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THE TWO-DIMENSIONAL CONTINUOUS RANDOM FIELD ISING MODEL

BY ADAM BOWDITCHa AND RONGFENG SUNb

Department of Mathematics, National University of Singapore, a.m.bowditch@gmail.com, b.matsr@nus.edu.sg

In this paper, we construct the two-dimensional continuum random field Ising model via scaling limits of a random field perturbation of the critical two-dimensional Ising model with diminishing disorder strength. Furthermore, we show that almost surely with respect to the continuum random field given by a white noise, the law of the magnetisation field is singular with respect to that of the two-dimensional continuum pure Ising model constructed by Camia, Garban and Newman (Ann. Probab. 43 (2015) 528–571).

REFERENCES


MSC2020 subject classifications. Primary 82B44; secondary 82B20, 82B27, 60G60, 60K35.

Key words and phrases. Random field Ising model, continuum scaling limit, magnetisation field.


MAXIMUM AND COUPLING OF THE SINE-GORDON FIELD

BY ROLAND BAUERSCHMIDT\textsuperscript{a} AND MICHAEL HOFSTETTER\textsuperscript{b}

Statistical Laboratory, DPMMS, University of Cambridge,\textsuperscript{a} rb812@cam.ac.uk, \textsuperscript{b} mh901@cam.ac.uk

For $0 < \beta < 6\pi$, we prove that the distribution of the centred maximum of the $\epsilon$-regularised continuum sine-Gordon field on the two-dimensional torus converges to a randomly shifted Gumbel distribution as $\epsilon \to 0$. Our proof relies on a strong coupling at all scales of the sine-Gordon field with the Gaussian free field, of independent interest, and extensions of existing methods for the maximum of the lattice Gaussian free field.

REFERENCES


\textit{MSC2020 subject classifications.} Primary 60G60; secondary 82B41.

\textit{Key words and phrases.} Maximum, sine-Gordon field, Gaussian free field, coupling.


EXTREMAL DISTANCE AND CONFORMAL RADIUS OF A CLE$_4$ LOOP

BY JUHAN ARU$^{1,a}$, TITUS LUPU$^{2,b}$ AND AVELIO SEPÚLVEDA$^{3,c}$

$^1$École Polytechnique Fédérale de Lausanne (EPFL), Institute of Mathematics, $^2$Université de Paris and Sorbonne Université, CNRS, LPSM, $^3$DIM / CMM, Universidad de Chile

Consider CLE$_4$ in the unit disk, and let $\ell$ be the loop of the CLE$_4$ surrounding the origin. Schramm, Sheffield and Wilson determined the law of the conformal radius seen from the origin of the domain surrounded by $\ell$. We complement their result by determining the law of the extremal distance between $\ell$ and the boundary of the unit disk. More surprisingly, we also compute the joint law of these conformal radius and extremal distance. This law involves first and last hitting times of a one-dimensional Brownian motion. Similar techniques also allow us to determine joint laws of some extremal distances in a critical Brownian loop-soup cluster.

REFERENCES


MSC2020 subject classifications. 31A15, 60G15, 60G60, 60J65, 60J67, 81T40.
Key words and phrases. Conformal loop ensemble, Gaussian free field, isomorphism theorems, local set, loop soup, metric graph, Schramm–Loewner evolution.


ON INHOMOGENEOUS POLYNUCLEAR GROWTH

BY KURT JOHANSSON¹,a AND MUSTAZEE RAHMAN²,b

¹Department of Mathematics, KTH Royal Institute of Technology, ²kurtj@kth.se
²Department of Mathematical Sciences, Durham University, bmustazee@gmail.com

This article studies the inhomogeneous geometric polynuclear growth model; the distribution of which is related to Schur functions. We explain a method to derive its distribution functions in both space-like and time-like directions, focusing on the two-time distribution. Asymptotics of the two-time distribution in the KPZ-scaling limit is then considered, extending to two times several single-time distributions in the KPZ universality class.

REFERENCES


MSC2020 subject classifications. Primary 60F05, 60K35, 82C23, 82C24; secondary 05E10, 15A15, 30E20.

