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Papers

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Empirical entropy, minimax regret and minimax risk

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We consider the random design regression model with square loss. We propose a method that aggregates empirical minimizers (ERM) over appropriately chosen random subsets and reduces to ERM in the extreme case, and we establish sharp oracle inequalities for its risk. We show that, under the ε^{-p} growth of the empirical ε-entropy, the excess risk of the proposed method attains the rate $n^{-2/(2+p)}$ for $p \in (0,2)$ and $n^{-1/p}$ for $p > 2$ where n is the sample size. Furthermore, for $p \in (0,2)$, the excess risk rate matches the behavior of the minimax risk of function estimation in regression problems under the well-specified model. This yields a conclusion that the rates of statistical estimation in well-specified models (minimax risk) and in misspecified models (minimax regret) are equivalent in the regime $p \in (0,2)$. In other words, for $p \in (0,2)$ the problem of statistical learning enjoys the same minimax rate as the problem of statistical estimation. On the contrary, for $p > 2$ we show that the rates of the minimax regret are, in general, slower than for the minimax risk. Our oracle inequalities also imply the $v \log(n/v)/n$ rates for Vapnik–Chervonenkis type classes of dimension v without the usual convexity assumption on the class; we show that these rates are optimal. Finally, for a slightly modified method, we derive a bound on the excess risk of s-sparse convex aggregation improving that of Lounici [*Math. Methods Statist.* **16** (2007) 246–259] and providing the optimal rate.

Keywords: aggregation; empirical risk minimization; entropy; minimax regret; minimax risk

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Semiparametric topographical mixture models with symmetric errors

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Motivated by the analysis of a Positron Emission Tomography (PET) imaging data considered in Bowen *et al*. [*Radiother. Oncol.* **105** (2012) 41–48], we introduce a semiparametric topographical mixture model able to capture the characteristics of dichotomous shifted response-type experiments. We propose a pointwise estimation procedure of the proportion and location functions involved in our model. Our estimation procedure is only based on the symmetry of the local noise and does not require any finite moments on the errors (e.g., Cauchy-type errors). We establish under mild conditions minimax properties and asymptotic normality of our estimators. Moreover, Monte Carlo simulations are conducted to examine their finite sample performance. Finally, a statistical analysis of the PET imaging data in Bowen *et al*. is illustrated for the proposed method.

Keywords: asymptotic normality; consistency; contrast estimators; finite mixture of regressions; Fourier transform; identifiability; inverse problem; mixture model; semiparametric; symmetric errors

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Type II chain graph models for categorical data: A smooth subclass

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The Probabilistic Graphical Models use graphs in order to represent the joint distribution of q variables. These models are useful for their ability to capture and represent the system of independence relationships among the variables involved, even when complex. This work concerns categorical variables and the possibility to represent symmetric and asymmetric dependences among categorical variables. For this reason we use the Chain Graphical Models proposed by Andersson, Madigan and Perlman (*Scand. J. Stat.* **28** (2001) 33–85), also known as Chain Graphical Models of type II (GMs II). The GMs II allow for symmetric relationships typical of log-linear models and, at the same time, asymmetric dependences typical of Graphical Models for Directed Acyclic Graphs. In general, GMs II are not smooth, however this work provides a subclass of smooth GMs II by parametrizing the probability function through marginal log-linear models. Furthermore, the proposed models are applied to a data-set from the European Value Study for the year 2008 (EVS (2010)).

Keywords: categorical variables; Chain Graph Models; conditional indipendence models; marginal models

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Lower bounds in the convolution structure density model

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The aim of the paper is to establish asymptotic lower bounds for the minimax risk in two generalized forms of the density deconvolution problem. The observation consists of an independent and identically distributed (i.i.d.) sample of n random vectors in $\mathbb{R}^d$. Their common probability distribution function $\mathfrak{p}$ can be written as $\mathfrak{p} = (1-\alpha)f + \alpha[f \star g]$, where f is the unknown function to be estimated, g is a known function, α is a known proportion, and $\star$ denotes the convolution product. The bounds on the risk are established in a very general minimax setting and for moderately ill posed convolutions. Our results show notably that neither the ill-posedness nor the proportion α play any role in the bounds whenever $\alpha \in [0, 1)$, and that a particular inconsistency zone appears for some values of the parameters. Moreover, we introduce an additional boundedness condition on f and we show that the inconsistency zone then disappears.

