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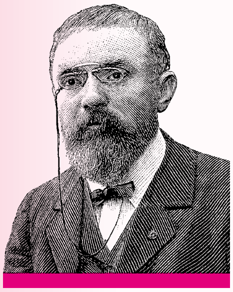

ANNALES DE L'INSTITUT HENRI POINCARÉ PROBABILITÉS ET STATISTIQUES

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Quantum triangles and imaginary geometry flow lines

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Abstract. We define a three-parameter family of random surfaces in Liouville quantum gravity (LQG) which can be viewed as the quantum version of triangles. These quantum triangles are natural in two senses. First, by our definition they produce the boundary three-point correlation functions of Liouville conformal field theory on the disk. Second, it turns out that the laws of the triangles bounded by flow lines in imaginary geometry coupled with LQG are given by these quantum triangles. In this paper we demonstrate the second point for boundary flow lines on a quantum disk. Our method has the potential to prove general conformal welding results with quantum triangles glued in an arbitrary way. Quantum triangles play a basic role in understanding the integrability of SLE and LQG via conformal welding. In this paper, we deduce integrability results for chordal SLE with three force points, using the conformal welding of a quantum triangle and a two-pointed quantum disk. Further applications will be explored in subsequent works.

Résumé. Nous définissons une famille à trois paramètres de surfaces aléatoires en gravité quantique de Liouville (LQG) qui peut être considérée comme la version quantique des triangles. Ces triangles quantiques sont naturels pour deux raisons. Premièrement, selon notre définition, ils produisent les fonctions de corrélation à trois points au bord de la théorie conforme des champs de Liouville sur le disque. Deuxièmement, il s'avère que les lois des triangles délimités par les lignes de flux dans la géométrie imaginaire couplée avec la LQG sont données par ces triangles quantiques. Dans cet article, nous démontrons le deuxième point pour les lignes de flot de bord sur un disque quantique. Notre méthode a le potentiel de prouver des résultats généraux de soudure conforme avec des triangles quantiques collés de manière arbitraire. Les triangles quantiques jouent un rôle fondamental dans la compréhension de l'intégrabilité de SLE et de LQG via la soudure conforme. Dans cet article, nous déduisons des résultats d'intégrabilité pour SLE chordale avec trois points de force, en utilisant la soudure conforme d'un triangle quantique et d'un disque quantique à deux points. D'autres applications seront explorées dans des travaux ultérieurs.

MSC2020 subject classifications: 60J67; 60G60

Keywords: Schramm–Loewner evolution; Liouville quantum gravity

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Scaling limits for fractional polyharmonic Gaussian fields

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Abstract. This work is concerned with fractional Gaussian fields, i.e., Gaussian fields whose covariance operator is given by the inverse fractional Laplacian $(-\Delta)^{-s}$ (where, in particular, we include the case $s > 1$). We define a lattice discretization of these fields and show that their scaling limits, with respect to the optimal Besov space topology (up to an endpoint case), are the original continuous fields. As a byproduct, in dimension $d < 2s$, we prove the convergence in distribution of the maximum of the fields. A key tool in the proof is a sharp error estimate for the natural finite difference scheme for $(-\Delta)^s$ under minimal regularity assumptions, which is also of independent interest.

Résumé. Ce travail traite des champs gaussiens fractionnaires, c'est-à-dire des champs gaussiens dont l'opérateur de covariance est donné par le Laplacien fractionnaire inverse $(-\Delta)^{-s}$ (où, en particulier, nous incluons le cas $s > 1$). Nous définissons une discrétisation en réseau de ces champs et montrons que leurs limites d'échelle, par rapport à la topologie optimale de l'espace de Besov (à l'exception d'un cas limite), sont les champs continus originaux. En conséquence, dans les dimensions $d < 2s$, nous obtenons la convergence en distribution du maximum des champs. Un outil clé dans la preuve est une estimation d'erreur optimale pour le schéma à différences finies naturel de $(-\Delta)^s$ dans le cadre d'hypothèses de régularité minimales, ce qui présente un intérêt indépendant.

MSC2020 subject classifications: Primary 60G15; secondary 35R11; 31B30; 60G60; 65N06

Keywords: Polyharmonic fractional Laplacian; Gaussian interface model; Scaling limit; Finite difference scheme; Besov spaces

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Lipschitz cutset for fractal graphs and applications to the spread of infections

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Abstract. We consider the fractal Sierpiński gasket or carpet graph in dimension $d \geq 2$, denoted by G. At time 0, we place a Poisson point process of particles onto the graph and let them perform independent simple random walks, which in this setting exhibit sub-diffusive behaviour. We generalise the concept of particle process dependent Lipschitz percolation to the (coarse graining of the) space-time graph $G \times \mathbb{R}$, where the opened/closed state of space-time cells is measurable with respect to the particle process inside the cell. We then provide an application of this generalised framework and prove the following: if particles can spread an infection when they share a site of G, and if they recover independently at some rate $\gamma > 0$, then if γ is sufficiently small, the infection started with a single infected particle survives indefinitely with positive probability.

Résumé. Nous considérons un graphe fractal G qui est un triangle ou un tapis de Sierpiński en dimension $d \geq 2$. À l'instant 0, nous plaçons un ensemble de particules sur le graphe, donné par un processus ponctuel de Poisson, et les laissons effectuer des marches aléatoires simples indépendantes, qui dans ce cadre présentent un comportement sous-diffusif. Nous généralisons le concept de percolation Lipschitz dépendante du processus de particules au graphe espace-temps $G \times \mathbb{R}$, où l'état ouvert/fermé des cellules espace-temps est mesurable par rapport au processus de particules à l'intérieur de la cellule. Nous fournissons ensuite une application de ce cadre généralisé et prouvons l'énoncé suivant : si les particules peuvent propager une infection lorsqu'elles partagent un site de G, et si elles se rétablissent indépendamment à un certain taux $\gamma > 0$, alors si γ est suffisamment petit, l'infection débutée avec une seule particule infectée survit indéfiniment avec une probabilité positive.

MSC2020 subject classifications: Primary 60K35; secondary 60G55

Keywords: Particle system; Fractal percolation; Sierpiński gasket; Infection spread; Sub-diffusive behaviour

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Diameter of uniform spanning trees on random weighted graphs

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Abstract. For any edge weight distribution, we consider the uniform spanning tree (UST) on finite graphs with i.i.d. random edge weights. We show that, for bounded degree expander graphs and finite boxes of $\mathbb{Z}^d$, the diameter of the UST is of order $n^{1/2+o(1)}$ with high probability, where n is the number of vertices.

