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CLASSIFICATION OF SCALING LIMITS OF UNIFORM QUADRANGULATIONS WITH A BOUNDARY

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We study noncompact scaling limits of uniform random planar quadrangulations with a boundary when their size tends to infinity. Depending on the asymptotic behavior of the boundary size and the choice of the scaling factor, we observe different limiting metric spaces. Among well-known objects like the Brownian plane or the self-similar continuum random tree, we construct two new one-parameter families of metric spaces that appear as scaling limits: the Brownian half-plane with skewness parameter θ and the infinite-volume Brownian disk of perimeter σ. We also obtain various coupling and limit results clarifying the relation between these objects.

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LARGE SCALE LIMIT OF INTERFACE FLUCTUATION MODELS

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We extend the weak universality of KPZ in Hairer and Quastel [*Forum Math. Pi* **6** (2018) e3] to weakly asymmetric interface models with general growth mechanisms beyond polynomials. A key new ingredient is a pointwise bound on correlations of trigonometric functions of Gaussians in terms of their polynomial counterparts. This enables us to reduce the problem of a general nonlinearity with sufficient regularity to that of a polynomial.

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Key words and phrases. KPZ, weak universality, interface fluctuation.

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SAMPLE PATH LARGE DEVIATIONS FOR LÉVY PROCESSES AND RANDOM WALKS WITH REGULARLY VARYING INCREMENTS

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Let X be a Lévy process with regularly varying Lévy measure ν. We obtain sample-path large deviations for scaled processes $\bar{X}_n(t) \triangleq X(nt)/n$ and obtain a similar result for random walks with regularly varying increments. Our results yield detailed asymptotic estimates in scenarios where multiple big jumps in the increment are required to make a rare event happen; we illustrate this through detailed conditional limit theorems. In addition, we investigate connections with the classical large deviations framework. In that setting, we show that a weak large deviation principle (with logarithmic speed) holds, but a full large deviation principle does not hold.

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GAUSSIAN FREE FIELD LIGHT CONES AND $\text{SLE}_\kappa(\rho)$

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Let h be an instance of the GFF. Fix $\kappa \in (0,4)$ and $\chi = 2/\sqrt{\kappa} - \sqrt{\kappa}/2$. Recall that an *imaginary geometry ray* is a flow line of $e^{i(h/\chi+\theta)}$ that looks locally like SLE_κ. The *light cone* with parameter $\theta \in [0,\pi]$ is the set of points reachable from the origin by a sequence of rays with angles in $[-\theta/2, \theta/2]$. It is known that when $\theta = 0$, the light cone looks like SLE_κ, and when $\theta = \pi$ it looks like the range of an $\text{SLE}_{16/\kappa}$ *counterflow line*. We find that when $\theta \in (0,\pi)$ the light cones are either fractal carpets with a dense set of holes or space-filling regions with no holes. We show that every non-space-filling light cone agrees in law with the range of an $\text{SLE}_\kappa(\rho)$ process with $\rho \in ((-2-\kappa/2) \vee (\kappa/2-4), -2)$. Conversely, the range of any such $\text{SLE}_\kappa(\rho)$ process agrees in law with a non-space-filling light cone. As a consequence of our analysis, we obtain the first proof that these $\text{SLE}_\kappa(\rho)$ processes are a.s. continuous curves and show that they can be constructed as natural path-valued functions of the GFF.

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FORMATION OF LARGE-SCALE RANDOM STRUCTURE BY COMPETITIVE EROSION

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We study the following one-dimensional model of annihilating particles. Beginning with all sites of $\mathbb{Z}$ uncolored, a blue particle performs simple random walk from 0 until it reaches a nonzero red or uncolored site, and turns that site blue; then a red particle performs simple random walk from 0 until it reaches a nonzero blue or uncolored site, and turns that site red. We prove that after n blue and n red particles alternately perform such walks, the total number of colored sites is of order $n^{1/4}$. The resulting random color configuration, after rescaling by $n^{1/4}$ and taking $n \to \infty$, has an explicit description in terms of alternating extrema of Brownian motion (the global maximum on a certain interval, the global minimum attained after that maximum, etc.).

