **32** (1A) (2004) 553–605. MR2040792 https://doi.org/10.1214/aop/1078415845
[4] Z. Bai and J. Yao. On the convergence of the spectral empirical process of Wigner matrices. *Bernoulli* **11** (6) (2005) 1059–1092. MR2189081 https://doi.org/10.3150/bj/1137421640

[5] M. Bender. Global fluctuations in general beta Dyson's Brownian motion. *Stochastic Process. Appl.* **118** (6) (2008) 1022–1042. MR2418256 https://doi.org/10.1016/j.spa.2007.07.010

[6] P. Biane and R. Speicher. Stochastic calculus with respect to free Brownian motion and analysis on Wigner space. *Probab. Theory Related Fields* **112** (3) (1998) 373–409. MR1660906 https://doi.org/10.1007/s004400050194

[7] P. Billingsley. *Convergence of Probability Measures*. John Wiley & Sons, New York, 2013. MR0233396

[8] G. Binotto, I. Nourdin and D. Nualart. Weak symmetric integrals with respect to the fractional Brownian motion. *Ann. Probab.* **46** (2016), 06. MR3813991 https://doi.org/10.1214/17-AOP1227

[9] T. Cabanal-Duvillard. Fluctuations de la loi empirique de grandes matrices aléatoires. *Ann. Inst. Henri Poincaré Probab. Stat.* **37** (3) (2001) 373–402. MR1831988 https://doi.org/10.1016/S0246-0203(00)01071-2

[10] G. Cébron and T. Kemp. Fluctuations of brownian motions on $\mathbb{GL}_N$. *Ann. Inst. Henri Poincaré Probab. Stat.* **58** (1) (2022) 524–547.

[11] E. Cépa and D. Lépingle. Diffusing particles with electrostatic repulsion. *Probab. Theory Related Fields* **107** (4) (1997) 429–449. MR1440140 https://doi.org/10.1007/s004400050092

[12] S. Chatterjee. Fluctuations of eigenvalues and second order Poincaré inequalities. *Probab. Theory Related Fields* **143** (1–2) (2009) 1–40. MR2449121 https://doi.org/10.1007/s00440-007-0118-6

[13] P. Cheridito and D. Nualart. Stochastic integral of divergence type with respect to fractional Brownian motion with Hurst parameter $H \in (0, \frac{1}{2})$. *Ann. Inst. Henri Poincaré Probab. Stat.* **41** (6) (2005) 1049–1081. MR2172209 https://doi.org/10.1016/j.anihpb.2004.09.004

[14] J. M. Corcuera, D. Nualart and J. H. C. Woerner. Power variation of some integral fractional processes. *Bernoulli* **12** (4) (2006) 713–735. MR2248234 https://doi.org/10.3150/bj/1155735933

[15] P. Diaconis and S. Evans. Linear functionals of eigenvalues of random matrices. *Trans. Amer. Math. Soc.* **353** (7) (2001) 2615–2633. MR1828463 https://doi.org/10.1090/S0002-9947-01-02800-8

[16] P. Diaconis and M. Shahshahani. On the eigenvalues of random matrices. *J. Appl. Probab.* **31** (A) (1994) 49–62. MR1274717 https://doi.org/10.2307/3214948

[17] M. Diaz, J. A. Mingo and S. Belinschi. On the global fluctuations of block Gaussian matrices. *Probab. Theory Related Fields* **176** (1) (2020) 599–648. MR4055196 https://doi.org/10.1007/s00440-019-00925-1

[18] V. L. Girko. *Theory of Stochastic Canonical Equations*, **2**. Springer, Berlin, 2001. MR1887676 https://doi.org/10.1007/978-94-010-0989-8

[19] A. Guionnet. Large deviations upper bounds and central limit theorems for non-commutative functionals of Gaussian large random matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **38** (3) (2002) 341–384. MR1899457 https://doi.org/10.1016/S0246-0203(01)01093-7

[20] D. Harnett, A. Jaramillo and D. Nualart. Symmetric stochastic integrals with respect to a class of self-similar Gaussian processes. *J. Theor. Probab.* (2017) 1–40. MR3979662 https://doi.org/10.1007/s10959-018-0833-1

[21] S. Israelsson. Asymptotic fluctuations of a particle system with singular interaction. *Stochastic Process. Appl.* **93** (1) (2001) 25–56. MR1819483 https://doi.org/10.1016/S0304-4149(00)00100-9

[22] A. Jaramillo and D. Nualart. Functional limit theorem for the self-intersection local time of the fractional Brownian motion. *Ann. Inst. Henri Poincaré Probab. Stat.* **55** (1) (2019) 480–527. MR3901653 https://doi.org/10.1214/18-aihp889

[23] A. Jaramillo and D. Nualart. Collision of eigenvalues for matrix-valued processes. *Random Matrices Theory Appl.* **9** (04) (2020), 2030001. MR4133067 https://doi.org/10.1142/S2010326320300016

[24] A. Jaramillo, J. C. Pardo and J. L. Pérez. Convergence of the empirical spectral distribution of Gaussian matrix-valued processes. *Electron. J. Probab.* **24** (2019). MR3916330 https://doi.org/10.1214/18-EJP203

[25] K. Johansson. On fluctuations of eigenvalues of random Hermitian matrices. *Duke Math. J.* **91** (1) (1998) 151–204. MR1487983 https://doi.org/10.1215/S0012-7094-98-09108-6

[26] T. Kemp, I. Nourdin, G. Peccati and R. Speicher. Wigner chaos and the fourth moment. *Ann. Probab.* **40** (4) (2012) 1577–1635. MR2978133 https://doi.org/10.1214/11-AOP657

[27] P. Lei and D. Nualart. Stochastic calculus for Gaussian processes and application to hitting times. *Commun. Stoch. Anal.* **6** (3) (2012) 11. MR2988698

[28] T. Lévy and M. Maïda. Central limit theorem for the heat kernel measure on the unitary group. *J. Funct. Anal.* **259** (12) (2010) 3163–3204. MR2727643 https://doi.org/10.1016/j.jfa.2010.08.005

[29] A. Lytova and L. Pastur. Central limit theorem for linear eigenvalue statistics of random matrices with independent entries. *Ann. Probab.* **37** (5) (2009) 1778–1840. MR2561434 https://doi.org/10.1214/09-AOP452

[30] J. A. Mingo, P. Śniady and R. Speicher. Second order freeness and fluctuations of random matrices: II. Unitary random matrices. *Adv. Math.* **209** (1) (2007) 212–240. MR2294222 https://doi.org/10.1016/j.aim.2006.05.003

[31] I. Nourdin and G. Peccati. *Normal Approximations with Malliavin Calculus: From Stein's Method to Universality*, **192**. Cambridge University Press, Cambridge, 2012. MR2962301 https://doi.org/10.1017/CBO9781139084659

[32] I. Nourdin, G. Peccati and G. Reinert. Second order Poincaré inequalities and CLTs on Wiener space. *J. Funct. Anal.* **257** (2) (2009) 593–609. MR2527030 https://doi.org/10.1016/j.jfa.2008.12.017

[33] I. Nourdin, G. Peccati and A. Réveillac. Multivariate normal approximation using Stein's method and Malliavin calculus. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** (1) (2010) 45–58. MR2641769 https://doi.org/10.1214/08-AIHP308

[34] D. Nualart. *The Malliavin Calculus and Related Topics*, **1995**. Springer, Berlin, 2006. MR1344217 https://doi.org/10.1007/978-1-4757-2437-0

[35] D. Nualart and G. Peccati. Central limit theorems for sequences of multiple stochastic integrals. *Ann. Probab.* **33** (1) (2005) 177–193. MR2118863 https://doi.org/10.1214/009117904000000621

[36] L. A. Pastur and M. Shcherbina. Eigenvalue distribution of large random matrices. In *Number*, **171**. Am. Math. Soc., Providence, 2011. MR2808038 https://doi.org/10.1090/surv/171

[37] V. Pérez-Abreu and C. Tudor. Functional limit theorems for trace processes in a Dyson Brownian motion. *Commun. Stoch. Anal.* **1** (3) (2007) 5. MR2403859 https://doi.org/10.31390/cosa.1.3.05

[38] A. Rényi. On mixing sequences of sets. *Acta Math. Acad. Sci. Hung.* **9** (1958) 215–228. MR0098161 https://doi.org/10.1007/BF02023873