Résumé. Pour toutes distributions des poids, on considère l'arbre couvrante uniforme (uniform spanning tree, UST) sur des graphes finis avec des poids i.i.d. sur les arêtes. On montre que, pour les graphes expanseurs avec degrés bornés et pour les boîtes finies de $\mathbb{Z}^d$, le diamètre de l'UST est d'ordre $n^{1/2+o(1)}$ avec grande probabilité, où n est le nombre de sommets du graphe.

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Keywords: Uniform spanning tree

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Percolation through isoperimetry

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Abstract. We provide a sufficient condition on the isoperimetric properties of a regular graph G of growing degree d, under which the random subgraph G_p typically undergoes a phase transition around $p = \frac{1}{d}$ which resembles the emergence of a giant component in the binomial random graph model $G(n, p)$. We further show that this condition is tight.

More precisely, let $d = \omega(1)$, let $\epsilon > 0$ be a small enough constant, and let $p \cdot d = 1 + \epsilon$. We show that if C is sufficiently large and G is a d-regular n-vertex graph where every subset $S \subseteq V(G)$ of order at most $\frac{n}{2}$ has edge-boundary of size at least $C|S|$, then G_p typically has a unique linear sized component, whose order is asymptotically $y(\epsilon)n$, where $y(\epsilon)$ is the survival probability of a Galton–Watson tree with offspring distribution $\mathrm{Po}(1 + \epsilon)$. We further give examples to show that this result is tight both in terms of its dependence on C, and with respect to the order of the second-largest component.

We also consider a more general setting, where we only control the expansion of sets up to size k. In this case, we show that if G is such that every subset $S \subseteq V(G)$ of order at most k has edge-boundary of size at least $d|S|$ and p is such that $p \cdot d \geq 1 + \epsilon$, then G_p typically contains a component of order $\Omega(k)$.

Résumé. Nous donnons une condition suffisante sur les propriétés isopérimétriques d'un graphe régulier G de degré croissant d, sous laquelle le sous-graphe aléatoire G_p subit typiquement une transition de phase autour de $p = \frac{1}{d}$, qui ressemble à l'apparition d'une composante géante dans le modèle de graphe aléatoire binomial $G(n, p)$. Nous montrons également que cette condition est optimale.

Plus précisément, soit $d = \omega(1)$, soit $\epsilon > 0$ une constante suffisamment petite, et soit $p \cdot d = 1 + \epsilon$. Nous montrons que, si C est suffisamment grand et que G est un graphe d-régulier à n sommets dans lequel chaque sous-ensemble $S \subseteq V(G)$ de taille au plus $\frac{n}{2}$ a un bord d'arêtes de longueur au moins $C|S|$, alors G_p possède typiquement une unique composante de taille linéaire, dont l'ordre est asymptotiquement $y(\epsilon)n$, où $y(\epsilon)$ est la probabilité de survie d'un arbre de Galton–Watson avec une loi de reproduction $\mathrm{Po}(1 + \epsilon)$. Nous donnons également des exemples montrant que ce résultat est optimal à la fois en termes de dépendance en C, et par rapport à l'ordre de la deuxième plus grande composante.

Nous considérons également un cadre plus général, où nous ne contrôlons que l'expansion des ensembles de taille maximale k. Dans ce cas, nous montrons que si G est tel que chaque sous-ensemble $S \subseteq V(G)$ de taille au plus k a un bord d'arêtes d'au moins $d|S|$, et que p est tel que $p \cdot d \geq 1 + \epsilon$, alors G_p contient typiquement une composante d'ordre $\Omega(k)$.

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Chaos, concentration and multiple valleys in first-passage percolation

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Abstract. A decade and a half ago Chatterjee established the first rigorous connection between anomalous fluctuations and a chaotic behaviour of the ground state in certain Gaussian disordered systems. The purpose of this paper is to show that Chatterjee's work gives evidence of a more general principle, by establishing an analogous connection between fluctuations and chaos in the context of first-passage percolation. The notion of 'chaos' here refers to the sensitivity of the time-minimising path between two points when exposed to a slight perturbation. More precisely, we resample a small proportion of the edge weights, and find that a vanishing fraction of the edges on the time-minimising path still belongs to the time-minimising path obtained after resampling. We also identify the point at which the system transitions from being stable to being chaotic in terms of the variance of the system. Finally we show that the chaotic behaviour implies the existence of a large number of almost-optimal paths that are almost disjoint from the time-minimising path, a phenomenon known as 'multiple valleys'.

Résumé. Il y a une quinzaine d'années, Chatterjee a établi la première connexion rigoureuse entre fluctuations anormales et comportement chaotique de l'état fondamental dans certains systèmes avec désordre gaussien. Le but de cet article est de montrer que le travail de Chatterjee apporte la preuve d'un principe plus général, en établissant une connexion analogue entre fluctuations et chaos dans le contexte de la percolation de premier passage. La notion de « chaos » fait ici référence à la sensibilité du chemin minimisant le temps entre deux points lorsqu'il est exposé à une légère perturbation. Plus précisément, nous ré-échantillonnons une petite proportion des poids des arêtes et constatons qu'une fraction nulle des arêtes sur le chemin minimisant le temps appartient toujours au chemin minimisant le temps obtenu après ré-échantillonnage. Nous identifions également le point auquel le système passe de stable à chaotique en termes de variance du système. Enfin, nous montrons que le comportement chaotique implique l'existence d'un grand nombre de chemins presque optimaux qui sont presque disjoints du chemin minimisant le temps, un phénomène connu sous le nom de « vallées multiples ».

MSC2020 subject classifications: 60K35

Keywords: First-passage percolation; Geodesics; Chaos; Noise sensitivity

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Brownian motion conditioned to have restricted L_2-norm

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Abstract. We condition a Brownian motion on having an atypically small L_2-norm on long time intervals. The obtained limiting process is a non-stationary Ornstein–Uhlenbeck process.

Résumé. On considère le mouvement Brownien conditionné à ce que sa norme L_2 sur un long intervalle de temps soit atypiquement petite. Le processus limite obtenu est un processus d'Ornstein–Uhlenbeck non-stationnaire.