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CUTOFF FOR THE SWENDSEN–WANG DYNAMICS ON THE LATTICE

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We study the Swendsen–Wang dynamics for the q-state Potts model on the lattice. Introduced as an alternative algorithm of the classical single-site Glauber dynamics, the Swendsen–Wang dynamics is a nonlocal Markov chain that recolors many vertices at once based on the random-cluster representation of the Potts model. In this work, we establish cutoff phenomenon for the Swendsen–Wang dynamics on the lattice at sufficiently high temperatures, proving that it exhibits a sharp transition from "unmixed" to "well mixed." In particular, we show that at high enough temperatures the Swendsen–Wang dynamics on the torus $(\mathbb{Z}/n\mathbb{Z})^d$ has cutoff at time $\frac{d}{2}(-\log(1-\gamma))^{-1}\log n$, where $\gamma(\beta)$ is the spectral gap of the infinite-volume dynamics.

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TOTAL VARIATION DISTANCE BETWEEN STOCHASTIC POLYNOMIALS AND INVARIANCE PRINCIPLES

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The goal of this paper is to estimate the total variation distance between two general stochastic polynomials. As a consequence, one obtains an invariance principle for such polynomials. This generalizes known results concerning the total variation distance between two multiple stochastic integrals on one hand, and invariance principles in Kolmogorov distance for multilinear stochastic polynomials on the other hand. As an application, we first discuss the asymptotic behavior of U-statistics associated to polynomial kernels. Moreover, we also give an example of CLT associated to quadratic forms.

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RANDOM GLUING OF METRIC SPACES[1]

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We construct random metric spaces by gluing together an infinite sequence of pointed metric spaces that we call *blocks*. At each step, we glue the next block to the structure constructed so far by randomly choosing a point on the structure and then identifying it with the distinguished point of the block. The random object that we study is the completion of the structure that we obtain after an infinite number of steps. In (*Ann. Inst. Fourier (Grenoble)* **67** (2017) 1963–2001), Curien and Haas study the case of segments, where the sequence of lengths is deterministic and typically behaves like $n^{-\alpha}$. They proved that for $\alpha > 0$, the resulting tree is compact and that the Hausdorff dimension of its set of leaves is α^{-1}. The aim of this paper is to handle a much more general case in which the blocks are i.i.d. copies of the same random metric space, scaled by deterministic factors that we call $(\lambda_n)_{n\geq 1}$. We work under some conditions on the distribution of the blocks ensuring that their Hausdorff dimension is almost surely d, for some $d \geq 0$. We also introduce a sequence $(w_n)_{n\geq 1}$ that we call the *weights* of the blocks. At each step, the probability that the next block is glued onto any of the preceding blocks is proportional to its weight. The main contribution of this paper is the computation of the Hausdorff dimension of the set $\mathcal{L}$ of points which appear during the completion procedure when the sequences $(\lambda_n)_{n\geq 1}$ and $(w_n)_{n\geq 1}$ typically behave like a power of n, say $n^{-\alpha}$ for the scaling factors and $n^{-\beta}$ for the weights, with $\alpha > 0$ and $\beta \in \mathbb{R}$. For a large domain of α and β, we have the same behaviour as the one observed in (*Ann. Inst. Fourier (Grenoble)* **67** (2017) 1963–2001), which is that $\dim_{\mathrm{H}}(\mathcal{L}) = \alpha^{-1}$. However, for $\beta > 1$ and $\alpha < 1/d$, our results reveal an interesting phenomenon: the dimension has a nontrivial dependence in α, β and d, namely

$$\dim_{\mathrm{H}}(\mathcal{L}) = \frac{2\beta - 1 - 2\sqrt{(\beta-1)(\beta-\alpha d)}}{\alpha}.$$

The computation of the dimension in the latter case involves new tools, which are specific to our model.

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FOUR-DIMENSIONAL LOOP-ERASED RANDOM WALK

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The loop-erased random walk (LERW) in $\mathbb{Z}^4$ is the process obtained by erasing loops chronologically for a simple random walk. We prove that the escape probability of the LERW renormalized by $(\log n)^{\frac{1}{3}}$ converges almost surely and in L^p for all $p > 0$. Along the way, we extend previous results by the first author building on slowly recurrent sets. We provide two applications for the escape probability. We construct the two-sided LERW, and we construct a ± 1 spin model coupled with the wired spanning forests on $\mathbb{Z}^4$ with the bi-Laplacian Gaussian field on $\mathbb{R}^4$ as its scaling limit.

