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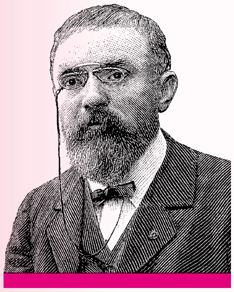

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On the spectrum of random simplicial complexes in thermodynamic regime

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Abstract. Linial–Meshulam complex is a random simplicial complex on n vertices with a complete $(d-1)$-dimensional skeleton and d-simplices occurring independently with probability p. Linial–Meshulam complex is one of the most studied generalizations of the Erdős–Rényi random graph in higher dimensions. In this paper, we study the spectrum of adjacency matrices of the Linial–Meshulam complex when $np \to \lambda \in (0,\infty)$. We prove the existence of a non-random limiting spectral distribution (LSD) for signed and unsigned adjacency matrices of Linial–Meshulam complex. We also prove that the line graph of Linial–Meshulam complex converges in Benjamini–Schramm sense to d-block Galton–Watson graph, a block version of Galton–Watson tree. A major achievement of this paper is proving that the limiting spectral distribution of signed adjacency matrix is a reflection of the spectral measure of a d-block Galton–Watson graph.

Résumé. Le complexe de Linial–Meshulam est un complexe simplicial aléatoire sur n sommets avec un squelette complet de dimension $(d-1)$ et avec des d-simplices apparaissant indépendamment avec une probabilité p. Le complexe de Linial–Meshulam est l'une des généralisations les plus étudiées du graphe aléatoire d'Erdős–Rényi en dimensions supérieures. Dans cet article, nous étudions le spectre des matrices d'adjacence du complexe de Linial–Meshulam lorsque $np \to \lambda \in (0,\infty)$. Nous prouvons l'existence d'une distribution spectrale limite non aléatoire pour les matrices d'adjacence signées et non signées du complexe de Linial–Meshulam. Nous prouvons également que le graphe linéaire du complexe de Linial–Meshulam converge au sens de Benjamini–Schramm vers le graphe de Galton–Watson à d blocs, une version en blocs de l'arbre de Galton–Watson. L'un des principaux résultats de cet article est de prouver que la distribution spectrale limite de la matrice d'adjacence signée est une réflexion de la mesure spectrale d'un graphe de Galton–Watson à d blocs.

MSC2020 subject classifications: Primary 60B20; 05C80; secondary 60B10

Keywords: Linial–Meshulam complex; Random simplicial complex; Limiting spectral distribution; Local weak convergence

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Visibility in Brownian interlacements, Poisson cylinders and Boolean models

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Abstract. We study visibility inside the vacant set of three models in $\mathbb{R}^d$ with slow decay of spatial correlations: Brownian interlacements, Poisson cylinders and Boolean model. For each of them, we obtain sharp asymptotic bounds on the probability of visibility to distance r in some direction in terms of the probability of visibility to distance r in a given direction. In dimensions $d \geq 4$, the ratio of the two probabilities has the same scaling $r^{2(d-1)}$ for all three models, but in lower dimensions the scalings are different. In particular, we improve some main results from (*C. R. Math. Acad. Sci. Paris* **347** (2009) 659–662, *ALEA Lat. Am. J. Probab. Math. Stat.* **16** (2019) 1007–1028).

Résumé. Nous étudions la visibilité dans l'ensemble vacant de trois modèles dans $\mathbb{R}^d$ avec une décroissance lente des corrélations spatiales : entrelacs Browniens, cylindres de Poisson et modèle Booléen. Pour chacun d'eux, nous obtenons des bornes asymptotiques nettes sur la probabilité de visibilité à la distance r en termes de probabilité de visibilité à la distance r dans une direction donnée. En dimension $d \geq 4$, le rapport des deux probabilités a la même échelle $r^{2(d-1)}$ pour les trois modèles, mais en dimension inférieure les échelles sont différentes. En particulier, nous améliorons certains des résultats de (*C. R. Math. Acad. Sci. Paris* **347** (2009) 659–662, *ALEA Lat. Am. J. Probab. Math. Stat.* **16** (2019) 1007–1028).

MSC2020 subject classifications: 60K35; 82B43

Keywords: Brownian interlacements; Poisson cylinders; Boolean model; Visibility

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Central limit theorem for tensor products of free variables via bi-free independence

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Abstract. In this paper, a connection between bi-free probability and the asymptotics of random quantum channels and tensor products of random matrices is established. Using bi-free matrix models, it is demonstrated that the spectral distribution of certain self-adjoint quantum channels and tensor products of random matrices tend to a distribution that can be obtained by an averaged sum of products of bi-freely independent pairs. Subsequently, using bi-free techniques, a Central Limit Theorem for such operator is established.

Résumé. Dans cet article, un lien entre la probabilité bi-libre, l'asymptotique des canaux quantiques aléatoires et les produits tensoriels de matrices aléatoires est établi. À l'aide de modèles matriciels bi-libres, il est démontré que la distribution spectrale de certains canaux quantiques auto-adjoints et les produits tensoriels de matrices aléatoires tend vers une distribution qui peut être obtenue par une somme moyenne de produits de paires bi-librement indépendantes. Par la suite, en utilisant des techniques bi-libres, un théorème central limite pour un tel opérateur est établi.

MSC2020 subject classifications: 46L53; 46L54

Keywords: Bi-free probability; Random Matrices; Quantum Channels; Central Limit Theorem

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Limit profile for the ASEP with one open boundary

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Abstract. We study the speed of convergence to equilibrium for the asymmetric simple exclusion process (ASEP) on a finite interval with one open boundary. We provide sharp estimates on the total-variation distance from equilibrium and verify that the limit profile undergoes a phase transition from a Gaussian to a KPZ profile.

Résumé. Nous étudions la vitesse de convergence vers l'équilibre du processus d'exclusion simple asymétrique (ASEP) sur un intervalle fini avec conditions au bord libres d'un côté. Nous obtenons des estimations précises de la distance en variation totale par rapport à l'équilibre, et vérifions l'existence d'une transition de phase à la limite, entre un profil gaussien et un profil de type KPZ.

MSC2020 subject classifications: 60K35; 60C05

Keywords: Limit profile; Mixing times; Asymmetric simple exclusion process; Type BC Hecke algebra

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Scaling limit of the TASEP speed process

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Abstract. The TASEP speed process introduced by Amir, Angel and Valkó in 2011 is a simultaneous coupling of all the translation-ergodic invariant distributions of multiclass totally asymmetric simple exclusion processes (TASEPs). It is defined as the process of limiting speeds of second-class particles started from each lattice site so that initially each particle sees a full lattice behind and an empty lattice ahead. We show that suitably scaled, the TASEP speed process converges weakly to the stationary horizon (SH), a stochastic process recently introduced and studied by the authors. Specifically, around each interior speed value, the family of continuously interpolated level curves of the TASEP speed process converges to a coupled family of Brownian motions with drift, and this limiting function-valued stochastic process is precisely SH. SH is believed to be the universal scaling limit of Busemann processes in the KPZ universality class. Our results add to the evidence for this universality by connecting SH with multiclass particle configurations. Previously SH has been associated with the exponential corner growth model, Brownian last-passage percolation, and the directed landscape (DL). As a consequence of the DL connection, we show that, in a certain technical sense, the set of speed process values converges weakly to the set of exceptional directions of DL, and the convoys of equal speed process values converge to the Busemann difference profiles.