MSC2020 subject classifications: Primary 60J65; 60J55; 60F99; secondary 60F10; 60G15

Keywords: Brownian motion; Conditioned process; L_2-norm; Ornstein–Uhlenbeck process

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Lévy measures on Banach spaces

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Abstract. In this work, we establish an explicit characterisation of Lévy measures on both L^p-spaces and UMD Banach spaces. In the case of L^p-spaces, Lévy measures are characterised by an integrability condition, which directly generalises the known description of Lévy measures on sequence spaces. The latter has been the only known description of Lévy measures on infinite dimensional Banach spaces that are not Hilbert. Lévy measures on UMD Banach spaces are characterised by the finiteness of the expectation of a random γ-radonifying norm. Although this description is more abstract, it reduces to simple integrability conditions in the case of L^p-spaces.

Résumé. Dans cet article, nous établissons une caractérisation explicite des mesures de Lévy à la fois sur les espaces L^p et les espaces de Banach UMD. Dans le cas des espaces L^p, les mesures de Lévy sont caractérisées par une condition d'intégrabilité, ce qui généralise directement la description connue des mesures de Lévy sur les espaces de suites. Cette dernière était la seule description connue des mesures de Lévy sur des espaces de Banach de dimension infinie qui ne sont pas des espaces de Hilbert. Les mesures de Lévy sur les espaces de Banach UMD sont caractérisées par la finitude de l'espérance d'une norme γ-radonifiante aléatoire. Bien que cette description soit plus abstraite, elle se réduit à des conditions d'intégrabilité simples dans le cas des espaces L^p.

MSC2020 subject classifications: Primary 60B05; secondary 46B09; 60E05; 60G57; 60H05

Keywords: Lévy measure; Poisson random measure; UMD-space

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A fixed-point equation approach for the superdiffusive elephant random walk

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Abstract. We study the elephant random walk in arbitrary dimension $d \geq 1$. Our main focus is the limiting random variable appearing in the superdiffusive regime. Building on a link between the elephant random walk and Pólya-type urn models, we prove a fixed-point equation (or system in dimension two and larger) for the limiting variable. Based on this, we deduce several properties of the limit distribution, such as the existence of a density with support on $\mathbb{R}^d$ for $d \in \{1, 2, 3\}$, and we bring evidence for a similar result for $d \geq 4$. We also investigate the moment-generating function of the limit and give, in dimension 1, a non-linear recurrence relation for the moments.

Résumé. Nous étudions la marche aléatoire de l'éléphant en toute dimension $d \geq 1$. Notre objectif principal est l'étude de la variable aléatoire limite apparaissant dans le régime superdiffusif. En nous appuyant sur un lien entre la marche aléatoire de l'éléphant et les modèles d'urnes de Pólya, nous prouvons une équation (ou un système en dimension deux et plus) de point fixe pour la variable limite. Nous en déduisons plusieurs propriétés de la distribution limite, comme l'existence d'une densité avec support sur tout $\mathbb{R}^d$ pour $d \in \{1, 2, 3\}$, et sommes proches d'un résultat similaire pour $d \geq 4$. Nous étudions également la fonction génératrice des moments de la limite et donnons, en dimension 1, une relation de récurrence non linéaire pour les moments.

MSC2020 subject classifications: 60E05; 60E10; 60J10; 60G50

Keywords: Elephant random walk; Multi-dimensional elephant random walk; Pólya-type urn models; Asymptotic distribution; Fixed-point equations; Krylov subspaces

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Double jump in the maximum of two-type reducible branching Brownian motion

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Abstract. Consider a two-type reducible branching Brownian motion in which particles' diffusion coefficients and branching rates are influenced by their types. Here reducible means that type 1 particles can produce particles of type 1 and type 2, but type 2 particles can only produce particles of type 2. The maximum of this process is determined by two parameters: the ratio of the diffusion coefficients and the ratio of the branching rates for particles of different types. Belloum and Mallein (Electron. J. Probab. **26** (2021) 61) identified three phases of the maximum and the extremal process, corresponding to three regions in the parameter space.

We investigate how the extremal process behaves asymptotically when the parameters lie on the boundaries between these regions. An interesting phenomenon is that a double jump occurs in the maximum when the parameters cross the boundary of the so called anomalous spreading region, while only a single jump occurs when the parameters cross the boundary between the remaining two regions.

Résumé. Considérons un mouvement brownien branchant à deux types, réductible, dans lequel les coefficients de diffusion et les taux de branchement des particules dépendent de leur type. Ici, "réductible" signifie que les particules de type 1 peuvent produire des particules de type 1 et de type 2, alors que les particules de type 2 ne peuvent produire que des particules de type 2. Le maximum de ce processus est déterminé par deux paramètres : le rapport des coefficients de diffusion et le rapport des taux de branchement pour les particules de différents types. Belloum et Mallein (Electron. J. Probab. **26** (2021) 61) ont identifié trois phases du maximum et du processus extrémal, correspondant à trois régions dans l'espace des paramètres.

Nous étudions le comportement asymptotique du processus extrémal lorsque les paramètres se situent sur les frontières entre ces régions. Un phénomène intéressant est qu'un double saut se produit dans le maximum lorsque les paramètres franchissent la frontière de la région dite de diffusion anormale, tandis qu'un simple saut se produit lorsque les paramètres franchissent la frontière entre les deux autres régions.

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Keywords: Branching Brownian motion; Double jump; Extremal process; Reducible branching process; Brownian motion

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Multifractal analysis and Erdös–Rényi laws of large numbers for branching random walks in $\mathbb{R}^d$

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Abstract. We revisit the multifractal analysis of $\mathbb{R}^d$-valued branching random walks averages by considering subsets of full Hausdorff dimension of the standard level sets, over each infinite branch of which a quantified version of the Erdös–Rényi law of large numbers holds. Assuming that the exponential moments of the increments of the walks are finite, we can indeed control simultaneously such sets when the levels belong to the interior of the compact convex domain I of possible levels, i.e. when they are associated to so-called Gibbs measures, as well as when they belong to the subset $(\partial I)_{\text{crit}}$ of ∂I made of levels associated to "critical" versions of these Gibbs measures. It turns out that given a level of one of these two types, the associated Erdös–Rényi LLN depends on the metric with which is endowed the boundary of the underlying Galton–Watson tree. To extend our control to all the boundary points in cases where $\partial I \neq (\partial I)_{\text{crit}}$, we slightly strengthen our assumption on the distribution of the increments to exhibit a natural decomposition of $\partial I \setminus (\partial I)_{\text{crit}}$ into at most countably many convex sets J of affine dimension $\leq d-1$ over each of which we can essentially reduce the study to that of interior points and critical points associated to some $\mathbb{R}^{\dim J}$-valued branching random walk.

