BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Volume Twenty Four Number Two May 2018 ISSN: 1350-7265

CONTENTS

(continued)

The papers published in Bernoulli are indexed or abstracted in *Current Index to Statistics*, *Mathematical Reviews*, *Statistical Theory and Method Abstracts-Zentralblatt (STMA-Z)*, and *Zentralblatt für Mathematik* (also avalaible on the *MATH via STN* database and *Compact MATH CD-ROM*). A list of forthcoming papers can be found online at http://www.bernoulli-society.org/index.php/publications/bernoulli-journal/bernoulli-journal-papers

BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Volume Twenty Four Number Two May 2018 ISSN: 1350-7265

CONTENTS

(continued)

BERNOULLI Volume 24 Number 2 May 2018 Pages 801–1612 ISI/BS

Volume 24 Number 2   May 2018   ISSN 1350-7265

BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Aims and Scope

BERNOULLI is the journal of the Bernoulli Society for Mathematical Statistics and Probability, issued four times per year. The journal provides a comprehensive account of important developments in the fields of statistics and probability, offering an international forum for both theoretical and applied work.

Bernoulli Society for Mathematical Statistics and Probability

The Bernoulli Society was founded in 1973. It is an autonomous Association of the International Statistical Institute, ISI. According to its statutes, the object of the Bernoulli Society is the advancement, through international contacts, of the sciences of probability (including the theory of stochastic processes) and mathematical statistics and of their applications to all those aspects of human endeavour which are directed towards the increase of natural knowledge and the welfare of mankind.

Meetings: http://www.bernoulli-society.org/index.php/meetings

The Society holds a World Congress every four years; more frequent meetings, coordinated by the Society's standing committees and often organised in collaboration with other organisations, are the European Meeting of Statisticians, the Conference on Stochastic Processes and their Applications, the CLAPEM meeting (Latin-American Congress on Probability and Mathematical Statistics), the European Young Statisticians Meeting, and various meetings on special topics – in the physical sciences in particular. The Society, as an association of the ISI, also collaborates with other ISI associations in the organization of the biennial ISI World Statistics Congresses (formerly ISI Sessions).

Executive Committee

The papers published in Bernoulli are indexed or abstracted in *Current Index to Statistics, Mathematical Reviews, Statistical Theory and Method Abstracts-Zentralblatt (STMA-Z), Thomson Scientific* and *Zentralblatt für Mathematik* (also available on the *MATH via STN* database and *Compact MATH CD-ROM*). A list of forthcoming papers can be found online at http://www.bernoulli-society.org/index.php/publications/bernoulli-journal/bernoulli-journal-papers

In 2018 Bernoulli consists of 4 issues published in February, May, August and November.

Bernoulli Society
for Mathematical Statistics
and Probability

Bernoulli **24**(2), 2018, 801–841
https://doi.org/10.3150/15-BEJ784

The minimum of a branching random walk outside the boundary case

JULIEN BARRAL[1,*], YUEYUN HU[1,**] and THOMAS MADAULE[2]

[1]*Département de Mathématiques, Université Paris XIII, Sorbonne Paris Cité, 99 avenue J-B Clément, F-93430 Villetaneuse, France. E-mail:* [*]*barral@math.univ-paris13.fr;* [**]*yueyun@math.univ-paris13.fr*
[2]*Institut de Mathématiques de Toulouse, Université Paul Sabatier, 118 route de Narbonne, F-31062 Toulouse Cedex 9, France. E-mail: thomas.madaule@math.univ-toulouse.fr*

This paper is a complement to the studies on the minimum of a real-valued branching random walk. In the boundary case [*Electron. J. Probab.* **10** (2005) 609–631], Aïdékon in a seminal paper [*Ann. Probab.* **41** (2013) 1362–1426] obtained the convergence in law of the minimum after a suitable renormalization. We study here the situation when the log-generating function of the branching random walk explodes at some positive point and it cannot be reduced to the boundary case. In the associated thermodynamics framework, this corresponds to a first-order phase transition, while the boundary case corresponds to a second-order phase transition.

Keywords: branching random walk; minimal position; phase transition

References

[1] Addario-Berry, L. and Reed, B. (2009). Minima in branching random walks. *Ann. Probab.* **37** 1044–1079. MR2537549
[2] Aïdékon, E. (2013). Convergence in law of the minimum of a branching random walk. *Ann. Probab.* **41** 1362–1426. MR3098680
[3] Alsmeyer, G., Biggins, J.D. and Meiners, M. (2012). The functional equation of the smoothing transform. *Ann. Probab.* **40** 2069–2105. MR3025711
[4] Attia, N. and Barral, J. (2014). Hausdorff and packing spectra, large deviations, and free energy for branching random walks in $\mathbb{R}^d$. *Comm. Math. Phys.* **331** 139–187. MR3231998
[5] Bacry, E. and Muzy, J.F. (2003). Log-infinitely divisible multifractal processes. *Comm. Math. Phys.* **236** 449–475. MR2021198
[6] Barral, J., Kupiainen, A., Nikula, M., Saksman, E. and Webb, C. (2014). Critical Mandelbrot cascades. *Comm. Math. Phys.* **325** 685–711. MR3148099
[7] Barral, J. and Mandelbrot, B.B. (2002). Multifractal products of cylindrical pulses. *Probab. Theory Related Fields* **124** 409–430. MR1939653
[8] Barral, J., Rhodes, R. and Vargas, V. (2012). Limiting laws of supercritical branching random walks. *C. R. Math. Acad. Sci. Paris* **350** 535–538. MR2929063
[9] Biggins, J.D. (1976). The first- and last-birth problems for a multitype age-dependent branching process. *Adv. in Appl. Probab.* **8** 446–459. MR0420890
[10] Biggins, J.D. (1977). Martingale convergence in the branching random walk. *J. Appl. Probab.* **14** 25–37. MR0433619
[11] Biggins, J.D. (1992). Uniform convergence of martingales in the branching random walk. *Ann. Probab.* **20** 137–151. MR1143415

1350-7265

[12] Biggins, J.D. and Kyprianou, A.E. (1997). Seneta–Heyde norming in the branching random walk. *Ann. Probab.* **25** 337–360. MR1428512
[13] Biggins, J.D. and Kyprianou, A.E. (2004). Measure change in multitype branching. *Adv. in Appl. Probab.* **36** 544–581. MR2058149
[14] Biggins, J.D. and Kyprianou, A.E. (2005). Fixed points of the smoothing transform: The boundary case. *Electron. J. Probab.* **10** 609–631. MR2147319
[15] Bramson, M., Ding, J. and Zeitouni, O. (2013). Convergence in law of the maximum of the two-dimensional discrete Gaussian free field. Preprint. Available at http://arxiv.org/abs/1301.6669.
[16] Bramson, M., Ding, J. and Zeitouni, O. (2014). Convergence in law of the maximum of nonlattice branching random walk. Preprint. Available at http://arxiv.org/abs/1404.3423.
[17] Bramson, M. and Zeitouni, O. (2009). Tightness for a family of recursion equations. *Ann. Probab.* **37** 615–653. MR2510018
[18] Chauvin, B., Rouault, A. and Wakolbinger, A. (1991). Growing conditioned trees. *Stochastic Process. Appl.* **39** 117–130. MR1135089
[19] Chen, X. (2015). A necessary and sufficient condition for the nontrivial limit of the derivative martingale in a branching random walk. *Adv. in Appl. Probab.* **47** 741–760. MR3406606
[20] Collet, P. and Koukiou, F. (1992). Large deviations for multiplicative chaos. *Comm. Math. Phys.* **147** 329–342. MR1174416
[21] Denisov, D., Dieker, A.B. and Shneer, V. (2008). Large deviations for random walks under subexponentiality: The big-jump domain. *Ann. Probab.* **36** 1946–1991. MR2440928
[22] Derrida, B. and Spohn, H. (1988). Polymers on disordered trees, spin glasses, and traveling waves. *J. Stat. Phys.* **51** 817–840. MR0971033
[23] Ding, J., Roy, R. and Zeitouni, O. (2015). Convergence of the centered maximum of log-correlated Gaussian fields. Preprint. Available at http://arxiv.org/abs/1503.04588.
[24] Durrett, R. and Liggett, T.M. (1983). Fixed points of the smoothing transformation. *Z. Wahrsch. Verw. Gebiete* **64** 275–301. MR0716487
[25] Fan, A.H. (1997). Sur les chaos de Lévy stables d'indice $0 < \alpha < 1$. *Ann. Sci. Math. Québec* **21** 53–66. MR1457064
[26] Gut, A. (2009). *Stopped Random Walks: Limit Theorems and Applications*, 2nd ed. *Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2489436
[27] Hammersley, J.M. (1974). Postulates for subadditive processes. *Ann. Probab.* **2** 652–680. MR0370721
[28] Hu, Y. and Shi, Z. (2009). Minimal position and critical martingale convergence in branching random walks, and directed polymers on disordered trees. *Ann. Probab.* **37** 742–789. MR2510023
[29] Kahane, J.-P. and Peyrière, J. (1976). Sur certaines martingales de Benoit Mandelbrot. *Adv. in Math.* **22** 131–145. MR0431355
[30] Kingman, J.F.C. (1975). The first birth problem for an age-dependent branching process. *Ann. Probab.* **3** 790–801. MR0400438
[31] Kyprianou, A.E. (2000). Martingale convergence and the stopped branching random walk. *Probab. Theory Related Fields* **116** 405–419. MR1749282
[32] Liu, Q. (1998). Fixed points of a generalized smoothing transformation and applications to the branching random walk. *Adv. in Appl. Probab.* **30** 85–112. MR1618888
[33] Lyons, R. (1997). A simple path to Biggins' martingale convergence for branching random walk. In *Classical and Modern Branching Processes* (*Minneapolis*, *MN*, 1994). *IMA Vol. Math. Appl.* **84** 217–221. New York: Springer. MR1601749
[34] Lyons, R., Pemantle, R. and Peres, Y. (1995). Conceptual proofs of $L \log L$ criteria for mean behavior of branching processes. *Ann. Probab.* **23** 1125–1138. MR1349164
[35] Madaule, T. (2011). Convergence in law for the branching random walk seen from its tip. Preprint. Available at http://arxiv.org/abs/1107.2543.

[36] Madaule, T. (2015). Maximum of a log-correlated Gaussian field. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** 1369–1431. MR3414451

[37] Madaule, T., Rhodes, R. and Vargas, V. (2016). Glassy phase and freezing of log-correlated Gaussian potentials. *Ann. Appl. Probab.* To appear. Available at arXiv:1310.5574.

[38] Mandelbrot, B. (1974). Multiplications aléatoires itérées et distributions invariantes par moyenne pondérée aléatoire. *C. R. Math. Acad. Sci. Paris* **278** 289–292. MR0431351

[39] Molchan, G.M. (1996). Scaling exponents and multifractal dimensions for independent random cascades. *Comm. Math. Phys.* **179** 681–702. MR1400758

[40] Mörters, P. and Ortgiese, M. (2008). Minimal supporting subtrees for the free energy of polymers on disordered trees. *J. Math. Phys.* **49** 125203, 21. MR2484334

[41] Ossiander, M. and Waymire, E.C. (2000). Statistical estimation for multiplicative cascades. *Ann. Statist.* **28** 1533–1560. MR1835030

[42] Rhodes, R., Sohier, J. and Vargas, V. (2014). Levy multiplicative chaos and star scale invariant random measures. *Ann. Probab.* **42** 689–724. MR3178471

[43] Shi, Z. (2016). *Branching Random Walks*. École D'été de Saint-Flour XLII. To appear.

[44] Stone, C. (1965). A local limit theorem for nonlattice multi-dimensional distribution functions. *Ann. Math. Stat.* **36** 546–551. MR0175166

[45] Vatutin, V.A. and Wachtel, V. (2009). Local probabilities for random walks conditioned to stay positive. *Probab. Theory Related Fields* **143** 177–217. MR2449127

[46] Waymire, E.C. and Williams, S.C. (1996). A cascade decomposition theory with applications to Markov and exchangeable cascades. *Trans. Amer. Math. Soc.* **348** 585–632. MR1322959

[47] Webb, C. (2011). Exact asymptotics of the freezing transition of a logarithmically correlated random energy model. *J. Stat. Phys.* **145** 1595–1619. MR2863721

Bernoulli **24**(2), 2018, 842–872
https://doi.org/10.3150/15-BEJ785

Uniform ergodicity of the iterated conditional SMC and geometric ergodicity of particle Gibbs samplers

CHRISTOPHE ANDRIEU[1], ANTHONY LEE[2] and MATTI VIHOLA[3]

[1]*School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, United Kingdom. E-mail: c.andrieu@bristol.ac.uk*
[2]*Department of Statistics, University of Warwick, Coventry CV4 7AL, United Kingdom. E-mail: anthony.lee@warwick.ac.uk*
[3]*Department of Mathematics and Statistics, University of Jyväskylä, P.O. Box 35, Finland. E-mail: matti.vihola@iki.fi*

We establish quantitative bounds for rates of convergence and asymptotic variances for iterated conditional sequential Monte Carlo (i-cSMC) Markov chains and associated particle Gibbs samplers [*J. R. Stat. Soc. Ser. B. Stat. Methodol.* **72** (2010) 269–342]. Our main findings are that the essential boundedness of potential functions associated with the i-cSMC algorithm provide necessary and sufficient conditions for the uniform ergodicity of the i-cSMC Markov chain, as well as quantitative bounds on its (uniformly geometric) rate of convergence. Furthermore, we show that the i-cSMC Markov chain cannot even be geometrically ergodic if this essential boundedness does not hold in many applications of interest. Our sufficiency and quantitative bounds rely on a novel non-asymptotic analysis of the expectation of a standard normalizing constant estimate with respect to a "doubly conditional" SMC algorithm. In addition, our results for i-cSMC imply that the rate of convergence can be improved arbitrarily by increasing N, the number of particles in the algorithm, and that in the presence of mixing assumptions, the rate of convergence can be kept constant by increasing N linearly with the time horizon. We translate the sufficiency of the boundedness condition for i-cSMC into sufficient conditions for the particle Gibbs Markov chain to be geometrically ergodic and quantitative bounds on its geometric rate of convergence, which imply convergence of properties of the particle Gibbs Markov chain to those of its corresponding Gibbs sampler. These results complement recently discovered, and related, conditions for the particle marginal Metropolis–Hastings (PMMH) Markov chain.

Keywords: geometric ergodicity; iterated conditional sequential Monte Carlo; Metropolis-within-Gibbs; particle Gibbs; uniform ergodicity

References

[1] Andrieu, C., Doucet, A. and Holenstein, R. (2010). Particle Markov chain Monte Carlo methods. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **72** 269–342. MR2758115
[2] Andrieu, C., Lee, A. and Vihola, M. Supplement to "Uniform ergodicity of the iterated conditional SMC and geometric ergodicity of particle Gibbs samplers." DOI:10.3150/15-BEJ785SUPP.
[3] Andrieu, C., Lee, A. and Vihola, M. (2013). Uniform ergodicity of the iterated conditional SMC and geometric ergodicity of particle Gibbs samplers. Preprint. Available at arXiv:1312.6432.
[4] Andrieu, C. and Roberts, G.O. (2009). The pseudo-marginal approach for efficient Monte Carlo computations. *Ann. Statist.* **37** 697–725. MR2502648

[5] Andrieu, C. and Vihola, M. (2015). Convergence properties of pseudo-marginal Markov chain Monte Carlo algorithms. *Ann. Appl. Probab.* **25** 1030–1077. MR3313762

[6] Bérard, J., Del-Moral, P. and Doucet, A. (2013). A lognormal central limit theorem for particle approximations of normalizing constants. *Electron. J. Probab.* **19**.

