THE ANNALS
of
PROBABILITY

AN OFFICIAL JOURNAL OF THE
INSTITUTE OF MATHEMATICAL STATISTICS

Articles

Color-position symmetry in interacting particle systems
ALEXEI BORODIN AND ALEXEY BUFETOV 1607

External diffusion-limited aggregation on a spanning-tree-weighted random planar map
EWAIN Gwynne and JOSHUA PFEFFER 1633

The Tutte embedding of the mated-CRT map converges to Liouville quantum gravity
EWAIN GWYNNE, JASON MILLER AND SCOTT SHEFFIELD 1677

Brownian absolute continuity of the KPZ fixed point with arbitrary initial condition
SOURAV SARKAR AND BÁLINT VIRÁG 1718

Bulk properties of the Airy line ensemble . . DUNCAN DAUVERGNE AND BÁLINT VIRÁG 1738

Eigenvector statistics of Lévy matrices
AMOL AGGARWAL, PATRICK LOPATTO AND JAKE MARCINEK 1778

Spectral edge in sparse random graphs: Upper and lower tail large deviations
BHASWAR B. BHATTACHARYA, SOHOM BHATTACHARYA AND SHIRSHENDU GANGULY 1847

On words of non-Hermitian random matrices
GUILLAUME DUBACH AND YUVAL PELED 1886

Asymptotics of the eigenvalues of the Anderson Hamiltonian with white noise potential in two dimensions ................KHALIL CHOUK AND WILLEM VAN ZUIJLEN 1917

Domino tilings of the Aztec diamond with doubly periodic weightings
TOMAS BERGGREN 1965

Emergence of extended states at zero in the spectrum of sparse random graphs
SIMON COSTE AND JUSTIN SALEZ 2012

Age evolution in the mean field forest fire model via multitype branching processes
EDWARD CRANE, BALÁzs RÁTH AND DOMINIC YEo 2031

Local and global geometry of the 2D Ising interface in critical prewetting
SHIRSHENDU GANGULY AND REZA GHEISSARI 2076
COLOR-POSITION SYMMETRY IN INTERACTING PARTICLE SYSTEMS

BY ALEXEI BORODIN¹ AND ALEXEY BUFETOV²

¹Department of Mathematics, MIT, borodin@math.mit.edu
²Hausdorff Center for Mathematics & Institute for Applied Mathematics, University of Bonn, alexey.bufetov@gmail.com

We prove a color-position symmetry for a class of ASEP-like interacting particle systems with discrete time on the one-dimensional lattice. The full space-time inhomogeneity of our systems allows to apply the result to colored (or multi-species) ASEP and stochastic vertex models for a certain class of initial/boundary conditions, generalizing previous results of Amir–Angel–Valko and Borodin–Wheeler. We are also able to use the symmetry, together with previously known results for uncolored models, to find novel asymptotic behavior of the second-class particles in several situations.

REFERENCES


MSC2020 subject classifications. 60K35.
Key words and phrases. Exclusion process, second-class particle, dissolving shock.


EXTERNAL DIFFUSION-LIMITED AGGREGATION ON A SPANNING-TREE-WEIGHTED RANDOM PLANAR MAP

BY EWAIN GWYNNE\textsuperscript{1} AND JOSHUA PFEFFER\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, University of Cambridge, eg558@cam.ac.uk
\textsuperscript{2}Department of Mathematics, Massachusetts Institute of Technology, pfeffer@mit.edu

Let $M$ be the infinite spanning-tree-weighted random planar map, which is the local limit of finite random planar maps sampled with probability proportional to the number of spanning trees they admit. We show that a.s. the graph-distance diameter of the external diffusion-limited aggregation (DLA) cluster on $M$ run for $m$ steps is of order $m^{2/d+o_m(1)}$, where $d$ is the metric ball volume growth exponent for $M$ (which was shown to exist by Ding and Gwynne (Comm. Math. Phys. 374 (2020) 1877–1934). By known bounds for $d$, one has $0.55051\ldots \leq 2/d \leq 0.563315\ldots$.