Résumé. Le processus de vitesse du TASEP introduit par Amir, Angel et Valkó en 2011 est un couplage simultané de toutes les lois invariantes ergodiques par translation des processus d'exclusion simples totalement asymétriques multi-classes (TASEPs). Il est défini comme le processus des vitesses limites des particules de seconde classe, initialement placées sur chaque site du réseau de telle manière que, initialement, chaque particule voit un réseau complet derrière elle et un réseau vide devant. Nous montrons que, après une renormalisation appropriée, le processus de vitesse du TASEP converge faiblement vers l'horizon stationnaire (SH), un processus stochastique récemment introduit et étudié par les auteurs. Plus précisément, autour de chaque valeur de vitesse intérieure, la famille des courbes de niveau interpolées continuement du processus de vitesse du TASEP converge vers une famille couplée de mouvements browniens avec dérive, et ce processus stochastique limite à valeurs fonctionnelles est précisément le SH. Le SH est conjecturé comme la limite d'échelle universelle des processus de Busemann dans la classe d'universalité KPZ (Kardar-Parisi-Zhang). Nos résultats renforcent cette conjecture d'universalité en connectant le SH avec les configurations de particules multiclasses. Auparavant, le SH a été associé au modèle de croissance exponentielle des coins, à la percolation de dernier passage brownienne et au paysage dirigé (DL). En conséquence de cette connexion avec le DL, nous montrons que, dans un certain sens technique, l'ensemble des valeurs du processus de vitesse converge faiblement vers l'ensemble des directions exceptionnelles du DL, et que les convois de valeurs égales du processus de vitesse convergent vers les profils de différence de Busemann.

MSC2020 subject classifications: 60K35; 60K37

Keywords: Busemann function; Directed landscape; Exclusion process; KPZ fixed point; KPZ universality; Multiclass process; Multitype invariant distribution; Particle system; Queueing theory; Second class particle; Stationary horizon; TASEP; TASEP speed process

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Occupation times and areas derived from random sampling

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Abstract. We consider the occupation area of spherical (fractional) Brownian motion, i.e. the area where the process is positive, and show that it is uniformly distributed. For the proof, we introduce a new simple combinatorial view on occupation times of stochastic processes that turns out to be surprisingly effective. A sampling method is used to relate the moments of occupation times to persistence probabilities of related stochastic processes (e.g. random walks) that again relate to combinatorial factors in the moments of beta distributions. This sampling method combined with Spitzer's formula and the use of Bell polynomials yields the first characterisation of the distribution of occupation times for all Lévy processes. Our approach also yields a new and completely elementary proof of Lévy's second arcsine law for Brownian motion.

Résumé. Nous considérons la zone d'occupation du mouvement brownien sphérique (fractionnaire), c'est-à-dire la zone où le processus est positif, et montrons qu'elle est uniformément distribuée. Pour la preuve, nous introduisons un point de vue combinatoire nouveau et simple sur les temps d'occupation des processus stochastiques qui s'avère étonnamment efficace. Une méthode d'échantillonnage est utilisée pour relier les moments des temps d'occupation aux probabilités de persistance des processus stochastiques associés (par exemple les marches aléatoires) qui sont à leur tour liées aux facteurs combinatoires dans les moments des distributions bêta. Cette méthode d'échantillonnage combinée à la formule de Spitzer et à l'utilisation de polynômes de Bell donne la première caractérisation de la distribution des temps d'occupation pour tous les processus de Lévy. Notre approche donne également une nouvelle preuve complètement élémentaire de la deuxième loi de Lévy de l'arc sinus pour le mouvement brownien.

MSC2020 subject classifications: Primary 60G22; secondary 60G09; 60G50; 60G51

Keywords: Occupation time; Spherical fractional Brownian motion; Bell polynomials; Lévy process; Fluctuation theory for random walks

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Additive functionals of exclusion processes from non-equilibrium

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Abstract. Consider the weakly asymmetric simple exclusion processes on the one-dimensional torus. We study the non-equilibrium fluctuations of a class of additive functionals, and show that its scaling limit is a Gaussian process. The proof is mainly based on the results obtained and techniques developed by Jara and Menezes [Non-equiliburim fluctuations of interacting particle systems, Preprint].

Résumé. Nous considérons les processus d'exclusion simples faiblement asymétriques sur le tore unidimensionnel. Nous étudions les fluctuations hors équilibre d'une classe de fonctionnelles additives, et montrons que sa limite d'échelle est un processus gaussien. La démonstration est principalement basée sur les résultats obtenus et les techniques développées par Jara et Menezes dans [Non-equiliburim fluctuations of interacting particle systems, Preprint].

MSC2020 subject classifications: Primary 60G15; 60F05; secondary 60J55

Keywords: Non-equilibrium fluctuations; Exclusion processes

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Last passage percolation in a product-type random environment

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Abstract. We consider a last passage percolation model in dimension $1+1$ with potential given by the product of a spatial i.i.d. potential with symmetric bounded distribution and an independent i.i.d. in time sequence of signs. We assume that the density of the spatial potential near the edge of its support behaves as a power, with exponent $\kappa > -1$. We investigate the linear growth rate of the actions of optimal point-to-point lazy random walk paths as a function of the path slope and describe the structure of the resulting shape function. It has a corner at 0 and, although its restriction to positive slopes cannot be linear, we prove that it has a flat edge near 0 if $\kappa > 0$.

For optimal point-to-line paths, we study their actions and locations of favorable edges that the paths tends to reach and stay at. Under an additional assumption on the time it takes for the optimal path to reach the favorable location, we prove that appropriately normalized actions converge to a limiting distribution that can be viewed as a counterpart of the Tracy–Widom law. Since the scaling exponent and the limiting distribution depend only on the parameter κ, our results provide a description of a new universality class.

Résumé. Nous considérons un modèle de percolation de dernier passage en dimension $1+1$ avec un potentiel donné par le produit d'un potentiel spatial i.i.d. de loi symétrique et bornée, et d'une suite i.i.d. indépendante de signes dans le temps. Nous supposons que la densité du potentiel spatial, près des bords de son support, se comporte comme une puissance, avec un exposant $\kappa > -1$. Nous étudions le taux de croissance linéaire des actions des chemins de marche aléatoire paresseuse optimaux reliant deux points, en fonction de la pente du chemin, et nous décrivons la structure de la fonction de forme résultante. Celle-ci présente un angle en 0 et, bien que sa restriction aux pentes positives ne puisse être linéaire, nous prouvons qu'elle possède une portion plate près de 0 si $\kappa > 0$.