[39] L. C. G. Rogers and Z. Shi. Interacting Brownian particles and the Wigner law. *Probab. Theory Related Fields* **95** (4) (1993) 555–570. MR1217451 https://doi.org/10.1007/BF01196734

[40] T. Tao. *Topics in Random Matrix Theory*, **132**. Am. Math. Soc., Providence, 2012. MR2906465 https://doi.org/10.1090/gsm/132

[41] J. Unterberger. Global fluctuations for 1D log-gas dynamics. *Stochastic Process. Appl.* **128**(12) (2018) 4104–4153. MR3906980 https://doi.org/10.1016/j.spa.2018.01.008

[42] D. Voiculescu. Limit laws for random matrices and free products. *Invent. Math.* **104** (1) (1991) 201–220. MR1094052 https://doi.org/10.1007/BF01245072

[43] E. P. Wigner. Characteristic vectors of bordered matrices with infinite dimensions. *Ann. of Math.* (1955) 548–564. MR0077805 https://doi.org/10.2307/1970079

[44] E. P. Wigner. Characteristic vectors of bordered matrices with infinite dimensions II. *Ann. of Math.* (1957) 203–207. MR0083848 https://doi.org/10.2307/1969956

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2250–2283
https://doi.org/10.1214/21-AIHP1229
© Association des Publications de l'Institut Henri Poincaré, 2022

Airy process with wanderers, KPZ fluctuations, and a deformation of the Tracy–Widom GOE distribution

Karl Liechty[1,a], Gia Bao Nguyen[2,b] and Daniel Remenik[3,c]

[1]*Department of Mathematical Sciences, DePaul University, Chicago, IL, 60614, USA.* [a]*kliechty@depaul.edu*
[2]*Department of Mathematics, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden.* [b]*nguyengb@kth.se*
[3]*Departamento de Ingeniería Matemática and Centro de Modelamiento Matemático (IRL-CNRS 2807), Universidad de Chile, Santiago, Chile.* [c]*dremenik@dim.uchile.cl*

Abstract. We study the distribution of the supremum of the Airy process with m wanderers minus a parabola, or equivalently the limit of the rescaled maximal height of a system of N non-intersecting Brownian bridges as $N \to \infty$, where the first $N - m$ paths start and end at the origin and the remaining m go between arbitrary positions. The distribution provides a $2m$-parameter deformation of the Tracy–Widom GOE distribution, which is recovered in the limit corresponding to all Brownian paths starting and ending at the origin.

We provide several descriptions of this distribution function: (i) A Fredholm determinant formula; (ii) A formula in terms of Painlevé II functions; (iii) A representation as a marginal of the KPZ fixed point with initial data given as the top path in a stationary system of reflected Brownian motions with drift; (iv) A characterization as the solution of a version of the Bloemendal–Virag PDE (*Probab. Theory Related Fields* **156** (2013) 795–825; *Ann. Probab.* **44** (2016) 2726–2769) for spiked Tracy–Widom distributions; (v) A representation as a solution of the KdV equation. We also discuss connections with a model of last passage percolation with boundary sources.

Résumé. Nous étudions la loi du supremum du processus d'Airy à m promeneurs, moins une parabole, ou de manière équivalente, la limite de la hauteur maximale rééchelonnée d'un système de N ponts browniens non intersectants, lorsque $N \to \infty$, où les $N - m$ premiers chemins commencent et terminent à l'origine et les m restants lient des positions arbitraires. Cette loi fournit une déformation à $2m$ paramètres de la loi GOE de Tracy–Widom, qui est retrouvée comme limite lorsque tous les chemins browniens qui commencent et terminent à l'origine. Nous fournissons plusieurs descriptions de cette loi : (i) une formule utilisant un déterminant de Fredholm ; (ii) une formule en termes de fonctions de Painlevé II ; (iii) une représentation en tant que marginal du point fixe KPZ, avec des données initiales données par le chemin supérieur d'un système stationnaire de mouvements browniens avec dérive réfléchis ; (iv) une caractérisation comme solution d'une version de l'EDP de Bloemendal–Virag (*Probab. Theory Related Fields* **156** (2013) 795–825 ; *Ann. Probab.* **44** (2016) 2726–2769) pour des lois de Tracy–Widom avec pointes ; (v) une représentation en tant que solution de l'équation KdV. Nous discutons aussi des liens avec un modèle de percolation de dernier passage avec source à la frontière.

MSC2020 subject classifications: 60K35; 60B20; 60J65

Keywords: Non-intersecting Brownian motions; Airy processes; KPZ fixed point; Random matrices; Painlevé II

References

[1] M. Adler, J. Delépine and P. van Moerbeke. Dyson's nonintersecting Brownian motions with a few outliers. *Comm. Pure Appl. Math.* **62** (3) (2009) 334–395. MR2487852 https://doi.org/10.1002/cpa.20264

[2] M. Adler, P. L. Ferrari and P. van Moerbeke. Airy processes with wanderers and new universality classes. *Ann. Probab.* **38** (2) (2010) 714–769. MR2642890 https://doi.org/10.1214/09-AOP493

[3] A. I. Aptekarev, P. M. Bleher and A. B. J. Kuijlaars. Large n limit of Gaussian random matrices with external source. II. *Comm. Math. Phys.* **259** (2) (2005) 367–389. MR2172687 https://doi.org/10.1007/s00220-005-1367-9

[4] J. Baik. Painlevé formulas of the limiting distributions for nonnull complex sample covariance matrices. *Duke Math. J.* **133** (2) (2006) 205–235. MR2225691 https://doi.org/10.1215/S0012-7094-06-13321-5

[5] J. Baik, G. Ben Arous and S. Péché. Phase transition of the largest eigenvalue for nonnull complex sample covariance matrices. *Ann. Probab.* **33** (5) (2005) 1643–1697. MR2165575 https://doi.org/10.1214/009117905000000233

[6] J. Baik, P. Deift and E. Strahov. Products and ratios of characteristic polynomials of random Hermitian matrices. *J. Math. Phys.* **44** (8) (2003) 3657–3670. MR2006773 https://doi.org/10.1063/1.1587875