[7] Cérou, F., Del Moral, P. and Guyader, A. (2011). A nonasymptotic theorem for unnormalized Feynman–Kac particle models. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** 629–649. MR2841068

[8] Chopin, N. and Singh, S.S. (2013). On the particle Gibbs sampler. *Bernoulli* **21** 1855–1883.

[9] Del Moral, P. (2004). *Feynman–Kac Formulae. Probability and Its Applications (New York)*. New York: Springer. MR2044973

[10] Del Moral, P., Kohn, R. and Patras, F. (2014). On Feynman–Kac and particle Markov chain Monte Carlo models. Preprint. Available at arXiv:1404.5733.

[11] Doucet, A., Pitt, M.K., Deligiannidis, G. and Kohn, R. (2015). Efficient implementation of Markov chain Monte Carlo when using an unbiased likelihood estimator. *Biometrika* **102** 295–313. MR3371005

[12] Hobert, J.P. and Robert, C.P. (2004). A mixture representation of π with applications in Markov chain Monte Carlo and perfect sampling. *Ann. Appl. Probab.* **14** 1295–1305. MR2071424

[13] Lee, A., Doucet, A. and Łatuszyński, K. (2014). Perfect simulation using atomic regeneration with application to sequential Monte Carlo. Available at arXiv:1407.5770.

[14] Lee, A. and Łatuszyński, K. (2014). Variance bounding and geometric ergodicity of Markov chain Monte Carlo kernels for approximate Bayesian computation. *Biometrika* **101** 655–671. MR3254907

[15] Lee, A., Yau, C., Giles, M.B., Doucet, A. and Holmes, C.C. (2010). On the utility of graphics cards to perform massively parallel simulation of advanced Monte Carlo methods. *J. Comput. Graph. Statist.* **19** 769–789.

[16] Lindsten, F., Douc, R. and Moulines, E. (2015). Uniform ergodicity of the particle Gibbs sampler. *Scand. J. Stat.* **42** 775–797. MR3391692

[17] Mengersen, K.L. and Tweedie, R.L. (1996). Rates of convergence of the Hastings and Metropolis algorithms. *Ann. Statist.* **24** 101–121. MR1389882

[18] Murdoch, D.J. and Green, P.J. (1998). Exact sampling from a continuous state space. *Scand. J. Stat.* **25** 483–502. MR1650023

[19] Propp, J.G. and Wilson, D.B. (1996). Exact sampling with coupled Markov chains and applications to statistical mechanics. In *Proceedings of the Seventh International Conference on Random Structures and Algorithms (Atlanta, GA*, 1995) **9** 223–252. Wiley. MR1611693

[20] Robert, C.P. and Casella, G. (1999). *Monte Carlo Statistical Methods. Springer Texts in Statistics*. New York: Springer. MR1707311

[21] Roberts, G.O. and Rosenthal, J.S. (1997). Geometric ergodicity and hybrid Markov chains. *Electron. Commun. Probab.* **2** 13–25 (electronic). MR1448322

[22] Roberts, G.O. and Rosenthal, J.S. (2001). Markov chains and de-initializing processes. *Scand. J. Stat.* **28** 489–504. MR1858413

[23] Roberts, G.O. and Tweedie, R.L. (1996). Geometric convergence and central limit theorems for multidimensional Hastings and Metropolis algorithms. *Biometrika* **83** 95–110. MR1399158

[24] Sherlock, C., Thiery, A.H., Roberts, G.O. and Rosenthal, J.S. (2015). On the efficiency of pseudo-marginal random walk Metropolis algorithms. *Ann. Statist.* **43** 238–275. MR3285606

[25] Whiteley, N. (2013). Stability properties of some particle filters. *Ann. Appl. Probab.* **23** 2500–2537. MR3127943

Bernoulli **24**(2), 2018, 873–894
https://doi.org/10.3150/15-BEJ786

Domains of attraction on countable alphabets

ZHIYI ZHANG

Department of Mathematics and Statistics, University of North Carolina at Charlotte, Charlotte, NC 28223, USA. E-mail: zzhang@uncc.edu

For each probability distribution on a countable alphabet, a sequence of positive functionals are developed as tail indices. By and only by the asymptotic behavior of these indices, domains of attraction for all probability distributions on the alphabet are defined. The three main domains of attraction are shown to contain distributions with thick tails, thin tails and no tails respectively, resembling in parallel the three main domains of attraction, Fréchet, Gumbel and Weibull families, for continuous random variables on the real line. In addition to the probabilistic merits associated with the domains, the tail indices are partially motivated by the fact that there exists an unbiased estimator for every index in the sequence, which is therefore statistically observable, provided that the sample is sufficiently large.

Keywords: distributions on alphabets; domains of attraction; tail index; Turing's formula

References

[1] de Haan, L. and Ferreira, A. (2006). *Extreme Value Theory*: *An Introduction*. *Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2234156
[2] Fisher, R.A. and Tippett, L.H.C. (1928). Limiting forms of the frequency-distribution of the largest or smallest member of a sample. *Math*. *Proc*. *Cambridge Philos*. *Soc*. **24** 180.
[3] Fréchet, M. (1927). Sur la loi de probabilité de l'écart maximum. *Annals de la Soc*. *Polonaise*. *de Math*. **6** 92.
[4] Gini, C. (1912). *Variabilità e Mutabilità*. *Contributo Allo Studiodelle Distribuzioni e delle Relazioni Statistiche*. Bologna: C. Cuppini.
[5] Gnedenko, B. (1943). Sur la distribution limite du terme maximum d'une série aléatoire. *Ann*. *of Math*. (2) **44** 423–453. MR0008655
[6] Good, I.J. (1953). The population frequencies of species and the estimation of population parameters. *Biometrika* **40** 237–264. MR0061330
[7] Simpsom, E.H. (1949). Measurement of diversity. *Nature* **163** 688.
[8] Smirnov, N.V. (1952). *Limit Distributions for the Terms of a Variational Series*. Am. Math. Soc., Providence, RI.
[9] Zhang, Z. (2012). Entropy estimation in Turing's perspective. *Neural Comput*. **24** 1368–1389. MR2952469
[10] Zhang, Z. and Grabchak, M. (2016). Entropic representation and estimation of Diversity indices. *J. Nonparametr. Stat*. **28** 563–575.
[11] Zhang, Z. and Zhou, J. (2010). Re-parameterization of multinomial distributions and diversity indices. *J. Statist*. *Plann*. *Inference* **140** 1731–1738. MR2606713

1350-7265

Bernoulli **24**(2), 2018, 895–928
https://doi.org/10.3150/15-BEJ790

Wavelet estimation for operator fractional Brownian motion

PATRICE ABRY[1] and GUSTAVO DIDIER[2]

[1]*Physics Lab, CNRS and École Normale Supérieure de Lyon, 46 allée d'Italie, F-69364, Lyon cedex 7, France. E-mail: patrice.abry@ens-lyon.fr*
[2]*Mathematics Department, Tulane University, 6823 St. Charles Avenue, New Orleans, LA 70118, USA. E-mail: gdidier@tulane.edu*

Operator fractional Brownian motion (OFBM) is the natural vector-valued extension of the univariate fractional Brownian motion. Instead of a scalar parameter, the law of an OFBM scales according to a Hurst matrix that affects every component of the process. In this paper, we develop the wavelet analysis of OFBM, as well as a new estimator for the Hurst matrix of bivariate OFBM. For OFBM, the univariate-inspired approach of analyzing the entry-wise behavior of the wavelet spectrum as a function of the (wavelet) scales is fraught with difficulties stemming from mixtures of power laws. Instead we consider the evolution along scales of the eigenstructure of the wavelet spectrum. This is shown to yield consistent and asymptotically normal estimators of the Hurst eigenvalues, and also of the eigenvectors under assumptions. A simulation study is included to demonstrate the good performance of the estimators under finite sample sizes.

Keywords: operator fractional Brownian motion; operator self-similarity; wavelets

References

[1] Abry, P., Baraniuk, R., Flandrin, P., Riedi, R. and Veitch, D. (2002). Multiscale network traffic analysis, modeling, and inference using wavelets, multifractals, and cascades. *IEEE Signal Process. Mag.* **3** (19) 28–46.
[2] Abry, P. and Didier, G. (2017). Supplement to "Wavelet estimation for operator fractional Brownian motion." DOI:10.3150/15-BEJ790SUPP.
[3] Achard, S. and Gannaz, I. (2016). Multivariate wavelet Whittle estimation in long-range dependence. *J. Time Series Anal.* **37** (4) 476–512.
[4] Amblard, P., Coeurjolly, J.-F., Lavancier, F. and Philippe, A. (2012). Basic properties of the multivariate fractional Brownian motion. *Bulletin de la Société Mathématique de France, Séminaires et Congrés* **28** 65–87.
[5] Amblard, P.-O. and Coeurjolly, J.-F. (2011). Identification of the multivariate fractional Brownian motion. *IEEE Trans. Signal Process.* **59** (11) 5152–5168. MR2883588
[6] Bardet, J.-M. (2000). Testing for the presence of self-similarity of Gaussian time series having stationary increments. *J. Time Series Anal.* **21** (5) 497–515. MR1794484
[7] Bardet, J.-M. (2002). Statistical study of the wavelet analysis of fractional Brownian motion. *IEEE Trans. Inform. Theory* **48** (4) 991–999. MR1908463
[8] Becker-Kern, P. and Pap, G. (2008). Parameter estimation of selfsimilarity exponents. *J. Multivariate Anal.* **99** 117–140. MR2408446
[9] Chung, C.-F. (2002). Sample means, sample autocovariances, and linear regression of stationary multivariate long memory processes. *Econometric Theory* **18** 51–78. MR1885350

1350-7265

[10] Coeurjolly, J.-F., Amblard, P.-O. and Achard, S. (2013). Wavelet analysis of the multivariate fractional Brownian motion. *ESAIM Probab. Stat*. **17** 592–604. MR3085634

[11] Cohen, S., Meerschaert, M.M. and Rosiński, J. (2010). Modeling and simulation with operator scaling. *Stochastic Process*. *Appl*. **120** 2390–2411. MR2728170

[12] Cramér, H. and Leadbetter, M.R. (1967). *Stationary and Related Stochastic Processes*: *Sample Function Properties and Their Applications*. Mineola, NY: Dover Publications.

[13] Daubechies, I. (1992). *Ten Lectures on Wavelets*. *CBMS-NSF Regional Conference Series in Applied Mathematics* **61**. Philadelphia, PA: SIAM. MR1162107

[14] Davidson, J. and De Jong, R. (2000). The Functional Central Limit Theorem and weak convergence to stochastic integrals. *Econometric Theory* **16** 643–666.

[15] Davidson, J. and Hashimzade, N. (2008). Alternative frequency and time domain versions of fractional Brownian motion. *Econometric Theory* **24** 256–293. MR2397267

[16] Delbeke, L. and Abry, P. (2000). Stochastic integral representation and properties of the wavelet coefficients of linear fractional stable motion. *Stochastic Process*. *Appl*. **86** (2) 177–182. MR1741803

[17] Delgado, R. (2007). A reflected fBm limit for fluid models with ON/OFF sources under heavy traffic. *Stochastic Process*. *Appl*. **117** 188–201. MR2290192

[18] Didier, G., Helgason, H. and Abry, P. (2015). Demixing multivariate-operator self-similar processes. In *IEEE International Conference on Acoustics*, *Speech and Signal Processing* (*ICASSP*) 1–5. Brisbane, Australia.

[19] Didier, G. and Pipiras, V. (2011). Integral representations and properties of operator fractional Brownian motions. *Bernoulli* **17** 1–33. MR2797980

[20] Didier, G. and Pipiras, V. (2012). Exponents, symmetry groups and classification of operator fractional Brownian motions. *J. Theoret*. *Probab*. **25** 353–395. MR2914433

[21] Dolado, J.J. and Marmol, F. (2004). Asymptotic inference results for multivariate long-memory processes. *Econom*. *J*. **7** 168–190. MR2076631

[22] Flandrin, P. (1992). Wavelet analysis and synthesis of fractional Brownian motion. *IEEE Trans*. *Inform*. *Theory* **38** 910–917. MR1162229

[23] Fox, R. and Taqqu, M.S. (1986). Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series. *Ann*. *Statist*. **14** 517–532. MR0840512

[24] Frisch, U. (1995). *Turbulence*. Cambridge: Cambridge Univ. Press. The legacy of A.N. Kolmogorov. MR1428905

[25] Helgason, H., Pipiras, V. and Abry, P. (2011). Fast and exact synthesis of stationary multivariate Gaussian time series using circulant embedding. *Signal Process*. **91** 1123–1133.

[26] Helgason, H., Pipiras, V. and Abry, P. (2011). Synthesis of multivariate stationary series with prescribed marginal distributions and covariance using circulant embedding. *Signal Process*. **91** 1741–1758.

[27] Hudson, W.N. and Mason, J.D. (1982). Operator-self-similar processes in a finite-dimensional space. *Trans*. *Amer. Math*. *Soc*. **273** 281–297. MR0664042

[28] Ivanov, P., Amaral, L.N., Goldberger, A., Havlin, S., Rosenblum, M., Struzik, Z. and Stanley, H. (1999). Multifractality in human heartbeat dynamics. *Nature* **399** 461–465.

[29] Kechagias, S. and Pipiras, V. (2015). Definitions and representations of multivariate long-range dependent time series. *J*. *Time Series Anal*. **36** 1–25. MR3300202

[30] Konstantopoulos, T. and Lin, S.-J. (1996). Fractional Brownian approximations of queueing networks. In *Stochastic Networks* (*New York*, 1995). *Lecture Notes in Statist*. **117** 257–273. New York: Springer. MR1466791

[31] Laha, R.G. and Rohatgi, V.K. (1982). Operator self-similar stochastic processes in $\mathbf{R}_d$. *Stochastic Process*. *Appl*. **12** 73–84. MR0632393

[32] Maejima, M. and Mason, J.D. (1994). Operator-self-similar stable processes. *Stochastic Process. Appl.* **54** 139–163. MR1302699

[33] Majewski, K. (2003). Large deviations for multi-dimensional reflected fractional Brownian motion. *Stoch. Stoch. Rep.* **75** 233–257. MR1994908

[34] Majewski, K. (2005). Fractional Brownian heavy traffic approximations of multiclass feedforward queueing networks. *Queueing Syst.* **50** 199–230. MR2143121

[35] Mallat, S. (1998). *A Wavelet Tour of Signal Processing*. San Diego, CA: Academic Press. MR1614527

[36] Marinucci, D. and Robinson, P.M. (2000). Weak convergence of multivariate fractional processes. *Stochastic Process. Appl.* **86** 103–120. MR1741198

[37] Meerschaert, M. and Scheffler, H.-P. (2003). Portfolio modeling with heavy-tailed random vectors. In *Handbook of Heavy-Tailed Distributions in Finance* (S.T. Rachev, ed.) 595–640. Amsterdam: Elsevier Science B.V.