Keywords: $\mathbb{L}_p$-risk; adaptive estimation; density estimation; generalized deconvolution model; minimax rates; Nikol'skii spaces

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Multilevel path simulation for weak approximation schemes with application to Lévy-driven SDEs

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In this paper, we discuss the possibility of using multilevel Monte Carlo (MLMC) approach for weak approximation schemes. It turns out that by means of a simple coupling between consecutive time discretisation levels, one can achieve the same complexity gain as under the presence of a strong convergence. We exemplify this general idea in the case of weak Euler schemes for Lévy-driven stochastic differential equations. The numerical performance of the new "weak" MLMC method is illustrated by several numerical examples.

Keywords: Lévy-driven stochastic differential equations; multilevel Monte Carlo; weak approximation schemes

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Two-sample smooth tests for the equality of distributions

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This paper considers the problem of testing the equality of two unspecified distributions. The classical omnibus tests such as the Kolmogorov–Smirnov and Cramér–von Mises are known to suffer from low power against essentially all but location-scale alternatives. We propose a new two-sample test that modifies the Neyman's smooth test and extend it to the multivariate case based on the idea of projection pursue. The asymptotic null property of the test and its power against local alternatives are studied. The multiplier bootstrap method is employed to compute the critical value of the multivariate test. We establish validity of the bootstrap approximation in the case where the dimension is allowed to grow with the sample size. Numerical studies show that the new testing procedures perform well even for small sample sizes and are powerful in detecting local features or high-frequency components.

Keywords: goodness-of-fit; high-frequency alternations; multiplier bootstrap; Neyman's smooth test; two-sample problem

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Nonparametric regression on hidden Φ-mixing variables: Identifiability and consistency of a pseudo-likelihood based estimation procedure

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This paper outlines a new nonparametric estimation procedure for unobserved Φ-mixing processes. It is assumed that the only information on the stationary hidden states $(X_k)_{k\geq 0}$ is given by the process $(Y_k)_{k\geq 0}$, where Y_k is a noisy observation of $f_\star(X_k)$. The paper introduces a maximum pseudo-likelihood procedure to estimate the function $f_\star$ and the distribution $\nu_{b,\star}$ of $(X_0,\dots,X_{b-1})$ using blocks of observations of length b. The identifiability of the model is studied in the particular cases $b=1$ and $b=2$ and the consistency of the estimators of $f_\star$ and of $\nu_{b,\star}$ as the number of observations grows to infinity is established.

Keywords: identifiability; maximum likelihood; nonparametric estimation; state space model

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Efficient estimation of functionals in nonparametric boundary models

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For nonparametric regression with one-sided errors and a boundary curve model for Poisson point processes, we consider the problem of efficient estimation for linear functionals. The minimax optimal rate is obtained by an unbiased estimation method which nevertheless depends on a Hölder condition or monotonicity assumption for the underlying regression or boundary function.

We first construct a simple blockwise estimator and then build up a nonparametric maximum-likelihood approach for exponential noise variables and the point process model. In that approach also non-asymptotic efficiency is obtained (UMVU: uniformly minimum variance among all unbiased estimators). The proofs rely essentially on martingale stopping arguments for counting processes and the point process geometry. The estimators are easily computable and a small simulation study confirms their applicability.

Keywords: completeness; frontier estimation; monotone boundary; nonparametric MLE; optional stopping; Poisson point process; shape constraint; sufficiency; support estimation; UMVU

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Perimeters, uniform enlargement and high dimensions

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We study the isoperimetric problem in product spaces equipped with the uniform distance. Our main result is a characterization of isoperimetric inequalities which, when satisfied on a space, are still valid for the product spaces, up a to a constant which does not depend on the number of factors. Such dimension free bounds have applications to the study of influences of variables.

Keywords: influences; isoperimetry

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Markovian growth-fragmentation processes

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Consider a Markov process X on $[0, \infty)$ which has only negative jumps and converges as time tends to infinity a.s. We interpret $X(t)$ as the size of a typical cell at time t, and each jump as a birth event. More precisely, if $\Delta X(s) = -y < 0$, then s is the birthtime of a daughter cell with size y which then evolves independently and according to the same dynamics, that is, giving birth in turn to great-daughters, and so on. After having constructed rigorously such cell systems as a general branching process, we define growth-fragmentation processes by considering the family of sizes of cells alive a some fixed time. We introduce the notion of excessive functions for the latter, whose existence provides a natural sufficient condition for the non-explosion of the system. We establish a simple criterion for excessiveness in terms of X. The case when X is self-similar is treated in details, and connexions with self-similar fragmentations and compensated fragmentations are emphasized.

Keywords: branching process; growth-fragmentation; self-similarity

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Automorphism groups of Gaussian Bayesian networks

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In this paper, we extend earlier work on groups acting on Gaussian graphical models to Gaussian Bayesian networks and more general Gaussian models defined by chain graphs with no induced subgraphs of the form $i \to j - k$. We fully characterise the maximal group of linear transformations which stabilises a given model and we provide basic statistical applications of this result. This includes equivariant estimation, maximal invariants for hypothesis testing and robustness. In our proof, we derive simple necessary and sufficient conditions on vanishing subminors of the concentration matrix in the model. The computation of the group requires finding the essential graph. However, by applying Stúdeny's theory of imsets, we show that computations for DAGs can be performed efficiently without building the essential graph.