[7] J. Baik, P. L. Ferrari and S. Péché. Limit process of stationary TASEP near the characteristic line. *Comm. Pure Appl. Math.* **63** (8) (2010) 1017–1070. MR2642384 https://doi.org/10.1002/cpa.20316
[8] J. Baik and E. M. Rains. Limiting distributions for a polynuclear growth model with external sources. *J. Stat. Phys.* **100** (3–4) (2000) 523–541. MR1788477 https://doi.org/10.1023/A:1018615306992
[9] J. Baik and E. M. Rains. Algebraic aspects of increasing subsequences. *Duke Math. J.* **109** (1) (2001) 1–65. MR1844203 https://doi.org/10.1215/S0012-7094-01-10911-3
[10] J. Baik and E. M. Rains. The asymptotics of monotone subsequences of involutions. *Duke Math. J.* **109** (2) (2001) 205–281. MR1845180 https://doi.org/10.1215/S0012-7094-01-10921-6
[11] D. Betea. On the combinatorics of last passage percolation in a quarter square and GOE^2 fluctuations. Preprint. Available at arXiv:1809.06792 [math-ph].
[12] A. Bloemendal and B. Virág. Limits of spiked random matrices I. *Probab. Theory Related Fields* **156** (3–4) (2013) 795–825. MR3078286 https://doi.org/10.1007/s00440-012-0443-2
[13] A. Bloemendal and B. Virág. Limits of spiked random matrices II. *Ann. Probab.* **44** (4) (2016) 2726–2769. MR3531679 https://doi.org/10.1214/15-AOP1033
[14] F. Bornemann. On the numerical evaluation of Fredholm determinants. *Math. Comp.* **79** (270) (2010) 871–915. MR2600548 https://doi.org/10.1090/S0025-5718-09-02280-7
[15] F. Bornemann, P. L. Ferrari and M. Prähofer. The Airy_1 process is not the limit of the largest eigenvalue in GOE matrix diffusion. *J. Stat. Phys.* **133** (3) (2008) 405–415. MR2448629 https://doi.org/10.1007/s10955-008-9621-0
[16] A. Borodin, I. Corwin and D. Remenik. Multiplicative functionals on ensembles of non-intersecting paths. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (1) (2015) 28–58. MR3300963 https://doi.org/10.1214/13-AIHP579
[17] A. Borodin and P. L. Ferrari. Large time asymptotics of growth models on space-like paths. I. PushASEP. *Electron. J. Probab.* **13** (50) (2008) 1380–1418. MR2438811 https://doi.org/10.1214/EJP.v13-541
[18] A. Borodin, P. L. Ferrari and M. Prähofer. Fluctuations in the discrete TASEP with periodic initial configurations and the Airy_1 process. *Int. Math. Res. Pap. IMRP* **2007** (2007) Art. ID rpm002. MR2334008
[19] A. Borodin, P. L. Ferrari, M. Prähofer and T. Sasamoto. Fluctuation properties of the TASEP with periodic initial configuration. *J. Stat. Phys.* **129** (5–6) (2007) 1055–1080. MR2363389 https://doi.org/10.1007/s10955-007-9383-0
[20] A. Borodin, P. L. Ferrari and T. Sasamoto. Transition between Airy_1 and Airy_2 processes and TASEP fluctuations. *Comm. Pure Appl. Math.* **61** (11) (2008) 1603–1629. MR2444377 https://doi.org/10.1002/cpa.20234
[21] A. Borodin, P. L. Ferrari and T. Sasamoto. Two speed TASEP. *J. Stat. Phys.* **137** (5–6) (2009) 936–977. MR2570757 https://doi.org/10.1007/s10955-009-9837-7
[22] A. Borodin and S. Péché. Airy kernel with two sets of parameters in directed percolation and random matrix theory. *J. Stat. Phys.* **132** (2) (2008) 275–290. MR2415103 https://doi.org/10.1007/s10955-008-9553-8
[23] E. Brézin and S. Hikami. Characteristic polynomials of random matrices. *Comm. Math. Phys.* **214** (1) (2000) 111–135. MR1794268 https://doi.org/10.1007/s002200000256
[24] R. Buckingham and K. Liechty. The k-tacnode process. *Probab. Theory Related Fields* **175** (1–2) (2019) 341–395. MR4009711 https://doi.org/10.1007/s00440-018-0885-2
[25] A. Budhiraja and C. Lee. Long time asymptotics for constrained diffusions in polyhedral domains. *Stochastic Process. Appl.* **117** (8) (2007) 1014–1036. MR2340877 https://doi.org/10.1016/j.spa.2006.11.007
[26] T. Claeys, A. B. J. Kuijlaars and M. Vanlessen. Multi-critical unitary random matrix ensembles and the general Painlevé II equation. *Ann. of Math. (2)* **168** (2) (2008) 601–641. MR2434886 https://doi.org/10.4007/annals.2008.168.601
[27] I. Corwin, P. L. Ferrari and S. Péché. Limit processes for TASEP with shocks and rarefaction fans. *J. Stat. Phys.* **140** (2) (2010) 232–267. MR2659279 https://doi.org/10.1007/s10955-010-9995-7
[28] I. Corwin and A. Hammond. Brownian Gibbs property for Airy line ensembles. *Invent. Math.* **2011** (2011) 1–68. MR3152753 https://doi.org/10.1007/s00222-013-0462-3
[29] I. Corwin, Z. Liu and D. Wang. Fluctuations of TASEP and LPP with general initial data. *Ann. Appl. Probab.* **26** (4) (2016) 2030–2082. MR3543889 https://doi.org/10.1214/15-AAP1139
[30] I. Corwin, J. Quastel and D. Remenik. Continuum statistics of the Airy_2 process. *Comm. Math. Phys.* **317** (2) (2013) 347–362. MR3010187 https://doi.org/10.1007/s00220-012-1582-0
[31] D. Dauvergne, J. Ortmann and B. Virág. The directed landscape. Preprint. Available at arXiv:1812.00309 [math.PR]. MR4035169
[32] A. B. Dieker and J. Moriarty. Reflected Brownian motion in a wedge: Sum-of-exponential stationary densities. *Electron. Commun. Probab.* **14** (2009) 1–16. MR2472171 https://doi.org/10.1214/ECP.v14-1437
[33] A. B. Dieker and J. Warren. Determinantal transition kernels for some interacting particles on the line. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** (6) (2008) 1162–1172. MR2469339 https://doi.org/10.1214/07-AIHP176
[34] I. Dumitriu and A. Edelman. Matrix models for beta ensembles. *J. Math. Phys.* **43** (11) (2002) 5830–5847. MR1936554 https://doi.org/10.1063/1.1507823
[35] P. Dupuis and R. J. Williams. Lyapunov functions for semimartingale reflecting Brownian motions. *Ann. Probab.* **22** (2) (1994) 680–702. MR1288127
[36] P. L. Ferrari. Slow decorrelations in Kardar–Parisi–Zhang growth. *J. Stat. Mech. Theory Exp.* **2008** (07) (2008) P07022.
[37] W. FitzGerald and J. Warren. Point-to-line last passage percolation and the invariant measure of a system of reflecting Brownian motions. *Probab. Theory Related Fields* **178** (1–2) (2020) 121–171. MR4146536 https://doi.org/10.1007/s00440-020-00972-z
[38] P. J. Forrester, S. N. Majumdar and G. Schehr. Non-intersecting Brownian walkers and Yang–Mills theory on the sphere. *Nuclear Phys. B* **844** (3) (2011) 500–526. MR2747559 https://doi.org/10.1016/j.nuclphysb.2010.11.013
[39] J. M. Harrison and R. J. Williams. Brownian models of open queueing networks with homogeneous customer populations. *Stochastics* **22** (2) (1987) 77–115. MR0912049 https://doi.org/10.1080/17442508708833469
[40] T. Imamura and T. Sasamoto. Fluctuations of the one-dimensional polynuclear growth model with external sources. *Nuclear Phys. B* **699** (3) (2004) 503–544. MR2098552 https://doi.org/10.1016/j.nuclphysb.2004.07.030
[41] T. Imamura and T. Sasamoto. Dynamics of a tagged particle in the asymmetric exclusion process with the step initial condition. *J. Stat. Phys.* **128** (4) (2007) 799–846. MR2344715 https://doi.org/10.1007/s10955-007-9326-9