[38] Meerschaert, M.M. and Scheffler, H.-P. (1999). Moment estimator for random vectors with heavy tails. *J. Multivariate Anal.* **71** 145–159. MR1721965

[39] Moulines, E., Roueff, F. and Taqqu, M.S. (2007). Central limit theorem for the log-regression wavelet estimation of the memory parameter in the Gaussian semi-parametric context. *Fractals* **15** 301–313. MR2396718

[40] Moulines, E., Roueff, F. and Taqqu, M.S. (2007). On the spectral density of the wavelet coefficients of long-memory time series with application to the log-regression estimation of the memory parameter. *J. Time Series Anal.* **28** 155–187. MR2345656

[41] Moulines, E., Roueff, F. and Taqqu, M.S. (2008). A wavelet Whittle estimator of the memory parameter of a nonstationary Gaussian time series. *Ann. Statist.* **36** 1925–1956. MR2435460

[42] Nielsen, F.S. (2011). Local Whittle estimation of multi-variate fractionally integrated processes. *J. Time Series Anal.* **32** (3) 317–335. MR2808259

[43] Robinson, P.M. (1995). Gaussian semiparametric estimation of long range dependence. *Ann. Statist.* **23** 1630–1661. MR1370301

[44] Robinson, P.M. (1995). Log-periodogram regression of time series with long range dependence. *Ann. Statist.* **23** 1048–1072. MR1345214

[45] Robinson, P.M. (2008). Multiple local Whittle estimation in stationary systems. *Ann. Statist.* **36** (5) 2508–2530. MR2458196

[46] Shimotsu, K. (2007). Gaussian semiparametric estimation of multivariate fractionally integrated processes. *J. Econometrics* **137** 277–310. MR2354946

[47] Taqqu, M.S. (2003). Fractional Brownian motion and long-range dependence. In *Theory and Applications of Long-Range Dependence* (P. Doukhan, G. Oppenheim and M.S. Taqqu, eds.) 5–38. Boston, MA: Birkhäuser. MR1956042

[48] Veitch, D. and Abry, P. (1999). A wavelet-based joint estimator of the parameters of long-range dependence. *IEEE Trans. Inform. Theory* **45** 878–897. MR1682517

[49] Veitch, D., Abry, P. and Taqqu, M.S. (2003). On the automatic selection of the onset of scaling. *Fractals* **11** (4) 377–390. MR2028959

[50] Vignat, C. (2012). A generalized Isserlis theorem for location mixtures of Gaussian random vectors. *Statist. Probab. Lett.* **82** 67–71. MR2863025

[51] Wendt, H., Scherrer, A., Abry, P. and Achard, S. (2009). Testing fractal connectivity in multivariate long memory processes. In *IEEE International Conference on Acoustics, Speech and Signal Processing* 2913–2916. Taipei, Taiwan.

[52] Wornell, G. and Oppenheim, A. (1992). Estimation of fractal signals from noisy measurements using wavelets. *IEEE Trans. Signal Process.* **40** 611–623.

Bernoulli **24**(2), 2018, 929–955
https://doi.org/10.3150/16-BEJ809

Asymptotics for the maximum sample likelihood estimator under informative selection from a finite population

DANIEL BONNÉRY[1], F. JAY BREIDT[2] and FRANÇOIS COQUET[3]

[1]*JPSM, 1218 LeFrak, College Park, MD 20742, USA.*
E-mail: dbonnery@umd.edu, url: jpsm.umd.edu/facultyprofile/Bonnéry/Daniel
[2]*Department of Statistics, Colorado State University, Fort Collins, CO 80523-1877.*
E-mail: jbreidt@stat.colostate.edu, url: www.stat.colostate.edu/statpeople/people/jbreidt.html
[3]*Ensai, Campus de Ker-Lann, Rue Blaise Pascal – BP 37203, 35172 Bruz - Cedex, France.*
E-mail: fcoquet@ensai.fr, url: www.ensai.com/francois-coquet-rub,30.html

Inference for the parametric distribution of a response given covariates is considered under informative selection of a sample from a finite population. Under this selection, the conditional distribution of a response in the sample, given the covariates and given that it was selected for observation, is not the same as the conditional distribution of the response in the finite population, given only the covariates. It is instead a weighted version of the conditional distribution of interest. Inference must be modified to account for this informative selection. An established approach in this context is maximum "sample likelihood", developing a weight function that reflects the informative sampling design, then treating the observations as if they were independently distributed according to the weighted distribution. While the sample likelihood methodology has been widely applied, its theoretical foundation has been less developed. A precise asymptotic description of a wide range of informative selection mechanisms is proposed. Under this framework, consistency and asymptotic normality of the maximum sample likelihood estimators are established. The theory allows for the possibility of nuisance parameters that describe the selection mechanism. The proposed regularity conditions are verifiable for various sample schemes, motivated by real problems in surveys. Simulation results for these examples illustrate the quality of the asymptotic approximations, and demonstrate a practical approach to variance estimation that combines aspects of model-based information theory and design-based variance estimation.

Keywords: complex survey; pseudo-likelihood; stratified sampling; weighted distribution

References

[1] Bonnéry, D. (2011). Asymptotic properties of the sample distribution under informative selection. Ph.D. thesis, Université de Rennes 1. Available at http://tel.archives-ouvertes.fr/tel-00658990.
[2] Bonnéry, D., Breidt, F.J. and Coquet, F. (2012). Uniform convergence of the empirical cumulative distribution function under informative selection from a finite population. *Bernoulli* **18** 1361–1385. MR2995800
[3] Breslow, N.E. and Cain, K.C. (1988). Logistic regression for two-stage case–control data. *Biometrika* **75** 11–20. MR0932812
[4] Cai, T. (2013). Investigation of ways to handle sampling weights for multilevel model analyses. *Sociological Methodology* **43** 178–219.

[5] Chambers, R.L., Steel, D.G., Wang, S. and Welsh, A. (2012). *Maximum Likelihood Estimation for Sample Surveys*. *Monographs on Statistics and Applied Probability* **125**. Boca Raton, FL: CRC Press. MR2963765
[6] Eideh, A. and Nathan, G. (2009). Two-stage informative cluster sampling-estimation and prediction with applications for small-area models. *J. Statist. Plann. Inference* **139** 3088–3101. MR2535185
[7] Eideh, A.A.H. and Nathan, G. (2006). The analysis of data from sampling surveys under informative sampling. *Acta Comment. Univ. Tartu. Math.* **10** 41–51. MR2309744
[8] Eideh, A.A.H. and Nathan, G. (2006). Fitting time series models for longitudinal survey data under informative sampling. *J. Statist. Plann. Inference* **136** 3052–3069. MR2256216
[9] Eideh, A.A.H. and Nathan, G. (2006). Corrigendum to: "Fitting time series models for longitudinal survey data under informative sampling" [*J. Statist. Plann. Inference* **136** (2006) 3052–3069]. *J. Statist. Plann. Inference* **137** 628. MR2297656
[10] Fuller, W.A. (2009). *Sampling Statistics* **560**. New York: Wiley.
[11] Ghosh, M. and Maiti, T. (2004). Small-area estimation based on natural exponential family quadratic variance function models and survey weights. *Biometrika* **91** 95–112. MR2050462
[12] Gong, G. and Samaniego, F.J. (1981). Pseudomaximum likelihood estimation: Theory and applications. *Ann. Statist.* **9** 861–869. MR0619289
[13] Horvitz, D.G. and Thompson, D.J. (1952). A generalization of sampling without replacement from a finite universe. *J. Amer. Statist. Assoc.* **47** 663–685. MR0053460
[14] Korn, E.L. and Graubard, B.I. (1999). *Analysis of Health Surveys*. *Wiley Series in Probability and Mathematical Statistics*. New York: Wiley.
[15] Krieger, A.M. and Pfeffermann, D. (1992). Maximum likelihood estimation from complex sample surveys. *Survey Methodology* **18** 225–239.
[16] Landsman, V. and Graubard, B.I. (2013). Efficient analysis of case–control studies with sample weights. *Stat. Med.* **32** 347–360. MR3041872
[17] Patil, G.P. and Rao, C.R. (1978). Weighted distributions and size-biased sampling with applications to wildlife populations and human families. *Biometrics* **34** 179–189. MR0507202
[18] Pfeffermann, D. (2011). Modelling of complex survey data: Why model? Why is it a problem? How can we approach it? *Survey Methodology* **37** 115–136.
[19] Pfeffermann, D., Krieger, A.M. and Rinott, Y. (1998). Parametric distributions of complex survey data under informative probability sampling. *Statist. Sinica* **8** 1087–1114. MR1666233
[20] Pfeffermann, D., Moura, F.A.D.S. and Silva, P.L.d.N. (2006). Multi-level modelling under informative sampling. *Biometrika* **93** 943–959. MR2285081
[21] Pfeffermann, D. and Sikov, A. (2011). Imputation and estimation under nonignorable nonresponse for household surveys with missing covariate information. *Journal of Official Statistics* **27** 181–209.
[22] Pfeffermann, D. and Sverchkov, M. (1999). Parametric and semi-parametric estimation of regression models fitted to survey data. *Sankhyā Ser. B* **61** 166–186. MR1720710
[23] Pfeffermann, D. and Sverchkov, M. (2007). Small-area estimation under informative probability sampling of areas and within the selected areas. *J. Amer. Statist. Assoc.* **102** 1427–1439. MR2412558
[24] Pfeffermann, D. and Sverchkov, M. (2009). Inference under informative sampling. In *Sample Surveys: Inference and Analysis* (D. Pfefermann, ed.). *Handbook of Statistics* **29B** 455–487. Elsevier/North-Holland, Amsterdam.
[25] Pfeffermann, D. and Sverchkov, M.Y. (2003). Fitting generalized linear models under informative sampling. In *Analysis of Survey Data* (*Southampton*, 1999). *Wiley Ser. Surv. Methodol.* 175–195. Wiley, Chichester. MR1978851
[26] Serfling, R.J. (1980). *Approximation Theorems of Mathematical Statistics*. New York: Wiley. MR0595165

[27] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247

[28] Yuan, K. and Jennrich, R.I. (2000). Estimating equations with nuisance parameters: Theory and applications. *Ann*. *Inst*. *Statist*. *Math*. **52** 343–350. MR1763567

Bernoulli **24**(2), 2018, 956–970
https://doi.org/10.3150/16-BEJ816

Hörmander-type theorem for Itô processes and related backward SPDEs

JINNIAO QIU

Department of Mathematics and Statistics, University of Calgary, 2500 University Drive NW, Calgary, AB T2N 1N4, Canada. E-mail: qiujinn@gmail.com

A Hörmander-type theorem is established for Itô processes and related backward stochastic partial differential equations (BSPDEs). A short self-contained proof is also provided for the L^2-theory of linear, possibly degenerate BSPDEs, in which new gradient estimates are obtained.

Keywords: backward stochastic partial differential equation; Hörmander theorem; Itô process; non-Markov

References

[1] Bender, C. and Dokuchaev, N. (2014). A first-order BSPDE for swing option pricing. *Math*. *Finance*.
[2] Bensoussan, A. (1983). Maximum principle and dynamic programming approaches of the optimal control of partially observed diffusions. *Stochastics* **9** 169–222. MR0705471
[3] Cass, T. and Friz, P. (2010). Densities for rough differential equations under Hörmander's condition. *Ann*. *of Math*. (2) **171** 2115–2141. MR2680405
[4] Du, K., Qiu, J. and Tang, S. (2012). L^p theory for super-parabolic backward stochastic partial differential equations in the whole space. *Appl*. *Math*. *Optim*. **65** 175–219. MR2891221
[5] Du, K., Tang, S. and Zhang, Q. (2013). $W^{m,p}$-solution ($p \geq 2$) of linear degenerate backward stochastic partial differential equations in the whole space. *J*. *Differential Equations* **254** 2877–2904. MR3017034
[6] Graewe, P., Horst, U. and Qiu, J. (2015). A non-Markovian liquidation problem and backward SPDEs with singular terminal conditions. *SIAM J*. *Control Optim*. **53** 690–711. MR3319844
[7] Hörmander, L. (1967). Hypoelliptic second order differential equations. *Acta Math*. **119** 147–171. MR0222474
[8] Hörmander, L. (1983). *The Analysis of Linear Partial Differential Operators*. *II*. *Grundlehren der Mathematischen Wissenschaften* **257**. Berlin: Springer. MR0705278
[9] Hu, Y., Ma, J. and Yong, J. (2002). On semi-linear degenerate backward stochastic partial differential equations. *Probab*. *Theory Related Fields* **123** 381–411. MR1918539
[10] Hu, Y. and Peng, S.G. (1991). Adapted solution of a backward semilinear stochastic evolution equation. *Stoch*. *Anal*. *Appl*. **9** 445–459. MR1133858
[11] Krylov, N.V. (2013). Hörmander's theorem for stochastic partial differential equations. Available at arXiv:1309.5543.
[12] Krylov, N.V. and Rozovskii, B.L. (1981). Stochastic evolution equations. *J*. *Sov*. *Math*. **16** 1233–1277.
[13] Ma, J. and Yong, J. (1999). On linear, degenerate backward stochastic partial differential equations. *Probab*. *Theory Related Fields* **113** 135–170. MR1676768
[14] Malliavin, P. (1978). Stochastic calculus of variation and hypoelliptic operators. In *Proceedings of the International Symposium on Stochastic Differential Equations* (*Res*. *Inst*. *Math*. *Sci*., *Kyoto Univ*., *Kyoto*, 1976) 195–263. New York: Wiley. MR0536013

1350-7265

[15] Mattingly, J.C. and Pardoux, É. (2006). Malliavin calculus for the stochastic 2D Navier–Stokes equation. *Comm. Pure Appl. Math.* **59** 1742–1790. MR2257860
[16] Peng, S.G. (1992). Stochastic Hamilton–Jacobi–Bellman equations. *SIAM J. Control Optim.* **30** 284–304. MR1149069
[17] Qiu, J. and Tang, S. (2012). Maximum principle for quasi-linear backward stochastic partial differential equations. *J. Funct. Anal.* **262** 2436–2480. MR2876411
[18] Qiu, J. and Wei, W. (2014). On the quasi-linear reflected backward stochastic partial differential equations. *J. Funct. Anal.* **267** 3598–3656. MR3266242
[19] Stroock, D.W. and Varadhan, S.R.S. (1979). *Multidimensional Diffusion Processes. Grundlehren der Mathematischen Wissenschaften* **233**. New York: Springer. MR0532498
[20] Tang, S. and Wei, W. (2016). On the Cauchy problem for backward stochastic partial differential equations in Hölder spaces. *Ann. Probab.* **44** 360–398. MR3456341
[21] Zhou, X.Y. (1992). A duality analysis on stochastic partial differential equations. *J. Funct. Anal.* **103** 275–293. MR1151549

Bernoulli **24**(2), 2018, 971–992
https://doi.org/10.3150/16-BEJ823

An upper bound on the convergence rate of a second functional in optimal sequence alignment

RAPHAEL HAUSER[1], HEINRICH MATZINGER[2] and IONEL POPESCU[3]

[1]*Mathematical Institute, University of Oxford, Radcliffe Observatory Quarter, Woodstock Road, Oxford OX2 6GG, United Kingdom, and Pembroke College, St Aldates, Oxford, OX1 1DW, United Kingdom. E-mail: hauser@maths.ox.ac.uk*

[2]*School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332-0160 USA. E-mail: matzi@math.gatech.edu*

[3]*School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332, USA, IMAR, 21 Calea Griviţei and FMI, Str. Academiei nr.14, Bucharest, Romania. E-mail: ipopescu@math.gatech.edu; ionel.popescu@imar.ro*

Consider finite sequences $X_{[1,n]} = X_1, \dots, X_n$ and $Y_{[1,n]} = Y_1, \dots, Y_n$ of length n, consisting of i.i.d. samples of random letters from a finite alphabet, and let S and T be chosen i.i.d. randomly from the unit ball in the space of symmetric scoring functions over this alphabet augmented by a gap symbol. We prove a probabilistic upper bound of linear order in $(\ln(n))^{1/4} n^{3/4}$ for the deviation of the score relative to T of optimal alignments with gaps of $X_{[1,n]}$ and $Y_{[1,n]}$ relative to S. It remains an open problem to prove a lower bound. Our result contributes to the understanding of the microstructure of optimal alignments relative to one given scoring function, extending a theory begun in (*J. Stat. Phys.* **153** (2013) 512–529).