Keywords: chain graphs; Gaussian graphical models; group action; equivariant estimator; invariant test; transformation family

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CLT for eigenvalue statistics of large-dimensional general Fisher matrices with applications

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Random Fisher matrices arise naturally in multivariate statistical analysis and understanding the properties of its eigenvalues is of primary importance for many hypothesis testing problems like testing the equality between two covariance matrices, or testing the independence between sub-groups of a multivariate random vector. Most of the existing work on random Fisher matrices deals with a particular situation where the population covariance matrices are equal. In this paper, we consider general Fisher matrices with arbitrary population covariance matrices and develop their spectral properties when the dimensions are proportionally large compared to the sample size. The paper has two main contributions: first the limiting distribution of the eigenvalues of a general Fisher matrix is found and second, a central limit theorem is established for a wide class of functionals of these eigenvalues. Applications of the main results are also developed for testing hypotheses on high-dimensional covariance matrices.

Keywords: central limit theorem; equality of covariance matrices; large-dimensional covariance matrices; large-dimensional Fisher matrix; linear spectral statistics

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Irreducibility of stochastic real Ginzburg–Landau equation driven by α-stable noises and applications

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We establish the irreducibility of stochastic real Ginzburg–Landau equation with α-stable noises by a maximal inequality and solving a control problem. As applications, we prove that the system converges to its equilibrium measure with exponential rate under a topology stronger than total variation and obeys the moderate deviation principle by constructing some Lyapunov test functions.

Keywords: α-stable noises; exponential ergodicity; irreducibility; moderate deviation principle; stochastic real Ginzburg–Landau equation

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Marginal likelihood and model selection for Gaussian latent tree and forest models

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Gaussian latent tree models, or more generally, Gaussian latent forest models have Fisher-information matrices that become singular along interesting submodels, namely, models that correspond to subforests. For these singularities, we compute the real log-canonical thresholds (also known as stochastic complexities or learning coefficients) that quantify the large-sample behavior of the marginal likelihood in Bayesian inference. This provides the information needed for a recently introduced generalization of the Bayesian information criterion. Our mathematical developments treat the general setting of Laplace integrals whose phase functions are sums of squared differences between monomials and constants. We clarify how in this case real log-canonical thresholds can be computed using polyhedral geometry, and we show how to apply the general theory to the Laplace integrals associated with Gaussian latent tree and forest models. In simulations and a data example, we demonstrate how the mathematical knowledge can be applied in model selection.

Keywords: algebraic statistics; Gaussian graphical model; latent tree models; marginal likelihood; multivariate normal distribution; singular learning theory

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Asymptotics of random processes with immigration I: Scaling limits

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Let $(X_1, \xi_1), (X_2, \xi_2), \ldots$ be i.i.d. copies of a pair (X, ξ) where X is a random process with paths in the Skorokhod space $D[0, \infty)$ and ξ is a positive random variable. Define $S_k := \xi_1 + \cdots + \xi_k$, $k \in \mathbb{N}_0$ and $Y(t) := \sum_{k\geq 0} X_{k+1}(t - S_k)\mathbb{1}_{\{S_k \leq t\}}$, $t \geq 0$. We call the process $(Y(t))_{t\geq 0}$ random process with immigration at the epochs of a renewal process. We investigate weak convergence of the finite-dimensional distributions of $(Y(ut))_{u>0}$ as $t \to \infty$. Under the assumptions that the covariance function of X is regularly varying in $(0, \infty) \times (0, \infty)$ in a uniform way, the class of limiting processes is rather rich and includes Gaussian processes with explicitly given covariance functions, fractionally integrated stable Lévy motions and their sums when the law of ξ belongs to the domain of attraction of a stable law with finite mean, and conditionally Gaussian processes with explicitly given (conditional) covariance functions, fractionally integrated inverse stable subordinators and their sums when the law of ξ belongs to the domain of attraction of a stable law with infinite mean.