[42] A. R. Its, A. B. J. Kuijlaars and J. Östensson. Critical edge behavior in unitary random matrix ensembles and the thirty-fourth Painlevé transcendent. *Int. Math. Res. Not. IMRN* **9** (2008) Art. ID rnn017, 67. MR2429248 https://doi.org/10.1093/imrn/rnn017
[43] K. Johansson. Discrete polynuclear growth and determinantal processes. *Comm. Math. Phys.* **242** (1–2) (2003) 277–329. MR2018275 https://doi.org/10.1007/s00220-003-0945-y
[44] K. Johansson. A multi-dimensional Markov chain and the Meixner ensemble. *Ark. Mat.* **48** (1) (2010) 79–95. MR2594587 https://doi.org/10.1007/s11512-008-0089-6
[45] O. Kallenberg. *Foundations of Modern Probability*, 2nd edition. *Probability and Its Applications (New York)*, xx+638. Springer-Verlag, New York, 2002. MR1876169 https://doi.org/10.1007/978-1-4757-4015-8
[46] M. Kardar, G. Parisi and Y.-C. Zhang. Dynamical scaling of growing interfaces. *Phys. Rev. Lett.* **56** (9) (1986) 889–892.
[47] S. Karlin and J. McGregor. Coincidence probabilities. *Pacific J. Math.* **9** (1959) 1141–1164. MR0114248
[48] C. E. Kenig, G. Ponce and L. Vega. Global solutions for the KdV equation with unbounded data. *J. Differ. Equ.* **139** (2) (1997) 339–364. MR1472352 https://doi.org/10.1006/jdeq.1997.3297
[49] R. Killip and M. Vişan. KdV is well-posed in H^{-1}. *Ann. of Math. (2)* **190** (1) (2019) 249–305. MR3990604 https://doi.org/10.4007/annals.2019.190.1.4
[50] K. Liechty. Nonintersecting Brownian motions on the half-line and discrete Gaussian orthogonal polynomials. *J. Stat. Phys.* **147** (3) (2012) 582–622. MR2923331 https://doi.org/10.1007/s10955-012-0485-y
[51] K. Liechty and D. Wang. Nonintersecting Brownian motions on the unit circle. *Ann. Probab.* **44** (2) (2016) 1134–1211. MR3474469 https://doi.org/10.1214/14-AOP998
[52] I. G. Macdonald. *Symmetric Functions and Hall Polynomials*, 2nd edition. *Oxford Mathematical Monographs. Oxford Science Publications*, x+475. The Clarendon Press Oxford University Press, New York, 1995. With contributions by A. Zelevinsky. MR1354144
[53] K. Matetski, J. Quastel and D. Remenik. The KPZ fixed point. *Acta Math.* **227** (2021) 115–203. MR4346267 https://doi.org/10.4310/acta.2021.v227.n1.a3
[54] K. Matetski and D. Remenik. TASEP and generalizations: Method for exact solution. Preprint. Available at arXiv:2107.07984 [math.PR].
[55] M. Y. Mo. Rank 1 real Wishart spiked model. *Comm. Pure Appl. Math.* **65** (11) (2012) 1528–1638. MR2969495 https://doi.org/10.1002/cpa.21415
[56] C. Nadal and S. N. Majumdar. A simple derivation of the Tracy–Widom distribution of the maximal eigenvalue of a Gaussian unitary random matrix. *J. Stat. Mech. Theory Exp.* **4** (2011) P04001, 29. MR2801166 https://doi.org/10.1088/1742-5468/2011/04/p04001
[57] G. B. Nguyen and D. Remenik. Non-intersecting Brownian bridges and the Laguerre orthogonal ensemble. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** (4) (2017) 2005–2029. MR3729645 https://doi.org/10.1214/16-AIHP781
[58] M. Nica, J. Quastel and D. Remenik. One-sided reflected Brownian motions and the KPZ fixed point. *Forum Math. Sigma* **8** (2020) Paper No. e63, 16. MR4190063 https://doi.org/10.1017/fms.2020.56
[59] M. Nica, J. Quastel and D. Remenik. Solution of the Kolmogorov equation for TASEP. *Ann. Probab.* **48** (5) (2020) 2344–2358. MR4152645 https://doi.org/10.1214/20-AOP1425
[60] S. Olver. A general framework for solving Riemann–Hilbert problems numerically. *Numer. Math.* **122** (2) (2012) 305–340. MR2969270 https://doi.org/10.1007/s00211-012-0459-7
[61] H. Oruç and G. M. Phillips. Explicit factorization of the Vandermonde matrix. *Linear Algebra Appl.* **315** (1–3) (2000) 113–123. MR1774962 https://doi.org/10.1016/S0024-3795(00)00124-5
[62] A. Perret and G. Schehr. Near-extreme eigenvalues and the first gap of Hermitian random matrices. *J. Stat. Phys.* **156** (5) (2014) 843–876. MR3231045 https://doi.org/10.1007/s10955-014-1044-5
[63] C. Pöppe. The Fredholm determinant method for the KdV equations. *Phys. D* **13** (1–2) (1984) 137–160. MR0775282 https://doi.org/10.1016/0167-2789(84)90274-4
[64] J. Quastel and D. Remenik. Local behavior and hitting probabilities of the Airy 1 process. *Probab. Theory Related Fields* **157** (3–4) (2013) 605–634. MR3129799 https://doi.org/10.1007/s00440-012-0466-8
[65] J. Quastel and D. Remenik. Supremum of the Airy 2 process minus a parabola on a half line. *J. Stat. Phys.* **150** (3) (2013) 442–456. MR3024136 https://doi.org/10.1007/s10955-012-0633-4
[66] J. Quastel and D. Remenik. Airy processes and variational problems. In *Topics in Percolative and Disordered Systems* 121–171. A. Ramírez, G. Ben Arous, P. A. Ferrari, C. Newman, V. Sidoravicius and M. E. Vares (Eds). *Springer Proceedings in Mathematics & Statistics* **69**, 2014. MR3229288 https://doi.org/10.1007/978-1-4939-0339-9_5
[67] J. Quastel and D. Remenik. How flat is flat in random interface growth? *Trans. Amer. Math. Soc.* **371** (9) (2019) 6047–6085. MR3937318 https://doi.org/10.1090/tran/7338
[68] J. Quastel and D. Remenik. KP governs random growth off a one dimensional substrate. *Forum Math. Pi*. To appear. Available at arXiv:1908.10353 [math.PR].
[69] J. Quastel and S. Sarkar. The KPZ equation converges to the KPZ fixed point. Preprint. Available at arXiv:2008.06584 [math.PR].
[70] A. Rákos and G. Schütz. Current distribution and random matrix ensembles for an integrable asymmetric fragmentation process. *J. Stat. Phys.* **118** (2005) 511–530. MR2123646 https://doi.org/10.1007/s10955-004-8819-z
[71] D. Revuz and M. Yor. *Continuous Martingales and Brownian Motion*, 3rd edition. *Fundamental Principles of Mathematical Sciences* **293**, xiv+602. Springer-Verlag, Berlin, 1999. MR1725357 https://doi.org/10.1007/978-3-662-06400-9
[72] A. Sarantsev. Reflected Brownian motion in a convex polyhedral cone: Tail estimates for the stationary distribution. *J. Theoret. Probab.* **30** (3) (2017) 1200–1223. MR3687255 https://doi.org/10.1007/s10959-016-0674-8
[73] T. Sasamoto. Spatial correlations of the 1D KPZ surface on a flat substrate. *J. Phys. A: Math. Gen.* **38** (33) (2005) L549. MR2165697 https://doi.org/10.1088/0305-4470/38/33/L01
[74] G. Schehr. Extremes of N vicious walkers for large N: Application to the directed polymer and KPZ interfaces. *J. Stat. Phys.* **149** (3) (2012) 385–410. MR2992794 https://doi.org/10.1007/s10955-012-0593-8
[75] G. M. Schütz. Exact solution of the master equation for the asymmetric exclusion process. *J. Stat. Phys.* **88** (1–2) (1997) 427–445. MR1468391 https://doi.org/10.1007/BF02508478
[76] B. Simon. *Trace Ideals and Their Applications*, 2nd edition. *Mathematical Surveys and Monographs* **120**, viii+150. American Mathematical Society, Providence, 2005. MR2154153 https://doi.org/10.1090/surv/120
[77] E. M. Stein and R. Shakarchi. *Complex Analysis*. *Princeton Lectures in Analysis* **2**, xviii+379. Princeton University Press, Princeton, NJ, 2003. MR1976398

[78] C. A. Tracy and H. Widom. Level-spacing distributions and the Airy kernel. *Comm. Math. Phys.* **159** (1) (1994) 151–174. MR1257246
[79] C. A. Tracy and H. Widom. On orthogonal and symplectic matrix ensembles. *Comm. Math. Phys.* **177** (3) (1996) 727–754. MR1385083
[80] B. Virág. The heat and the landscape I. Preprint. Available at arXiv:2008.07241 [math.PR].
[81] D. Wang. The largest sample eigenvalue distribution in the rank 1 quaternionic spiked model of Wishart ensemble. *Ann. Probab.* **37** (4) (2009) 1273–1328. MR2546746 https://doi.org/10.1214/08-AOP432
[82] R. J. Williams. Semimartingale reflecting Brownian motions in the orthant. In *Stochastic Networks*. *IMA Vol. Math. Appl.* **71**, 125–137. Springer, New York, 1995. MR1381009 https://doi.org/10.1007/978-1-4757-2418-9_7
[83] S.-X. Xu and Y.-Q. Zhao. Painlevé XXXIV asymptotics of orthogonal polynomials for the Gaussian weight with a jump at the edge. *Stud. Appl. Math.* **127** (1) (2011) 67–105. MR2849293 https://doi.org/10.1111/j.1467-9590.2010.00512.x

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2284–2320
https://doi.org/10.1214/21-AIHP1225
© Association des Publications de l'Institut Henri Poincaré, 2022

Spectrum and pseudospectrum for quadratic polynomials in Ginibre matrices

Nicholas A. Cook[1,a], Alice Guionnet[2,b] and Jonathan Husson[2,c]

[1]*Duke University, Durham, NC 27708, USA.* [a]*nickcook@math.duke.edu*
[2]*Université de Lyon, ENSL, CNRS, France.* [b]*Alice.Guionnet@ens-lyon.fr,* [c]*Jonathan.Husson@ens-lyon.fr*

Abstract. For a fixed quadratic polynomial $\mathfrak{p}$ in n non-commuting variables, and n independent $N \times N$ complex Ginibre matrices $X_1^N, \dots, X_n^N$, we establish the convergence of the empirical measure of the eigenvalues of $P^N = \mathfrak{p}(X_1^N, \dots, X_n^N)$ to the Brown measure of $\mathfrak{p}$ evaluated at n freely independent circular elements $c_1, \dots, c_n$ in a non-commutative probability space. As in previous works on non-normal random matrices, a key step is to obtain quantitative control on the pseudospectrum of P^N. Via a linearization trick of Haagerup–Thorbjørnsen for lifting non-commutative polynomials to tensors, we obtain this as a consequence of a lower tail estimate for the smallest singular value of patterned block matrices with strongly dependent entries. This reduces to establishing anti-concentration for determinants of random walks in a matrix space of bounded dimension, for which we encounter novel structural obstacles of an algebro-geometric nature.

