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HITTING TIMES OF INTERACTING DRIFTED BROWNIAN MOTIONS AND THE VERTEX REINFORCED JUMP PROCESS

BY CHRISTOPHE SABOT[1] AND XIAOLIN ZENG[2]

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Consider a negatively drifted one-dimensional Brownian motion starting at positive initial position, its first hitting time to 0 has the inverse Gaussian law. Moreover, conditionally on this hitting time, the Brownian motion up to that time has the law of a three-dimensional Bessel bridge. In this paper, we give a generalization of this result to a family of Brownian motions with interacting drifts, indexed by the vertices of a conductance network. The hitting times are equal in law to the inverse of a random potential that appears in the analysis of a self-interacting process called the vertex reinforced jump process (*Ann. Probab.* **45** (2017) 3967–3986; *J. Amer. Math. Soc.* **32** (2019) 311–349). These Brownian motions with interacting drifts have remarkable properties with respect to restriction and conditioning, showing hidden Markov properties. This family of processes are closely related to the martingale that plays a crucial role in the analysis of the vertex reinforced jump process and edge reinforced random walk (*J. Amer. Math. Soc.* **32** (2019) 311–349) on infinite graphs.

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THE TWO-DIMENSIONAL KPZ EQUATION IN THE ENTIRE SUBCRITICAL REGIME

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We consider the KPZ equation in space dimension 2 driven by space-time white noise. We showed in previous work that if the noise is mollified in space on scale ε and its strength is scaled as $\hat{\beta}/\sqrt{|\log \varepsilon|}$, then a transition occurs with explicit critical point $\hat{\beta}_c = \sqrt{2\pi}$. Recently Chatterjee and Dunlap showed that the solution admits subsequential scaling limits as $\varepsilon \downarrow 0$, for sufficiently small $\hat{\beta}$. We prove here that the limit exists in the entire subcritical regime $\hat{\beta} \in (0, \hat{\beta}_c)$ and we identify it as the solution of an additive stochastic heat equation, establishing so-called Edwards–Wilkinson fluctuations. The same result holds for the directed polymer model in random environment in space dimension 2.

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EXCHANGEABLE INTERVAL HYPERGRAPHS AND LIMITS OF ORDERED DISCRETE STRUCTURES

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A hypergraph (V, E) is called an interval hypergraph if there exists a linear order l on V such that every edge $e \in E$ is an interval w.r.t. l; we also assume that $\{j\} \in E$ for every $j \in V$. Our main result is a de Finetti-type representation of random exchangeable interval hypergraphs on $\mathbb{N}$ (EIHs): the law of every EIH can be obtained by sampling from some random compact subset K of the triangle $\{(x, y) : 0 \le x \le y \le 1\}$ at i.i.d. uniform positions $U_1, U_2, \dots,$ in the sense that, restricted to the node set $[n] := \{1, \dots, n\}$ every nonsingleton edge is of the form $e = \{i \in [n] : x < U_i < y\}$ for some $(x, y) \in K$. We obtain this result via the study of a related class of stochastic objects: erased-interval processes (EIPs). These are certain transient Markov chains $(I_n, \eta_n)_{n\in\mathbb{N}}$ such that I_n is an interval hypergraph on $V = [n]$ w.r.t. the usual linear order (called interval system). We present an almost sure representation result for EIPs. Attached to each transient Markov chain is the notion of Martin boundary. The points in the boundary of EIPs can be seen as limits of growing interval systems. We obtain a one-to-one correspondence between these limits and compact subsets K of the triangle with $(x, x) \in K$ for all $x \in [0, 1]$.

Interval hypergraphs are a generalizations of hierarchies and as a consequence we obtain a representation result for exchangeable hierarchies, which is close to a result of Forman, Haulk and Pitman in (*Probab. Theory Related Fields* **172** (2018) 1–29). Several ordered discrete structures can be seen as interval systems with additional properties, that is, Schröder trees (rooted, ordered, no node has outdegree one) or even more special: binary trees. We describe limits of Schröder trees as certain tree-like compact sets. These can be seen as an ordered counterpart to real trees, which are widely used to describe limits of discrete unordered trees. Considering binary trees, we thus obtain a homeomorphic description of the Martin boundary of Rémy's tree growth chain, which has been analyzed by Evans, Grübel and Wakolbinger in (*Ann. Probab.* **45** (2017) 225–277).