Résumé. On revisite l'analyse multifractale des moyennes de marches aléatoires de branchement à valeurs dans $\mathbb{R}^d$ en considérant des sous-ensembles de dimension de Hausdorff pleine des ensembles de niveaux standards, sur chaque branche infinie desquels on observe une version quantifiée de la loi des grands nombres d'Erdös–Rényi. En supposant que les moments exponentiels des accroissements des marches sont finis, on peut en effet contrôler simultanément de tels ensembles lorsque les niveaux sont à l'intérieur du domaine convexe compact I des niveaux possibles, c'est-à-dire lorsqu'ils sont associés à des mesures dites de Gibbs, ainsi que lorsqu'ils sont dans le sous-ensemble $(\partial I)_{\text{crit}}$ de ∂I constitué des niveaux associés aux versions « critiques » de ces mesures de Gibbs ; étant donné un niveau d'un de ces deux types, la loi d'Erdös–Rényi associée dépend de la métrique dont est munie la frontière de l'arbre de Galton–Watson sous-jacent. Pour étendre le contrôle à tous les points limites dans les cas où $\partial I \neq (\partial I)_{\text{crit}}$, on renforce légèrement l'hypothèse sur la distribution des incréments afin d'exhiber une décomposition naturelle de $\partial I \setminus (\partial I)_{\text{crit}}$ en un nombre au plus dénombrable d'ensembles convexes J de dimension affine $\leq d-1$ sur chacun desquels on peut essentiellement réduire l'étude à celle des points intérieurs et des points critiques associés à une marche aléatoire de branchement à valeurs dans $\mathbb{R}^{\dim J}$.

MSC2020 subject classifications: Primary 28A78; 28A80; secondary 60F10; 60G50; 60G57

Keywords: Hausdorff dimension; Large deviations; Branching random walk; Percolation; Multiplicative chaos

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Limit theorems for first passage times of multivariate perpetuity sequences

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Abstract. We study the first passage time $\tau_u = \inf\{n \geq 1 : |V_n| > u\}$ for the multivariate perpetuity sequence $V_n = Q_1 + M_1 Q_2 + \cdots + (M_1 \ldots M_{n-1}) Q_n$, where (M_n, Q_n) is a sequence of independent and identically distributed random variables with M_1 a $d \times d$ ($d \geq 1$) random matrix with nonnegative entries, and Q_1 a nonnegative random vector in $\mathbb{R}^d$. Here $|\cdot|$ denotes the vector norm. The exact asymptotic for the probability $\mathbb{P}(\tau_u < \infty)$ as $u \to \infty$ has been found by Kesten (*Acta Math.* **131** (1973) 207–248). In this paper we prove a conditioned weak law of large numbers for τ_u: conditioned on the event $\{\tau_u < \infty\}$, $\frac{\tau_u}{\log u}$ converges in probability to a certain constant $\rho > 0$ as $u \to \infty$. A conditioned central limit theorem for τ_u is also obtained. We further establish precise large deviation asymptotics for the lower probability $\mathbb{P}(\tau_u \leq (\beta - l)\log u)$ as $u \to \infty$, where $\beta \in (0, \rho)$ and $l \geq 0$ is a vanishing perturbation satisfying $l \to 0$ as $u \to \infty$. Our results extend those of Buraczewski et al. (*Ann. Probab.* **44** (2016) 3688–3739) from the univariate case ($d = 1$) to the multivariate case ($d > 1$). As consequences, we deduce exact asymptotics for the pointwise probability $\mathbb{P}(\tau_u = \lfloor (\beta - l)\log u \rfloor)$ and the local probability $\mathbb{P}(\tau_u - (\beta - l)\log u \in (a, a + m])$, where $a < 0$ and $m \in \mathbb{Z}_+$. We also establish analogous results for the first passage time $\tau_u^y = \inf\{n \geq 1 : \langle y, V_n \rangle > u\}$, where y is a nonnegative vector in $\mathbb{R}^d$ with $|y| = 1$.

Résumé. Nous étudions le temps de premier passage $\tau_u = \inf\{n \geq 1 : |V_n| > u\}$ pour la suite de perpétuité multivariée $V_n = Q_1 + M_1 Q_2 + \cdots + (M_1 \ldots M_{n-1}) Q_n$, où (M_n, Q_n) est une séquence de variables aléatoires indépendantes et identiquement distribuées, avec M_1 une matrice aléatoire $d \times d$ ($d \geq 1$) à coefficients non négatifs, et Q_1 un vecteur aléatoire non négatif dans $\mathbb{R}^d$. Ici, $|\cdot|$ désigne la norme du vecteur. L'asymptotique exacte de la probabilité $\mathbb{P}(\tau_u < \infty)$ lorsque $u \to \infty$ a été trouvée par Kesten (*Acta Math.* **131** (1973) 207–248). Dans cet article, nous prouvons une loi faible des grands nombres conditionnée pour τ_u : conditionnellement à l'événement $\{\tau_u < \infty\}$, $\frac{\tau_u}{\log u}$ converge en probabilité vers une certaine constante $\rho > 0$ lorsque $u \to \infty$. Un théorème limite central conditionné pour τ_u est également obtenu. Nous établissons en outre des asymptotiques précises de grandes déviations pour la probabilité inférieure $\mathbb{P}(\tau_u \leq (\beta - l)\log u)$ lorsque $u \to \infty$, où $\beta \in (0, \rho)$ et $l \geq 0$ est une perturbation tendant vers zéro lorsque $u \to \infty$. Nos résultats étendent ceux de Buraczewski et al. (*Ann. Probab.* **44** (2016) 3688–3739) du cas univarié ($d = 1$) au cas multivarié ($d > 1$). Comme conséquences, nous déduisons les asymptotiques exactes de la probabilité ponctuelle $\mathbb{P}(\tau_u = \lfloor (\beta - l)\log u \rfloor)$ et la probabilité locale $\mathbb{P}(\tau_u - (\beta - l)\log u \in (a, a + m])$, où $a < 0$ et $m \in \mathbb{Z}_+$. Nous établissons également des résultats analogues pour le temps de premier passage $\tau_u^y = \inf\{n \geq 1 : \langle y, V_n \rangle > u\}$, où y est un vecteur non négatif dans $\mathbb{R}^d$ avec $|y| = 1$.