Résumé. Pour un polynôme quadratique $\mathfrak{p}$ en n variables non-commutatives, et n matrices $N \times N$ de Ginibre complexes $X_1^N, \dots, X_n^N$, nous établissons la convergence de la mesure empirique des valeurs propres de $P^N = \mathfrak{p}(X_1^N, \dots, X_n^N)$ vers la mesure de Brown de $\mathfrak{p}$ évaluée en n éléments circulaires librement indépendants $c_1, \dots, c_n$ dans un espace de probabilité non-commutatif. Comme dans de précédents travaux portant sur des matrices aléatoires non normales, une étape clé est d'obtenir un contrôle qualitatif sur le pseudo-spectre de P^N. Par une méthode de linéarisation due à Haagerup et Thorbjørnsen qui permet de relever les polynômes non-commutatifs en des tenseurs, nous obtenons ce contrôle comme conséquence d'une estimée de la queue de la loi de la plus petite valeur singulière d'une matrice dotée d'une structure par blocs avec des coefficients fortement dépendants. Cela nous ramène à établir l'anti-concentration pour les déterminants de marches aléatoires dans un espace de matrices de dimension bornée, pour lesquelles nous rencontrons de nouveaux obstacles de nature algèbrico-géométriques.

MSC2020 subject classifications: 60B20; 46L54

Keywords: Random matrix; Small-ball estimate; Brown measure; Non-Hermitian matrix; Smallest singular value

References

[1] J. Alt, L. Erdős and T. Krüger. Local inhomogeneous circular law. *Ann. Appl. Probab.* **28** (2018) 148–203. MR3770875 https://doi.org/10.1214/17-AAP1302
[2] J. Alt and T. Krüger Inhomogeneous circular law for correlated matrices. Preprint. Available at arXiv:2005.13533. MR4271784 https://doi.org/10.1016/j.jfa.2021.109120
[3] G. W. Anderson. A local limit law for the empirical spectral distribution of the anticommutator of independent Wigner matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015) 809–841. MR3365962 https://doi.org/10.1214/14-AIHP602
[4] G. W. Anderson, A. Guionnet and O. Zeitouni. *An Introduction to Random Matrices*. *Cambridge Studies in Advanced Mathematics* **118**. Cambridge University Press, Cambridge, 2010. MR2760897
[5] J. Banks, A. Kulkarni, S. Mukherjee and N. Srivastava Gaussian regularization of the pseudospectrum and davies' conjecture. Preprint. Available at arXiv:1906.11819. MR4303015 https://doi.org/10.1002/cpa.22017
[6] J. Banks, J. G. Vargas, A. Kulkarni and N. Srivastava Pseudospectral shattering, the sign function, and diagonalization in nearly matrix multiplication time. Preprint. Available at arXiv:1912.08805. MR4232064 https://doi.org/10.1109/FOCS46700.2020.00056
[7] M. Banna and T. Mai. Hölder continuity of cumulative distribution functions for noncommutative polynomials under finite free Fisher information. *J. Funct. Anal.* **279** (2020), 108710. MR4124857 https://doi.org/10.1016/j.jfa.2020.108710
[8] A. Basak, N. Cook and O. Zeitouni. Circular law for the sum of random permutation matrices. *Electron. J. Probab.* **23** (2018) Paper No. 33, 51. MR3798243 https://doi.org/10.1214/18-EJP162
[9] A. Basak and A. Dembo. Limiting spectral distribution of sums of unitary and orthogonal matrices. *Electron. Commun. Probab.* **18** (69) (2013) 19. MR3091727 https://doi.org/10.1214/ECP.v18-2466

[10] A. Basak, E. Paquette and O. Zeitouni. Spectrum of random perturbations of Toeplitz matrices with finite symbols. *Trans. Amer. Math. Soc.* **373** (2020) 4999–5023. MR4127869 https://doi.org/10.1090/tran/8040

[11] S. T. Belinschi, T. Mai and R. Speicher. Analytic subordination theory of operator-valued free additive convolution and the solution of a general random matrix problem. *J. Reine Angew. Math.* **732** (2017) 21–53. MR3717087 https://doi.org/10.1515/crelle-2014-0138

[12] S. T. Belinschi, P. Śniady and R. Speicher. Eigenvalues of non-Hermitian random matrices and brown measure of non-normal operators: Hermitian reduction and linearization method. *Linear Algebra Appl.* **537** (2018) 48–83. MR3716236 https://doi.org/10.1016/j.laa.2017.09.024

[13] P. Biane and F. Lehner. Computation of some examples of Brown's spectral measure in free probability. *Colloq. Math.* **90** (2001) 181–211. MR1876844 https://doi.org/10.4064/cm90-2-3

[14] C. Bordenave and D. Chafaï. Around the circular law. *Probab. Surv.* **9** (2012) 1–89. MR2908617 https://doi.org/10.1214/11-PS183

[15] L. G. Brown. Lidskiĭ's theorem in the type II case. In *Geometric Methods in Operator Algebras* 1–35. *Kyoto*, *1983*. *Pitman Res. Notes Math. Ser.* **123**. Longman Sci. Tech, Harlow, 1986. MR0866489

[16] M. Campos, M. Jenssen, M. Michelen and J. Sahasrabudhe The singularity probability of a random symmetric matrix is exponentially small. Preprint. Available at arXiv:arXiv:2105.11384.

[17] M. Campos, L. Mattos, R. Morris and N. Morrison On the singularity of random symmetric matrices. Preprint, 2019. Available at arXiv:1904.11478. MR4255046 https://doi.org/10.1215/00127094-2020-0054

[18] A. Carbery and J. Wright. Distributional and L_q norm inequalities for polynomials over convex bodies in $\mathbb{R}^n$. *Math. Res. Lett.* **8** (2001) 233–248. MR1839474 https://doi.org/10.4310/MRL.2001.v8.n3.a1

[19] N. Cook, W. Hachem, J. Najim and D. Renfrew. Non-Hermitian random matrices with a variance profile (I): Deterministic equivalents and limiting ESDs. *Electron. J. Probab.* **23** (2018) Paper No. 61, 110. MR3878135 https://doi.org/10.1214/18-ejp230

[20] K. P. Costello, T. Tao and V. Vu. Random symmetric matrices are almost surely nonsingular. *Duke Math. J.* **135** (2006) 395–413. MR2267289 https://doi.org/10.1215/S0012-7094-06-13527-5

[21] A. Edelman. Eigenvalues and condition numbers of random matrices. *SIAM J. Matrix Anal. Appl.* **9** (1988) 543–560. MR0964668 https://doi.org/10.1137/0609045

[22] L. Erdős, T. Krüger and Y. Nemish. Local laws for polynomials of Wigner matrices. *J. Funct. Anal.* **278** (2020), 108507, 59. MR4078529 https://doi.org/10.1016/j.jfa.2020.108507

[23] J. Feinberg and A. Zee. Non-Gaussian non-Hermitian random matrix theory: Phase transition and addition formalism. *Nuclear Phys. B* **501** (1997) 643–669. MR1477381 https://doi.org/10.1016/S0550-3213(97)00419-7

[24] A. Ferber and V. Jain. Singularity of random symmetric matrices – a combinatorial approach to improved bounds. *Forum Math. Sigma* **7** (2019) Paper No. e22, 29. MR3993806 https://doi.org/10.1017/fms.2019.21

[25] J. Fox, M. Kwan and L. Sauermann Combinatorial anti-concentration inequalities, with applications. Preprint. Available at arXiv:1905.12142. MR4299587 https://doi.org/10.1017/s0305004120000183