Pour les chemins optimaux reliant un point à une ligne, nous examinons leurs actions et les emplacements des arêtes favorables que les chemins tendent à atteindre et où ils ont tendance à rester. Sous une hypothèse supplémentaire sur le temps nécessaire pour que le chemin optimal atteigne l'emplacement favorable, nous prouvons que les actions normalisées de manière appropriée convergent vers une loi limite qui peut être considérée comme un analogue de la loi de Tracy–Widom. Étant donné que l'exposant d'échelle et la distribution limite dépendent uniquement du paramètre κ, nos résultats fournissent une description d'une nouvelle classe d'universalité.

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Keywords: Directed polymers; Last passage percolation

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Local times of anisotropic Gaussian random fields and stochastic heat equation

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Abstract. We study the local times of a large class of Gaussian random fields satisfying a property of strong local nondeterminism with respect to an anisotropic metric. We establish moment estimates and Hölder conditions for the local times of the Gaussian random fields. Our key estimates rely on the geometric properties of Voronoi partitions with respect to the anisotropic metric and the use of Besicovitch's covering theorem. Consequently, we deduce sample path properties of the Gaussian random fields related to Chung's law of the iterated logarithm and modulus of non-differentiability. Moreover, we apply our results to systems of stochastic heat equations with additive Gaussian noise and determine the exact Hausdorff measure function with respect to the parabolic metric for the level sets of the solutions.

Résumé. Nous étudions les temps locaux d'une grande classe de champs aléatoires gaussiens satisfaisant une propriété de non-déterminisme local fort par rapport à une métrique anisotrope. Nous établissons des estimations de moments et des conditions de Hölder pour les temps locaux des champs aléatoires gaussiens. Nos estimations clés reposent sur les propriétés géométriques des partitions de Voronoi par rapport à la métrique anisotrope et sur l'utilisation du théorème de recouvrement de Besicovitch. Par conséquent, nous déduisons des propriétés des trajectoires des champs aléatoires gaussiens liées à la loi de Chung du logarithme itéré et une borne inférieure sur le module de continuité. De plus, nous appliquons nos résultats à des systèmes d'équations de la chaleur stochastiques avec bruit gaussien additif et déterminons la fonction de mesure de Hausdorff exacte par rapport à la métrique parabolique pour les ensembles de niveaux des solutions.

MSC2020 subject classifications: 60G15; 60G60; 60J55; 60G17

Keywords: Local times; Gaussian random fields; Anisotropy; Stochastic heat equation; Strong local nondeterminism; Hausdorff measure; Voronoi partition; Besicovitch covering theorem

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On absolutely continuous curves in the Wasserstein space over $\mathbb{R}$ and their representation by an optimal Markov process

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Abstract. Let $\boldsymbol{\mu} = (\mu_t)_{t\in\mathbb{R}}$ be a 1-parameter family of probability measures on $\mathbb{R}$. In (*J. Éc. Polytech. Math.* **9** (2022) 1–62) we introduced its "Markov-quantile" process: a process $X = (X_t)_{t\in\mathbb{R}}$ that resembles as much as possible the quantile process attached to μ, among the Markov processes attached to μ, *i.e.* whose family of marginal laws is $\boldsymbol{\mu}$.

In this article we look at the case where $\boldsymbol{\mu}$ is absolutely continuous in the Wasserstein space $\mathbb{P}_2(\mathbb{R})$. Then X is solution of a dynamical transport problem with marginals $(\mu_t)_t$. It provides a *Markov* minimal Lagrangian probabilistic representative of $\boldsymbol{\mu}$, which is moreover unique among the processes obtained as certain types of limits: limits for the finite dimensional topology of quantile processes where the past is made independent of the future conditionally on the present at finitely many times, or limits of processes linearly interpolating $\boldsymbol{\mu}$.

This raises new questions about ways to obtain *Markov* Lagrangian representatives, and to seek uniqueness properties in this framework.

Résumé. Soit $\boldsymbol{\mu} = (\mu_t)_{t\in\mathbb{R}}$ une famille à un paramètre de mesures de probabilité sur $\mathbb{R}$. Dans (*J. Éc. Polytech. Math.* **9** (2022) 1–62) nous introduisons le processus "Markov-quantile" qui lui est attaché : c'est le processus $X = (X_t)_{t\in\mathbb{R}}$ qui ressemble le plus possible au processus quantile associé à $\boldsymbol{\mu}$, parmi les processus markoviens associés à $\boldsymbol{\mu}$, c'est-à-dire dont la famille de marginales est $\boldsymbol{\mu}$.

Dans cet article nous considérons le cas où $\boldsymbol{\mu}$ est absolument continue dans l'espace de Wasserstein $\mathbb{P}_2(\mathbb{R})$. Alors X est solution d'un problème de transport dynamique, de marginales $(\mu_t)_t$. Il fournit un représentant probabiliste lagrangien minimal *markovien* de $\boldsymbol{\mu}$. Il est en outre unique parmi les processus obtenus comme certains types de limites : limites pour la topologie de dimension finie de processus quantiles dont le passé est rendu indépendant du futur, conditionnellement au présent, en un nombre fini d'instants, ou limites de processus interpolant linéairement $\boldsymbol{\mu}$.

Ceci soulève de nouvelles questions sur les manières d'obtenir des représentants lagrangiens *markoviens*, et de demander des propriétés d'unicité dans ce cadre.

MSC2020 subject classifications: Primary 60A10; 49Q22; 60J25; secondary 35Q35; 28A33; 49J55

Keywords: Markov process; Lagrangian action; optimal transport

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Moment bounds for Gaussian multiplicative chaos with higher-dimensional singularities

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Abstract. We determine the exact threshold of the extended Seiberg bound for the existence of correlation functions in the boundary Liouville conformal field theory in the unit disk. In probabilistic terms, our result is a toolbox yielding the threshold for the finiteness of positive moments of Gaussian multiplicative chaos measure, applicable to cases where singularities of arbitrary (co-)dimension in the background metric are present. We improve previous results of this type for 0-dimensional singularities in (Comm. Math. Phys. **342** (2016) 869–907) and a sufficient condition for the 1-dimensional singularity in an unpublished appendix of (Ann. Inst. Henri Poincaré Probab. Stat. **54** (2018) 1694–1730). In particular, we prove the optimality of the moment bound threshold for boundary Gaussian multiplicative chaos conjectured in (Ann. Inst. Henri Poincaré Probab. Stat. **54** (2018) 1694–1730), which is equivalent to the so-called unit volume Seiberg bound of the boundary Liouville conformal field theory.