Keywords: convex geometry; large deviations; percolation theory; sequence alignment

References

[1] Azuma, K. (1967). Weighted sums of certain dependent random variables. *Tôhoku Math. J.* (2) **19** 357–367. MR0221571
[2] Chvatal, V. and Sankoff, D. (1975). Longest common subsequences of two random sequences. *J. Appl. Probab.* **12** 306–315. MR0405531
[3] Evans, L.C. and Gariepy, R.F. (1992). *Measure Theory and Fine Properties of Functions*. *Studies in Advanced Mathematics*. Boca Raton, FL: CRC Press. MR1158660
[4] Hauser, R. and Matzinger, H. (2013). Letter change bias and local uniqueness in optimal sequence alignments. *J. Stat. Phys.* **153** 512–529. MR3107656
[5] Kesten, H., ed. (2004). *Probability on Discrete Structures*. *Encyclopaedia of Mathematical Sciences* **110**. Berlin: Springer. MR2023649
[6] Lember, J. and Matzinger, H. (2012). Detecting the homology of DNA-sequences based on the variety of optimal alignments: A case study. Available at arXiv:1210.3771 [stat.AP].
[7] Needleman, S.B. and Wunsch, C.D. (1970). A general method applicable to the search for similarities in the amino acid sequence of two proteins. *J. Mol. Biol.* **48** 443–453.
[8] Waterman, M.S. and Vingron, M. (1994). Sequence comparison significance and Poisson approximation. *Statist. Sci.* **9** 367–381. MR1325433

1350-7265

Bernoulli **24**(2), 2018, 993–1009
https://doi.org/10.3150/16-BEJ832

Mixing time and cutoff for a random walk on the ring of integers mod n

MICHAEL BATE* and STEPHEN CONNOR**

Department of Mathematics, University of York, York, YO10 5DD, UK.
*E-mail: *michael.bate@york.ac.uk; **stephen.connor@york.ac.uk*

We analyse a random walk on the ring of integers mod n, which at each time point can make an additive 'step' or a multiplicative 'jump'. When the probability of making a jump tends to zero as an appropriate power of n, we prove the existence of a total variation pre-cutoff for this walk. In addition, we show that the process obtained by subsampling our walk at jump times exhibits a true cutoff, with mixing time dependent on whether the step distribution has zero mean.

Keywords: cutoff phenomenon; group representation theory; mixing time; pre-cutoff; random number generation; random walk

References

[1] Apostol, T.M. (1974). *Mathematical Analysis*, 2nd ed. Reading, MA: Addison-Wesley. MR0344384
[2] Chung, F.R.K., Diaconis, P. and Graham, R.L. (1987). Random walks arising in random number generation. *Ann. Probab.* **15** 1148–1165. MR0893921
[3] Connor, S.B. (2010). Separation and coupling cutoffs for tuples of independent Markov processes. *ALEA Lat. Am. J. Probab. Math. Stat.* **7** 65–77. MR2644042
[4] Diaconis, P. (1988). *Group Representations in Probability and Statistics. Institute of Mathematical Statistics Lecture Notes – Monograph Series* **11**. Hayward, CA: IMS. MR0964069
[5] Diaconis, P. (2011). The mathematics of mixing things up. *J. Stat. Phys.* **144** 445–458. MR2826629
[6] Diaconis, P. and Shahshahani, M. (1981). Generating a random permutation with random transpositions. *Z. Wahrsch. Verw. Gebiete* **57** 159–179. MR0626813
[7] Hildebrand, M. (1990). Rates of convergence of some random processes on finite groups. Ph.D. thesis, Dept. Mathematics, Harvard University.
[8] Hildebrand, M. (1993). Random processes of the form $X_{n+1} = a_n X_n + b_n \pmod p$. *Ann. Probab.* **21** 710–720. MR1217562
[9] Hildebrand, M. (1994). Random walks supported on random points of $\mathbf{Z}/n\mathbf{Z}$. *Probab. Theory Related Fields* **100** 191–203. MR1296428
[10] Hildebrand, M. (1994). Some random processes related to affine random walks. IMA preprint 1210.
[11] Levin, D.A., Peres, Y. and Wilmer, E.L. (2009). *Markov Chains and Mixing Times*. Providence, RI: Amer. Math. Soc. With a chapter by James G. Propp and David B. Wilson. MR2466937
[12] Mossel, E., Peres, Y. and Sinclair, A. (2004). Shuffling by semi-random transpositions. In *Proceedings of the 45th Annual IEEE Symposium on Foundations of Computer Science (FOCS'04)* 572–581. Washington, DC, USA: IEEE Computer Society.
[13] Saloff-Coste, L. (2004). Random walks on finite groups. In *Probability on Discrete Structures. Encyclopaedia Math. Sci.* **110** 263–346. Berlin: Springer. MR2023654
[14] Smith, A. (2014). A Gibbs sampler on the n-simplex. *Ann. Appl. Probab.* **24** 114–130. MR3161643

1350-7265

[15] Subag, E. (2013). A lower bound for the mixing time of the random-to-random insertions shuffle. *Electron. J. Probab.* **18** 1–20. MR3035748

[16] Terras, A. (1999). *Fourier Analysis on Finite Groups and Applications*. *London Mathematical Society Student Texts* **43**. Cambridge: Cambridge Univ. Press. MR1695775

Bernoulli **24**(2), 2018, 1010–1032
https://doi.org/10.3150/16-BEJ845

Exponential mixing properties for time inhomogeneous diffusion processes with killing

PIERRE DEL MORAL[1] and DENIS VILLEMONAIS[2]

[1] *INRIA Bordeaux Research Center, University of Bordeaux, 200 Avenue de la Vieille Tour, 33405 Talence, France. E-mail: pierre.del-moral@inria.fr*
[2] *TOSCA project-team, INRIA Nancy – Grand Est, IECL – UMR 7502, Université de Lorraine B.P. 70239, 54506 Vandoeuvre-lès-Nancy Cedex, France. E-mail: denis.villemonais@univ-lorraine.fr*

We consider an elliptic and time-inhomogeneous diffusion process with time-periodic coefficients evolving in a bounded domain of $\mathbb{R}^d$ with a smooth boundary. The process is killed when it hits the boundary of the domain (hard killing) or after an exponential time (soft killing) associated with some bounded rate function. The branching particle interpretation of the non absorbed diffusion again behaves as a set of interacting particles evolving in an absorbing medium. Between absorption times, the particles evolve independently one from each other according to the diffusion evolution operator; when a particle is absorbed, another selected particle splits into two offsprings. This article is concerned with the stability properties of these non absorbed processes. Under some classical ellipticity properties on the diffusion process and some mild regularity properties of the hard obstacle boundaries, we prove an uniform exponential strong mixing property of the process conditioned to not be killed. We also provide uniform estimates w.r.t. the time horizon for the interacting particle interpretation of these non-absorbed processes, yielding what seems to be the first result of this type for this class of diffusion processes evolving in soft and hard obstacles, both in homogeneous and non-homogeneous time settings.

Keywords: process with absorption; time-inhomogeneous diffusion process; uniform mixing property

References

[1] Burdzy, K., Holyst, R., Ingerman, D. and March, P. (1996). Configurational transition in a Fleming–Viot-type model and probabilistic interpretation of Laplacian eigenfunctions. *J. Phys. A* **29** 2633–2642.

[2] Burdzy, K., Hołyst, R. and March, P. (2000). A Fleming–Viot particle representation of the Dirichlet Laplacian. *Comm. Math. Phys.* **214** 679–703. MR1800866

[3] Cattiaux, P., Collet, P., Lambert, A., Martínez, S., Méléard, S. and San Martín, J. (2009). Quasi-stationary distributions and diffusion models in population dynamics. *Ann. Probab.* **37** 1926–1969. MR2561437

[4] Cattiaux, P. and Méléard, S. (2010). Competitive or weak cooperative stochastic Lotka–Volterra systems conditioned on non-extinction. *J. Math. Biol.* **60** 797–829. MR2606515

[5] Champagnat, N. and Villemonais, D. (2016). Exponential convergence to quasi-stationary distribution and Q-process. *Probab. Theory Related Fields* **164** 243–283. MR3449390

[6] Delfour, M.C. and Zolésio, J.-P. (2011). *Shapes and Geometries*: *Metrics*, *Analysis*, *Differential Calculus*, *and Optimization*, 2nd ed. *Advances in Design and Control* **22**. Philadelphia, PA: Society for Industrial and Applied Mathematics (SIAM). MR2731611

[7] Del Moral, P. (2013). *Mean Field Simulation for Monte Carlo Integration*. *Monographs on Statistics and Applied Probability* **126**. Boca Raton, FL: CRC Press. MR3060209

[8] Del Moral, P. and Doucet, A. (2004). Particle motions in absorbing medium with hard and soft obstacles. *Stoch*. *Anal*. *Appl*. **22** 1175–1207. MR2089064

[9] Del Moral, P. and Guionnet, A. (2001). On the stability of interacting processes with applications to filtering and genetic algorithms. *Ann*. *Inst*. *Henri Poincaré Probab*. *Stat*. **37** 155–194. MR1819122

[10] Del Moral, P. and Miclo, L. (2000). Branching and interacting particle systems approximations of Feynman–Kac formulae with applications to non-linear filtering. In *Séminaire de Probabilités*, *XXXIV*. *Lecture Notes in Math*. **1729** 1–145. Berlin: Springer. MR1768060

[11] Del Moral, P. and Miclo, L. (2000). A Moran particle system approximation of Feynman–Kac formulae. *Stochastic Process*. *Appl*. **86** 193–216. MR1741805

[12] Del Moral, P. and Miclo, L. (2002). On the stability of nonlinear Feynman–Kac semigroups. *Ann*. *Fac*. *Sci*. *Toulouse Math*. (6) **11** 135–175. MR1988460

[13] Del Moral, P. and Miclo, L. (2003). Particle approximations of Lyapunov exponents connected to Schrödinger operators and Feynman–Kac semigroups. *ESAIM Probab*. *Stat*. **7** 171–208. MR1956078

[14] Ethier, S.N. and Krone, S.M. (1995). Comparing Fleming–Viot and Dawson–Watanabe processes. *Stochastic Process*. *Appl*. **60** 171–190. MR1376800

[15] Ethier, S.N. and Kurtz, T.G. (1986). *Markov Processes*: *Characterization and Convergence*. *Wiley Series in Probability and Mathematical Statistics*: *Probability and Mathematical Statistics*. New York: Wiley. MR0838085

[16] Ferrari, P.A. and Marić, N. (2007). Quasi stationary distributions and Fleming–Viot processes in countable spaces. *Electron*. *J*. *Probab*. **12** 684–702. MR2318407

[17] Fleming, W.H. and Viot, M. (1979). Some measure-valued Markov processes in population genetics theory. *Indiana Univ*. *Math*. *J*. **28** 817–843. MR0542340

[18] Gong, G.L., Qian, M.P. and Zhao, Z.X. (1988). Killed diffusions and their conditioning. *Probab*. *Theory Related Fields* **80** 151–167. MR0970476

[19] Graham, C. and Méléard, S. (1997). Stochastic particle approximations for generalized Boltzmann models and convergence estimates. *Ann*. *Probab*. **25** 115–132. MR1428502

[20] Grigorescu, I. and Kang, M. (2004). Hydrodynamic limit for a Fleming–Viot type system. *Stochastic Process*. *Appl*. **110** 111–143. MR2052139

[21] Ikeda, N. and Watanabe, S. (1981). *Stochastic Differential Equations and Diffusion Processes*. *North-Holland Mathematical Library* **24**. Amsterdam–New York: North-Holland; Tokyo: Kodansha, Ltd. MR0637061

[22] Knobloch, R. and Partzsch, L. (2010). Uniform conditional ergodicity and intrinsic ultracontractivity. *Potential Anal*. **33** 107–136. MR2658978

[23] Kolb, M. and Wübker, A. (2011). Spectral analysis of diffusions with jump boundary. *J*. *Funct*. *Anal*. **261** 1992–2012. MR2822321

[24] Lindvall, T. and Rogers, L.C.G. (1986). Coupling of multidimensional diffusions by reflection. *Ann*. *Probab*. **14** 860–872. MR0841588

[25] Littin C., J. (2012). Uniqueness of quasistationary distributions and discrete spectra when ∞ is an entrance boundary and 0 is singular. *J*. *Appl*. *Probab*. **49** 719–730. MR3012095

[26] Méléard, S. and Villemonais, D. (2012). Quasi-stationary distributions and population processes. *Probab*. *Surv*. **9** 340–410. MR2994898

[27] Miura, Y. (2014). Ultracontractivity for Markov semigroups and quasi-stationary distributions. *Stoch*. *Anal*. *Appl*. **32** 591–601. MR3219695

[28] Pinsky, R.G. (1985). On the convergence of diffusion processes conditioned to remain in a bounded region for large time to limiting positive recurrent diffusion processes. *Ann. Probab.* **13** 363–378. MR0781410

[29] Priola, E. and Wang, F.-Y. (2006). Gradient estimates for diffusion semigroups with singular coefficients. *J. Funct. Anal.* **236** 244–264. MR2227134

[30] Rousset, M. (2006). On the control of an interacting particle estimation of Schrödinger ground states. *SIAM J. Math. Anal.* **38** 824–844 (electronic). MR2262944

[31] van Doorn, E.A. and Pollett, P.K. (2013). Quasi-stationary distributions for discrete-state models. *European J. Oper. Res.* **230** 1–14. MR3063313

[32] Villemonais, D. (2011). Interacting particle systems and Yaglom limit approximation of diffusions with unbounded drift. *Electron. J. Probab.* **16** 1663–1692. MR2835250

[33] Villemonais, D. (2013). Uniform tightness for time-inhomogeneous particle systems and for conditional distributions of time-inhomogeneous diffusion processes. *Markov Process. Related Fields* **19** 543–562. MR3156965

[34] Villemonais, D. (2014). General approximation method for the distribution of Markov processes conditioned not to be killed. *ESAIM Probab. Stat.* **18** 441–467. MR3333998

[35] Wang, F.-Y. (2004). Gradient estimates of Dirichlet heat semigroups and application to isoperimetric inequalities. *Ann. Probab.* **32** 424–440. MR2040788

Bernoulli **24**(2), 2018, 1033–1052
https://doi.org/10.3150/16-BEJ847

Functional central limit theorems in $L^2(0, 1)$ for logarithmic combinatorial assemblies

KOJI TSUKUDA

Biostatistics Center, Kurume University, 67 Asahi-machi, Kurume-city, Fukuoka, 830-0011, Japan. E-mail: k.tsukuda@gmail.com

Functional central limit theorems in $L^2(0, 1)$ for logarithmic combinatorial assemblies are presented. The random elements argued in this paper are viewed as elements taking values in $L^2(0, 1)$ whereas the Skorokhod space is argued as a framework of weak convergences in functional central limit theorems for random combinatorial structures in the literature. It enables us to treat other standardized random processes which converge weakly to a corresponding Gaussian process with additional assumptions.