[26] O. Friedland and S. Sodin. Bounds on the concentration function in terms of the Diophantine approximation. *C. R. Math. Acad. Sci. Paris* **345** (2007) 513–518. MR2375113 https://doi.org/10.1016/j.crma.2007.10.006

[27] V. L. Girko. Circular law. *Theory Probab. Appl.* **29** (1984) 694–706. MR0773436

[28] F. Götze and A. Tikhomirov. The circular law for random matrices. *Ann. Probab.* **38** (2010) 1444–1491. MR2663633 https://doi.org/10.1214/09-AOP522

[29] A. Guionnet, M. Krishnapur and O. Zeitouni. The single ring theorem. *Ann. of Math. (2)* **174** (2011) 1189–1217. MR2831116 https://doi.org/10.4007/annals.2011.174.2.10

[30] A. Guionnet, P. M. Wood and O. Zeitouni. Convergence of the spectral measure of non-normal matrices. *Proc. Amer. Math. Soc.* **142** (2014) 667–679. MR3134007 https://doi.org/10.1090/S0002-9939-2013-11761-2

[31] U. Haagerup and S. Thorbjørnsen. A new application of random matrices: $\mathrm{Ext}(C^*_{\mathrm{red}}(F_2))$ is not a group. *Ann. of Math. (2)* **162** (2005) 711–775. MR2183281 https://doi.org/10.4007/annals.2005.162.711

[32] H. Kösters and A. Tikhomirov. Limiting spectral distributions of sums of products of non-Hermitian random matrices. *Probab. Math. Statist.* **38** (2018) 359–384. On table of contents: Vol. 33 (2013). MR3896715 https://doi.org/10.4153/cjm-1986-016-7

[33] J. E. Littlewood and A. C. Offord. On the number of real roots of a random algebraic equation. III. *Rec. Math. [Mat. Sbornik] N.S.* **12** (54) (1943) 277–286. MR0009656

[34] L. Lovász. Singular spaces of matrices and their application in combinatorics. *Bol. Soc. Bras. Mat.* **20** (1989) 87–99. MR1129080 https://doi.org/10.1007/BF02585470

[35] V. A. Marčenko and L. A. Pastur. Distribution of eigenvalues in certain sets of random matrices. *Math. USSR, Sb.* **1** (1967) 457–483. English translation of Mat. Sbornik 72 507–536. MR0208649

[36] R. Meka, O. Nguyen and V. Vu. Anti-concentration for polynomials of independent random variables. *Theory Comput.* **12** (2016) Paper No. 11, 16. MR3542863 https://doi.org/10.4086/toc.2016.v012a011

[37] J. A. Mingo and R. Speicher. *Free Probability and Random Matrices*, **35**. The Fields Institute for Research in the Mathematical Sciences, New York, NY, 2017. MR3585560 https://doi.org/10.1007/978-1-4939-6942-5

[38] H. H. Nguyen. Inverse Littlewood–Offord problems and the singularity of random symmetric matrices. *Duke Math. J.* **161** (2012) 545–586. MR2891529 https://doi.org/10.1215/00127094-1548344

[39] H. H. Nguyen and S. O'Rourke. The elliptic law. *Int. Math. Res. Not. IMRN* (2015) 7620–7689. MR3403996 https://doi.org/10.1093/imrn/rnu174

[40] H. H. Nguyen and S. O'Rourke. On the concentration of random multilinear forms and the universality of random block matrices. *Probab. Theory Related Fields* **162** (2015) 97–154. MR3350042 https://doi.org/10.1007/s00440-014-0567-7

[41] H. H. Nguyen and V. H. Vu. Small ball probability, inverse theorems, and applications. In *Erdös centennial* 409–463. *Bolyai Soc. Math. Stud., János Bolyai Math.* **25**. Soc., Budapest, 2013. MR3203607 https://doi.org/10.1007/978-3-642-39286-3_16

[42] S. O'Rourke, D. Renfrew, A. Soshnikov and V. Vu. Products of independent elliptic random matrices. *J. Stat. Phys.* **160** (2015) 89–119. MR3357969 https://doi.org/10.1007/s10955-015-1246-5

[43] S. O'Rourke and A. Soshnikov. Products of independent non-Hermitian random matrices. *Electron. J. Probab.* **16** (81) (2011) 2219–2245. MR2861673 https://doi.org/10.1214/EJP.v16-954

[44] M. Rudelson and R. Vershynin. The Littlewood-Offord problem and invertibility of random matrices. *Adv. Math.* **218** (2008) 600–633. MR2407948 https://doi.org/10.1016/j.aim.2008.01.010

[45] M. Rudelson and R. Vershynin. Smallest singular value of a random rectangular matrix. *Comm. Pure Appl. Math.* **62** (2009) 1707–1739. MR2569075 https://doi.org/10.1002/cpa.20294

[46] D. Shlyakhtenko and P. Skoufranis. Freely independent random variables with non-atomic distributions. *Trans. Amer. Math. Soc.* **367** (2013) 6267–6291. MR3356937 https://doi.org/10.1090/S0002-9947-2015-06434-4

[47] P. Śniady. Random regularization of Brown spectral measure. *J. Funct. Anal.* **193** (2002) 291–313. MR1929504 https://doi.org/10.1006/jfan.2001.3935

[48] M. Strzelecka, M. Strzelecki and T. Tkocz. On the convex infimum convolution inequality with optimal cost function. *ALEA Lat. Am. J. Probab. Math. Stat.* **14** (2017) 903–915. MR3731797 https://doi.org/10.30757/alea.v14-39

[49] T. Tao, V. Vu and M. Krishnapur. Random matrices: Universality of ESDs and the circular law. *Ann. Probab.* **38** (2010) 2023–2065. MR2722794 https://doi.org/10.1214/10-AOP534

[50] T. Tao and V. H. Vu. Random matrices: The circular law. *Commun. Contemp. Math.* **10** (2008) 261–307. MR2409368 https://doi.org/10.1142/S0219199708002788

[51] T. Tao and V. H. Vu. Inverse Littlewood-Offord theorems and the condition number of random discrete matrices. *Ann. of Math. (2)* **169** (2009) 595–632. MR2480613 https://doi.org/10.4007/annals.2009.169.595

[52] A. N. Tikhomirov. On the asymptotics of the spectrum of the product of two rectangular random matrices. *Sibirsk. Mat. Zh.* **52** (2011) 936–954. MR2883224 https://doi.org/10.1134/S0037446611040161

[53] L. N. Trefethen and M. Embree. *Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators*. Princeton University Press, Princeton, N.J, 2005. MR2155029

[54] R. Vershynin. Invertibility of symmetric random matrices. *Random Structures Algorithms* **44** (2014) 135–182. MR3158627 https://doi.org/10.1002/rsa.20429

[55] M. Vogel and O. Zeitouni Deterministic equivalence for noisy perturbations. Preprint. Available at arXiv:2001.09024. MR4291588 https://doi.org/10.1090/proc/15499

[56] D. Voiculescu. Limit laws for random matrices and free products. *Invent. Math.* **104** (1991) 201–220. MR1094052 https://doi.org/10.1007/BF01245072

[57] E. P. Wigner. On the distribution of the roots of certain symmetric matrices. *Annals Math.* **67** (1958) 325–327. MR0095527 https://doi.org/10.2307/1970008

[58] P. M. Wood. Universality of the ESD for a fixed matrix plus small random noise: A stability approach. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 1877–1896. MR3573299 https://doi.org/10.1214/15-AIHP702

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2321–2346
https://doi.org/10.1214/21-AIHP1221
© Association des Publications de l'Institut Henri Poincaré, 2022

A zero-one law for invariant measures and a local limit theorem for coefficients of random walks on the general linear group

Ion Grama[1,a], Jean-François Quint[2,b] and Hui Xiao[3,c]

[1] *Univ Bretagne Sud, CNRS UMR 6205 LMBA, Campus de Tohannic 56017, Vannes, France.* [a]*ion.grama@univ-ubs.fr*
[2] *Univ. Bordeaux, CNRS, Bordeaux INP, IMB, UMR 5251, F-33400 Talence, France.* [b]*Jean-Francois.Quint@math.u-bordeaux.fr*
[3] *Universität Hildesheim, Institut für Mathematik und Angewandte Informatik, Hildesheim, Germany.* [c]*xiao@uni-hildesheim.de*

Abstract. We prove a zero-one law for the stationary measure for algebraic sets generalizing the results of Furstenberg (*Proc. Sympos. Pure Math.* **26** (1973) 193–229) and Guivarc'h and Le Page (*Ann. Inst. Henri Poincaré Probab. Stat.* **52**(2) (2016) 503–574). As an application, we establish a local limit theorem for the coefficients of random walks on the general linear group.