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Keywords: Multivariate perpetuity sequences; First passage time; Law of large numbers; Central limit theorem; Large deviations; Products of random matrices

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Multi-scale McKean–Vlasov SDEs: Moderate deviation principle in different regimes

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Abstract. The main aim of this paper is to study the moderate deviation principle for McKean–Vlasov stochastic differential equations with multiple scales. Specifically, we are interested in the asymptotic estimates of the deviation processes $\frac{X^\delta-\bar{X}}{\lambda(\delta)}$ as $\delta \to 0$ in different regimes (i.e. $\varepsilon = o(\delta)$ and $\varepsilon = O(\delta)$), where δ stands for the intensity of the noise and $\varepsilon := \varepsilon(\delta)$ stands for the time scale separation. The rate functions in two regimes are different, in particular, we show that it is strongly affected by the noise of the fast component in latter regime, which is essentially different from the former one and the case of large deviations (cf. (*Probab. Theory Related Fields* **187** (2023) 133–201)). As a by-product, the explicit representation formulas of the rate functions in all of regimes are also given. The main techniques are based on the weak convergence approach and the functional occupation measure approach.

Résumé. Le principal objectif de cet article est d'étudier le principe de déviations modérées pour les équations différentielles stochastiques de McKean–Vlasov à multiples échelles. Plus précisément, nous nous intéressons aux estimations asymptotiques des processus de déviation $\frac{X^\delta-\bar{X}}{\lambda(\delta)}$ lorsque $\delta \to 0$ dans différents régimes (c'est-à-dire $\varepsilon = o(\delta)$ et $\varepsilon = O(\delta)$), où δ représente l'intensité du bruit et $\varepsilon := \varepsilon(\delta)$ représente la séparation des échelles de temps. Les fonctions de taux dans les deux régimes sont différentes ; en particulier, nous montrons qu'elles sont fortement influencées par le bruit de la composante rapide dans le second régime, ce qui est fondamentalement différent du premier régime et du cas des grandes déviations (cf. (*Probab. Theory Related Fields* **187** (2023) 133–201)). En outre, des formules explicites de représentation des fonctions de taux dans tous les régimes sont également données. Les principales techniques reposent sur les approches de la convergence faible et de la mesure d'occupation fonctionnelle.

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Keywords: McKean–Vlasov SDEs; Moderate deviation principle; Multi-scale; Weak convergence approach; Occupation measure

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The constant coefficient in precise Laplace asymptotics for gPAM

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Abstract. This article resumes the analysis of precise Laplace asymptotics for the generalised Parabolic Anderson Model (gPAM) initiated by Peter Friz and the author. More precisely, we provide an explicit formula for the constant coefficient in the asymptotic expansion in terms of traces and Carleman–Fredholm determinants of certain explicit operators under only slightly stronger assumptions. The proof combines classical Gaussian analysis in abstract Wiener spaces with arguments from the theory of regularity structures. As an ingredient, we prove that the minimiser in the (extended) phase functional of gPAM has better than just Cameron–Martin regularity.

Résumé. Cet article reprend l'analyse des asymptotiques précises de Laplace pour le modèle parabolique d'Anderson généralisé (gPAM), initiée par Peter Friz et l'auteur. Plus précisément, nous fournissons une formule explicite pour le coefficient constant dans le développement asymptotique en termes de traces et de déterminants de Carleman–Fredholm de certains opérateurs explicites, sous des hypothèses seulement légèrement renforcées. La démonstration combine l'analyse gaussienne classique dans des espaces de Wiener abstraits avec des arguments issus de la théorie des structures de régularité. Un ingrédient de la preuve consiste à montrer que le minimiseur dans la fonctionnelle de phase (étendue) du gPAM possède une régularité supérieure à celle de Cameron–Martin.

MSC2020 subject classifications: Primary 60F10; 60H17; 60L30; secondary 60L20

Keywords: Large deviations; Laplace asymptotics; Regularity structures; Singular stochastic PDEs

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On particle systems and critical strengths of general singular interactions

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Dedicated to the memory of Yu. A. Semënov

Abstract. For finite interacting particle systems with strong repulsing-attracting or general interactions, we prove global weak well-posedness almost up to the critical threshold of the strengths of attracting interactions (independent of the number of particles), and establish other regularity results, such as a heat kernel bound in the regions where strongly attracting particles are close to each other. Our main analytic instruments are a variant of De Giorgi's method in L^p and an abstract desingularization theorem.

Résumé. Pour les systèmes finis de particules avec de fortes interactions attractives-répulsives ou plus générales, on montre l'existence globale et l'unicité essentiellement jusqu'à la valeur critique de l'intensité des interactions attractives (indépendant du nombre des particules) et obtient d'autres résultats sur la régularité du système, y compris une estimation sur le noyau de la chaleur dans les régions où les particules sont proches les unes aux autres. Nos instruments analytiques principaux sont la méthode de De Giorgi dans L^p et un théorème abstrait sur la desingularisation.

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On the regularity of solutions of some linear parabolic path-dependent PDEs

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Abstract. We study a class of linear parabolic path-dependent PDEs (PPDEs) defined on the space of càdlàg paths $\mathbf{x} \in D([0, T])$, in which the coefficient functions at time t depend on $\mathbf{x}(t)$ and $\int_0^t \mathbf{x}(s)\, dA_s$, for some (deterministic) continuous function A with bounded variations. Under uniform ellipticity and Hölder regularity conditions on the coefficients, together with some technical conditions on A, we obtain the existence of a smooth solution to the PPDE by appealing to the notion of Dupire's derivatives. It provides a generalization to the existing literature studying the case where $A_t = t$, and complements our recent work in (*Ann. Appl. Probab.* **33** (2023) 5781–5809) on the regularity of approximate viscosity solutions for parabolic PPDEs. As a by-product, we also obtain existence and uniqueness of weak solutions for a class of path-dependent SDEs.