Keywords: functional central limit theorem; logarithmic assembly; Poisson approximation; random mappings; the Ewens sampling formula

References

[1] Arratia, R., Barbour, A.D. and Tavaré, S. (1992). Poisson process approximations for the Ewens sampling formula. *Ann. Appl. Probab.* **2** 519–535. MR1177897

[2] Arratia, R., Barbour, A.D. and Tavaré, S. (2000). Limits of logarithmic combinatorial structures. *Ann. Probab.* **28** 1620–1644. MR1813836

[3] Arratia, R., Stark, D. and Tavaré, S. (1995). Total variation asymptotics for Poisson process approximations of logarithmic combinatorial assemblies. *Ann. Probab.* **23** 1347–1388. MR1349176

[4] Arratia, R. and Tavaré, S. (1992). Limit theorems for combinatorial structures via discrete process approximations. *Random Structures Algorithms* **3** 321–345. MR1164844

[5] Arratia, R. and Tavaré, S. (1994). Independent process approximations for random combinatorial structures. *Adv. Math.* **104** 90–154. MR1272071

[6] DeLaurentis, J.M. and Pittel, B.G. (1985). Random permutations and Brownian motion. *Pacific J. Math.* **119** 287–301. MR0803120

[7] Donnelly, P.J., Ewens, W.J. and Padmadisastra, S. (1991). Functionals of random mappings: Exact and asymptotic results. *Adv. in Appl. Probab.* **23** 437–455. MR1122869

[8] Ewens, W.J. (1972). The sampling theory of selectively neutral alleles. *Theoret. Population Biology* **3** 87–112; erratum, ibid. **3** (1972), 240; erratum, ibid. **3** (1972), 376. MR0325177

[9] Flajolet, P. and Soria, M. (1990). Gaussian limiting distributions for the number of components in combinatorial structures. *J. Combin. Theory Ser. A* **53** 165–182. MR1041444

[10] Hansen, J.C. (1989). A functional central limit theorem for random mappings. *Ann. Probab.* **17** 317–332. Correction: *Ann. Probab.* **19** (1991) 1393–1396. MR0972788

[11] Hansen, J.C. (1990). A functional central limit theorem for the Ewens sampling formula. *J. Appl. Probab.* **27** 28–43. MR1039182

[12] Prohorov, Yu.V. (1956). Convergence of random processes and limit theorems in probability theory. *Teor. Veroyatn. Primen.* **1** 177–238. MR0084896

1350-7265

[13] Teicher, H. (1955). An inequality on Poisson probabilities. *Ann*. *Math*. *Stat*. **26** 147–149. MR0067384
[14] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247
[15] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes*: *With Applications to Statistics*. *Springer Series in Statistics*. New York: Springer. MR1385671

Bernoulli **24**(2), 2018, 1053–1071
https://doi.org/10.3150/16-BEJ864

Inference for a two-component mixture of symmetric distributions under log-concavity

FADOUA BALABDAOUI[1] and CHARLES R. DOSS[2]

[1]*CEREMADE, Université Paris-Dauphine, 75775 Paris, France and Seminar für Statistik, ETH Zürich, 8092 Zürich, Schweiz. E-mail: fadoua.balabdaoui@stat.math.ethz.ch*
[2]*University of Minnesota, School of Statistics, 313 Ford Hall, 224 Church Street SE, Minneapolis, MN 55455, USA. E-mail: cdoss@stat.umn.edu*

In this article, we revisit the problem of estimating the unknown zero-symmetric distribution in a two-component location mixture model, considered in previous works, now under the assumption that the zero-symmetric distribution has a log-concave density. When consistent estimators for the shift locations and mixing probability are used, we show that the nonparametric log-concave Maximum Likelihood estimator (MLE) of both the mixed density and that of the unknown zero-symmetric component are consistent in the Hellinger distance. In case the estimators for the shift locations and mixing probability are $\sqrt{n}$-consistent, we establish that these MLE's converge to the truth at the rate $n^{-2/5}$ in the L_1 distance. To estimate the shift locations and mixing probability, we use the estimators proposed by (*Ann. Statist.* **35** (2007) 224–251). The unknown zero-symmetric density is efficiently computed using the R package `logcondens.mode`.

Keywords: bracketing entropy; consistency; empirical processes; global rate; Hellinger metric; log-concave; mixture; symmetric

References

[1] Azzalini, A. and Bowman, A.W. (1990). A look at some data on the Old Faithful geyser. *J. R. Stat. Soc., Ser. C* **39** 357–365.
[2] Balabdaoui, F. and Butucea, C. (2014). On location mixtures with Pólya frequency components. *Statist. Probab. Lett.* **95** 144–149. MR3262962
[3] Balabdaoui, F. and Doss, C.R. (2016). Supplement to "Inference for a two-component mixture of symmetric distributions under log-concavity." DOI:10.3150/16-BEJ864SUPP.
[4] Balabdaoui, F., Jankowski, H., Rufibach, K. and Pavlides, M. (2013). Asymptotics of the discrete log-concave maximum likelihood estimator and related applications. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **75** 769–790. MR3091658
[5] Balabdaoui, F., Rufibach, K. and Wellner, J.A. (2009). Limit distribution theory for maximum likelihood estimation of a log-concave density. *Ann. Statist.* **37** 1299–1331. MR2509075
[6] Bordes, L., Mottelet, S. and Vandekerkhove, P. (2006). Semiparametric estimation of a two-component mixture model. *Ann. Statist.* **34** 1204–1232. MR2278356
[7] Butucea, C. and Vandekerkhove, P. (2014). Semiparametric mixtures of symmetric distributions. *Scand. J. Stat.* **41** 227–239. MR3181141
[8] Chang, G.T. and Walther, G. (2007). Clustering with mixtures of log-concave distributions. *Comput. Statist. Data Anal.* **51** 6242–6251. MR2408591
[9] Chee, C.-S. and Wang, Y. (2013). Estimation of finite mixtures with symmetric components. *Stat. Comput.* **23** 233–249. MR3016941

1350-7265

[10] Chen, Y. and Samworth, R.J. (2013). Smoothed log-concave maximum likelihood estimation with applications. *Statist*. *Sinica* **23** 1373–1398. MR3114718

[11] Ciullo, M. (2009). Genetic park of Cilento and Vallo di Dano project. Available at http://www.igb.cnr.it/cilentoisolates.

[12] Colonna, V., Nutile, T., Astore, M., Guardiola, O., Antoniol, G., Ciullo, M. and Persico, M.G. (2007). Campora: A young genetic isolate in South Italy. *Hum*. *Hered*. **64** 123–135.

[13] Cule, M. and Samworth, R. (2010). Theoretical properties of the log-concave maximum likelihood estimator of a multidimensional density. *Electron*. *J*. *Stat*. **4** 254–270. MR2645484

[14] Cule, M., Samworth, R. and Stewart, M. (2010). Maximum likelihood estimation of a multi-dimensional log-concave density. *J*. *R*. *Stat*. *Soc*. *Ser*. *B Stat*. *Methodol*. **72** 545–607. MR2758237

[15] Dempster, A.P., Laird, N.M. and Rubin, D.B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *J*. *Roy*. *Statist*. *Soc*. *Ser*. *B* **39** 1–38. MR0501537

[16] Doss, C.R. and Wellner, J.A. (2016). Global rates of convergence of the MLEs of log-concave and s-concave densities. *Ann*. *Statist*. **44** 954–981. MR3485950

[17] Dümbgen, L., Hüsler, A. and Rufibach, K. (2010). Active set and EM algorithms for log-concave densities based on complete and censored data. Technical report, Univ. Bern. Available at arXiv:0707.4643.

[18] Dümbgen, L. and Rufibach, K. (2009). Maximum likelihood estimation of a log-concave density and its distribution function: Basic properties and uniform consistency. *Bernoulli* **15** 40–68. MR2546798

[19] Dümbgen, L. and Rufibach, K. (2011). logcondens: Computations related to univariate log-concave density estimation. *J*. *Stat*. *Softw*. **39** 1–28.

[20] Dümbgen, L., Samworth, R. and Schuhmacher, D. (2011). Approximation by log-concave distributions, with applications to regression. *Ann*. *Statist*. **39** 702–730. MR2816336

[21] Dümbgen, L., Samworth, R. and Schuhmacher, D. (2011). Approximation by log-concave distributions, with applications to regression. Available at arXiv:1002.3448v3.

[22] Eilers, P.H.C. and Borgdorff, M.W. (2007). Non-parametric log-concave mixtures. *Comput*. *Statist*. *Data Anal*. **51** 5444–5451. MR2370883

[23] Everitt, B.S. and Hand, D.J. (1981). *Finite Mixture Distributions*. London: Chapman & Hall. MR0624267

[24] Fraley, C. and Raftery, A.E. (2002). Model-based clustering, discriminant analysis, and density estimation. *J*. *Amer*. *Statist*. *Assoc*. **97** 611–631. MR1951635

[25] Has'minskiĭ, R.Z. (1978). A lower bound for risks of nonparametric density estimates in the uniform metric. *Teor*. *Veroyatnost*. *i Primenen*. **23** 824–828. MR0516279

[26] Hunter, D.R., Wang, S. and Hettmansperger, T.P. (2007). Inference for mixtures of symmetric distributions. *Ann*. *Statist*. **35** 224–251. MR2332275

[27] Kim, A.K.H. and Samworth, R.J. (2014). Global rates of convergence in log-concave density estimation. Available at arXiv.org:1404.2298v1.

[28] Koenker, R. and Mizera, I. (2010). Quasi-concave density estimation. *Ann*. *Statist*. **38** 2998–3027. MR2722462

[29] McLachlan, G. and Peel, D. (2000). *Finite Mixture Models*. *Wiley Series in Probability and Statistics*: *Applied Probability and Statistics*. New York: Wiley. MR1789474

[30] Niculescu, C.P. and Persson, L.-E. (2006). *Convex Functions and Their Applications*: *A Contemporary Approach*. *CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC*, 23. New York: Springer. MR2178902

[31] Pal, J.K., Woodroofe, M. and Meyer, M. (2007). Estimating a Polya frequency function$_2$. In *Complex Datasets and Inverse Problems*. *Institute of Mathematical Statistics Lecture Notes—Monograph Series* **54** 239–249. Beachwood, OH: IMS. MR2459192

[32] Rockafellar, R.T. (1970). *Convex Analysis*. *Princeton Mathematical Series*, *No*. 28. Princeton, NJ: Princeton Univ. Press. MR0274683

[33] Rufibach, K. (2007). Computing maximum likelihood estimators of a log-concave density function. *J. Stat. Comput. Simul.* **77** 561–574. MR2407642

[34] Schoenberg, I.J. (1951). On Pólya frequency functions. I. The totally positive functions and their Laplace transforms. *J. Anal. Math.* **1** 331–374. MR0047732

[35] Schuhmacher, D. and Dümbgen, L. (2010). Consistency of multivariate log-concave density estimators. *Statist. Probab. Lett.* **80** 376–380. MR2593576

[36] Silverman, B.W. (1982). On the estimation of a probability density function by the maximum penalized likelihood method. *Ann. Statist.* **10** 795–810. MR0663433

[37] Stone, C.J. (1980). Optimal rates of convergence for nonparametric estimators. *Ann. Statist.* **8** 1348–1360. MR0594650

[38] Titterington, D.M., Smith, A.F.M. and Makov, U.E. (1985). *Statistical Analysis of Finite Mixture Distributions*. *Wiley Series in Probability and Mathematical Statistics*: *Applied Probability and Statistics*. Chichester: Wiley. MR0838090

[39] Walther, G. (2002). Detecting the presence of mixing with multiscale maximum likelihood. *J. Amer. Statist. Assoc.* **97** 508–513. MR1941467

[40] Weisberg, S. (2005). *Applied Linear Regression*, 3rd ed. *Wiley Series in Probability and Statistics*. Wiley: Hoboken, NJ. MR2112740

Bernoulli **24**(2), 2018, 1072–1100
https://doi.org/10.3150/16-BEJ865

On matrix estimation under monotonicity constraints

SABYASACHI CHATTERJEE[1], ADITYANAND GUNTUBOYINA[2] and BODHISATTVA SEN[3]

[1]*5747 S. Ellis Avenue, Chicago, IL 60637, USA. E-mail: sabyasachi@galton.uchicago.edu*
[2]*423 Evans Hall, Berkeley, CA 94720, USA. E-mail: aditya@stat.berkeley.edu*
[3]*1255 Amsterdam Avenue, New York, NY 10027, USA. E-mail: bodhi@stat.columbia.edu*

We consider the problem of estimating an unknown $n_1 \times n_2$ matrix $\boldsymbol{\theta}^*$ from noisy observations under the constraint that $\boldsymbol{\theta}^*$ is nondecreasing in both rows and columns. We consider the least squares estimator (LSE) in this setting and study its risk properties. We show that the worst case risk of the LSE is $n^{-1/2}$, up to multiplicative logarithmic factors, where $n = n_1 n_2$ and that the LSE is minimax rate optimal (up to logarithmic factors). We further prove that for some special $\boldsymbol{\theta}^*$, the risk of the LSE could be much smaller than $n^{-1/2}$; in fact, it could even be parametric, that is, n^{-1} up to logarithmic factors. Such parametric rates occur when the number of "rectangular" blocks of $\boldsymbol{\theta}^*$ is bounded from above by a constant. We also derive an interesting adaptation property of the LSE which we term variable adaptation – the LSE adapts to the "intrinsic dimension" of the problem and performs as well as the oracle estimator when estimating a matrix that is constant along each row/column. Our proofs, which borrow ideas from empirical process theory, approximation theory and convex geometry, are of independent interest.