Résumé. Nous prouvons une loi zéro-un pour la mesure stationnaire pour des ensembles algébriques en généralisant les résultats de Furstenberg (*Proc. Sympos. Pure Math.* **26** (1973) 193–229) et Guivarc'h et Le Page (*Ann. Inst. Henri Poincaré Probab. Stat.* **52**(2) (2016) 503–574). Comme application, nous établissons un théorème local limite pour les coefficients de marches aléatoires sur le groupe linéaire général.

MSC2020 subject classifications: Primary 60B15; 15B52; 37A30; secondary 60B20

Keywords: General linear group; Zero-one law; Stationary measure; Random matrices; Regularity; Algebraic set

References

[1] R. Aoun. Transience of algebraic varieties in linear groups-applications to generic Zariski density. *Ann. Inst. Fourier (Grenoble)* **63** (5) (2013) 2049–2080. MR3203113 https://doi.org/10.5802/aif.2822
[2] Y. Benoist and J. F. Quint. Stationary measures and invariant subsets of homogeneous spaces (II). *J. Amer. Math. Soc.* **26** (3) (2013) 659–734. MR3037785 https://doi.org/10.1090/S0894-0347-2013-00760-2
[3] Y. Benoist and J. F. Quint. Random walks on projective spaces. *Compos. Math.* **150** (9) (2014) 1579–1606. MR3260142 https://doi.org/10.1112/S0010437X1400726X
[4] Y. Benoist and J. F. Quint. Central limit theorem for linear groups. *Ann. Probab.* **44** (2) (2016) 1308–1340. MR3474473 https://doi.org/10.1214/15-AOP1002
[5] Y. Benoist and J. F. Quint. *Random Walks on Reductive Groups*. Springer, Berlin, 2016. MR3560700
[6] F. F. Bonsall. *Lectures on Some Fixed Point Theorems of Functional Analysis*. Tata, Bombay, 1962. MR0198173
[7] A. Borel. *Linear Algebraic Groups*. Springer, Berlin, 1991. MR0837722
[8] A. A. Borovkov and K. A. Borovkov. *Asymptotic Analysis of Random Walks*. Cambridge University Press, Cambridge, 2008. MR2424161 https://doi.org/10.1017/CBO9780511721397
[9] P. Bougerol and J. Lacroix. *Products of Random Matrices with Applications to Schrödinger Operators*. Birkhäuser, Boston, 1985. MR0886674 https://doi.org/10.1007/978-1-4684-9172-2
[10] J. Bourgain, A. Furman, E. Lindenstrauss and S. Mozes. Stationary measures and equidistribution for orbits of nonabelian semigroups on the torus. *J. Amer. Math. Soc.* **24** (1) (2011) 231–280. MR2726604 https://doi.org/10.1090/S0894-0347-2010-00674-1
[11] E. Breuillard. Distributions diophantiennes et théorème limite local sur $\mathbb{R}^d$. *Probab. Theory Related Fields* **132** (1) (2005) 13–38. MR2136866 https://doi.org/10.1007/s00440-004-0388-1
[12] H. Furstenberg. Noncommuting random products. *Trans. Amer. Math. Soc.* **108** (3) (1963) 377–428. MR0163345 https://doi.org/10.2307/1993589
[13] H. Furstenberg. Boundary theory and stochastic processes on homogeneous spaces. *Proc. Sympos. Pure Math.* **26** (1973) 193–229. MR0352328
[14] H. Furstenberg and H. Kesten. Products of random matrices. *Ann. Math. Stat.* **31** (2) (1960) 457–469. MR0121828 https://doi.org/10.1214/aoms/1177705909
[15] B. V. Gnedenko. On a local limit theorem of the theory of probability. *Uspekhi Mat. Nauk* **3** (3) (1948) 187–194. MR0026275
[16] I. Y. Goldsheid and Y. Guivarc'h. Zariski closure and the dimension of the Gaussian law of the product of random matrices. *Probab. Theory Related Fields* **105** (1) (1996) 109–142. MR1389734 https://doi.org/10.1007/BF01192073
[17] I. Grama, R. Lauvergnat and É. Le Page. Conditioned local limit theorems for random walks defined on finite Markov chains. *Probab. Theory Related Fields* **176** (1–2) (2020) 669–735. MR4055198 https://doi.org/10.1007/s00440-019-00948-8

[18] Y. Guivarc'h. Produits de matrices aléatoires et applications aux propriétés géométriques des sous-groupes du groupe linéaire. *Ergodic Theory Dynam. Systems* **10** (3) (1990) 483–512. MR1074315 https://doi.org/10.1017/S0143385700005708

[19] Y. Guivarc'h. Spectral gap properties and limit theorems for some random walks and dynamical systems. *Proc. Sympos. Pure Math.* **89** (2015) 279–310. MR3309101 https://doi.org/10.1090/pspum/089/01488

[20] Y. Guivarc'h and É. Le Page. Simplicité de spectres de Lyapunov et propriété d'isolation spectrale pour une famille d'opérateurs de transfert sur l'espace projectif. In *Random Walks and Geometry*, V. A. Kaimanovich (Ed.). Proceedings of a workshop at the Erwin Schrödinger Institute, *Vienna*, *June 18–July 13, 2001*. de Gruyter, Berlin, 2001. In collaboration with Klaus Schmidt and Wolfgang Woess. Collected papers, (ISBN 3-11-017237-2/hbk). 181–259, 2004. MR2087783

[21] Y. Guivarc'h and É. Le Page. Spectral gap properties for linear random walks and Pareto's asymptotics for affine stochastic recursions. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2) (2016) 503–574. MR3498000 https://doi.org/10.1214/15-AIHP668

[22] Y. Guivarc'h and A. Raugi. Frontiere de Furstenberg, propriétés de contraction et théorèmes de convergence. *Probab. Theory Related Fields* **69** (2) (1985) 187–242. MR0779457 https://doi.org/10.1007/BF02450281

[23] R. Hartshorne. *Algebraic Geometry*, **52**. Springer, Berlin, 2013. MR0463157

[24] H. Hennion and L. Hervé. *Limit Theorems for Markov Chains and Stochastic Properties of Dynamical Systems by Quasi-Compactness*. *Lecture Notes in Mathematics* **1766**. Springer, Berlin, 2001. MR1862393 https://doi.org/10.1007/b87874

[25] É. Le Page. Théorèmes limites pour les produits de matrices aléatoires. In *Probability Measures on Groups* 258–303. Springer, Berlin, 1982. MR0669072

[26] P. Mattila. *Geometry of Sets and Measures in Euclidean Spaces: Fractals and Rectifiability*. *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2015. MR1333890 https://doi.org/10.1017/CBO9780511623813

[27] C. Stone. A local limit theorem for nonlattice multi-dimensional distribution functions. *Ann. Math. Stat.* **36** (2) (1965) 546–551. MR0175166 https://doi.org/10.1214/aoms/1177700165

[28] H. Xiao, I. Grama and Q. Liu. Berry-Esseen bound and precise moderate deviations for products of random matrices. *J. Eur. Math. Soc. (JEMS)* **24** (8) (2022) 2691–2750.

[29] H. Xiao, I. Grama and Q. Liu. Precise large deviation asymptotics for products of random matrices. *Stochastic Process. Appl.* **130** (9) (2020) 5213–5242. MR4127328 https://doi.org/10.1016/j.spa.2020.03.005

[30] H. Xiao, I. Grama and Q. Liu. Large deviation expansions for the coefficients of random walks on the general linear group, 2020. Preprint. Available at arXiv:2010.00553.

[31] H. Xiao, I. Grama and Q. Liu. Moderate deviations and local limit theorems for the coefficients of random walks on the general linear group, 2022. Preprint. Available at arXiv:2209.04628.