Résumé. Nous étudions une classe d'EDP dépendant du chemin définies sur l'espace de Skorokhod des trajectoires càdlàg $\mathbf{x} \in D([0, T])$, où les coefficients à l'instant $t \in [0, T]$ dépendent de $\mathbf{x}(t)$ et $\int_0^t \mathbf{x}(s)\, dA_s$, pour une fonction continue A à variation finie. Sous des conditions d'ellipticité et de régularité Hölder sur les coefficients, et sur les variations de A, nous obtenons l'existence d'une (unique) solution régulière au sens de Dupire. Ceci étend de manière non triviale le cas $A_t = t$ déjà étudié, et complémente le travail récent de (*Ann. Appl. Probab.* **33** (2023) 5781–5809) sur la régularité des *solutions de viscosité approchées* pour les EDP dépendant du chemin. Comme sous-produit, nous obtenons l'existence et l'unicité de la solution faible pour une classe d'EDS avec coefficients dépendant du chemin.

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Keywords: Path-dependent PDE; Degenerate parabolic PDE; Dupire's functional calculus

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On optimal error rates for strong approximation of SDEs with a drift coefficient of fractional Sobolev regularity

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Abstract. We study strong approximation of scalar additive noise driven stochastic differential equations (SDEs) at time point 1 in the case that the drift coefficient is bounded and has Sobolev regularity $s \in (0,1)$. Recently, it has been shown in (*Ann. Appl. Probab.* **33** (2023) 2291–2323) that for such SDEs the equidistant Euler approximation achieves an L^2-error rate of at least $(1+s)/2$, up to an arbitrary small ε, in terms of the number of evaluations of the driving Brownian motion W. In the present article we prove a matching lower error bound for $s \in (1/2,1)$. More precisely we show that, for every $s \in (1/2,1)$, the L^2-error rate $(1+s)/2$ can, up to a logarithmic term, not be improved in general by no numerical method based on finitely many evaluations of W at fixed time points. Up to now, this result was known in the literature only for the cases $s = 1/2-$ and $s = 1-$.

For the proof we employ the coupling of noise technique recently introduced in (*Ann. Appl. Probab.* **33** (2023) 902–935) to bound the L^2-error of an arbitrary approximation from below by the L^2-distance of two occupation time functionals provided by a specifically chosen drift coefficient with Sobolev regularity s and two solutions of the corresponding SDE with coupled driving Brownian motions. For the analysis of the latter distance we employ a transformation of the original SDE to overcome the problem of correlated increments of the difference of the two coupled solutions, occupation time estimates to cope with the lack of regularity of the chosen drift coefficient around the point 0 and scaling properties of the drift coefficient.

Résumé. Nous étudions l'approximation forte des équations différentielles stochastiques (EDS) scalaires à bruit additif à l'instant $t = 1$, dans le cas où le coefficient de dérive est borné et possède une régularité de Sobolev $s \in (0,1)$. Récemment, il a été démontré dans (*Ann. Appl. Probab.* **33** (2023) 2291–2323) que, pour de telles EDS, l'approximation d'Euler équidistante atteint un taux d'erreur L^2 d'au moins $(1+s)/2$, à une petite constante arbitraire ε près, en fonction du nombre d'évaluations du mouvement brownien W qui dirige l'équation. Dans cet article, nous établissons une borne inférieure correspondante pour $s \in (1/2,1)$. Plus précisément, nous montrons que, pour tout $s \in (1/2,1)$, le taux $(1+s)/2$ d'erreur L^2 ne peut, en général, être amélioré (à un terme logarithmique près) par aucune méthode numérique basée sur un nombre fini d'évaluations de W à des instants fixes. Jusqu'à présent, ce résultat n'était connu dans la littérature que pour les cas $s = 1/2-$ et $s = 1-$.

Pour la preuve, nous utilisons la technique de couplage de bruit récemment introduite dans (*Ann. Appl. Probab.* **33** (2023) 902–935) pour minorer l'erreur L^2 d'une approximation quelconque par la distance L^2 entre deux fonctionnelles du temps d'occupation, fournies par un coefficient de dérive spécialement choisi ayant une régularité de Sobolev s et deux solutions de l'EDS correspondante dirigées par des mouvements browniens couplés. Pour analyser la dite distance, nous utilisons une transformation de l'EDS originale pour surmonter le problème des incréments corrélés de la différence entre les deux solutions couplées, des estimations du temps d'occupation pour gérer le manque de régularité du coefficient de dérive choisi autour du point 0, et des propriétés d'échelle du coefficient de dérive.

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Keywords: Stochastic differential equations; Sobolev regularity; Strong approximation; Lower error bounds

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Determinantal structures for Bessel fields

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Abstract. A Bessel field $\mathcal{B} = \{\mathcal{B}_i(\alpha, t), \alpha \in \mathbb{N}_0, t \in \mathbb{R}, i \in \mathbb{N}\}$ is a random process with three parameters (α, t, i). When the parameters α and t are fixed, a Bessel field has the law of a Bessel point process with index α. The Bessel fields arise as hard edge scaling limits of the Laguerre field, a natural extension of the classical Laguerre unitary ensemble. It is recently proved in (*Electron. J. Probab.* **28** (2023) Paper No. 77) that for fixed α, $\{\mathcal{B}_i(\alpha, t), t \in \mathbb{R}, i \in \mathbb{N}\}$ is a squared Bessel Gibbsian line ensemble. In this paper, we discover rich integrable structures for the Bessel fields: along a time-like or a space-like path, $\mathcal{B}$ is a determinantal point process with an explicit correlation kernel; for fixed t, $\{\mathcal{B}_i(\alpha, t), \alpha \in \mathbb{N}_0, i \in \mathbb{N}\}$ is an exponential Gibbsian line ensemble.