Keywords: adaptation; bivariate isotonic regression; metric entropy bounds; minimax lower bound; oracle inequalities; tangent cone; variable adaptation

References

[1] Ayer, M., Brunk, H.D., Ewing, G.M., Reid, W.T. and Silverman, E. (1955). An empirical distribution function for sampling with incomplete information. *Ann. Math. Stat.* **26** 641–647. MR0073895
[2] Barlow, R.E., Bartholomew, D.J., Bremner, J.M. and Brunk, H.D. (1972). *Statistical Inference Under Order Restrictions. The Theory and Application of Isotonic Regression*. New York: Wiley. MR0326887
[3] Bellec, P. (2015). Sharp oracle inequalities for least squares estimators in shape restricted regression. Preprint. Available at arXiv:1510.08029.
[4] Birgé, L. and Massart, P. (1993). Rates of convergence for minimum contrast estimators. *Probab. Theory Related Fields* **97** 113–150. MR1240719
[5] Brunk, H.D. (1955). Maximum likelihood estimates of monotone parameters. *Ann. Math. Stat.* **26** 607–616. MR0073894
[6] Chatterjee, S. (2014). A new perspective on least squares under convex constraint. *Ann. Statist.* **42** 2340–2381. MR3269982
[7] Chatterjee, S. (2015). Matrix estimation by universal singular value thresholding. *Ann. Statist.* **43** 177–214. MR3285604
[8] Chatterjee, S., Guntuboyina, A. and Sen, B. (2015). On risk bounds in isotonic and other shape restricted regression problems. *Ann. Statist.* **43** 1774–1800. MR3357878

1350-7265

[9] Chatterjee, S., Guntuboyina, A. and Sen, B. (2015). On matrix estimation under monotonicity constraints. Available at http://arxiv.org/abs/1506.03430.
[10] Chatterjee, S. and Mukherjee, S. (2016). On estimation in tournaments and graphs under monotonicity constraints. Preprint. Available at arXiv:1603.04556.
[11] Donoho, D. (1991). Gelfand n-widths and the method of least squares Technical report, University of California, Berkeley.
[12] Gao, F. and Wellner, J.A. (2007). Entropy estimate for high-dimensional monotonic functions. *J. Multivariate Anal*. **98** 1751–1764. MR2392431
[13] Gebhardt, F. (1970). An algorithm for monotone regression with one or more independent variables. *Biometrika* **57** 263–271.
[14] Hanson, D.L., Pledger, G. and Wright, F.T. (1973). On consistency in monotonic regression. *Ann. Statist*. **1** 401–421. MR0353540
[15] Makowski, G.G. (1977). Consistency of an estimator of doubly nondecreasing regression functions. *Z. Wahrsch. Verw. Gebiete* **39** 263–268. MR0652723
[16] Meyer, M. and Woodroofe, M. (2000). On the degrees of freedom in shape-restricted regression. *Ann. Statist*. **28** 1083–1104. MR1810920
[17] Oymak, S. and Hassibi, B. (2016). Sharp MSE Bounds for Proximal Denoising. *Found. Comput. Math*. **16** 965–1029. MR3529131
[18] Robertson, T. and Wright, F.T. (1975). Consistency in generalized isotonic regression. *Ann. Statist*. **3** 350–362. MR0365871
[19] Robertson, T., Wright, F.T. and Dykstra, R.L. (1988). *Order Restricted Statistical Inference*. *Wiley Series in Probability and Mathematical Statistics*: *Probability and Mathematical Statistics*. Chichester: Wiley. MR0961262
[20] Shah, N.B., Balakrishnan, S., Guntuboyina, A. and Wainright, M.J. (2015). Stochastically transitive models for pairwise comparisons: Statistical and computational issues. Preprint. Available at arXiv:1510.05610.
[21] van Eeden, C. (1958). *Testing and Estimating Ordered Parameters of Probability Distributions* Amsterdam: Mathematical Centre. MR0102874
[22] van de Geer, S. (1990). Estimating a regression function. *Ann. Statist*. **18** 907–924. MR1056343
[23] van de Geer, S. (1993). Hellinger-consistency of certain nonparametric maximum likelihood estimators. *Ann. Statist*. **21** 14–44. MR1212164
[24] van de Geer, S. and Wainwright, M. (2015). On concentration for (regularized) empirical risk minimization. Preprint. Available at arXiv:1512.00677.
[25] van de Geer, S.A. (2000). *Applications of Empirical Process Theory*. *Cambridge Series in Statistical and Probabilistic Mathematics* **6**. Cambridge: Cambridge Univ. Press. MR1739079
[26] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247
[27] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes*: *With Applications to Statistics*. *Springer Series in Statistics*. New York: Springer. MR1385671
[28] Wang, Y. (1996). The L_2 risk of an isotonic estimate. *Comm. Statist. Theory Methods* **25** 281–294. MR1379445
[29] Zhang, C.-H. (2002). Risk bounds in isotonic regression. *Ann. Statist*. **30** 528–555. MR1902898

Bernoulli **24**(2), 2018, 1101–1127
https://doi.org/10.3150/16-BEJ873

Efficient estimation for generalized partially linear single-index models

LI WANG[1] and GUANQUN CAO[2]

[1]*Department of Statistics and the Statistical Laboratory, Iowa State University, Ames, IA 50011, USA.*
E-mail: lilywang@iastate.edu
[2]*Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849, USA.*
E-mail: gzc0009@auburn.edu

In this paper, we study the estimation for generalized partially linear single-index models, where the systematic component in the model has a flexible semi-parametric form with a general link function. We propose an efficient and practical approach to estimate the single-index link function, single-index coefficients as well as the coefficients in the linear component of the model. The estimation procedure is developed by applying quasi-likelihood and polynomial spline smoothing. We derive large sample properties of the estimators and show the convergence rate of each component of the model. Asymptotic normality and semiparametric efficiency are established for the coefficients in both the single-index and linear components. By making use of spline basis approximation and Fisher score iteration, our approach has numerical advantages in terms of computing efficiency and stability in practice. Both simulated and real data examples are used to illustrate our proposed methodology.

Keywords: asymptotic normality; generalized linear model; polynomial splines; quasi-likelihood; semi-parametric regression; single-index model

References

[1] Bickel, P.J., Klaassen, C.A.J., Ritov, Y. and Wellner, J.A. (1993). *Efficient and Adaptive Estimation for Semiparametric Models*. *Johns Hopkins Series in the Mathematical Sciences*. Baltimore, MD: Johns Hopkins Univ. Press. MR1245941

[2] Carroll, R.J., Fan, J., Gijbels, I. and Wand, M.P. (1997). Generalized partially linear single-index models. *J. Amer. Statist. Assoc.* **92** 477–489. MR1467842

[3] Chen, H. (1991). Estimation of a projection-pursuit type regression model. *Ann. Statist.* **19** 142–157. MR1091843

[4] Chen, J., Li, D., Liang, H. and Wang, S. (2015). Semiparametric GEE analysis in partially linear single-index models for longitudinal data. *Ann. Statist.* **43** 1682–1715. MR3357875

[5] de Boor, C. (2001). *A Practical Guide to Splines*, Revised ed. *Applied Mathematical Sciences* **27**. New York: Springer. MR1900298

[6] Friedman, J.H. and Stuetzle, W. (1981). Projection pursuit regression. *J. Amer. Statist. Assoc.* **76** 817–823. MR0650892

[7] Hall, P. (1989). On projection pursuit regression. *Ann. Statist.* **17** 573–588. MR0994251

[8] Härdle, W., Hall, P. and Ichimura, H. (1993). Optimal smoothing in single-index models. *Ann. Statist.* **21** 157–178. MR1212171

[9] Härdle, W. and Stoker, T.M. (1989). Investigating smooth multiple regression by the method of average derivatives. *J. Amer. Statist. Assoc.* **84** 986–995. MR1134488

1350-7265

[10] Hastie, T.J. and Tibshirani, R.J. (1990). *Generalized Additive Models*. Boca Raton: CRC Press.
[11] Hristache, M., Juditsky, A. and Spokoiny, V. (2001). Direct estimation of the index coefficient in a single-index model. *Ann. Statist*. **29** 595–623. MR1865333
[12] Huang, J.Z. and Yang, L. (2004). Identification of non-linear additive autoregressive models. *J. R. Stat. Soc. Ser. B. Stat. Methodol*. **66** 463–477. MR2062388
[13] Huang, J.Z., Zhang, L. and Zhou, L. (2007). Efficient estimation in marginal partially linear models for longitudinal/clustered data using splines. *Scand. J. Stat*. **34** 451–477. MR2368793
[14] Huber, P.J. (1985). Projection pursuit. *Ann. Statist*. **13** 435–525. MR0790553
[15] Ichimura, H. (1993). Semiparametric least squares (SLS) and weighted SLS estimation of single-index models. *J. Econometrics* **58** 71–120. MR1230981
[16] Klein, R.W. and Spady, R.H. (1993). An efficient semiparametric estimator for binary response models. *Econometrica* **61** 387–421. MR1209737
[17] Küchenhoff, H. and Carroll, R.J. (1997). Segmented regression with errors in predictors: Semi-parametric and parametric methods. *Stat. Med*. **16** 169–188.
[18] Lai, P., Li, G. and Lian, H. (2013). Quadratic inference functions for partially linear single-index models with longitudinal data. *J. Multivariate Anal*. **118** 115–127. MR3054094
[19] Lai, P., Tian, Y. and Lian, H. (2014). Estimation and variable selection for generalised partially linear single-index models. *J. Nonparametr. Stat*. **26** 171–185. MR3174315
[20] Liang, H., Liu, X., Li, R. and Tsai, C.-L. (2010). Estimation and testing for partially linear single-index models. *Ann. Statist*. **38** 3811–3836. MR2766869
[21] Ma, S., Liang, H. and Tsai, C.-L. (2014). Partially linear single index models for repeated measurements. *J. Multivariate Anal*. **130** 354–375. MR3229543
[22] Mammen, E., Linton, O. and Nielsen, J. (1999). The existence and asymptotic properties of a backfitting projection algorithm under weak conditions. *Ann. Statist*. **27** 1443–1490. MR1742496
[23] McCullagh, P. and Nelder, J.A. (1989). *Generalized Linear Models*, 2nd ed. *Monographs on Statistics and Applied Probability*. London: Chapman & Hall. MR3223057
[24] Powell, J.L., Stock, J.H. and Stoker, T.M. (1989). Semiparametric estimation of index coefficients. *Econometrica* **57** 1403–1430. MR1035117
[25] Sheng, W. and Yin, X. (2013). Direction estimation in single-index models via distance covariance. *J. Multivariate Anal*. **122** 148–161. MR3189314
[26] Wang, J.-L., Xue, L., Zhu, L. and Chong, Y.S. (2010). Estimation for a partial-linear single-index model. *Ann. Statist*. **38** 246–274. MR2589322
[27] Wang, L. and Cao, G. (2016). Supplement to "Efficient estimation for generalized partially linear single-index models." DOI:10.3150/16-BEJ873SUPP.
[28] Wang, L., Liu, X., Liang, H. and Carroll, R.J. (2011). Estimation and variable selection for generalized additive partial linear models. *Ann. Statist*. **39** 1827–1851. MR2893854
[29] Wang, L. and Yang, L. (2007). Spline-backfitted kernel smoothing of nonlinear additive autoregression model. *Ann. Statist*. **35** 2474–2503. MR2382655
[30] Wang, L. and Yang, L. (2009). Spline estimation of single-index models. *Statist. Sinica* **19** 765–783. MR2514187
[31] Wood, S.N. (2006). *Generalized Additive Models: An Introduction with R*. *Texts in Statistical Science Series*. Boca Raton, FL: Chapman & Hall/CRC. MR2206355
[32] Xia, Y. and Härdle, W. (2004). Semi-parametric estimation of generalized partially linear single-index models. Technical report, Humboldt Universities, Berlin.
[33] Xia, Y., Li, W.K., Tong, H. and Zhang, D. (2004). A goodness-of-fit test for single-index models. *Statist. Sinica* **14** 1–39. MR2036761
[34] Xia, Y., Tong, H., Li, W.K. and Zhu, L.-X. (2002). An adaptive estimation of dimension reduction space. *J. R. Stat. Soc. Ser. B. Stat. Methodol*. **64** 363–410. MR1924297

[35] Xue, L. and Yang, L. (2006). Additive coefficient modeling via polynomial spline. *Statist*. *Sinica* **16** 1423–1446. MR2327498

[36] Yu, Y. and Ruppert, D. (2002). Penalized spline estimation for partially linear single-index models. *J*. *Amer*. *Statist*. *Assoc*. **97** 1042–1054. MR1951258

Bernoulli **24**(2), 2018, 1128–1170
https://doi.org/10.3150/16-BEJ874

Critical points of multidimensional random Fourier series: Central limits

LIVIU I. NICOLAESCU[1]

[1]*Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556-4618, USA.*
E-mail: lnicolae@nd.edu; url: http://www3.nd.edu/~lnicolae/

We investigate certain families $X^{\hbar}$, $0 < \hbar \ll 1$ of stationary Gaussian random smooth functions on the m-dimensional torus $\mathbb{T}^m := \mathbb{R}^m/\mathbb{Z}^m$ approaching the white noise as $\hbar \to 0$. We show that there exists universal constants $c_1, c_2 > 0$ such that for any cube $B \subset \mathbb{R}^m$ of size $r \leq 1/2$, the number of critical points of $X^{\hbar}$ in the region $B \bmod \mathbb{Z}^m \subset \mathbb{T}^m$ has mean $\sim c_1 \operatorname{vol}(B)\hbar^{-m}$, variance $\sim c_2 \operatorname{vol}(B)\hbar^{-m}$, and satisfies a central limit theorem as $\hbar \searrow 0$.

Keywords: central limit theorem; critical points; Gaussian Hilbert spaces; Gaussian random functions; Kac–Rice formula; Wiener chaos

References

[1] Adler, R.J. and Naizat, G. (2015). A central limit theorem for the Euler integral of a Gaussian random field. Preprint. Available at arXiv:1506.08772.
[2] Adler, R.J. and Taylor, J.E. (2007). *Random Fields and Geometry*. *Springer Monographs in Mathematics*. New York: Springer. MR2319516
[3] Arcones, M.A. (1994). Limit theorems for nonlinear functionals of a stationary Gaussian sequence of vectors. *Ann. Probab.* **22** 2242–2274. MR1331224
[4] Arnold, V.I. (2006). Statistics and classification of topologies of periodic functions and trigonometric polynomials. *Proc. Steklov Inst. Math.* S13–S23. MR2246984
[5] Arnold, V.I. (2007). Topological classification of trigonometric polynomials of the affine Coxeter group $\tilde{A}_2$. *Tr. Mat. Inst. Steklova* **258** 7–16. MR2400519
[6] Arnold, V.I. (2007). Topological classification of Morse functions and generalisations of Hilbert's 16-th problem. *Math. Phys. Anal. Geom.* **10** 227–236. MR2368960
[7] Arnold, V.I. (2015). *Experimental Mathematics*. *MSRI Mathematical Circles Library* **16**. Providence, RI: Amer. Math. Soc. MR3309308
[8] Azaïs, J.-M., Dalmao, F. and León, J.R. (2016). CLT for the zeros of classical random trigonometric polynomials. *Ann. Inst. Henri Poincaré Probab. Stat.* **52** 804–820. MR3498010
[9] Azaïs, J.-M., Dalmao, F., León, J.R., Nourdin, I. and Poly, G. (2015). Local universality of the number of zeros of random trigonometric polynomials with continuous coefficients. Preprint. Available at arXiv:1512.05583.
[10] Azaïs, J.-M. and León, J.R. (2013). CLT for crossings of random trigonometric polynomials. *Electron. J. Probab.* **18** no. 68, 17. MR3084654
[11] Azaïs, J.-M. and Wschebor, M. (2009). *Level Sets and Extrema of Random Processes and Fields*. Hoboken, NJ: Wiley. MR2478201
[12] Bernstein, D.N. (1975). The number of roots of a system of equations. *Funct. Anal. Appl.* **9** 1–4. MR0435072