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ON THE TOPOLOGICAL BOUNDARY OF THE RANGE OF SUPER-BROWNIAN MOTION

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We show that if $\partial\mathcal{R}$ is the boundary of the range of super-Brownian motion and dim denotes Hausdorff dimension, then with probability one, for any open set U, $U \cap \partial\mathcal{R} \neq \varnothing$ implies

$$\dim(U \cap \partial\mathcal{R}) = \begin{cases} 4 - 2\sqrt{2} \approx 1.17 & \text{if } d = 2, \\ \dfrac{9 - \sqrt{17}}{2} \approx 2.44 & \text{if } d = 3. \end{cases}$$

This improves recent results of the last two authors by working with the actual topological boundary, rather than the boundary of the zero set of the local time, and establishing a local result for the dimension.

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NORMAL APPROXIMATION FOR WEIGHTED SUMS UNDER A SECOND-ORDER CORRELATION CONDITION

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Under correlation-type conditions, we derive an upper bound of order $(\log n)/n$ for the average Kolmogorov distance between the distributions of weighted sums of dependent summands and the normal law. The result is based on improved concentration inequalities on high-dimensional Euclidean spheres. Applications are illustrated on the example of log-concave probability measures.

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ENTRANCE AND EXIT AT INFINITY FOR STABLE JUMP DIFFUSIONS

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In his seminal work from the 1950s, William Feller classified all one-dimensional diffusions on $-\infty \le a < b \le \infty$ in terms of their ability to access the boundary (Feller's test for explosions) and to enter the interior from the boundary. Feller's technique is restricted to diffusion processes as the corresponding differential generators allow explicit computations and the use of Hille–Yosida theory. In the present article, we study exit and entrance from infinity for the most natural generalization, that is, jump diffusions of the form

$$dZ_t = \sigma(Z_{t-})\, dX_t,$$

driven by stable Lévy processes for $\alpha \in (0, 2)$. Many results have been proved for jump diffusions, employing a variety of techniques developed after Feller's work but exit and entrance from infinite boundaries has long remained open. We show that the presence of jumps implies features not seen in the diffusive setting without drift. Finite time explosion is possible for $\alpha \in (0, 1)$, whereas entrance from different kinds of infinity is possible for $\alpha \in [1, 2)$. Accordingly, we derive necessary and sufficient conditions on σ.

Our proofs are based on very recent developments for path transformations of stable processes via the Lamperti–Kiu representation and new Wiener–Hopf factorisations for Lévy processes that lie therein. The arguments draw together original and intricate applications of results using the Riesz–Bogdan–Żak transformation, entrance laws for self-similar Markov processes, perpetual integrals of Lévy processes and fluctuation theory, which have not been used before in the SDE setting, thereby allowing us to employ classical theory such as Hunt–Nagasawa duality and Getoor's characterisation of transience and recurrence.

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ERGODIC POISSON SPLITTINGS

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In this paper, we study splittings of a Poisson point process which are equivariant under a conservative transformation. We show that, if the Cartesian powers of this transformation are all ergodic, the only ergodic splitting is the obvious one, that is, a collection of independent Poisson processes. We apply this result to the case of a marked Poisson process: under the same hypothesis, the marks are necessarily independent of the point process and i.i.d. Under additional assumptions on the transformation, a further application is derived, giving a full description of the structure of a random measure invariant under the action of the transformation.

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OPERATOR LIMIT OF THE CIRCULAR BETA ENSEMBLE

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We provide a precise coupling of the finite circular beta ensembles and their limit process via their operator representations. We prove explicit bounds on the distance of the operators and the corresponding point processes. We also prove an estimate on the beta-dependence of the Sine_β process.

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ENTROPIC REPULSION FOR THE OCCUPATION-TIME FIELD OF RANDOM INTERLACEMENTS CONDITIONED ON DISCONNECTION

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We investigate percolation of the vacant set of random interlacements on $\mathbb{Z}^d$, $d \geq 3$, in the strongly percolative regime. We consider the event that the interlacement set at level u disconnects the discrete blow-up of a compact set $A \subseteq \mathbb{R}^d$ from the boundary of an enclosing box. We derive asymptotic large deviation upper bounds on the probability that the local averages of the occupation times deviate from a specific function depending on the harmonic potential of A, when disconnection occurs. If certain critical levels coincide, which is plausible but open at the moment, these bounds imply that conditionally on disconnection, the occupation-time profile undergoes an entropic push governed by a specific function depending on A. Similar entropic repulsion phenomena conditioned on disconnection by level-sets of the discrete Gaussian free field on $\mathbb{Z}^d$, $d \geq 3$, have been obtained by the authors in (Chiarini and Nitzschner (2018)). Our proofs rely crucially on the "solidification estimates" developed in (Nitzschner and Sznitman (2017)).