Résumé. Un champ de Bessel $\mathcal{B} = \{\mathcal{B}_i(\alpha, t), \alpha \in \mathbb{N}_0, t \in \mathbb{R}, i \in \mathbb{N}\}$ est un processus aléatoire à trois paramètres (α, t, i). Lorsque les paramètres α et t sont fixés, un champ de Bessel suit la loi d'un processus ponctuel de Bessel d'indice α. Les champs de Bessel apparaissent comme limites d'échelle au bord dur du champ de Laguerre, une extension naturelle de l'ensemble unitaire de Laguerre classique. Il a été récemment démontré dans (*Electron. J. Probab.* **28** (2023) Paper No. 77) que pour un α fixé, $\{\mathcal{B}_i(\alpha, t), t \in \mathbb{R}, i \in \mathbb{N}\}$ est un ensemble de lignes gibbsien du carré de Bessel. Dans cet article, nous mettons en évidence de riches structures intégrables pour les champs de Bessel : le long d'un chemin de type temps ou de type espace, $\mathcal{B}$ est un processus ponctuel déterminantal avec un noyau de corrélation explicite ; pour un t fixé, $\{\mathcal{B}_i(\alpha, t), \alpha \in \mathbb{N}_0, i \in \mathbb{N}\}$ est un ensemble de lignes gibbsien exponentiel.

MSC2020 subject classifications: Primary 60B20; 60G55; secondary 60B10

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Stochastic equations for interacting particle systems with continuous spins

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Abstract. We study a general class of interacting particle systems over a countable state space V where on each site $x \in V$, the particle mass $\eta(x) \geq 0$ follows a stochastic differential equation. We construct the corresponding Markovian dynamics in terms of strong solutions to an infinite coupled system of stochastic differential equations and prove a comparison principle with respect to the initial configuration as well as the drift of the process. Using this comparison principle, we provide sufficient conditions for the existence and uniqueness of an invariant measure in the subcritical regime and prove convergence of the transition probabilities in the Wasserstein-1-distance. Finally, we establish a linear growth theorem for sublinear drifts showing that the spatial spread is at most linear in time. Our results cover a large class of finite and infinite branching particle systems with interactions among different sites.

Résumé. Nous étudions une classe générale de systèmes de particules en interaction sur un espace d'états dénombrable V où, sur chaque site $x \in V$, la masse de la particule $\eta(x) \geq 0$ suit une équation différentielle stochastique. Nous construisons la dynamique markovienne correspondante en termes de solutions fortes d'un système couplé infini d'équations différentielles stochastiques et prouvons un principe de comparaison en ce qui concerne la configuration initiale ainsi que la dérive du processus. En utilisant ce principe de comparaison, nous fournissons des conditions suffisantes pour l'existence et l'unicité d'une mesure invariante dans le régime sous-critique et prouvons la convergence des probabilités de transition dans la distance de Wasserstein-1. Enfin, nous établissons un théorème de croissance linéaire pour les dérives sous-linéaires montrant que la propagation spatiale est au plus linéaire en temps. Nos résultats couvrent une large classe de systèmes de particules à ramifications finies et infinies avec des interactions entre différents sites.

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Invariant measure of gaps in degenerate competing three-particle systems

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Abstract. We study the gap processes in a degenerate system of three particles interacting through their ranks. We obtain the Laplace transform of the invariant measure of these gaps, and an explicit expression for the corresponding invariant density. To derive these results, we start from the basic adjoint relationship characterizing the invariant measure, and apply a combination of two approaches: the invariance methodology of W. Tutte, thanks to which we compute the Laplace transform in closed form; and a recursive compensation approach, which leads to the density of the invariant measure as an infinite sum of exponential functions. As in the case of Brownian motion with reflection or killing at the endpoints of an interval, certain Jacobi theta functions play a crucial role in our computations.

Résumé. Nous étudions le processus des écarts entre particules ordonnées dans un système dégénéré de trois particules en interaction. Nous obtenons la transformée de Laplace de la mesure invariante de ces écarts ainsi qu'une expression explicite de la densité invariante correspondante. Pour obtenir ces résultats, nous partons de la "relation adjointe de base" qui caractérise la mesure invariante, et nous appliquons une combinaison de deux approches : d'abord le concept d'invariants de W. Tutte, grâce auquel nous obtenons une formule explicite pour la transformée de Laplace ; ensuite une approche récursive dite de compensation, qui conduit à la densité de la mesure invariante comme somme infinie de fonctions exponentielles. Comme pour le mouvement brownien réfléchi ou tué aux bords d'un intervalle, certaines fonctions thêta de Jacobi jouent un rôle crucial dans nos calculs.

MSC2020 subject classifications: Primary 60J65; 60K35; secondary 35Q70; 60J70

Keywords: Competing particle systems; Reflected planar Brownian motion; Invariant measure; Tutte's invariant method; Recursive compensation approach

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Stable distributions and domains of attraction for unitarily invariant Hermitian random matrix ensembles

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Abstract. We consider random matrix ensembles on the set of Hermitian matrices that are heavy tailed, in particular not all moments exist, and that are invariant under the conjugate action of the unitary group. The latter property entails that the eigenvectors are Haar distributed and, therefore, factorise from the eigenvalue statistics. We prove a classification for stable matrix ensembles of this kind of matrices represented in terms of matrices, their eigenvalues and their diagonal entries with the help of the classification of the multivariate stable distributions and the harmonic analysis on symmetric matrix spaces. Moreover, we identify sufficient and necessary conditions for their domains of attraction. To illustrate our findings we discuss for instance elliptical invariant random matrix ensembles and Pólya ensembles, the latter playing a particular role in matrix convolutions. As a byproduct we generalise the derivative principle on the Hermitian matrices to general tempered distributions. This principle relates the joint probability density of the eigenvalues and the diagonal entries of the random matrix.

Résumé. Nous considérons des ensembles de matrices aléatoires sur l'ensemble des matrices hermitiennes à queue lourde, pour lesquelles en particulier certains moments n'existent pas, et qui sont invariantes sous l'action conjuguée du groupe unitaire. Cette dernière propriété implique que les vecteurs propres sont distribués selon la mesure de Haar et, par conséquent, se factorisent de la statistique des valeurs propres. Nous prouvons une classification pour les ensembles de matrices stables de ce type de matrices, représentées en termes de matrices, de leurs valeurs propres et de leurs entrées diagonales à l'aide de la classification des distributions stables multivariées et de l'analyse harmonique sur les espaces matriciels symétriques. De plus, nous identifions les conditions nécessaires et suffisantes pour leurs domaines d'attraction. Pour illustrer nos résultats, nous discutons par exemple des ensembles de matrices aléatoires invariantes elliptiques et des ensembles de Pólya, ces derniers jouant un rôle particulier dans les convolutions matricielles. Comme sous-produit, nous généralisons aux distributions tempérées générales le principe de dérivation sur les matrices hermitiennes. Ce principe relie la densité de probabilité conjointe des valeurs propres et les entrées diagonales de la matrice aléatoire.