1350-7265

[13] Breuer, P. and Major, P. (1983). Central limit theorems for nonlinear functionals of Gaussian fields. *J. Multivariate Anal.* **13** 425–441. MR0716933
[14] Chambers, D. and Slud, E. (1989). Central limit theorems for nonlinear functionals of stationary Gaussian processes. *Probab. Theory Related Fields* **80** 323–346. MR0976529
[15] Cuzick, J. (1976). A central limit theorem for the number of zeros of a stationary Gaussian process. *Ann. Probab.* **4** 547–556. MR0420809
[16] Dalmao, F. (2015). Asymptotic variance and CLT for the number of zeros of Kostlan Shub Smale random polynomials. *C. R. Math. Acad. Sci. Paris* **353** 1141–1145. MR3427922
[17] Estrade, A. and León, J.R. (2015). A central limit theorem for the Euler characteristic of a Gaussian excursion set. *Ann. Probab.* **44** 3849–3878.
[18] Gelfand, I.M. and Vilenkin, N.Ya. (1964 [1977]). *Generalized Functions*, *Vol.* 4. New York: Academic Press. MR0435834
[19] Granville, A. and Wigman, I. (2011). The distribution of the zeros of random trigonometric polynomials. *Amer. J. Math.* **133** 295–357. MR2797349
[20] Hörmander, L. (2003). *The Analysis of Linear Partial Differential Operators*. *i*. *Classics in Mathematics*. Berlin: Springer. MR1996773
[21] Horn, R.A. and Johnson, C.R. (1985). *Matrix Analysis*. Cambridge: Cambridge Univ. Press. MR0832183
[22] Janson, S. (1997). *Gaussian Hilbert Spaces*. *Cambridge Tracts in Mathematics* **129**. Cambridge: Cambridge Univ. Press. MR1474726
[23] Kouchnirenko, A.G. (1976). Polyèdres de Newton et nombres de Milnor. *Invent. Math.* **32** 1–31. MR0419433
[24] Kratz, M.F. and León, J.R. (1997). Hermite polynomial expansion for non-smooth functionals of stationary Gaussian processes: Crossings and extremes. *Stochastic Process. Appl.* **66** 237–252. MR1440400
[25] Kratz, M.F. and León, J.R. (2001). Central limit theorems for level functionals of stationary Gaussian processes and fields. *J. Theoret. Probab.* **14** 639–672. MR1860517
[26] Major, P. (1981). *Multiple Wiener–Itô Integrals*. *Lecture Notes in Math.* **849**. Berlin: Springer. MR0611334
[27] Malevich, T.L. (1969). Asymptotic normality of the number of crossings of the zero level by a Gaussian process. *Theory Probab. Appl.* **14** 287–295.
[28] Malliavin, P. (1995). *Integration and Probability*. *Graduate Texts in Mathematics* **157**. New York: Springer. MR1335234
[29] Meschenmoser, D. and Shashkin, A. (2013). Functional central limit theorem for the measures of level surfaces of the Gaussian random field. *Theory Probab. Appl.* **57** 162–172. MR3201646
[30] Nicolaescu, L.I. (2012). Random Morse functions and spectral geometry. Preprint. Available at arXiv:1209.0639.
[31] Nicolaescu, L.I. (2015). Critical points of multidimensional random Fourier series: Variance estimates. *J. Math. Phys.* **57** no. 8, 083304, 42 pp.
[32] Nicolaescu, L.I. (2015). A CLT concerning critical points of random functions on a Euclidean space. *Stochastic Process. Appl.* To appear. Available at DOI:10.1016/j.spa.2017.02.009.
[33] Nourdin, I. and Peccati, G. (2009). Stein's method on Wiener chaos. *Probab. Theory Related Fields* **145** 75–118. MR2520122
[34] Nourdin, I. and Peccati, G. (2012). *Normal Approximations with Malliavin Calculus*. *from Stein's Method to Universality*. *Cambridge Tracts in Mathematics* **192**. Cambridge: Cambridge Univ. Press. MR2962301
[35] Nourdin, I., Peccati, G. and Podolskij, M. (2011). Quantitative Breuer–Major theorems. *Stochastic Process. Appl.* **121** 793–812. MR2770907

[36] Nualart, D. and Peccati, G. (2005). Central limit theorems for sequences of multiple stochastic integrals. *Ann. Probab.* **33** 177–193. MR2118863
[37] Peccati, G. and Tudor, C.A. (2005). Gaussian limits for vector-valued multiple stochastic integrals. In *Séminaire de Probabilités XXXVIII*. *Lecture Notes in Math*. **1857** 247–262. Berlin: Springer. MR2126978
[38] Slud, E. (1991). Multiple Wiener–Itô integral expansions for level-crossing-count functionals. *Probab. Theory Related Fields* **87** 349–364. MR1084335
[39] Slud, E.V. (1994). MWI representation of the number of curve-crossings by a differentiable Gaussian process, with applications. *Ann. Probab.* **22** 1355–1380. MR1303648
[40] Sodin, M. and Tsirelson, B. (2004). Random complex zeroes. I. Asymptotic normality. *Israel J. Math*. **144** 125–149. MR2121537

Bernoulli **24**(2), 2018, 1171–1201
https://doi.org/10.3150/16-BEJ896

Determinantal point process models on the sphere

JESPER MØLLER[1,*], MORTEN NIELSEN[1,**], EMILIO PORCU[2] and EGE RUBAK[1,†]

[1]*Department of Mathematical Sciences, Aalborg University, Denmark.*
E-mail: [*]*jm@math.aau.dk;* [**]*mnielsen@math.aau.dk;* [†]*rubak@math.aau.dk*
[2]*Department of Mathematics, University Federico Santa Maria, Chile. E-mail:* *emilio.porcu@uv.cl*

We consider determinantal point processes on the d-dimensional unit sphere $\mathbb{S}^d$. These are finite point processes exhibiting repulsiveness and with moment properties determined by a certain determinant whose entries are specified by a so-called kernel which we assume is a complex covariance function defined on $\mathbb{S}^d \times \mathbb{S}^d$. We review the appealing properties of such processes, including their specific moment properties, density expressions and simulation procedures. Particularly, we characterize and construct isotropic DPPs models on $\mathbb{S}^d$, where it becomes essential to specify the eigenvalues and eigenfunctions in a spectral representation for the kernel, and we figure out how repulsive isotropic DPPs can be. Moreover, we discuss the shortcomings of adapting existing models for isotropic covariance functions and consider strategies for developing new models, including a useful spectral approach.

Keywords: isotropic covariance function; joint intensities; quantifying repulsiveness; Schoenberg representation; spatial point process density; spectral representation

References

[1] Abramowitz, M. and Stegun, I. (1965). *Handbook of Mathematical Functions*. New York: Dover Publications.
[2] Baddeley, A., Rubak, E. and Turner, R. (2015). *Spatial Point Patterns*: *Methodology and Applications with R*. London: Chapman & Hall.
[3] Berg, C. and Porcu, E. (2016). From Schoenberg coefficients to Schoenberg functions. Constructive approximation. *Constr. Approx*. To appear.
[4] Biscio, C.A.N. and Lavancier, F. (2016). Quantifying repulsiveness of determinantal point processes. *Bernoulli* **22** 2001–2028. MR3498021
[5] Cavoretto, R. and De Rossi, A. (2010). Fast and accurate interpolation of large scattered data sets on the sphere. *J. Comput*. *Appl*. *Math*. **234** 1505–1521. MR2610367
[6] Dai, F. and Xu, Y. (2013). *Approximation Theory and Harmonic Analysis on Spheres and Balls*. *Springer Monographs in Mathematics*. New York: Springer. MR3060033
[7] Daley, D.J. and Porcu, E. (2013). Dimension walks through Schoenberg spectral measures. *Proc*. *Amer. Math*. *Soc*. **42** 1813–1824.
[8] Ginibre, J. (1965). Statistical ensembles of complex, quaternion, and real matrices. *J. Math*. *Phys*. **6** 440–449. MR0173726
[9] Gneiting, T. (2002). Compactly supported correlation functions. *J. Multivariate Anal*. **83** 493–508. MR1945966

1350-7265

[10] Gneiting, T. (2013). Strictly and non-strictly positive definite functions on spheres. *Bernoulli* **19** 1327–1349. MR3102554
[11] Golubov, B.I. (1981). On Abel–Poisson type and Riesz means. *Anal. Math.* **7** 161–184. MR0635483
[12] Guinness, J. and Fuentes, M. (2016). Isotropic covariance functions on spheres: Some properties and modeling considerations. *J. Multivariate Anal.* **143** 143–152. MR3431424
[13] Higuchi, A. (1987). Symmetric tensor spherical harmonics on the N-sphere and their application to the de Sitter group SO$(N, 1)$. *J. Math. Phys.* **28** 1553–1566. MR0894848
[14] Hitczenko, M. and Stein, M.L. (2012). Some theory for anisotropic processes on the sphere. *Stat. Methodol.* **9** 211–227. MR2863609
[15] Hough, J.B., Krishnapur, M., Peres, Y. and Virág, B. (2006). Determinantal processes and independence. *Probab. Surv.* **3** 206–229. MR2216966
[16] Hough, J.B., Krishnapur, M., Peres, Y. and Virág, B. (2009). *Zeros of Gaussian Analytic Functions and Determinantal Point Processes*. *University Lecture Series* **51**. Providence, RI: Amer. Math. Soc. MR2552864
[17] Jones, R.H. (1963). Stochastic processes on a sphere. *Ann. Math. Statist.* **34** 213–218. MR0170378
[18] Kim, D.S., Kim, T. and Rim, S.-H. (2012). Some identities involving Gegenbauer polynomials. *Adv. Difference Equ.* 219. MR3017365
[19] Lavancier, F., Møller, J. and Rubak, E. (2014). Determinantal point process models and statistical inference: Extended version. Technical report. Available at arXiv:1205.4818.
[20] Lavancier, F., Møller, J. and Rubak, E. (2015). Determinantal point process models and statistical inference. *J. R. Stat. Soc. Ser. B.* (*Stat. Methodol.*) **77** 853–877. MR3382600
[21] Marinucci, D. and Peccati, G. (2011). *Random Fields on the Sphere*: *Representation*, *Limit Theorems and Cosmological Applications*. *London Mathematical Society Lecture Note Series* **389**. Cambridge: Cambridge Univ. Press. MR2840154
[22] Matheron, G. (1973). The intrinsic random functions and their applications. *Adv. in Appl. Probab.* **5** 439–468. MR0356209
[23] Porcu, E., Bevilacqua, M. and Genton, M.G. (2016). Spatio-temporal covariance and cross-covariance functions of the great circle distance on a sphere. *J. Amer. Statist. Assoc.* **111** 888–898. MR3538713
[24] R Core Team (2015). R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing.
[25] Rainville, E.D. (1971). *Special Functions*, 1st ed. Bronx, NY: Chelsea. MR0393590
[26] Riesz, F. and Sz.-Nagy, B. (1990). *Functional Analysis*. *Dover Books on Advanced Mathematics*. New York: Dover. MR1068530
[27] Schoenberg, I.J. (1942). Positive definite functions on spheres. *Duke Math. J.* **9** 96–108. MR0005922
[28] Shirai, T. and Takahashi, Y. (2003). Random point fields associated with certain Fredholm determinants. I. Fermion, Poisson and boson point processes. *J. Funct. Anal.* **205** 414–463. MR2018415
[29] Szegő, G. (1975). *Orthogonal Polynomials*, 4th ed. Providence, RI: Amer. Math. Soc. MR0372517
[30] Wickham, H. (2009). *ggplot2*: *Elegant Graphics for Data Analysis*. New York: Springer.
[31] Zastavnyi, V.P. (2000). On positive definiteness of some functions. *J. Multivariate Anal.* **73** 55–81. MR1766121
[32] Zastavnyĭ, V.P. (2006). On some properties of the Buhmann functions. *UkraïN. Mat. Zh.* **58** 1045–1067. MR2345078

Bernoulli **24**(2), 2018, 1202–1232
https://doi.org/10.3150/16-BEJ897

Baxter's inequality for finite predictor coefficients of multivariate long-memory stationary processes

AKIHIKO INOUE[1], YUKIO KASAHARA[2] and MOHSEN POURAHMADI[3]

[1]*Department of Mathematics, Hiroshima University, Higashi-Hiroshima 739-8526, Japan.*
E-mail: inoue100@hiroshima-u.ac.jp
[2]*Department of Mathematics, Hokkaido University, Sapporo 060-0810, Japan.*
E-mail: y-kasa@math.sci.hokudai.ac.jp
[3]*Department of Statistics, Texas A&M University, College Station, TX 77843, USA.*
E-mail: pourahm@stat.tamu.edu

For a multivariate stationary process, we develop explicit representations for the finite predictor coefficient matrices, the finite prediction error covariance matrices and the partial autocorrelation function (PACF) in terms of the Fourier coefficients of its phase function in the spectral domain. The derivation is based on a novel alternating projection technique and the use of the forward and backward innovations corresponding to predictions based on the infinite past and future, respectively. We show that such representations are ideal for studying the rates of convergence of the finite predictor coefficients, prediction error covariances, and the PACF as well as for proving a multivariate version of Baxter's inequality for a multivariate FARIMA process with a common fractional differencing order for all components of the process.