Along the way, we also prove that loop-erased random walk (LERW) on $M$ typically travels graph distance $m^{2/d+o_m(1)}$ in $m$ units of time and that the graph-distance diameter of a finite spanning-tree-weighted random planar map with $n$ edges, with or without boundary, is of order $n^{1/d+o_n(1)}$ except on an event with probability decaying faster than any negative power of $n$.

Our proofs are based on a special relationship between DLA and LERW on spanning-tree-weighted random planar maps as well as estimates for distances in such maps which come from the theory of Liouville quantum gravity.

REFERENCES


MSC2020 subject classifications. 82B24, 82B41, 60J67.
Key words and phrases. Diffusion-limited aggregation, loop-erased random walk, random planar maps, Schramm–Loewner evolution, Liouville quantum gravity.


THE TUTTE EMBEDDING OF THE MATED-CRT MAP CONVERGES TO LIOUVILLE QUANTUM GRAVITY

BY EWAIN GWYNNE¹,*, JASON MILLER¹,† AND SCOTT SHEFFIELD²

¹Department of Mathematics, University of Cambridge, *eg558@cam.ac.uk; †jpmiller@statslab.cam.ac.uk
²Department of Mathematics, Massachusetts Institute of Technology, sheffield@math.mit.edu

We prove that the Tutte embeddings (a.k.a. harmonic/barycentric embeddings) of certain random planar maps converge to γ-Liouville quantum gravity (γ-LQG). Specifically, we treat mated-CRT maps, which are discretized matings of correlated continuum random trees, and γ ranges from 0 to 2 as one varies the correlation parameter. We also show that the associated space-filling path on the embedded map converges to space-filling SLEκ for κ = 16/γ² (in the annealed sense) and that simple random walk on the embedded map converges to Brownian motion (in the quenched sense).

This work constitutes the first proof that a discrete conformal embedding of a random planar map converges to LQG. Many more such statements have been conjectured. Since the mated-CRT map can be viewed as a coarse-grained approximation to other random planar maps (the UIPT, tree-weighted maps, bipolar-oriented maps, etc.), our results indicate a potential approach for proving that embeddings of these maps converge to LQG as well.

To prove the main result, we establish several (independently interesting) theorems about LQG surfaces decorated by space-filling SLE. There is a natural way to use the SLE curve to divide the plane into “cells” corresponding to vertices of the mated-CRT map. We study the law of the shape of the origin-containing cell, in particular proving moments for the ratio of its squared diameter to its area. We also give bounds on the degree of the origin-containing cell and establish a form of ergodicity for the entire configuration. Ultimately, we use these properties to show (with the help of a general theorem proved in a separate paper) that random walk on these cells converges to a time change of Brownian motion, which in turn leads to the Tutte embedding result.

REFERENCES


MSC2020 subject classifications. 60J67, 60D05, 60J65.

Key words and phrases. Random planar maps, mated-CRT map, Tutte embedding, Liouville quantum gravity, Schramm–Loewner evolution, random walk.


[58] Miller, J. and Sheffield, S. (2016). Imaginary geometry II: Reversibility of SLE$_{\kappa}(\rho_1; \rho_2)$ for $\kappa \in (0, 4)$. *Ann. Probab.* **44** 1647–1722. MR3502592 https://doi.org/10.1214/14-AOP943


BROWNIAN ABSOLUTE CONTINUITY OF THE KPZ FIXED POINT WITH ARBITRARY INITIAL CONDITION

BY SOURAV SARKAR\textsuperscript{1} AND BáLINT VIRÁG\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, University of Toronto, ssarkar@math.toronto.edu
\textsuperscript{2}Departments of Mathematics and Statistics, University of Toronto, balint@math.toronto.edu

We show that the law of the KPZ fixed point starting from arbitrary initial condition is absolutely continuous with respect to the law of Brownian motion $B$ on every compact interval. In particular, the Airy\textsubscript{1} process is absolutely continuous with respect to $B$ on any compact interval.

REFERENCES


\textit{MSC2020} subject classifications. 82B23, 82C22, 60H15.

\textit{Key words and phrases}. KPZ fixed point, locally Brownian, absolute continuity, Brownian on compacts, finitary initial conditions.
BULK PROPERTIES OF THE AIRY LINE ENSEMBLE

BY DUNCAN DAUVERGNE\textsuperscript{1} AND BÁLINT VIRÁG\textsuperscript{2}

\textsuperscript{1}Department of Mathematics, Princeton University, dd18@math.princeton.edu
\textsuperscript{2}Department of Mathematics, University of Toronto, balint@math.toronto.edu

The Airy line ensemble is a central object in random matrix theory and last passage percolation defined by a determinantal formula. The goal of this paper is to provide a set of tools, which allow for precise probabilistic analysis of the Airy line ensemble. The two main theorems are a representation in terms of independent Brownian bridges connecting a fine grid of points, and a modulus of continuity result for all lines. Along the way, we give tail bounds and moduli of continuity for nonintersecting Brownian ensembles, and a quick proof of tightness for Dyson’s Brownian motion converging to the Airy line ensemble.

REFERENCES


MSC2020 subject classifications. Primary 60K35; secondary 60B20.

Key words and phrases. Airy line ensemble, Airy process, KPZ universality class, last passage percolation, Brownian Gibbs property, modulus of continuity, Dyson’s Brownian motion.


EIGENVECTOR STATISTICS OF LÉVY MATRICES

BY AMOL AGGARWAL1, PATRICK LOPATTO2 AND JAKE MARCINEK3

1Department of Mathematics, Columbia University, amolaggarwal@math.columbia.edu
2School of Mathematics, Institute for Advanced Study, lopatto@ias.edu
3Department of Mathematics, Harvard University, marcinek@math.harvard.edu

We analyze statistics for eigenvector entries of heavy-tailed random symmetric matrices (also called Lévy matrices) whose associated eigenvalues are sufficiently small. We show that the limiting law of any such entry is non-Gaussian, given by the product of a normal distribution with another random variable that depends on the location of the corresponding eigenvalue. Although the latter random variable is typically nonexplicit, for the median eigenvector it is given by the inverse of a one-sided stable law. Moreover, we show that different entries of the same eigenvector are asymptotically independent, but that there are nontrivial correlations between eigenvectors with nearby eigenvalues. Our findings contrast sharply with the known eigenvector behavior for Wigner matrices and sparse random graphs.

REFERENCES


MSC2020 subject classifications. 15B52, 60B20.
Key words and phrases. Lévy matrices, eigenvector statistics, eigenvector moment flow, resolvent, Poisson weighted infinite tree.


In this paper, we consider the problem of estimating the joint upper and lower tail large deviations of the edge eigenvalues of an Erdös–Rényi random graph $G_{n,p}$, in the regime of $p$ where the edge of the spectrum is no longer governed by global observables, such as the number of edges, but rather by localized statistics, such as high degree vertices. Going beyond the recent developments in mean-field approximations of related problems, this paper provides a comprehensive treatment of the large deviations of the spectral edge in this entire regime, which notably includes the well-studied case of constant average degree. In particular, for $r \geq 1$ fixed, we pin down the asymptotic probability that the top $r$ eigenvalues are jointly greater/less than their typical values by multiplicative factors bigger/smaller than 1, in the regime mentioned above. The proof for the upper tail relies on a novel structure theorem, obtained by building on estimates in (Combin. Probab. Comput. 12 (2003) 61–72), followed by an iterative cycle removal process, which shows, conditional on the upper tail large deviation event, with high probability the graph admits a decomposition in to a disjoint union of stars and a spectrally negligible part. On the other hand, the key ingredient in the proof of the lower tail is a Ramsey-type result which shows that if the $K$th largest degree of a graph is not atypically small (for some large $K$ depending on $r$), then either the top eigenvalue or the $r$th largest eigenvalue is larger than that allowed by the lower tail event on the top $r$ eigenvalues, thus forcing a contradiction. The above arguments reduce the problems to developing a large deviation theory for the extremal degrees which could be of independent interest.

REFERENCES


MSC2020 subject classifications. 60F10, 05C80, 60B20, 15A18.

Key words and phrases. Erdös–Rényi random graph, large deviations, random matrices, extreme eigenvalues.


ON WORDS OF NON-HERMITIAN RANDOM MATRICES

BY GUILLAUME DUBACH¹ AND YUVAL PELED²

¹IST Austria, guillaume.dubach@ist.ac.at
²Courant Institute, New York University, yuval.peled@cims.nyu.edu

We consider words $G_{i_1} \cdots G_{i_m}$ involving i.i.d. complex Ginibre matrices and study tracial expressions of their eigenvalues and singular values. We show that the limit distribution of the squared singular values of every word of length $m$ is a Fuss–Catalan distribution with parameter $m + 1$. This generalizes previous results concerning powers of a complex Ginibre matrix and products of independent Ginibre matrices. In addition, we find other combinatorial parameters of the word that determine the second-order limits of the spectral statistics. For instance, the so-called coperiod of a word characterizes the fluctuations of the eigenvalues. We extend these results to words of general non-Hermitian matrices with i.i.d. entries under moment-matching assumptions, band matrices, and sparse matrices.

These results rely on the moments method and genus expansion, relating Gaussian matrix integrals to the counting of compact orientable surfaces of a given genus. This allows us to derive a central limit theorem for the trace of any word of complex Ginibre matrices and their conjugate transposes, where all parameters are defined topologically.

REFERENCES


MSC2020 subject classifications. 60B20, 57K20.

Key words and phrases. Complex Ginibre ensemble, Fuss–Catalan distribution, genus expansion, mixed moments of non-Hermitian matrices, second order freeness, words of random matrices.


ASYMPTOTICS OF THE EIGENVALUES OF THE ANDERSON HAMILTONIAN WITH WHITE NOISE POTENTIAL IN TWO DIMENSIONS

BY KHALIL CHOUK\textsuperscript{1} AND WILLEM VAN ZUIJLEN\textsuperscript{2}

\textsuperscript{1}School of Mathematics, University of Edinburgh, khalil.chouk@gmail.com
\textsuperscript{2}Weierstrass Institute for Applied Analysis and Stochastics, Berlin, vanzuijlen@wias-berlin.de

In this paper we consider the Anderson Hamiltonian with white noise potential on the box $[0, L]^2$ with Dirichlet boundary conditions. We show that all of the eigenvalues divided by $\log L$, converge as $L \to \infty$, almost surely to the same deterministic constant which is given by a variational formula.

REFERENCES


MSC2020 subject classifications. Primary 60H25, 60F15, 35J10, 35J15; secondary 60F10.
Key words and phrases. Anderson Hamiltonian, white noise, paracontrolled distributions, operators with Dirichlet boundary conditions.


DOMINO TILINGS OF THE AZTEC DIAMOND WITH DOUBLY PERIODIC WEIGHTINGS

BY TOMAS BERGGREN

Department of Mathematics, University of Michigan, berggren@umich.edu

In this paper we consider domino tilings of the Aztec diamond with doubly periodic weightings. In particular, a family of models which, for any \( k \in \mathbb{N} \), includes models with \( k \) smooth regions is analyzed as the size of the Aztec diamond tends to infinity. We use a nonintersecting paths formulation and give a double integral formula for the correlation kernel of the Aztec diamond of finite size. By a classical steepest descent analysis of the correlation kernel, we obtain the local behavior in the smooth and rough regions, as the size of the Aztec diamond tends to infinity. From the mentioned limit the macroscopic picture, such as the arctic curves and, in particular, the number of smooth regions, is deduced. Moreover, we compute the limit of the height function, and, as a consequence, we confirm in the setting of this paper that the limit in the rough region fulfills the complex Burgers’ equation, as stated by Kenyon and Okounkov.

REFERENCES


MSC2020 subject classifications. Primary 60G55; secondary 60B10, 52C20, 30E20. 
Key words and phrases. Determinantal point processes, periodically weighted random tilings.
EMERGENCE OF EXTENDED STATES AT ZERO IN THE SPECTRUM OF SPARSE RANDOM GRAPHS

BY SIMON COSTE$^1$ AND JUSTIN SALEZ$^2$

$^1$INRIA Paris, simon.coste@inria.fr

$^2$CEREMADE, University Paris-Dauphine & PSL, justin.salez@dauphine.psl.eu

We confirm the long-standing prediction that $c = e \approx 2.718$ is the threshold for the emergence of a nonvanishing absolutely continuous part (extended states) at zero in the limiting spectrum of the Erdős–Rényi random graph with average degree $c$. This is achieved by a detailed second-order analysis of the resolvent $(A - z)^{-1}$ near the singular point $z = 0$, where $A$ is the adjacency operator of the Poisson–Galton–Watson tree with mean offspring $c$. More generally, our method applies to arbitrary unimodular Galton–Watson trees, yielding explicit criteria for the presence or absence of extended states at zero in the limiting spectral measure of a variety of random graph models, in terms of the underlying degree distribution.

REFERENCES


MSC2020 subject classifications. 05C80, 60B20, 47A10.

Key words and phrases. Spectrum, sparse Erdős–Rényi random graphs, unimodular Galton–Watson trees, extended states.


AGE EVOLUTION IN THE MEAN FIELD FOREST FIRE MODEL VIA
MULTITYPE BRANCHING PROCESSES

BY EDWARD CRANE¹, BALÁZS RÁTH² AND DOMINIC YEO³

¹School of Mathematics, University of Bristol, Edward.Crane@bristol.ac.uk
²MTA-BME Stochastics Research Group, Budapest University of Technology and Economics, rathb@math.bme.hu
³Department of Statistics, University of Oxford, dominicjyeo@gmail.com

We study the distribution of ages in the mean field forest fire model introduced by Ráth and Tóth. This model is an evolving random graph whose dynamics combine Erdős–Rényi edge-addition with a Poisson rain of lightning strikes. All edges in a connected component are deleted when any of its vertices is struck by lightning. We consider the asymptotic regime of lightning rates for which the model displays self-organized criticality. The age of a vertex increases at unit rate, but it is reset to zero at each burning time. We show that the empirical age distribution converges as a process to a deterministic solution of an autonomous measure-valued differential equation. The main technique is to observe that, conditioned on the vertex ages, the graph is an inhomogeneous random graph in the sense of Bollobás, Janson and Riordan. We then study the evolution of the ages via the multitype Galton–Watson trees that arise as the limit in law of the component of an identified vertex at any fixed time. These trees are critical from the gelation time onwards.

REFERENCES


MSC2020 subject classifications. 05C80, 60J80, 46N55, 35Q82.

Key words and phrases. Inhomogeneous random graph, multitype branching process, self-organized criticality, Perron–Frobenius theory, differential equations.


LOCAL AND GLOBAL GEOMETRY OF THE 2D ISING INTERFACE IN CRITICAL PREWETTING

BY SHIRSHENDU GANGULY* AND REZA GHEISSARI†

Department of Statistics, University of California, Berkeley, *sganguly@berkeley.edu; †gheissari@berkeley.edu

Consider the Ising model at low temperatures and positive external field \(\lambda\) on an \(N \times N\) box with Dobrushin boundary conditions that are plus on the north, east and west boundaries and minus on the south boundary. If \(\lambda = 0\), the interface separating the plus and minus phases is diffusive, having \(O(\sqrt{N})\) height fluctuations, and the model is fully wetted. Under an order one field, the interface fluctuations are \(O(1)\), and the interface is only partially wetted, being pinned to its southern boundary. We study the critical prewetting regime of \(\lambda N^{-1/3}\), where the height fluctuations are expected to scale as \(\lambda^{-1/3}\) and the rescaled interface is predicted to converge to the Ferrari–Spohn diffusion. Velenik (Probab. Theory Related Fields 129 (2004) 83–112) identified the order of the area under the interface up to logarithmic corrections. Since then, more refined features of such interfaces have only been identified in simpler models of random walks under area tilts.

In this paper we resolve several conjectures of Velenik regarding the refined features of the Ising interface in the critical prewetting regime. Our main result is a sharp bound on the one-point height fluctuation, proving \(e^{-\Theta(x^{3/2})}\) upper tails reminiscent of the Tracy–Widom distribution, capturing a tradeoff between the locally Brownian oscillations and the global field effect. We further prove a concentration estimate for the number of points above which the interface attains a large height. These are used to deduce various geometric properties of the interface, including the order and tails of the area it confines and the polylogarithmic prefactor governing its maximum height fluctuation. Our arguments combine classical inputs from the random-line representation of the Ising interface with novel local resampling and coupling schemes.

REFERENCES


MSC2020 subject classifications. Primary 60K35, 82B24; secondary 82B20, 82B41.

Key words and phrases. Ising model, wetting transition, entropic repulsion, interface, cube-root fluctuations.


The Annals of Probability

Vol. 49 September 2021 No. 5

Articles

Universality for critical KCM: Finite number of stable directions
IVA I L OHARTARSKY, FABIO MARTINELLI AND CRISTINA TONINELLI

Symmetries of stochastic colored vertex models
PAVEL GALASHIN

The height of Mallows trees
LOUIGI ADDARIO-BERRY AND BENOÎT CORSINI

Frozen percolation on the binary tree is nonendogenous
BALÁZS RÁTH, JAN M. SWART AND TAMÁS TERPAI

Metastability and exit problems for systems of stochastic reaction–diffusion equations
MICHAEL SALINS AND KONSTANTINOS SPILOPOULOS

On stochastic equations with drift in $L^d$
N.V. KRYLOV

Sharp threshold for the Ising perceptron model
CHANGJI XU

A geometric representation of fragmentation processes on stable trees
PAUL THÉVENIN

Characterization of Brownian Gibbsian line ensembles
EVGENI DIMITROV AND KONSTANTIN MATETSKI

Chase-escape with death on trees
ERIN BECKMAN, KEISHA COOK, NICOLE EIKMEIER, SARAI HERNANDEZ-TORRES AND MATTHEW JUNGE

Small ball probabilities and a support theorem for the stochastic heat equation
SIVA ATHREYA, MATHEW JOSEPH AND CARL MUELLER

Polarity of almost all points for systems of nonlinear stochastic heat equations in the critical dimension
ROBERT C. DALANG, CARL MUELLER AND YIMIN XIAO

Moment estimates for some renormalized parabolic Anderson models
XIA CHEN, AURÉLIEN DEYA, CHENG OUYYANG AND SAMY TINDEL

To fixate or not to fixate in two-type annihilating branching random walks
DANIEL AHLBERG, SIMON GRIFFITHS AND SVANTE JANSON

Second errata to “Distance covariance in metric spaces”
RUSSELL LYONS
**Probability on Graphs**

*Random Processes on Graphs and Lattices*

Geoffrey Grimmett

This introduction to some of the principal models in the theory of disordered systems leads the reader through the basics, to the very edge of contemporary research, with the minimum of technical fuss. Topics covered include random walk, percolation, self-avoiding walk, interacting particle systems, uniform spanning tree, random graphs, as well as the Ising, Potts, and random-cluster models for ferromagnetism, and the Lorentz model for motion in a random medium. Schramm–Löwner evolutions (SLE) arise in various contexts. The choice of topics is strongly motivated by modern applications and focuses on areas that merit further research. Special features include a simple account of Smirnov’s proof of Cardy’s formula for critical percolation, and a fairly full account of the theory of influence and sharp-thresholds. Accessible to a wide audience of mathematicians and physicists, this book can be used as a graduate course text. Each chapter ends with a range of exercises.