Résumé. On détermine la valeur exacte de la borne de Seiberg étendue pour l'existence des fonctions de corrélation de la théorie du champ conforme de Liouville dans le cas avec bord. En termes probabilistes, notre résultat est une boîte à outils, qui sert à déterminer le seuil exact pour la finitude des moments positifs de la measure du chaos multiplicatif Gaussien. Notre technique est applicable aux cas où des singularités de (co-)dimensions arbitraires sont présentes dans la métrique du fond. On améliore plusieurs résultats précédents : la borne de Seiberg étendue avec des singularités de dimension 0 dans (Comm. Math. Phys. **342** (2016) 869–907), et une condition suffisante avec des singularités de dimension 1 dans un annexe non-publié de (Ann. Inst. Henri Poincaré Probab. Stat. **54** (2018) 1694–1730). En particulier, on montre que la borne pour la finitude des moments positifs du chaos multiplicative Gaussien avec bord, conjecturée dans (Ann. Inst. Henri Poincaré Probab. Stat. **54** (2018) 1694–1730), est optimale. La détermination de cette borne dans ce cas est équivalente à la détermination de la borne de Seiberg de la théorie du champ conforme de Liouville avec bord et à volume constant.

MSC2020 subject classifications: Primary 60D05; secondary 30F10; 81T40; 81T20

Keywords: Log-correlated Gaussian field; Gaussian multiplicative chaos; Extended Seiberg bounds; Boundary Liouville conformal field theory; Gaussian processes and inequalities

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Thick trace at infinity for the hyperbolic Radial Spanning Tree

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Abstract. Since the works of Howard & Newman (2001), it is known that in *straight* radial rooted trees, with probability 1, infinite paths all have an asymptotic direction and each asymptotic direction is reached by (at least) an infinite path. Moreover, there exists a set of'exceptionnal' directions reached by (at least) two infinite paths which is random, dense and only countable in dimension 2. Howard & Newman's method says nothing about (random) directions reached by more than two infinite paths and, in particular, if such'very exceptionnal' directions exist in dimension 2. The answer is conjectured to be no for a large class of bi-dimensional random trees and this has been stated in only one case, namely the Directed Last-Passage Percolation Tree (with exponential weights) in $\mathbb{N}^2$.

In this paper, we prove the conjecture for the hyperbolic Radial Spanning Tree (RST): in dimension 2, this tree does not contain three infinite paths with the same asymptotic direction with probability 1. Turned in another way, this means that there is no infinite but thin subtree in the hyperbolic RST, i.e. whose infinite paths would all have the same asymptotic direction. We actually prove a stronger result in dimension $d+1$, $d \geq 1$, stating that any infinite subtree of the hyperbolic RST a.s. generates a thick trace at infinity, i.e. the set of asymptotic directions reached by its infinite paths has a positive measure.

Résumé. Depuis le travail de Howard & Newman (2001), il est connu que pour des arbres radiaux *tendus*, avec probabilité 1, tous les chemins infinis admettent une direction asymptotique et que, chaque direction asymptotique est atteinte par (au moins) un chemin infini. De plus, il existe un ensemble de directions'exceptionnelles' atteintes par (au moins) deux chemins infinis qui est aléatoire, dense et seulement dénombrable en dimension 2. La méthode de Howard & Newman ne dit rien au sujet des directions (aléatoires) atteintes par plus de deux chemins infinis et, en particulier, si de telles directions'très exceptionnelles' existent en dimension 2. On conjecture que la réponse est non pour une grande famille d'arbres aléatoires bidimensionnels et cela n'a été établi que dans un seul cas, celui du Directed Last-Passage Percolation Tree (avec poids exponentiels) dans $\mathbb{N}^2$.

Dans cet article, nous établissons la conjecture pour le Radial Spanning Tree (RST) hyperbolique : en dimension 2, cet arbre ne contient pas trois chemins infinis ayant la même direction asymptotitque avec probabilité 1. Dit autrement, cela signifie qu'il n'existe pas dans le RST hyperbolique de sous-arbre infini mais fin, i.e. dont tous les chemins infinis auraient la même direction asymptotique. Nous prouvons en fait un résultat plus fort en dimension $d+1$, $d \geq 1$, stipulant que tout sous-arbre infini du RST hyperbolique génère p.s. une trace épaisse à l'infini, i.e. l'ensemble des directions asymptotiques atteintes par ses chemins infinis a un volume positif.

MSC2020 subject classifications: 60D05; 60K35; 82B21

Keywords: Hyperbolic space; Stochastic geometry; Random geometric tree; Radial Spanning Tree; Continuum percolation; Poisson point process

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Periodic homogenization for singular Lévy SDEs

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Abstract. With the motivation of understanding the long time behaviour of a Brox (jump) diffusion with periodic white noise drift, we generalize the theory of periodic homogenization for multidimensional SDEs with additive Brownian and stable Lévy noise for $\alpha \in (1,2)$ (cf. (*Asymptotic Analysis for Periodic Structures* (1978) North-Holland Publishing Co., *J. Theoret. Probab.* **20** (2007) 1087–1100)) to the setting of singular periodic Besov drifts $F \in (\mathscr{C}^\beta(\mathbb{T}^d))^d$ for $\beta \in ((2-2\alpha)/3, 0)$. For the periodic martingale solution from (*Bernoulli* **28** (2022) 1757–1783) we prove existence and uniqueness of an invariant probability measure π with strictly positive Lebesgue density exploiting the theory of paracontrolled distributions and a strict maximum principle for the singular Fokker–Planck equation. Furthermore, we prove a spectral gap estimate and solve the Poisson equation with singular right-hand side. Provided there is a Brownian noise, the recentered diffusion converges in CLT scaling to a Brownian motion with constant diffusion matrix. In the pure α-stable noise case, we rescale in the scaling of the α-stable process and show convergence of the recentered process to the stable process itself. We conclude with a periodic homogenization result for the singular parabolic PDE.

Résumé. Dans le but de comprendre le comportement à long terme d'une diffusion (saut) de Brox avec un drift périodique de bruit blanc, nous généralisons la théorie de l'homogénéisation périodique pour les EDS multidimensionnelles avec un bruit brownien additif et un bruit de Lévy stable pour $\alpha \in (1,2)$ (cf. (*Asymptotic Analysis for Periodic Structures* (1978) North-Holland Publishing Co., *J. Theoret. Probab.* **20** (2007) 1087–1100)) au cadre des dérives de Besov périodiques singulières $F \in (\mathscr{C}^\beta(\mathbb{T}^d))^d$ pour $\beta \in ((2-2\alpha)/3, 0)$. Pour la solution martingale périodique de (*Bernoulli* **28** (2022) 1757–1783), nous prouvons l'existence et l'unicité d'une mesure de probabilité invariante π avec une densité de Lebesgue strictement positive en exploitant la théorie des distributions paracontrôlées et un principe de maximum strict pour l'équation singulière de Fokker–Planck. En outre, nous prouvons une estimation du trou spectral et nous résolvons l'équation de Poisson avec un côté droit singulier. S'il y a un bruit brownien, la diffusion converge dans l'échelle CLT vers un mouvement brownien avec une matrice de diffusion constante. Dans le cas d'un bruit pur α-stable, nous changeons d'échelle dans l'échelle du processus α-stable et montrons la convergence vers le processus stable lui-même. Nous concluons par un résultat d'homogénéisation périodique pour l'EDP parabolique singulière.

MSC2020 subject classifications: 35A21; 35B27; 60H10; 60G51; 60L40; 60K37

Keywords: Periodic homogenization; Singular diffusion; Stable Lévy noise; Poisson equation; Singular Fokker–Planck equation; Paracontrolled distributions

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Insertion pre-Lie products and translation of rough paths based on multi-indices

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Abstract. We use the diagram-free approach to regularity structures introduced by Otto et. al. to build rough paths based on multi-indices. We identify the analogue of the insertion pre-Lie algebra of trees and use it to build the corresponding group of translations of rough paths. We make this identification precise by showing that the natural dictionary between trees and multi-indices is a pre-Lie morphism under insertion, which in turn yields a Hopf algebra morphism to the rooted tree Hopf algebra equipped with the extraction-contraction coproduct.

Résumé. Nous utilisons l'approche sans diagramme des structures de régularité introduite par Otto et al. pour construire des chemins rugueux basés sur des multi-indices. Nous identifions l'analogue de l'algèbre pré-Lie d'insertion des arbres et l'utilisons pour construire le groupe correspondant de translations des chemins rugueux. Nous précisons cette identification en montrant que le dictionnaire naturel entre les arbres et les multi-indices définit un morphisme pré-Lie sous l'insertion, ce qui induit à son tour un morphisme d'algèbre de Hopf vers l'algèbre de Hopf des arbres enracinés, équipée du coproduit d'extraction-contraction.

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Keywords: Rough paths; Multi-indices; Pre-Lie algebras; Hopf algebras; Extraction-contraction

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Rate of convergence of the smoothed empirical Wasserstein distance

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Abstract. Consider an empirical measure $\mathbb{P}_n$ induced by n i.i.d. samples from a d-dimensional K-subgaussian distribution $\mathbb{P}$ and let $\gamma = \mathcal{N}(0, \sigma^2 I_d)$ be the isotropic Gaussian measure. We study the speed of convergence of the smoothed Wasserstein distance $W_2(\mathbb{P}_n * \gamma, \mathbb{P} * \gamma) = n^{-\alpha + o(1)}$ with $*$ being the convolution of measures. For $K < \sigma$ and in any dimension $d \geq 1$ we show that $\alpha = \frac{1}{2}$. For $K > \sigma$ in dimension $d = 1$ we show that the rate is slower and is given by $\alpha = \frac{(\sigma^2+K^2)^2}{4(\sigma^4+K^4)} < 1/2$. This resolves several open problems in (*IEEE Trans. Inf. Theory* **66** (2020) 4368–4391), and in particular precisely identifies the amount of smoothing σ needed to obtain a parametric rate. In addition, for any d-dimensional K-subgaussian distribution $\mathbb{P}$, we also establish that $D_{\mathrm{KL}}(\mathbb{P}_n * \gamma \,\|\, \mathbb{P} * \gamma)$ has rate $O(1/n)$ for $K < \sigma$ but only slows down to $O(\frac{(\log n)^{d+1}}{n})$ for $K > \sigma$. The surprising difference of the behavior of W_2^2 and KL implies the failure of T_2-transportation inequality when $\sigma < K$. Consequently, it follows that for $K > \sigma$ and $d = 1$ the log-Sobolev inequality (LSI) for the Gaussian mixture $\mathbb{P} * \gamma$ cannot hold. This closes an open problem in (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 898–914), who established the LSI under the condition $K < \sigma$ and asked if their bound can be improved.

Résumé. Soit $\mathbb{P}_n$ la mesure empirique associée à un échantillon de taille n tiré selon la loi $\mathbb{P}$ en dimension d supposée K-sous-gaussienne. Soit $\gamma = \mathcal{N}(0, \sigma^2 I_d)$ la mesure gaussienne isotrope sur le même espace. Nos résultats établissent la vitesse de convergence de $\mathbb{P}_n$ vers $\mathbb{P}$ pour la distance Wasserstein lissée $W_2(\mathbb{P}_n * \gamma, \mathbb{P} * \gamma) = n^{-\alpha + o(1)}$, où $*$ représente l'opérateur de convolution. Ici α est $\alpha = \frac{1}{2}$ lorsque $\sigma > K$ pour toute dimension d et lorsque que $\sigma < K$, notre résultat démontre l'existence d'une vitesse lente characterisée par $\alpha = \frac{(\sigma^2+K^2)^2}{4(\sigma^4+K^4)} < 1/2$ en dimension $d = 1$. Ensemble, ces résultats répondent a plusieurs questions laissées ouvertes dans (*IEEE Trans. Inf. Theory* **66** (2020) 4368–4391), en particulier la détermination de la valeur minimale du paramètre de lissage σ nécessaire pour l'obtention de la vitesse paramétrique.

D'autre part, pour la divergence de Kullback–Leibler (KL) $D_{\mathrm{KL}}(\mathbb{P}_n * \gamma \,\|\, \mathbb{P} * \gamma)$, nous obtenons une vitesse de l'ordre $\frac{1}{n}$ lorsque $\sigma > K$, qui se dégrade en $\frac{(\log n)^{d+1}}{n}$ quand $\sigma < K$. Cette différence de comportement entre W_2^2 et KL implique que l'inégalité de transport T_2 n'est pas satisfaite lorsque $K > \sigma$. Par conséquent, le mélange de gaussiennes $\mathbb{P} * \mathcal{N}(0, \sigma^2)$ ne vérifie pas l'inégalité log-Sobolev (LSI) quand $\sigma < K$ et $d = 1$, répondant ainsi à une question de (*Ann. Inst. Henri Poincaré Probab. Stat.* **52** (2016) 898–914) où une LSI est démontrée pour le cas $\sigma > K$.

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Keywords: Wasserstein distance; KL divergence; Log-Sobolev inequality

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Annealed transition density of simple random walk on a high-dimensional loop-erased random walk

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Abstract. We derive sub-Gaussian bounds for the annealed transition density of the simple random walk on a high-dimensional loop-erased random walk. The walk dimension that appears in these is the exponent governing the space-time scaling of the process with respect to the extrinsic Euclidean distance, which contrasts with the exponent given by the intrinsic graph distance that appears in the corresponding quenched bounds. We prove our main result using novel pointwise Gaussian estimates on the distribution of the high-dimensional loop-erased random walk.

Résumé. Nous dérivons des limites sous-gaussiennes pour la densité de transition recuite de la marche aléatoire simple sur une marche aléatoire à boucle effacée de grande dimension. La dimension de la marche qui apparaît dans ces limites est l'exposant gouvernant la mise à l'échelle spatio-temporelle du processus par rapport à la distance euclidienne extrinsèque, qui contraste avec l'exposant donné par la distance intrinsèque du graphe qui apparaît dans les limites gelées correspondantes. Nous prouvons notre résultat principal en utilisant de nouvelles estimations gaussiennes ponctuelles sur la distribution de la marche aléatoire à boucle effacée de grande dimension.

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Keywords: Heat kernel estimates; Loop-erased random walk; Random walk in random environment

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A note on Sarnak processes

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Abstract. Basic properties of stationary processes called *Sarnak processes* are studied. As an application, a combinatorial reformulation of Sarnak's conjecture on Möbius orthogonality is provided.

Résumé. Nous étudions des propriétés d'une classe de processus stationnaires appelés *processus de Sarnak*. En application, nous donnons une reformulation combinatoire de la conjecture de Sarnak sur l'orthogonalité de la fonction de Möbius.

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Keywords: Sarnak processes; Remote past of a stationary process; Sarnak conjecture; Chowla conjecture

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Concentration for the nodal component count of Gaussian Laplace eigenfunctions

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Abstract. We study the nodal component count of the following Gaussian Laplace eigenfunctions: monochromatic random waves (MRW) on $\mathbb{R}^2$, arithmetic random waves (ARW) on $\mathbb{T}^2$, and random spherical harmonics (RSH) on $\mathbb{S}^2$. Exponential concentration for the nodal component count of RSH on $\mathbb{S}^2$ and ARW on $\mathbb{T}^2$ was established in (*Amer. J. Math.* **131** (2009) 1337–1357) and (*Int. Math. Res. Not. IMRN* **22** (2017) 6990–7027) respectively. We prove exponential concentration of the nodal component count in the following three cases: MRW on growing Euclidean balls in $\mathbb{R}^2$; RSH and ARW on geodesic balls, in $\mathbb{S}^2$ and $\mathbb{T}^2$ respectively, whose radius is slightly larger than the wavelength scale.

Résumé. Nous étudions le nombre de composantes nodales des fonctions propres gaussiennes de Laplace suivantes : ondes aléatoires monochromatiques (MRW) sur $\mathbb{R}^2$, ondes aléatoires arithmétiques (ARW) sur $\mathbb{T}^2$ et harmoniques sphériques aléatoires (RSH) sur $\mathbb{S}^2$. Une concentration exponentielle pour le nombre de composantes nodales de RSH sur $\mathbb{S}^2$ et ARW sur $\mathbb{T}^2$ a été établie dans (*Amer. J. Math.* **131** (2009) 1337–1357) et (*Int. Math. Res. Not. IMRN* **22** (2017) 6990–7027) respectivement. Nous démontrons une concentration exponentielle du nombre de composantes nodales dans les trois cas suivants : MRW sur des boules euclidiennes croissantes dans $\mathbb{R}^2$; RSH et ARW sur des boules géodésiques, dans $\mathbb{S}^2$ et $\mathbb{T}^2$ respectivement, dont le rayon est légèrement plus grand que l'échelle de longueur d'onde.

MSC2020 subject classifications: Primary 58C40; 60F10; 60G60; secondary 33C45

Keywords: Gaussian Laplace eigenfunctions; Nodal domains; Exponential concentration

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A stochastic analysis approach to tensor field theories

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Abstract. We present two different arguments using stochastic analysis to construct super-renormalizable tensor field theories, namely the T_3^4 and T_4^4 models. The first approach is the construction of a Langevin dynamic (*Sci. Sin.* **24** (1981) 483–496; *Comm. Math. Phys.* **384** (2021) 1–75) combined with a PDE energy estimate while the second is an application of the variational approach of Barashkov and Gubinelli (*Duke Math. J.* **169** (2020) 3339–3415). By leveraging the melonic structure of divergences, regularizing properties of non-local products, and controlling certain random operators, we demonstrate that for tensor field theories these approaches can be significantly simplified in comparison to what is required for Φ_d^4 models.

Résumé. Nous présentons deux arguments utilisant l'analyse stochastique pour construire des théories tensorielles des champs super-renormalisables, les théories T_3^4 et T_4^4. La première approche est la construction de la dynamique de Langevin (*Sci. Sin.* **24** (1981) 483–496 ; *Comm. Math. Phys.* **384** (2021) 1–75) combinée avec des estimées d'énergie sur la solution de l'EDP, tandis que la seconde approche est une application de la méthode variationnelle introduite par Barashkov et Gubinelli (*Duke Math. J.* **169** (2020) 3339–3415). En s'appuyant sur la structure melonique des divergences, sur les propriétés régularisantes du produit non local, et en contrôlant plusieurs opérateurs aléatoires, nous démontrons que pour les théories tensorielles des champs ces approches sont substantiellement simplifiées par rapport au cas des théories Φ_d^4.

MSC2020 subject classifications: 60H30; 81T08

Keywords: Stochastic partial differential equations; Euclidean quantum field theories; Tensor field theories

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A strong law of large numbers for real roots of random polynomials

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Abstract. We consider random polynomials $p_n(x) = \xi_0 + \xi_1 + \cdots + \xi_n x^n$ whose coefficients are independent and identically distributed with zero mean, unit variance, and bounded $(2+\epsilon)^{\text{th}}$ moment (for some $\epsilon > 0$), also known as the Kac polynomials. Let N_n denote the number of real roots of p_n. In this paper, motivated by a question from Igor Pritsker, we prove that almost surely the following convergence holds:

$$\lim_{n\to\infty} \frac{N_n([-1,1])}{\log n} = \frac{1}{\pi}.$$

This convergence could be viewed as a local strong law for the real roots. The main ingredient in the proof is a set of maximal inequalities that reduces the proof to proving convergence along lacunary subsequences, which in turn follows from a recent concentration estimate of Can–Nguyen.

Résumé. Nous considérons des polynômes aléatoires $p_n(x) = \xi_0 + \xi_1 + \cdots + \xi_n x^n$ dont les coefficients sont indépendants et identiquement distribués, centrés, de variance unitaire et d'un moment $(2+\epsilon)^{\text{th}}$-borné (pour un certain $\epsilon > 0$), également connus sous le nom de polynômes de Kac. Soit N_n le nombre de racines réelles de p_n. Dans cet article, motivés par une question d'Igor Pritsker, nous prouvons que, presque sûrement,

$$\lim_{n\to\infty} \frac{N_n([-1,1])}{\log n} = \frac{1}{\pi}.$$

Cette convergence pourrait être vue comme une loi forte locale pour les racines réelles. L'élément principal de la démonstration est un ensemble d'inégalités maximales qui réduit la preuve à celle de la convergence le long de sous-séquences lacunaires, ce qui découle à son tour d'une estimation récente de concentration due à Can–Nguyen.

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Keywords: Random polynomials; Real roots; Strong law of large numbers; Almost sure convergence

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Spectrum of Laplacian matrices associated with large random elliptic matrices

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Abstract. A Laplacian matrix is a square matrix whose row sums are zero. We study the limiting eigenvalue distribution of a Laplacian matrix formed by taking a random elliptic matrix and subtracting the diagonal matrix containing its row sums. Under some mild assumptions, we show that the empirical spectral distribution of the Laplacian matrix converges to a deterministic probability distribution as the size of the matrix tends to infinity. The limiting measure can be interpreted as the Brown measure of the sum of an elliptic operator and a freely independent normal operator with a Gaussian distribution.

Résumé. Une matrice laplacienne est une matrice carrée dont les sommes de lignes sont nulles. Nous étudions la distribution limite des valeurs propres d'une matrice laplacienne formée en prenant une matrice elliptique aléatoire et en soustrayant la matrice diagonale contenant ses sommes de lignes. Sous certaines hypothèses modérées, nous montrons que la distribution spectrale empirique de la matrice laplacienne converge vers une distribution de probabilité déterministe lorsque la taille de la matrice tend vers l'infini. La mesure limite peut être interprétée comme la mesure de Brown de la somme d'un opérateur elliptique et d'un opérateur normal librement indépendant avec une distribution gaussienne.

MSC2020 subject classifications: 60B20

Keywords: Random matrices; Laplacian matrix; Elliptic random matrices; Empirical spectral measure; Free probability theory; Brown measure; Least singular value

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The all-time maximum for branching Brownian motion with absorption conditioned on long-time survival

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Abstract. We consider branching Brownian motion in which initially there is one particle at x, particles produce a random number of offspring with mean $m+1$ at the time of branching events, and each particle branches at rate $\beta = 1/2m$. Particles independently move according to Brownian motion with drift -1 and are killed at the origin. It is well-known that this process eventually dies out with positive probability. We condition this process to survive for an unusually large time t and study the behavior of the process at small times $s \ll t$ using a spine decomposition. We show, in particular, that the time when a particle gets furthest from the origin is of the order $t^{5/6}$.

Résumé. Nous considérons un mouvement brownien branchant dans lequel il y a initialement une particule à x, les particules produisent un nombre aléatoire de descendants avec une moyenne de $m+1$ au moment des événements de ramification, et chaque particule branche au taux $\beta = 1/2m$. Les particules se déplacent indépendamment selon des mouvements browniens avec une dérive de -1 et sont tuées à l'origine. Il est bien connu que ce processus finit par s'éteindre avec une probabilité positive. Nous conditionnons ce processus à survivre pendant un temps inhabituellement grand t et nous étudions le comportement du processus à de petits temps $s \ll t$ en utilisant une décomposition épinale. Nous montrons en particulier que le temps où une particule s'éloigne le plus de l'origine est de l'ordre de $t^{5/6}$.

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Keywords: Branching Brownian motion; Yaglom limit theorem; Brownian taboo process; Excursion theory

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Fractional kinetics equation from a Markovian system of interacting Bouchaud trap models

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Abstract. We consider a partial exclusion process evolving on $\mathbb{Z}^d$ in a random trapping environment. In dimension $d \geq 2$, we derive the fractional kinetics equation

$$\frac{\partial^\beta \rho_t}{\partial t^\beta} = \Delta \rho_t$$

as a hydrodynamic limit of the particle system. Here, $\frac{\partial^\beta}{\partial t^\beta}$, $\beta \in (0, 1)$, denotes the fractional derivative in the Caputo sense. We thus exhibit a Markovian interacting particle system whose frequency field rescales to a sub-diffusive equation corresponding to a non-Markovian process, the Fractional Kinetics process. In contrast, we show that, when $d = 1$, the system rescales to the solution to

$$\frac{\partial \rho_t}{\partial t} = \mathcal{L}_\beta \rho_t,$$

where $\mathcal{L}_\beta$ is the random generator of the singular quasi-diffusion known as the FIN diffusion.

Résumé. Nous étudions un processus d'exclusion partiel évoluant sur $\mathbb{Z}^d$ dans un environnement aléatoire avec des pièges. En dimension $d \geq 2$, nous obtenons l'équation cinétique fractionnaire suivante :

$$\frac{\partial^\beta \rho_t}{\partial t^\beta} = \Delta \rho_t,$$

comme limite hydrodynamique du système de particules. Ici, $\frac{\partial^\beta}{\partial t^\beta}$, avec $\beta \in (0, 1)$, désigne la dérivée fractionnaire au sens de Caputo. Ainsi, nous trouvons un système de particules markoviennes en interaction dont le champ de fréquence tend vers une équation sous-diffusive correspondant à un processus non markovien. Nous montrons que, lorsque $d = 1$, le système converge à la solution de

$$\frac{\partial \rho_t}{\partial t} = \mathcal{L}_\beta \rho_t,$$

où $\mathcal{L}_\beta$ est le générateur aléatoire de la quasi-diffusion singulière connue sous le nom de diffusion FIN.

MSC2020 subject classifications: 60K35; 60K37; 60G57; 60G22; 35B27

Keywords: Interacting particle systems; Hydrodynamic limit; Fractional kinetics equation; Bouchaud trap model; Random environment

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Gelation in cluster coagulation processes

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Abstract. We consider the problem of gelation in the cluster coagulation model introduced by Norris (*Comm. Math. Phys.* **209**(2) (2000) 407–435), where pairs of clusters of types (x, y) taking values in a measure space E, merge to form a new particle of type $z \in E$ according to a transition kernel $K(x, y, \mathrm{d}z)$. This model possesses enough generality to accommodate inhomogeneities in the evolution of clusters, including variations in their shape or spatial distribution. We derive general, sufficient criteria for stochastic gelation in this model. As particular cases, we extend results related to the classical Marcus–Lushnikov coagulation process, showing that reasonable 'homogenous' coagulation processes with exponent $\gamma > 1$ yield gelation; and also, coagulation processes with kernel $\bar{K}(m, n) \geq (m \wedge n) \log (m \wedge n)^{3+\epsilon}$ for $\epsilon > 0$.

Résumé. Nous considérons le problème de la gélification dans le modèle de *cluster coagulation* introduit par Norris (*Comm. Math. Phys.* **209**(2) (2000) 407–435), où une paire de particules de types (x, y) prenant des valeurs dans un espace de mesure E, fusionne pour former une nouvelle particule de type $z \in E$ selon un noyau de transition $K(x, y, \mathrm{d}z)$. Ce modèle est suffisamment général pour intégrer plusieurs inhomogénéités dans l'évolution des particules, y compris des variations dans leur forme ou leur distribution spatiale. Nous dérivons des critères généraux et suffisants pour la gélification stochastique dans ce modèle. En tant que cas particuliers, nous étendons des résultats liés au processus de coagulation classique de Marcus–Lushnikov en montrant que des raisonnables processus de coagulation 'homogènes' avec un exposant $\gamma > 1$ entraînent la gélification ; et aussi, des processus de coagulation avec un noyau $\bar{K}(m, n) \geq (m \wedge n) \log (m \wedge n)^{3+\epsilon}$ for $\epsilon > 0$.