Annales de l'Institut Henri Poincaré - Probabilités et Statistiques
2022, Vol. 58, No. 4, 2347–2381
https://doi.org/10.1214/21-AIHP1210
© Association des Publications de l'Institut Henri Poincaré, 2022

The Erdős-Rényi-Shepp law of large numbers for ballistic random walk in random environment

Darcy Camargo[1,a], Yuri Kifer[2,c] and Ofer Zeitouni[1,b]

[1]*Department of Mathematics, Weizmann Institute, POB 26, Rehovot 7610001, Israel.* [a]*darcygcamargo@gmail.com,* [b]*ofer.zeitouni@weizmann.ac.il*
[2]*Department of Mathematics, Hebrew University, Givat Ram, Jerusalem 9190401, Israel.* [c]*yuri.kifer@mail.huji.ac.il*

Abstract. We consider a one dimensional ballistic nearest-neighbor random walk in a random environment. We prove an Erdős-Rényi–Shepp strong law for the increments.

Résumé. Nous considérons une marche aléatoire unidimensionnelle au plus proche voisin, en milieu aléatoire. Nous démontrons une loi forte de grands nombres de type Erdős-Rényi–Shepp pour les accroissements.

MSC2020 subject classifications: Primary 60G50; 60F15; secondary 60F10

Keywords: Random walks; Strong limit theorems; Large deviations

References

[1] F. Comets, N. Gantert and O. Zeitouni. Quenched, annealed and functional large deviations for one-dimensional random walk in random environment. *Probab. Theory Related Fields* **118** (2000) 65–114. MR1785454 https://doi.org/10.1007/s004400000074
[2] F. Comets, M. Menshikov and S. Popov. Lyapunov functions for random walks and strings in random environment. *Ann. Probab.* **26** (1998) 1433–1445. MR1675023 https://doi.org/10.1214/aop/1022855869
[3] A. Dembo and O. Zeitouni. *Large Deviation Techniques and Applications*, 2nd edition. Springer, Berlin, 1998. MR1619036 https://doi.org/10.1007/978-1-4612-5320-4
[4] P. Erdös and A. Rényi. On a new law of large numbers. *J. Anal. Math.* **23** (1970) 103–111. MR0272026 https://doi.org/10.1007/BF02795493
[5] L. A. Shepp. A limit law concerning moving averages. *Ann. Math. Stat.* **35** (1964) 424–428. MR0166818 https://doi.org/10.1214/aoms/1177703767
[6] F. Solomon. Random walks in a random environment. *Ann. Probab.* **3** (1975) 1–31. MR0362503 https://doi.org/10.1214/aop/1176996444
[7] S. R. S. Varadhan. *Probability Theory*. Am. Math. Soc., Providence, 2001. MR1852999 https://doi.org/10.1090/cln/007
[8] O. Zeitouni. In *Random Walks in Random Environment* 190–312. *Lecture Notes in Mathematics* **1837**, 2004. MR2071631 https://doi.org/10.1007/978-3-540-39874-5_2

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

Recommandations aux auteurs
Instructions to authors

Les *Annales de l'I.H.P., Probabilités et Statistiques*, sont une revue internationale publiant des articles originaux en français et en anglais. Le journal publie des articles de qualité reflétant les différents aspects des processus stochastiques, de la statistique mathématique et des domaines contigus.

A compter du 14 avril 2008, les Annales ont adopté le système EJMS pour soumettre et traiter les articles. Les auteurs sont encouragés à utiliser ce système et peuvent y accéder à l'adresse http://www.e-publications.org/ims/submission/. Des informations complémentaires se trouvent à http://www.imstat.org/aihp/mansub.html. Les articles peuvent être écrits en français ou en anglais. Sous le titre, les auteurs indiqueront leurs prénoms et noms ainsi qu'une désignation succincte de leur laboratoire – notamment l'adresse. Afin de faciliter la communication, il est aussi souhaitable que les auteurs fournissent une adresse de courrier électronique.

Les articles doivent être accompagnés d'un résumé précisant clairement les points essentiels développés dans l'article. Pour les articles en français, l'auteur est invité à fournir la traduction en anglais de ce résumé. Pour les articles en anglais, l'auteur est invité à fournir un résumé en français. Le comité éditorial pourra effectuer ces traductions le cas échéant.

Les références seront numérotées continûment, renvoyant à la liste bibliographique indiquant l'initiale du prénom + le nom de l'auteur, le titre de la publication, le titre de la Revue, l'année, la tomaison ou le cas échéant le numéro, les pages de début et de fin d'article et, dans le cas d'un livre, l'éditeur, le lieu et l'année d'édition.

Les articles acceptés pour publication sont considérés comme ne varietur. Les auteurs recevront une seule épreuve de leur article : celle-ci devra être retournée à l'éditeur dans le délai maximal d'une semaine. Toutes modifications ou corrections excessives autres que celles provenant d'erreurs typographiques peuvent être facturées aux auteurs. La publication des articles ou mémoires est gratuite, la facturation des pages est optionnelle. L'auteur correspondant recevra un fichier pdf de leur article final par courrier electronique.

Les auteurs sont encouragés à préparer leurs manuscrits au moyen de l'un des logiciels Plain TeX, LaTeX ou AMS TeX. Un support LATEX se trouve à l'adresse http://www.e-publications.org/ims/support/

The *Annales de l'I.H.P., Probabilites et Statistiques* is an international Journal publishing original articles in French or in English. The Journal publishes papers of high quality representing different aspects of the theory of stochastic processes, mathematical statistics and related fields.

Effective April 14, 2008, the Annales has adopted an Electronic Journal Management System (EJMS) for submission and handling of papers by the editorial board and reviewers for its journal. Authors are encouraged to use this system and should access EJMS at http://www.e-publications.org/ims/submission/. Please see http://www.imstat.org/aihp/mansub.html for additional details. Papers may be written in French or in English. The authors give their name below the title, together with their current affiliation and address. In order to facilitate communication, the authors should also provide an e-mail address.

Articles should begin with a summary explaining the basic points developed in the article. For papers in English, the authors should provide the translation in French of this summary. The Editorial committee will supply the translation if necessary. The authors of articles in French should provide the translation in English of this summary.

References should be numbered, referring to the bibliography giving for each reference the initial and name of the author followed by the title of the publication, the name of the Journal, the volume, the year, the pages of the article, and in the case of a book, the editor, the place and year of edition.

Papers accepted for publication are considered as ne variatur. The corresponding author will receive email regarding proofs, which should be sent back to the publisher within one week.

A charge may be made for any excessive corrections other than those due to typographical errors.

The publication of articles is free, page charges are optional. Every corresponding author will receive a pdf file via email of the final article. Paper offprints may be purchased by using the IMS Offprint Purchase Order Forms below.

The authors are encouraged to prepare their manuscripts using Plain TeX, LaTeX or AMS TeX. A LaTeX support page is available at http://www.e-publications.org/ims/support/

• **Editors-in-chief**
Giambattista Giacomin, *Université de Paris*
Yueyun Hu, *Université Sorbonne Paris Nord*

• **Editorial Board**
E. Aïdékon, *Fudan University*
S. Arlot, *Université Paris-Sud*
J. Bertoin, *Universität Zürich*
P. Bourgade, *New York Univ.*
F. Caravenna, *Univ. Milano-Bicocca*
D. Chafai, *Ecole Normale Supérieure, Paris*
I. Corwin, *Columbia University*
A. Debussche, *École Normale Supérieure de Rennes*
I. Dumitriu, *UC San Diego*
B. Gess, *Universität Bielefeld*
S. Gouëzel, *Université de Nantes*
A. Guillin, *Clermont-Auvergne University*
M. Hairer, *Imperial College London*
M. Hoffmann, *Univ. Paris-Dauphine*
N. Holden, *ETH Zurich*
T. Hutchcroft, *Cambridge University*
A. Nachmias, *Tel Aviv University*
J. Norris, *Cambridge University*
R. Rhodes, *Université Aix-Marseille*
J. Rousseau, *University of Oxford*
M. Sasada, *University of Tokyo*
P. Sousi, *Cambridge University*
B. de Tilière, *Univ. Paris-Dauphine*
V. Wachtel, *Universität München*
H. Weber, *Univ. of Warwick*

Indexations: *Current Contents (PC&ES), Zentralblatt für Mathematik, Inspec, Current Index to statistics, Pascal (INIST), Science Citation Index, SciSearch®, Research Alert®, Compu Math Citation Index®, MathSciNet. Also covered in the abstract and citation database SCOPUS®.*