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MODULUS OF CONTINUITY OF POLYMER WEIGHT PROFILES IN BROWNIAN LAST PASSAGE PERCOLATION

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In last passage percolation models lying in the KPZ universality class, the energy of long energy-maximizing paths may be studied as a function of the paths' pair of endpoint locations. Scaled coordinates may be introduced, so that these maximizing paths, or polymers, now cross unit distances with unit-order fluctuations, and have scaled energy, or weight, of unit order. In this article, we consider Brownian last passage percolation in these scaled coordinates. In the narrow wedge case, one endpoint of such polymers is fixed, say at $(0, 0) \in \mathbb{R}^2$, and the other is varied horizontally, over $(z, 1)$, $z \in \mathbb{R}$, so that the polymer weight profile is a function of $z \in \mathbb{R}$. This profile is known to manifest a one-half power law, having $1/2$-Hölder continuity. The polymer weight profile may be defined beginning from a much more general initial condition. In this article, we present a more general assertion of this one-half power law, as well as a bound on the polylogarithmic correction. The polymer weight profile admits a modulus of continuity of order $x^{1/2}(\log x^{-1})^{2/3}$, with a high degree of uniformity in the scaling parameter and over a very broad class of initial data.

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THE SCALING LIMIT OF THE MEMBRANE MODEL

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On the integer lattice, we consider the discrete membrane model, a random interface in which the field has Laplacian interaction. We prove that, under appropriate rescaling, the discrete membrane model converges to the continuum membrane model in $d \geq 2$. Namely, it is shown that the scaling limit in $d = 2, 3$ is a Hölder continuous random field, while in $d \geq 4$ the membrane model converges to a random distribution. As a by-product of the proof in $d = 2, 3$, we obtain the scaling limit of the maximum. This work complements the analogous results of Caravenna and Deuschel (*Ann. Probab.* **37** (2009) 903–945) in $d = 1$.

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THE STRUCTURE OF LOW-COMPLEXITY GIBBS MEASURES ON PRODUCT SPACES

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Let $K_1, \dots, K_n$ be bounded, complete, separable metric spaces. Let λ_i be a Borel probability measure on K_i for each i. Let $f : \prod_i K_i \longrightarrow \mathbb{R}$ be a bounded and continuous potential function, and let

$$\mu(\mathrm{d}\boldsymbol{x}) \propto \mathrm{e}^{f(\boldsymbol{x})}\lambda_1(\mathrm{d}x_1)\cdots\lambda_n(\mathrm{d}x_n)$$

be the associated Gibbs distribution.

At each point $\boldsymbol{x} \in \prod_i K_i$, one can define a 'discrete gradient' $\nabla f(\boldsymbol{x}, \cdot)$ by comparing the values of f at all points which differ from $\boldsymbol{x}$ in at most one coordinate. In case $\prod_i K_i = \{-1, 1\}^n \subset \mathbb{R}^n$, the discrete gradient $\nabla f(\boldsymbol{x}, \cdot)$ is naturally identified with a vector in $\mathbb{R}^n$.

This paper shows that a 'low-complexity' assumption on ∇f implies that μ can be approximated by a mixture of other measures, relatively few in number, and most of them close to product measures in the sense of optimal transport. This implies also an approximation to the partition function of f in terms of product measures, along the lines of Chatterjee and Dembo's theory of 'nonlinear large deviations'.

An important precedent for this work is a result of Eldan in the case $\prod_i K_i = \{-1, 1\}^n$. Eldan's assumption is that the discrete gradients $\nabla f(\boldsymbol{x}, \cdot)$ all lie in a subset of $\mathbb{R}^n$ that has small Gaussian width. His proof is based on the careful construction of a diffusion in $\mathbb{R}^n$ which starts at the origin and ends with the desired distribution on the subset $\{-1, 1\}^n$. Here our assumption is a more naive covering-number bound on the set of gradients $\{\nabla f(\boldsymbol{x}, \cdot) : \boldsymbol{x} \in \prod_i K_i\}$, and our proof relies only on basic inequalities of information theory. As a result, it is shorter, and applies to Gibbs measures on arbitrary product spaces.