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LOCALITY OF THE CRITICAL PROBABILITY FOR TRANSITIVE GRAPHS OF EXPONENTIAL GROWTH

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Around 2008, Schramm conjectured that the critical probabilities for Bernoulli bond percolation satisfy the following continuity property: If $(G_n)_{n\geq 1}$ is a sequence of transitive graphs converging locally to a transitive graph G and $\limsup_{n\to\infty} p_c(G_n) < 1$, then $p_c(G_n) \to p_c(G)$ as $n \to \infty$. We verify this conjecture under the additional hypothesis that there is a uniform exponential lower bound on the volume growth of the graphs in question. The result is new even in the case that the sequence of graphs is uniformly nonamenable.

In the unimodular case, this result is obtained as a corollary to the following theorem of independent interest: For every $g > 1$ and $M < \infty$, there exist positive constants $C = C(g, M)$ and $\delta = \delta(g, M)$ such that if G is a transitive unimodular graph with degree at most M and growth $\mathrm{gr}(G) := \inf_{r\geq 1} |B(o,r)|^{1/r} \geq g$, then

$$\mathbf{P}_{p_c}(|K_o| \geq n) \leq Cn^{-\delta}$$

for every $n \geq 1$, where K_o is the cluster of the root vertex o. The proof of this inequality makes use of new universal bounds on the probabilities of certain two-arm events, which hold for every unimodular transitive graph.

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RANDOM MATRIX PRODUCTS: UNIVERSALITY AND LEAST SINGULAR VALUES

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We establish, under a moment matching hypothesis, the local universality of the correlation functions associated with products of M independent i.i.d. random matrices, as M is fixed, and the sizes of the matrices tend to infinity. This generalizes an earlier result of Tao and the third author for the case $M = 1$.

We also prove Gaussian limits for the centered linear spectral statistics of products of M independent i.i.d. random matrices. This is done in two steps. First, we establish the result for product random matrices with Gaussian entries, and then extend to the general case of non-Gaussian entries by another moment matching argument. Prior to our result, Gaussian limits were known only for the case $M = 1$. In a similar fashion, we establish Gaussian limits for the centered linear spectral statistics of products of independent truncated random unitary matrices. In both cases, we are able to obtain explicit expressions for the limiting variances.

The main difficulty in our study is that the entries of the product matrix are no longer independent. Our key technical lemma is a lower bound on the least singular value of the translated linearization matrix associated with the product of M normalized independent random matrices with independent and identically distributed sub-Gaussian entries. This lemma is of independent interest.

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PERCOLATION FOR LEVEL-SETS OF GAUSSIAN FREE FIELDS ON METRIC GRAPHS

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We study level-set percolation for Gaussian free fields on metric graphs. In two dimensions, we give an upper bound on the chemical distance between the two boundaries of a macroscopic annulus. Our bound holds with high probability conditioned on connectivity and is sharp up to a poly-logarithmic factor with an exponent of one-quarter. This substantially improves a previous result by Li and the first author. In three dimensions and higher, we provide rather precise estimates of percolation probabilities in different regimes which altogether describe a sharp phase transition.

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LARGE DEVIATIONS FOR THE LARGEST EIGENVALUE OF RADEMACHER MATRICES

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In this article, we consider random Wigner matrices, that is, symmetric matrices such that the subdiagonal entries of X_n are independent, centered and with variance one except on the diagonal where the entries have variance two. We prove that, under some suitable hypotheses on the laws of the entries, the law of the largest eigenvalue satisfies a large deviation principle with the same rate function as in the Gaussian case. The crucial assumption is that the Laplace transform of the entries must be bounded above by the Laplace transform of a centered Gaussian variable with same variance. This is satisfied by the Rademacher law and the uniform law on $[-\sqrt{3}, \sqrt{3}]$. We extend our result to complex entries Wigner matrices and Wishart matrices.

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THE ALMOST-SURE ASYMPTOTIC BEHAVIOR OF THE SOLUTION TO THE STOCHASTIC HEAT EQUATION WITH LÉVY NOISE

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We examine the almost-sure asymptotics of the solution to the stochastic heat equation driven by a Lévy space-time white noise. When a spatial point is fixed and time tends to infinity, we show that the solution develops unusually high peaks over short time intervals, even in the case of additive noise, which leads to a breakdown of an intuitively expected strong law of large numbers. More precisely, if we normalize the solution by an increasing nonnegative function, we either obtain convergence to 0, or the limit superior and/or inferior will be infinite. A detailed analysis of the jumps further reveals that the strong law of large numbers can be recovered on discrete sequences of time points increasing to infinity. This leads to a necessary and sufficient condition that depends on the Lévy measure of the noise and the growth and concentration properties of the sequence at the same time. Finally, we show that our results generalize to the stochastic heat equation with a multiplicative nonlinearity that is bounded away from zero and infinity.