Key words and phrases. KPZ universality, last passage percolation, polynuclear growth, two-time distribution.


THE DIRICHLET–FERGUSON DIFFUSION ON THE SPACE OF PROBABILITY MEASURES OVER A CLOSED RIEMANNIAN MANIFOLD

BY LORENZO DELLO SCHIAVO

IST Austria, lorenzo.delloschiavo@ist.ac.at

We construct a recurrent diffusion process with values in the space of probability measures over an arbitrary closed Riemannian manifold of dimension $d \geq 2$. The process is associated with the Dirichlet form defined by integration of the Wasserstein gradient w.r.t. the Dirichlet–Ferguson measure, and is the counterpart on multidimensional base spaces to the modified massive Arratia flow over the unit interval described in V. Konarovskyi and M.-K. von Renesse (Comm. Pure Appl. Math. 72 (2019) 764–800). Together with two different constructions of the process, we discuss its ergodicity, invariant sets, finite-dimensional approximations, and Varadhan short-time asymptotics.

REFERENCES


MSC2020 subject classifications. Primary 60J46, 60J60; secondary 47D07, 60G57.

Key words and phrases. Dirichlet–Ferguson measure, Dirichlet–Ferguson diffusion, Wasserstein diffusion, modified massive Arratia flow.


Precise Local Estimates for Differential Equations Driven by Fractional Brownian Motion: Hypoelliptic Case

By Xi Geng\textsuperscript{1,a}, Cheng Ouyang\textsuperscript{2,b} and Samy Tindel\textsuperscript{3,c}

\textsuperscript{1}School of Mathematics and Statistics, The University of Melbourne, \textsuperscript{a}xi.geng@unimelb.edu.au
\textsuperscript{2}Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, \textsuperscript{b}couyang@math.uic.edu
\textsuperscript{3}Department of Mathematics, Purdue University, \textsuperscript{c}stindel@purdue.edu

This article is concerned with stochastic differential equations driven by a \(d\) dimensional fractional Brownian motion with Hurst parameter \(H > 1/4\) and understood in the rough paths sense. Whenever the coefficients of the equation satisfy a uniform hypoellipticity condition, we establish a sharp local estimate on the associated control distance function and a sharp local lower estimate on the density of the solution. Our methodology relies heavily on the rough paths structure of the equation.

REFERENCES


MSC2020 subject classifications. 60H10, 60H07, 60G15.

Key words and phrases. Rough paths, Malliavin calculus, fractional Brownian motion.


HYPERCONTRACTIVITY AND LOWER DEVIATION ESTIMATES IN NORMED SPACES

BY GRIGORIS PAOURIS\(^1, a\), KONSTANTIN TIKHOMIROV\(^2, b\) AND PETROS VALETTAS\(^3, c\)

\(^1\)Department of Mathematics, Texas A&M University, \(^a\)grigorios.paouris@gmail.com
\(^2\)Department of Mathematics, Georgia Institute of Technology, \(^b\)ktikhomirov6@gatech.edu
\(^3\)Departments of Mathematics and EE&CS, University of Missouri-Columbia, \(^c\)valettasp@missouri.edu

We consider the problem of estimating small ball probabilities \(\mathbb{P}\{f(G) \leq \delta \mathbb{E} f(G)\}\) for subadditive, positively homogeneous functions \(f\) with respect to the Gaussian measure. We establish estimates that depend on global parameters of the underlying function, which take into account analytic and statistical measures, such as the variance and the \(L^1\)-norms of its partial derivatives. This leads to dimension-dependent bounds for small ball and lower small deviation estimates for seminorms when the linear structure is appropriately chosen to optimize the aforementioned parameters. Our bounds are best possible up to numerical constants. In all regimes, \(\|G\|_\infty = \max_{i \leq n} |g_i|\) arises as an extremal case in this study. The proofs exploit the convexity and hypercontractivity properties of the Gaussian measure.

REFERENCES


MSC2020 subject classifications. Primary 46B09; secondary 52A21.

Key words and phrases. Talagrand’s \(L^1 \rightarrow L^2\) bound, Alon–Milman theorem, Ornstein–Uhlenbeck semigroup, Gaussian convexity, hypercontractivity, superconcentration.