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DIRECTED POLYMERS IN HEAVY-TAIL RANDOM ENVIRONMENT[1]

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We study the directed polymer model in dimension $1+1$ when the environment is heavy-tailed, with a decay exponent $\alpha \in (0,2)$. We give all possible scaling limits of the model in the *weak-coupling* regime, that is, when the inverse temperature temperature $\beta = \beta_n$ vanishes as the size of the system n goes to infinity. When $\alpha \in (1/2, 2)$, we show that all possible transversal fluctuations $\sqrt{n} \le h_n \le n$ can be achieved by tuning properly β_n, allowing to interpolate between all superdiffusive scales. Moreover, we determine the scaling limit of the model, answering a conjecture by Dey and Zygouras [*Ann. Probab.* **44** (2016) 4006–4048]—we actually identify five different regimes. On the other hand, when $\alpha < 1/2$, we show that there are only two regimes: the transversal fluctuations are either $\sqrt{n}$ or n. As a key ingredient, we use the *Entropy-controlled Last-Passage Percolation* (E-LPP), introduced in a companion paper [*Ann. Appl. Probab.* **29** (2019) 1878–1903].

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HEAVY BERNOULLI-PERCOLATION CLUSTERS ARE INDISTINGUISHABLE

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We prove that the heavy clusters are indistinguishable for Bernoulli percolation on quasi-transitive nonunimodular graphs. As an application, we show that the uniqueness threshold of any quasi-transitive graph is also the threshold for connectivity decay. This resolves a question of Lyons and Schramm (*Ann. Probab.*27 (1999) 1809–1836) in the Bernoulli percolation case and confirms a conjecture of Schonmann (*Comm. Math. Phys.* **219** (2001) 271–322). We also prove that every infinite cluster of Bernoulli percolation on a nonamenable quasi-transitive graph is transient almost surely.

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Key words and phrases. Bernoulli percolation, unimodularity, indistinguishability, heavy clusters, quasi-transitive graph.

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STRICT MONOTONICITY OF PERCOLATION THRESHOLDS UNDER COVERING MAPS

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We answer a question of Benjamini and Schramm by proving that under reasonable conditions, quotienting a graph strictly increases the value of its percolation critical parameter p_c. More precisely, let $\mathcal{G} = (V, E)$ be a quasi-transitive graph with $p_c(\mathcal{G}) < 1$, and let G be a nontrivial group that acts freely on V by graph automorphisms. Assume that $\mathcal{H} := \mathcal{G}/G$ is quasi-transitive. Then one has $p_c(\mathcal{G}) < p_c(\mathcal{H})$.

We provide results beyond this setting: we treat the case of general covering maps and provide a similar result for the uniqueness parameter p_u, under an additional assumption of boundedness of the fibres. The proof makes use of a coupling built by lifting the exploration of the cluster, and an exploratory counterpart of Aizenman–Grimmett's essential enhancements.

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A STOCHASTIC TELEGRAPH EQUATION FROM THE SIX-VERTEX MODEL[1]

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A stochastic telegraph equation is defined by adding a random inhomogeneity to the classical (second-order linear hyperbolic) telegraph differential equation. The inhomogeneities we consider are proportional to the two-dimensional white noise, and solutions to our equation are two-dimensional random Gaussian fields. We show that such fields arise naturally as asymptotic fluctuations of the height function in a certain limit regime of the stochastic six-vertex model in a quadrant. The corresponding law of large numbers—the limit shape of the height function—is described by the (deterministic) homogeneous telegraph equation.

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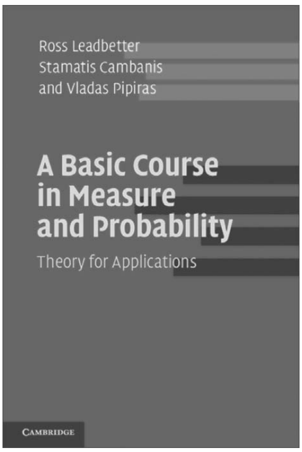

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