Keywords: random process with immigration; renewal theory; shot noise processes; weak convergence of finite-dimensional distributions

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Asymptotics of random processes with immigration II: Convergence to stationarity

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Let $X_1, X_2, \dots$ be random elements of the Skorokhod space $D(\mathbb{R})$ and $\xi_1, \xi_2, \dots$ positive random variables such that the pairs $(X_1, \xi_1), (X_2, \xi_2), \dots$ are independent and identically distributed. We call the random process $(Y(t))_{t\in\mathbb{R}}$ defined by $Y(t) := \sum_{k\geq 0} X_{k+1}(t - \xi_1 - \cdots - \xi_k)\mathbb{1}_{\{\xi_1+\cdots+\xi_k\leq t\}}$, $t \in \mathbb{R}$ random process with immigration at the epochs of a renewal process. Assuming that X_k and ξ_k are independent and that the distribution of ξ_1 is nonlattice and has finite mean we investigate weak convergence of $(Y(t))_{t\in\mathbb{R}}$ as $t \to \infty$ in $D(\mathbb{R})$ endowed with the J_1-topology. The limits are stationary processes with immigration.

Keywords: random point process; renewal shot noise process; stationary renewal process; weak convergence in the Skorokhod space

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Parametric estimation of pairwise Gibbs point processes with infinite range interaction

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This paper is concerned with statistical inference for infinite range interaction Gibbs point processes, and in particular for the large class of Ruelle superstable and lower regular pairwise interaction models. We extend classical statistical methodologies such as the pseudo-likelihood and the logistic regression methods, originally defined and studied for finite range models. Then we prove that the associated estimators are strongly consistent and satisfy a central limit theorem, provided the pairwise interaction function tends sufficiently fast to zero. To this end, we introduce a new central limit theorem for almost conditionally centered triangular arrays of random fields.

Keywords: central limit theorem; Lennard–Jones potential; pseudo-likelihood

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Nonparametric tests for detecting breaks in the jump behaviour of a time-continuous process

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This paper is concerned with tests for changes in the jump behaviour of a time-continuous process. Based on results on weak convergence of a sequential empirical tail integral process, asymptotics of certain test statistics for breaks in the jump measure of an Itô semimartingale are constructed. Whenever limiting distributions depend in a complicated way on the unknown jump measure, empirical quantiles are obtained using a multiplier bootstrap scheme. An extensive simulation study shows a good performance of our tests in finite samples.

Keywords: change points; Lévy measure; multiplier bootstrap; sequential empirical processes; weak convergence

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Uniformly and strongly consistent estimation for the Hurst function of a Linear Multifractional Stable Motion

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Since the middle of the 90s, multifractional processes have been introduced for overcoming some limitations of the classical Fractional Brownian Motion model. In their context, the Hurst parameter becomes a Hölder continuous function $H(\cdot)$ of the time variable t. Linear Multifractional Stable Motion (LMSM) is the most known one of them with heavy-tailed distributions. Generally speaking, global and local sample path roughness of a multifractional process are determined by values of its parameter $H(\cdot)$; therefore, since about two decades, several authors have been interested in their statistical estimation, starting from discrete variations of the process. Because of complex dependence structures of variations, in order to show consistency of estimators one has to face challenging problems.

The main goal of our article is to introduce, in the setting of the symmetric α-stable non-anticipative moving average LMSM, where $\alpha \in (1, 2)$, a new strategy for dealing with such kind of problems. It can also be useful in other contexts. In contrast with previously developed strategies, this new one does not require to look for sharp estimates of covariances related to functionals of variations. Roughly speaking, it consists of expressing variations in such a way that they become independent random variables up to negligible remainders. Thanks to it, we obtain, an almost surely and $L^p(\Omega)$, $p \in (0, 4]$, consistent estimator of the whole function $H(\cdot)$, which converges, uniformly in t, and even for some Hölder norms. Also, we obtain estimates for the rates of convergence. Such kind of strong consistency results in uniform and Hölder norms are rather unusual in the literature on statistical estimation of functions.

Keywords: discrete variations; heavy-tailed distributions; laws of large numbers; statistical estimation of functions; time changing Hurst parameter

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A robust approach for estimating change-points in the mean of an AR(1) process

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We consider the problem of multiple change-point estimation in the mean of an AR(1) process. Taking into account the dependence structure does not allow us to use the dynamic programming algorithm, which is the only algorithm giving the optimal solution in the independent case. We propose a robust estimator of the autocorrelation parameter, which is consistent and satisfies a central limit theorem in the Gaussian case. Then, we propose to follow the classical inference approach, by plugging this estimator in the criteria used for change-points estimation. We show that the asymptotic properties of these estimators are the same as those of the classical estimators in the independent framework. The same plug-in approach is then used to approximate the modified BIC and choose the number of segments. This method is implemented in the R package `AR1seg` and is available from the Comprehensive R Archive Network (CRAN). This package is used in the simulation section in which we show that for finite sample sizes taking into account the dependence structure improves the statistical performance of the change-point estimators and of the selection criterion.

Keywords: auto-regressive model; change-points; model selection; robust estimation of the AR(1) parameter; time series

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