CONCENTRATION INEQUALITIES FOR LOG-CONCAVE DISTRIBUTIONS
WITH APPLICATIONS TO RANDOM SURFACE FLUCTUATIONS

BY ALEXANDER MAGAZINOV\textsuperscript{a} AND RON PELED\textsuperscript{b}

School of Mathematical Sciences, Tel Aviv University, \textsuperscript{a}magazinov-al@yandex.ru, \textsuperscript{b}peledron@tauex.tau.ac.il

We derive two concentration inequalities for linear functions of log-concave distributions: an enhanced version of the classical Brascamp–Lieb concentration inequality and an inequality quantifying log-concavity of marginals in a manner suitable for obtaining variance and tail probability bounds.

These inequalities are applied to the statistical mechanics problem of estimating the fluctuations of random surfaces of the $\nabla \phi$ type. The classical Brascamp–Lieb inequality bounds the fluctuations whenever the interaction potential is uniformly convex. We extend these bounds to the case of convex potentials whose second derivative vanishes only on a zero measure set, when the underlying graph is a $d$-dimensional discrete torus. The result applies, in particular, to potentials of the form $U(x) = |x|^p$ with $p > 1$ and answers a question discussed by Brascamp–Lieb–Lebowitz (In Statistical Mechanics (1975) 379–390, Springer). Additionally, new tail probability bounds are obtained for the family of potentials $U(x) = |x|^p + x^2$, $p > 2$. This result answers a question mentioned by Deuschel and Giacomin (Stochastic Process. Appl. 89 (2000) 333–354).

REFERENCES


MSC2020 subject classifications. Primary 60E15, 60K35, 82C24; secondary 82B20.

Key words and phrases. Brascamp–Lieb concentration inequality, log-concave distributions, random surfaces of the $\nabla \phi$ type, effective interface models, localization, tail probability bounds.


ON MULTIPLE SLE FOR THE FK–ISING MODEL

BY KONSTANTIN IZYUROV

Department of Mathematics and Statistics, University of Helsinki, konstantin.izyurov@helsinki.fi

We prove the convergence of multiple interfaces in the critical planar $q = 2$ random cluster model and provide an explicit description of the scaling limit. Remarkably, the expression for the partition function of the resulting multiple SLE_{16/3} coincides with the bulk spin correlation in the critical Ising model in the half-plane, after formally replacing a position of each spin and its complex conjugate with a pair of points on the real line. As a corollary we recover Belavin–Polyakov–Zamolodchikov equations for the spin correlations.

REFERENCES


MSC2020 subject classifications. 60J67, 82B20.

Key words and phrases. Schramm–Loewner evolution, Ising model, random cluster models.
We study the McKean–Vlasov optimal control problem with common noise which allow the law of the control process to appear in the state dynamics under various formulations: strong and weak ones, Markovian or non-Markovian. By interpreting the controls as probability measures on an appropriate canonical space with two filtrations, we then develop the classical measurable selection, conditioning and concatenation arguments in this new context, and establish the dynamic programming principle under general conditions.

REFERENCES


MSC2020 subject classifications. Primary 49L20, 93E20; secondary 60K35, 60H30.

Key words and phrases. McKean–Vlasov optimal control, dynamic programming principle, measurable selection.


The Annals of Probability

Vol. 50       May 2022       No. 3

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A Basic Course in Measure and Probability: Theory for Applications

Ross Leadbetter, Stamatis Cambanis, and Vladas Pipiras

Originating from the authors’ own graduate course at the University of North Carolina, this material has been thoroughly tried and tested over many years, making the book perfect for a two-term course or for self-study. It provides a concise introduction that covers all of the measure theory and probability most useful for statisticians, including Lebesgue integration, limit theorems in probability, martingales, and some theory of stochastic processes. Readers can test their understanding of the material through the 300 exercises provided.

The book is especially useful for graduate students in statistics and related fields of application (biostatistics, econometrics, finance, meteorology, machine learning, and so on) who want to shore up their mathematical foundation. The authors establish common ground for students of varied interests which will serve as a firm ‘take-off point’ for them as they specialize in areas that exploit mathematical machinery.

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