Keywords: Baxter's inequality; long memory; multivariate stationary processes; partial autocorrelation functions; phase functions; predictor coefficients

References

[1] Baillie, R.T. and Kapetanios, G. (2013). Estimation and inference for impulse response functions from univariate strongly persistent processes. *Econom. J.* **16** 373–399. MR3146771
[2] Baxter, G. (1962). An asymptotic result for the finite predictor. *Math. Scand.* **10** 137–144. MR0149584
[3] Berk, K.N. (1974). Consistent autoregressive spectral estimates. *Ann. Statist.* **2** 489–502. MR0421010
[4] Bhatia, R. (1997). *Matrix Analysis*. New York: Springer. MR1477662
[5] Bingham, N.H. (2012). Multivariate prediction and matrix Szegö theory. *Probab. Surv.* **9** 325–339. MR2956574
[6] Bingham, N.H., Goldie, C.M. and Teugels, J.L. (1989). *Regular Variation*. Cambridge: Cambridge Univ. Press. MR1015093
[7] Bingham, N.H., Inoue, A. and Kasahara, Y. (2012). An explicit representation of Verblunsky coefficients. *Statist. Probab. Lett.* **82** 403–410. MR2875229
[8] Brockwell, P.J. and Davis, R.A. (1991). *Time Series: Theory and Methods*, 2nd ed. New York: Springer. MR1093459
[9] Bühlmann, P. (1995). Moving-average representation of autoregressive approximations. *Stochastic Process. Appl.* **60** 331–342. MR1376807

1350-7265

[10] Bühlmann, P. (1997). Sieve bootstrap for time series. *Bernoulli* **3** 123–148. MR1466304
[11] Cheng, R. and Pourahmadi, M. (1993). Baxter's inequality and convergence of finite predictors of multivariate stochastic processes. *Probab. Theory Related Fields* **95** 115–124. MR1207310
[12] Chung, C.-F. (2001). Calculating and analyzing impulse responses for the vector ARFIMA model. *Econom. Lett.* **71** 17–25. MR1821722
[13] Damanik, D., Pushnitski, A. and Simon, B. (2008). The analytic theory of matrix orthogonal polynomials. *Surv. Approx. Theory* **4** 1–85. MR2379691
[14] Dègerine, S. (1990). Canonical partial autocorrelation function of a multivariate time series. *Ann. Statist.* **18** 961–971. MR1056346
[15] Ginovian, M.S. (1999). Asymptotic behavior of the prediction error for stationary random sequences. *Izv. Nats. Akad. Nauk Armenii Mat.* **34** 18–36. MR1854056
[16] Granger, C.W.J. and Joyeux, R. (1980). An introduction to long-memory time series models and fractional differencing. *J. Time Series Anal.* **1** 15–29. MR0605572
[17] Hannan, E.J. and Deistler, M. (1988). *The Statistical Theory of Linear Systems*. New York: Wiley. MR0940698
[18] Helson, H. and Lowdenslager, D. (1961). Prediction theory and Fourier series in several variables. II. *Acta Math.* **106** 175–213. MR0176287
[19] Hosking, J.R.M. (1981). Fractional differencing. *Biometrika* **68** 165–176. MR0614953
[20] Ibragimov, I.A. and Solev, V.N. (1968). Asymptotic behavoir of the prediction error of a stationary sequence with a spectral density of special form. *Theory Probab. Appl.* **13** 703–707.
[21] Ing, C.-K., Chiou, H.-T. and Guo, M. (2016). Estimation of inverse autocovariance matrices for long memory processes. *Bernoulli* **22** 1301–1330. MR3474817
[22] Inoue, A. (2000). Asymptotics for the partial autocorrelation function of a stationary process. *J. Anal. Math.* **81** 65–109. MR1785278
[23] Inoue, A. (2002). Asymptotic behavior for partial autocorrelation functions of fractional ARIMA processes. *Ann. Appl. Probab.* **12** 1471–1491. MR1936600
[24] Inoue, A. (2008). AR and MA representation of partial autocorrelation functions, with applications. *Probab. Theory Related Fields* **140** 523–551. MR2365483
[25] Inoue, A. and Kasahara, Y. (2004). Partial autocorrelation functions of the fractional ARIMA processes with negative degree of differencing. *J. Multivariate Anal.* **89** 135–147. MR2041213
[26] Inoue, A. and Kasahara, Y. (2006). Explicit representation of finite predictor coefficients and its applications. *Ann. Statist.* **34** 973–993. MR2283400
[27] Inoue, A., Kasahara, Y. and Pourahmadi, M. (2016). The intersection of past and future for multivariate stationary processes. *Proc. Amer. Math. Soc.* **144** 1779–1786. MR3451253
[28] Kasahara, Y. and Bingham, N.H. (2014). Verblunsky coefficients and Nehari sequences. *Trans. Amer. Math. Soc.* **366** 1363–1378. MR3145734
[29] Kasahara, Y., Inoue, A. and Pourahmadi, M. (2016). Rigidity for matrix-valued Hardy functions. *Integral Equations Operator Theory* **84** 289–300. MR3456943
[30] Katsnelson, V.E. and Kirstein, B. (1997). On the theory of matrix-valued functions belonging to the Smirnov class. In *Topics in Interpolation Theory* (*Leipzig*, 1994). *Oper. Theory Adv. Appl.* **95** 299–350. Basel: Birkhäuser. MR1473261
[31] Kreiss, J.-P., Paparoditis, E. and Politis, D.N. (2011). On the range of validity of the autoregressive sieve bootstrap. *Ann. Statist.* **39** 2103–2130. MR2893863
[32] Masani, P. (1960). The prediction theory of multivariate stochastic processes. III. Unbounded spectral densities. *Acta Math.* **104** 141–162. MR0121952
[33] Meyer, M., Jentsch, C. and Kreiss, J.-P. (2017). Baxter's inequality and sieve bootstrap for random fields. *Bernoulli* **23** 2988–3020. MR3654797

[34] Meyer, M., McMurry, T. and Politis, D. (2015). Baxter's inequality for triangular arrays. *Math. Methods Statist.* **24** 135–146. MR3366950
[35] Peller, V.V. (2003). *Hankel Operators and Their Applications*. New York: Springer. MR1949210
[36] Poskitt, D.S., Grose, S.D. and Martin, G.M. (2015). Higher-order improvements of the sieve bootstrap for fractionally integrated processes. *J. Econometrics* **188** 94–110. MR3371662
[37] Pourahmadi, M. (2001). *Foundations of Time Series Analysis and Prediction Theory*. New York: Wiley. MR1849562
[38] Rozanov, Yu.A. (1967). *Stationary Random Processes*. San Francisco: Holden-Day. MR0214134
[39] Rupasinghe, M. and Samaranayake, V.A. (2012). Asymptotic properties of sieve bootstrap prediction intervals for *FARIMA* processes. *Statist. Probab. Lett.* **82** 2108–2114. MR2979746
[40] Sarason, D. (1978). *Function Theory on the Unit Circle*. Blacksburg, VA: Virginia Polytechnic Institute and State Univ. Notes for lectures given at a Conference at Virginia Polytechnic Institute and State University, Blacksburg, Va., June 19–23, 1978. MR0521811

Bernoulli **24**(2), 2018, 1233–1265
https://doi.org/10.3150/16-BEJ898

Smooth backfitting for additive modeling with small errors-in-variables, with an application to additive functional regression for multiple predictor functions

KYUNGHEE HAN[1], HANS-GEORG MÜLLER[2] and BYEONG U. PARK[1,*]

[1]*Department of Statistics, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Republic of Korea. E-mail:* [*]*bupark@stats.snu.ac.kr*
[2]*Department of Statistics, University of California, 1 Shields Avenue, Davis, CA 95616, USA*

We study smooth backfitting when there are errors-in-variables, which is motivated by functional additive models for a functional regression model with a scalar response and multiple functional predictors that are additive in the functional principal components of the predictor processes. The development of a new smooth backfitting technique for the estimation of the additive component functions in functional additive models with multiple functional predictors requires to address the difficulty that the eigenfunctions and therefore the functional principal components of the predictor processes, which are the arguments of the proposed additive model, are unknown and need to be estimated from the data. The available estimated functional principal components contain an error that is small for large samples but nevertheless affects the estimation of the additive component functions. This error-in-variables situation requires to develop new asymptotic theory for smooth backfitting. Our analysis also pertains to general situations where one encounters errors in the predictors for an additive model, when the errors become smaller asymptotically. We also study the finite sample properties of the proposed method for the application in functional additive regression through a simulation study and a real data example.

Keywords: errors in predictors; functional additive model; functional data analysis; functional principal component; kernel smoothing; smooth backfitting

References

[1] Bosq, D. (2000). *Linear Processes in Function Spaces*: *Theory and Applications*. New York: Springer. MR1783138

[2] Cardot, H., Ferraty, F. and Sarda, P. (1999). Functional linear model. *Statist*. *Probab*. *Lett*. **45** 11–22. MR1718346

[3] Carroll, R.J., Maity, A., Mammen, E. and Yu, K. (2009). Nonparametric additive regression for repeatedly measured data. *Biometrika* **96** 383–398. MR2507150

[4] Chen, D., Hall, P. and Müller, H.-G. (2011). Single and multiple index functional regression models with nonparametric link. *Ann*. *Statist*. **39** 1720–1747. MR2850218

[5] Fan, Y., James, G.M. and Radchenko, P. (2013). Functional additive regression. *Ann*. *Statist*. **40** 2296–2325. MR3396986

[6] Fanaee-T, H. and Gama, J. (2014). Event labeling combining ensemble detectors and background knowledge. *J. Prog. Artif. Intell.* **2** 113–127.
[7] Febrero-Bande, M. and González-Manteiga, W. (2013). Generalized additive models for functional data. *TEST* **22** 278–292. MR3062258
[8] Ferraty, F., Goia, A., Salinelli, E. and Vieu, P. (2013). Functional projection pursuit regression. *TEST* **22** 293–320. MR3062259
[9] Hildebrandt, T., Bissantz, N. and Dette, H. (2014). Additive inverse regression models with convolution-type operators. *Electron. J. Stat.* **8** 1–40. MR3161732
[10] James, G.M. and Silverman, B.W. (2005). Functional adaptive model estimation. *J. Amer. Statist. Assoc.* **100** 565–576. MR2160560
[11] Jiang, C.-R. and Wang, J.-L. (2011). Functional single index model. *Ann. Statist.* **39** 362–388. MR2797850
[12] Kneip, A. and Utikal, K.J. (2001). Inference for density families using functional principal component analysis. *J. Amer. Statist. Assoc.* **96** 519–542. MR1946423
[13] Lee, Y.K., Mammen, E. and Park, B.U. (2010). Backfitting and smooth backfitting for additive quantile models. *Ann. Statist.* **38** 2857–2883. MR2722458
[14] Lee, Y.K., Mammen, E. and Park, B.U. (2012). Flexible generalized varying coefficient regression models. *Ann. Statist.* **40** 1906–1933. MR3015048
[15] Ma, S. (2014). Estimation and inference in functional single-index models. *Ann. Inst. Statist. Math.* **68** 181–208 1–28. MR3440219
[16] Mammen, E., Linton, O. and Nielsen, J. (1999). The existence and asymptotic properties of a backfitting projection algorithm under weak conditions. *Ann. Statist.* **27** 1443–1490. MR1742496
[17] McLean, M.W., Hooker, G., Staicu, A.-M., Scheipl, F. and Ruppert, D. (2014). Functional generalized additive models. *J. Comput. Graph. Statist.* **23** 249–269. MR3173770
[18] Müller, H.-G., Wu, Y. and Yao, F. (2013). Continuously additive models for nonlinear functional regression. *Biometrika* **100** 607–622. MR3094440
[19] Müller, H.-G. and Yao, F. (2008). Functional additive models. *J. Amer. Statist. Assoc.* **103** 1534–1544. MR2504202
[20] Ramsay, J.O. and Silverman, B.W. (2005). *Functional Data Analysis*, 2nd ed. New York: Springer. MR2168993
[21] Stone, C.J. (1985). Additive regression and other nonparametric models. *Ann. Statist.* **13** 689–705. MR0790566
[22] Yao, F. and Müller, H.-G. (2010). Functional quadratic regression. *Biometrika* **97** 49–64. MR2594416
[23] Yu, K., Park, B.U. and Mammen, E. (2008). Smooth backfitting in generalized additive models. *Ann. Statist.* **36** 228–260. MR2387970
[24] Zhang, X., Park, B.U. and Wang, J.-L. (2013). Time-varying additive models for longitudinal data. *J. Amer. Statist. Assoc.* **108** 983–998. MR3174678
[25] Zhu, H., Yao, F. and Zhang, H.H. (2014). Structured functional additive regression in reproducing kernel Hilbert spaces. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **76** 581–603. MR3210729

Bernoulli **24**(2), 2018, 1266–1306
https://doi.org/10.3150/16-BEJ899

Bump detection in heterogeneous Gaussian regression

FARIDA ENIKEEVA[1,2,*], AXEL MUNK[3,4,**] and FRANK WERNER[3,4,†]

[1]*Laboratoire de Mathématiques et Applications, CNRS UMR 7348, Université de Poitiers, Téléport 2 – BP 30179, Boulevard Marie et Pierre Curie, 86962 Futuroscope Chasseneuil Cedex, France. E-mail:* [*]*farida.enikeeva@math.univ-poitiers.fr*

[2]*Institute for Information Transmission Problems, Russian Academy of Science, Bolshoy Karentny per. 19-1, 127051 Moscow, Russia*

[3]*Felix Bernstein Institute for Mathematical Statistics in the Bioscience, University of Göttingen, Goldschmidtstraße 7, 37077 Göttingen, Germany. E-mail:* [**]*munk@math.uni-goettingen.de*

[4]*Statistical Inverse Problems in Biophysics Group, Max Planck Institute for Biophysical Chemistry, Am Faßberg 11, 37077 Göttingen, Germany. E-mail:* [†]*frank.werner@mpibpc.mpg.de*

We analyze the effect of a heterogeneous variance on bump detection in a Gaussian regression model. To this end, we allow for a simultaneous bump in the variance and specify its impact on the difficulty to detect the null signal against a single bump with known signal strength. This is done by calculating lower and upper bounds, both based on the likelihood ratio.

Lower and upper bounds together lead to explicit characterizations of the detection boundary in several subregimes depending on the asymptotic behavior of the bump heights in mean and variance. In particular, we explicitly identify those regimes, where the additional information about a simultaneous bump in variance eases the detection problem for the signal. This effect is made explicit in the constant and/or the rate, appearing in the detection boundary.

We also discuss the case of an unknown bump height and provide an adaptive test and some upper bounds in that case.

Keywords: change point detection; heterogeneous Gaussian regression; minimax testing theory

References

[1] Arias-Castro, E., Candès, E.J., Helgason, H. and Zeitouni, O. (2008). Searching for a trail of evidence in a maze. *Ann. Statist.* **36** 1726–1757. MR2435454

[2] Arias-Castro, E. and Wang, M. (2013). Distribution-free tests for sparse heterogeneous mixtures. Preprint. Available at arXiv:1308.0346.

[3] Arlot, S. and Celisse, A. (2011). Segmentation of the mean of heteroscedastic data via cross-validation. *Stat. Comput.* **21** 613–632. MR2826696

[4] Bai, J. and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica* **66** 47–78. MR1616121

[5] Bai, J. and Perron, P. (2003). Computation and analysis of multiple structural change models. *J. Appl. Econometrics* **18** 1–22.

[6] Baraud, Y. (2002). Non-asymptotic minimax rates of testing in signal detection. *Bernoulli* **8** 577–606. MR1935648

1350-7265

[7] Ben-Tal, A., El Ghaoui, L. and Nemirovski, A. (2009). *Robust Optimization*. *Princeton Series in Applied Mathematics*. Princeton, NJ: Princeton Univ. Press. MR2546839

[8] Birgé, L. (2001). An alternative point of view on Lepski's method. In *State of the Art in Probability and Statistics* (*Leiden*, 1999). *Institute of Mathematical Statistics Lecture Notes—Monograph Series* **36** 113–133. Beachwood, OH: IMS. MR1836557

[9] Birgé, L. and Massart, P. (2001). Gaussian model selection. *J. Eur. Math. Soc.* (*JEMS*) **3** 203–268. MR1848946

[10] Boucheron, S., Lugosi, G. and Massart, P. (2013). *Concentration Inequalities*: *A Nonasymptotic Theory of Independence*. Oxford: Oxford Univ. Press. MR3185193

[11] Boutahar, M. (2012). Testing for change in mean of independent multivariate observations with time varying covariance. *J. Probab. Stat*. Art. ID 969753, 17. MR2897334

[12] Boysen, L., Kempe, A., Liebscher, V., Munk, A. and Wittich, O. (2009). Consistencies and rates of convergence of jump-penalized least squares estimators. *Ann. Statist*. **37** 157–183. MR2488348

[13] Braun, J.V., Braun, R.K. and Müller, H.-G. (2000). Multiple changepoint fitting via quasilikelihood, with application to DNA sequence segmentation. *Biometrika* **87** 301–314. MR1782480

[14] Cai, T.T., Jeng, X.J. and Jin, J. (2011). Optimal detection of heterogeneous and heteroscedastic mixtures. *J. R. Stat. Soc. Ser. B. