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THE WIENER CONDITION AND THE CONJECTURES OF EMBRECHTS AND GOLDIE

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We show that the class of convolution equivalent distributions and the class of locally subexponential distributions are not closed under convolution roots. It gives a negative answer to the classical conjectures of Embrechts and Goldie. Moreover, we establish two sufficient conditions in order that the class of convolution equivalent distributions is closed under convolution roots.

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BIPOLAR ORIENTATIONS ON PLANAR MAPS AND SLE_{12}

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We give bijections between bipolar-oriented (acyclic with unique source and sink) planar maps and certain random walks, which show that the uniformly random bipolar-oriented planar map, decorated by the "peano curve" surrounding the tree of left-most paths to the sink, converges in law with respect to the peanosphere topology to a $\sqrt{4/3}$-Liouville quantum gravity surface decorated by an independent Schramm–Loewner evolution with parameter $\kappa = 12$ (i.e., SLE_{12}). This result is universal in the sense that it holds for bipolar-oriented triangulations, quadrangulations, k-angulations and maps in which face sizes are mixed.

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LOCAL SINGLE RING THEOREM ON OPTIMAL SCALE

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Let U and V be two independent N by N random matrices that are distributed according to Haar measure on $U(N)$. Let Σ be a nonnegative deterministic N by N matrix. The *single ring theorem* [*Ann. of Math. (2)* **174** (2011) 1189–1217] asserts that the empirical eigenvalue distribution of the matrix $X := U\Sigma V^*$ converges weakly, in the limit of large N, to a deterministic measure which is supported on a single ring centered at the origin in $\mathbb{C}$. Within the bulk regime, that is, in the interior of the single ring, we establish the convergence of the empirical eigenvalue distribution on the optimal local scale of order $N^{-1/2+\varepsilon}$ and establish the optimal convergence rate. The same results hold true when U and V are Haar distributed on $O(N)$.

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LARGE DEVIATION PRINCIPLE FOR RANDOM MATRIX PRODUCTS[1]

BY CAGRI SERT

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Under a Zariski density assumption, we extend the classical theorem of Cramér on large deviations of sums of i.i.d. real random variables to random matrix products.

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REGENERATIVE RANDOM PERMUTATIONS OF INTEGERS

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Motivated by recent studies of large Mallows(q) permutations, we propose a class of random permutations of $\mathbb{N}_+$ and of $\mathbb{Z}$, called *regenerative permutations*. Many previous results of the limiting Mallows(q) permutations are recovered and extended. Three special examples: blocked permutations, p-shifted permutations and p-biased permutations are studied.

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FOUR MOMENTS THEOREMS ON MARKOV CHAOS

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We obtain quantitative four moments theorems establishing convergence of the laws of elements of a Markov chaos to a Pearson distribution, where the only assumption we make on the Pearson distribution is that it admits four moments. These results are obtained by first proving a general carré du champ bound on the distance between laws of random variables in the domain of a Markov diffusion generator and invariant measures of diffusions, which is of independent interest, and making use of the new concept of chaos grade. For the heavy-tailed Pearson distributions, this seems to be the first time that sufficient conditions in terms of (finitely many) moments are given in order to converge to a distribution that is not characterized by its moments.

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CAPACITY OF THE RANGE OF RANDOM WALK ON $\mathbb{Z}^4$

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We study the scaling limit of the capacity of the range of a random walk on the integer lattice in dimension four. We establish a strong law of large numbers and a central limit theorem with a non-Gaussian limit. The asymptotic behaviour is analogous to that found by Le Gall in '86 [*Comm. Math. Phys.* **104** (1986) 471–507] for the *volume* of the range in dimension two.

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SEPARATING CYCLES AND ISOPERIMETRIC INEQUALITIES IN THE UNIFORM INFINITE PLANAR QUADRANGULATION[1]

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We study geometric properties of the infinite random lattice called the uniform infinite planar quadrangulation or UIPQ. We establish a precise form of a conjecture of Krikun stating that the minimal size of a cycle that separates the ball of radius R centered at the root vertex from infinity grows linearly in R. As a consequence, we derive certain isoperimetric bounds showing that the boundary size of any simply connected set A consisting of a finite union of faces of the UIPQ and containing the root vertex is bounded below by a (random) constant times $|A|^{1/4}(\log|A|)^{-(3/4)-\delta}$, where the volume $|A|$ is the number of faces in A.

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CUTOFF PHENOMENON FOR THE ASYMMETRIC SIMPLE EXCLUSION PROCESS AND THE BIASED CARD SHUFFLING

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We consider the biased card shuffling and the Asymmetric Simple Exclusion Process (ASEP) on the segment. We obtain the asymptotic of their mixing times: our results show that these two continuous-time Markov chains display cutoff. Our analysis combines several ingredients including: a study of the hydrodynamic profile for ASEP, the use of monotonic eigenfunctions, stochastic comparisons and concentration inequalities.

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SUBOPTIMALITY OF LOCAL ALGORITHMS FOR A CLASS OF MAX-CUT PROBLEMS

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We show that in random K-uniform hypergraphs of constant average degree, for even $K \geq 4$, local algorithms defined as factors of i.i.d. can not find nearly maximal cuts, when the average degree is sufficiently large. These algorithms have been used frequently to obtain lower bounds for the max-cut problem on random graphs, but it was not known whether they could be successful in finding nearly maximal cuts. This result follows from the fact that the overlap of any two nearly maximal cuts in such hypergraphs does not take values in a certain nontrivial interval—a phenomenon referred to as the overlap gap property—which is proved by comparing diluted models with large average degree with appropriate fully connected spin glass models and showing the overlap gap property in the latter setting.

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INFINITELY RAMIFIED POINT MEASURES AND BRANCHING LÉVY PROCESSES

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We call a random point measure *infinitely ramified* if for every $n \in \mathbb{N}$, it has the same distribution as the nth generation of some branching random walk. On the other hand, *branching Lévy processes* model the evolution of a population in continuous time, such that individuals move in space independently, according to some Lévy process, and further beget progenies according to some Poissonian dynamics, possibly on an everywhere dense set of times. Our main result connects these two classes of processes much in the same way as in the case of infinitely divisible distributions and Lévy processes: the value at time 1 of a branching Lévy process is an infinitely ramified point measure, and conversely, any infinitely ramified point measure can be obtained as the value at time 1 of some branching Lévy process.

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LARGEST EIGENVALUES OF SPARSE INHOMOGENEOUS ERDŐS–RÉNYI GRAPHS

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We consider inhomogeneous Erdős–Rényi graphs. We suppose that the maximal mean degree d satisfies $d \ll \log n$. We characterise the asymptotic behaviour of the $n^{1-o(1)}$ largest eigenvalues of the adjacency matrix and its centred version. We prove that these extreme eigenvalues are governed at first order by the largest degrees and, for the adjacency matrix, by the nonzero eigenvalues of the expectation matrix. Our results show that the extreme eigenvalues exhibit a novel behaviour which in particular rules out their convergence to a nondegenerate point process. Together with the companion paper [Benaych-Georges, Bordenave and Knowles (2017)], where we analyse the extreme eigenvalues in the complementary regime $d \gg \log n$, this establishes a crossover in the behaviour of the extreme eigenvalues around $d \sim \log n$. Our proof relies on a tail estimate for the Poisson approximation of an inhomogeneous sum of independent Bernoulli random variables, as well as on an estimate on the operator norm of a pruned graph due to Le, Levina, and Vershynin from [*Random Structures Algorithms* **51** (2017) 538–561].

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ON THE ALMOST EIGENVECTORS OF RANDOM REGULAR GRAPHS

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Let $d \geq 3$ be fixed and G be a large random d-regular graph on n vertices. We show that if n is large enough then the entry distribution of every almost eigenvector of G (with entry sum 0 and normalized to have length $\sqrt{n}$) is close to some Gaussian distribution $N(0, \sigma)$ in the weak topology where $0 \leq \sigma \leq 1$. Our theorem holds even in the stronger sense when many entries are looked at simultaneously in small random neighborhoods of the graph. Furthermore, we also get the Gaussianity of the joint distribution of several almost eigenvectors if the corresponding eigenvalues are close. Our proof uses graph limits and information theory. Our results have consequences for factor of i.i.d. processes on the infinite regular tree.

In particular, we obtain that if an invariant eigenvector process on the infinite d-regular tree is in the weak closure of factor of i.i.d. processes then it has Gaussian distribution.

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IRREDUCIBLE CONVEX PAVING FOR DECOMPOSITION OF MULTIDIMENSIONAL MARTINGALE TRANSPORT PLANS[1]

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Martingale transport plans on the line are known from Beiglböck and Juillet (*Ann. Probab.* **44** (2016) 42–106) to have an irreducible decomposition on a (at most) countable union of intervals. We provide an extension of this decomposition for martingale transport plans in $\mathbb{R}^d$, $d \geq 1$. Our decomposition is a partition of $\mathbb{R}^d$ consisting of a possibly uncountable family of relatively open convex components, with the required measurability so that the disintegration is well defined. We justify the relevance of our decomposition by proving the existence of a martingale transport plan filling these components. We also deduce from this decomposition a characterization of the structure of polar sets with respect to all martingale transport plans.

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A NONLINEAR WAVE EQUATION WITH FRACTIONAL PERTURBATION

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We study a d-dimensional wave equation model ($2 \leq d \leq 4$) with quadratic nonlinearity and stochastic forcing given by a space-time fractional noise. Two different regimes are exhibited, depending on the Hurst parameter $H = (H_0, \dots, H_d) \in (0, 1)^{d+1}$ of the noise: If $\sum_{i=0}^{d} H_i > d - \frac{1}{2}$, then the equation can be treated directly, while in the case $d - \frac{3}{4} < \sum_{i=0}^{d} H_i \leq d - \frac{1}{2}$, the model must be interpreted in the Wick sense, through a renormalization procedure.

Our arguments essentially rely on a fractional extension of the considerations of [*Trans. Amer. Math. Soc.* **370** (2017) 7335–7359] for the two-dimensional white-noise situation, and more generally follow a series of investigations related to stochastic wave models with polynomial perturbation.

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WEAK TAIL CONDITIONS FOR LOCAL MARTINGALES[1]

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The following conditions are necessary and jointly sufficient for an arbitrary càdlàg local martingale to be a uniformly integrable martingale: (A) The weak tail of the supremum of its modulus is zero; (B) its jumps at the first-exit times from compact intervals converge to zero in L^1 on the events that those times are finite; and (C) its almost sure limit is an integrable random variable.

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