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CONNECTIVITY PROPERTIES OF THE ADJACENCY GRAPH OF SLE_κ BUBBLES FOR $\kappa \in (4, 8)$

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We study the adjacency graph of bubbles, that is, complementary connected components of a SLE_κ curve for $\kappa \in (4, 8)$, with two such bubbles considered to be adjacent if their boundaries intersect. We show that this adjacency graph is a.s. connected for $\kappa \in (4, \kappa_0]$, where $\kappa_0 \approx 5.6158$ is defined explicitly. This gives a partial answer to a problem posed by Duplantier, Miller and Sheffield (2014). Our proof in fact yields a stronger connectivity result for $\kappa \in (4, \kappa_0]$, which says that there is a Markovian way of finding a path from any fixed bubble to ∞. We also show that there is a (nonexplicit) $\kappa_1 \in (\kappa_0, 8)$ such that this stronger condition does not hold for $\kappa \in [\kappa_1, 8)$.

Our proofs are based on an encoding of SLE_κ in terms of a pair of independent $\kappa/4$-stable processes, which allows us to reduce our problem to a problem about stable processes. In fact, due to this encoding, our results can be rephrased as statements about the connectivity of the adjacency graph of loops when one glues together an independent pair of so-called $\kappa/4$-stable looptrees, as studied, for example, by Curien and Kortchemski (2014).

The above encoding comes from the theory of Liouville quantum gravity (LQG), but the paper can be read without any knowledge of LQG if one takes the encoding as a black box.

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MEAN FIELD SYSTEMS ON NETWORKS, WITH SINGULAR INTERACTION THROUGH HITTING TIMES

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Building on the line of work (*Ann. Appl. Probab.* **25** (2015) 2096–2133; *Stochastic Process. Appl.* **125** (2015) 2451–2492; *Ann. Appl. Probab.* **29** (2019) 89–129; *Arch. Ration. Mech. Anal.* **233** (2019) 643–699; *Ann. Appl. Probab.* **29** (2019) 2338–2373; *Finance Stoch.* **23** (2019) 535–594), we continue the study of particle systems with singular interaction through hitting times. In contrast to the previous research, we (i) consider very general driving processes and interaction functions, (ii) allow for inhomogeneous connection structures and (iii) analyze a game in which the particles determine their connections strategically. Hereby, we uncover two completely new phenomena. First, we characterize the "times of fragility" of such systems (e.g., the times when a macroscopic part of the population defaults or gets infected simultaneously, or when the neuron cells "synchronize") explicitly in terms of the dynamics of the driving processes, the current distribution of the particles' values and the topology of the underlying network (represented by its Perron–Frobenius eigenvalue). Second, we use such systems to describe a dynamic credit-network game and show that, in equilibrium, the system regularizes, that is, the times of fragility never occur, as the particles avoid them by adjusting their connections strategically. Two auxiliary mathematical results, useful in their own right, are uncovered during our investigation: a generalization of Schauder's fixed-point theorem for the Skorokhod space with the M1 topology, and the application of max-plus algebra to the equilibrium version of the network flow problem.

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FINITARY CODINGS FOR SPATIAL MIXING MARKOV RANDOM FIELDS

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It has been shown by van den Berg and Steif (*Ann. Probab.* **27** (1999) 1501–1522) that the subcritical and critical Ising model on $\mathbb{Z}^d$ is a finitary factor of an i.i.d. process (ffiid), whereas the super-critical model is not. In fact, they showed that the latter is a general phenomenon in that a phase transition presents an obstruction for being ffiid. The question remained whether this is the only such obstruction. We make progress on this, showing that certain spatial mixing conditions (notions of weak dependence on boundary conditions, not to be confused with other notions of mixing in ergodic theory) imply ffiid. Our main result is that weak spatial mixing implies ffiid with power-law tails for the coding radius, and that strong spatial mixing implies ffiid with exponential tails for the coding radius. The weak spatial mixing condition can be relaxed to a condition which is satisfied by some critical two-dimensional models. Using a result of the author (Spinka (2018)), we deduce that strong spatial mixing also implies ffiid with stretched-exponential tails from a *finite-valued* i.i.d. process.

We give several applications to models such as the Potts model, proper colorings, the hard-core model, the Widom–Rowlinson model and the beach model. For instance, for the ferromagnetic q-state Potts model on $\mathbb{Z}^d$ at inverse temperature β, we show that it is ffiid with exponential tails if β is sufficiently small, it is ffiid if $\beta < \beta_c(q,d)$, it is not ffiid if $\beta > \beta_c(q,d)$ and, when $d = 2$ and $\beta = \beta_c(q,d)$, it is ffiid if and only if $q \leq 4$.

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FLOWS, COALESCENCE AND NOISE. A CORRECTION

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