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Keywords: Random matrices; Heavy tails; Central limit theorems; Spherical transform

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Edgeworth-type expansion for the one-point distribution of the KPZ fixed point with a large height at a prior location

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Abstract. We consider the Kardar–Parisi–Zhang (KPZ) fixed point $\mathrm{H}(x,\tau)$ with the narrow-wedge initial condition and investigate the distribution of $\mathrm{H}(x,\tau)$ conditioned on a large height at an earlier space-time point $\mathrm{H}(x',\tau')$. As $\mathrm{H}(x',\tau')$ tends to infinity, we prove that the conditional one-point distribution of $\mathrm{H}(x,\tau)$ in the regime $\tau>\tau'$ converges to the Gaussian Unitary Ensemble (GUE) Tracy–Widom distribution and that the next two lower-order error terms can be expressed as derivatives of the Tracy–Widom distribution. The lowe order expansion here is analogue to the Edgeworth expansion in the central limit theorem. These KPZ-type limiting behaviors are different from the Gaussian-type ones obtained in (Liu and Wang (2022)) where they study the finite-dimensional distribution of $\mathrm{H}(x,\tau)$ conditioned on a large height at a later space-time point $\mathrm{H}(x',\tau')$. They show, with the narrow-wedge initial condition, that the conditional random field $\mathrm{H}(x,\tau)$ in the regime $\tau<\tau'$ converges to the minimum of two independent Brownian bridges modified by linear drifts as $\mathrm{H}(x',\tau')$ goes to infinity. The two results stated above provide the phase diagram of the asymptotic behaviors of a conditional law of KPZ fixed point in the regimes $\tau>\tau'$ and $\tau<\tau'$ when $\mathrm{H}(x',\tau')$ goes to infinity.

Résumé. Nous considérons le point fixe de Kardar–Parisi–Zhang (KPZ) $\mathrm{H}(x,\tau)$ avec la condition initiale de type "narrow-wedge" et étudions la distribution de $\mathrm{H}(x,\tau)$ conditionnée par une grande hauteur à un point d'espace-temps antérieur $\mathrm{H}(x',\tau')$. Comme $\mathrm{H}(x',\tau')$ tend vers l'infini, nous prouvons que la distribution conditionnelle à un point de $\mathrm{H}(x,\tau)$ dans le régime $\tau>\tau'$ converge vers la distribution de Tracy–Widom de l'ensemble unitaire gaussien (GUE) et que les deux termes d'erreur d'ordre inférieur suivants peuvent être exprimés comme des dérivées de la distribution de Tracy–Widom. Le développement d'ordre inférieur ici est analogue au développement d'Edgeworth dans le théorème central limite. Ces comportements limites de type KPZ sont différents de ceux de type gaussien obtenus dans (Liu and Wang (2022)) où ils étudient la distribution de dimension finie de $\mathrm{H}(x,\tau)$ conditionnée par une grande hauteur à un point d'espace-temps ultérieur $\mathrm{H}(x',\tau')$. Ils montrent, avec la condition initiale de type "narrow-wedge", que le champ aléatoire conditionnel $\mathrm{H}(x,\tau)$ dans le régime $\tau<\tau'$ converge vers le minimum de deux ponts browniens indépendants modifiés par des dérives linéaires lorsque $\mathrm{H}(x',\tau')$ tend vers l'infini. Les deux résultats énoncés ci-dessus fournissent le diagramme de phase des comportements asymptotiques d'une loi conditionnelle du point fixe de KPZ dans les régimes $\tau>\tau'$ et $\tau<\tau'$ lorsque $\mathrm{H}(x',\tau')$ tend vers l'infini.

MSC2020 subject classifications: 60K35; 60F99

Keywords: Tracy–Widom distribution; Edgeworth-type expansion; KPZ fixed point

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On Orlicz spaces satisfying the Hoffmann-Jørgensen inequality

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Abstract. Building on Talagrand's proof of the Hoffmann-Jørgensen inequality for L_p spaces and its version for the exponential Orlicz spaces we provide a full characterization of Orlicz functions Ψ for which an analogous inequality holds in the Orlicz space $L_\Psi(F)$, where F is an arbitrary Banach space.

As an application we present a characterization of Talagrand-type concentration inequality for suprema of empirical processes with envelope in L_Ψ (equivalently for sums of independent random variables in $L_\Psi(F)$). This result generalizes in particular an inequality by the first-named author concerning exponentially integrable summands and a recent inequality due to Chamakh–Gobet–Liu on summands with β-heavy tails. Another corollary concerns concentration for convex functions of independent, unbounded random variables, generalizing recent results due to Klochkov–Zhivotovskiy and Sambale.

We also obtain a corollary concerning boundedness in $L_\Psi(F)$ of partial sums of a series of independent random variables, generalizing the original result by Hoffmann-Jørgensen.

Résumé. En nous appuyant sur la preuve de Talagrand de l'inégalité de Hoffmann-Jørgensen pour les espaces L_p et sa version pour les espaces d'Orlicz exponentiels, nous obtenons une caractérisation complète des fonctions d'Orlicz Ψ pour lesquelles une inégalité analogue a lieu pour l'espace d'Orlicz $L_\Psi(F)$, où F est un espace de Banach arbitraire.

Comme application, nous proposons une caractérisation d'inégalités de concentration à la Talagrand pour des suprema de processus empiriques avec enveloppe dans L_Ψ, de manière équivalente pour des sommes de variables aléatoires indépendantes dans $L_\Psi(F)$. Ce résultat généralise en particulier une inégalité du premier auteur concernant l'intégrabilité exponentielle de sommes de variables intégrables, et une inégalité récente de Chamakh–Gobet–Liu sur les sommes de variables à queues β-lourdes. Un corollaire supplémentaire concerne la concentration pour des fonctions convexes de variables aléatoires indépendantes non-bornées, généralisant les résultats récents de Klochkov–Zhivotovskiy et Sambale.

Nous obtenons également un corollaire concernant la bornitude dans $L_\Psi(F)$ de sommes partielles de séries aléatoires de variables aléatoires indépendantes, généralisant le résultat originel de Hoffmann-Jørgensen.

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Keywords: Orlicz norm; Banach space valued random variables; Hoffmann-Jørgensen inequality; Talagrand inequality

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