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Keywords: Cluster coagulation; Marcus–Lushnikov process; Stochastic gelation; Strong gelation; Inhomogeneous random graphs

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Geodesic trees in last passage percolation and some related problems

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Abstract. For the exactly solvable model of exponential last passage percolation on $\mathbb{Z}^2$, it is known that given any non-axial direction, all the semi-infinite geodesics starting from points in $\mathbb{Z}^2$ in that direction almost surely coalesce, thereby forming a geodesic tree which has only one end. It is widely understood that the geodesic trees are important objects in understanding the geometry of the LPP landscape. In this paper we study several natural questions about these geodesic trees and their intersections. In particular, we obtain optimal (up to constants) upper and lower bounds for the (power law) tails of the height and the volume of the backward sub-tree rooted at a fixed point. We also obtain bounds for the probability that the sub-tree contains a specific vertex, e.g. the sub-tree in the direction $(1, 1)$ rooted at the origin contains the vertex $-(n, n)$, which answers a question analogous to the well-known midpoint problem in the context of semi-infinite geodesics. Furthermore, we obtain bounds for the probability that a pair of intersecting geodesics both pass through a given vertex. These results are interesting in their own right as well as useful in several other applications.

Résumé. Pour le modèle exactement soluble de percolation de dernier passage exponentielle sur $\mathbb{Z}^2$, il est connu que, pour toute direction oblique, toutes les géodésiques semi-infinies partant de points de $\mathbb{Z}^2$ dans cette direction se rejoignent presque sûrement, formant ainsi un arbre géodésique n'ayant qu'une seule extrémité. Il est largement admis que ces arbres géodésiques sont des objets clés pour comprendre la géométrie du paysage de percolation de dernier passage (LPP). Dans cet article, nous étudions plusieurs questions naturelles concernant ces arbres géodésiques et leurs intersections. En particulier, nous obtenons des bornes supérieures et inférieures optimales (à des constantes près) pour les queues de loi (de type loi de puissance) de la hauteur et du volume du sous-arbre rétrograde enraciné en un point fixé. Nous obtenons également des bornes pour la probabilité que ce sous-arbre contienne un sommet donné, par exemple, la probabilité que le sous-arbre dans la direction $(1, 1)$ enraciné à l'origine contienne le sommet (n, n), ce qui répond à une question analogue au célèbre problème « du milieu » dans le contexte des géodésiques semi-infinies. De plus, nous établissons des bornes pour la probabilité qu'une paire de géodésiques intersectantes passent toutes deux par un sommet donné. Ces résultats sont intéressants en eux-mêmes, et aussi utiles dans plusieurs autres applications.

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Keywords: Last Passage Percolation; Semi-infinite geodesic; Geodesic Tree

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Disorder chaos in short-range, diluted, and Lévy spin glasses

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Abstract. In a recent breakthrough, Chatterjee proved site disorder chaos in the Edwards–Anderson (EA) short-range spin glass model (*J. Phys. F, Met. Phys.* **5** (1975), 965) utilizing the Hermite spectral method. In this paper, we demonstrate the further usefulness of this Hermite spectral approach by extending the validity of site disorder chaos in three related spin glass models. The first, called the mixed even p-spin short-range model, is a generalization of the EA model where the underlying graph is a deterministic bounded degree hypergraph consisting of hyperedges with even number of vertices. The second model is the diluted mixed p-spin model, which is allowed to have hyperedges with both odd and even number of vertices. For both models, our results hold under general symmetric disorder distributions. The main novelty of our argument is played by an elementary algebraic equation for the Fourier–Hermite series coefficients for the two-spin correlation functions. It allows us to deduce necessary geometric conditions to determine the contributing coefficients in the overlap function, which in spirit is the same as the crucial Lemma 1 in (Chatterjee (2023)). Finally, we also establish disorder chaos in the Lévy model with stable index $\alpha \in (1, 2)$.

Résumé. Dans une percée récente, Chatterjee a démontré l'existence d'un chaos de désordre de site dans le modèle de verre de spin à courte portée d'Edwards–Anderson (EA) en utilisant la méthode spectrale d'Hermite. Dans cet article, nous démontrons l'utilité de cette approche spectrale d'Hermite en étendant la validité du chaos de désordre de site à trois modèles de verre de spin apparentés. Le premier, appelé modèle mixte à courte portée à p-spin pair, est une généralisation du modèle EA où le graphe sous-jacent est un hypergraphe déterministe à degré borné, constitué d'hyperarêtes à nombre pair de sommets. Le second modèle est le modèle mixte à p-spin dilué, qui permet d'avoir des hyperarêtes à nombre pair et impair de sommets. Pour les deux modèles, nos résultats sont valables sous des distributions de désordre symétriques générales. La principale nouveauté de notre argument réside dans une équation algébrique élémentaire pour les coefficients de la série de Fourier–Hermite pour les fonctions de corrélation à deux spins. Cela nous permet de déduire les conditions géométriques nécessaires à la détermination des coefficients qui contribuent à la fonction de chevauchement, ce qui, dans l'esprit, est identique au lemme crucial 1 de (Chatterjee (2023)). Enfin, nous établissons également le chaos de désordre dans le modèle de Lévy avec un indice stable $\alpha \in (1, 2)$.

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Bernstein's inequalities for general Markov chains

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Abstract. We establish Bernstein's inequalities for functions of general (general-state-space and possibly non-reversible) Markov chains. These inequalities achieve sharp variance proxies and encompass the classical Bernstein inequality for independent random variables as special cases. The key analysis lies in bounding the operator norm of a perturbed Markov transition kernel by the exponential of sum of two convex functions. One coincides with what delivers the classical Bernstein inequality, and the other reflects the influence of the Markov dependence. A convex analysis on these two functions then derives our Bernstein inequalities. As applications, we apply our Bernstein inequalities to the Markov chain Monte Carlo integral estimation problem and the robust mean estimation problem with Markov-dependent samples, and achieve tight deviation bounds that previous inequalities can not.

Résumé. Nous établissons des inégalités de Bernstein pour des fonctions de chaînes de Markov générales (à espace d'états général et possiblement non réversibles). Ces inégalités atteignent des bornes précises pour la variance et englobent l'inégalité classique de Bernstein pour des variables aléatoires indépendantes comme cas particulier. L'analyse principale consiste à borner la norme d'opérateur d'un noyau de transition de Markov perturbé par l'exponentielle de la somme de deux fonctions convexes. L'une correspond à celle qui apparaît dans l'inégalité classique de Bernstein, et l'autre reflète l'influence de la dépendance propre aux chaînes de Markov. Une analyse convexe sur ces deux fonctions permet alors d'établir nos inégalités de Bernstein. Comme applications, nous utilisons ces inégalités pour le problème d'estimation d'intégrale par la méthode de Monte Carlo par chaîne de Markov ainsi que pour l'estimation robuste de la moyenne avec des échantillons dépendants selon une chaîne de Markov, et nous obtenons des bornes de déviation précises que les inégalités précédentes ne permettent pas d'atteindre.

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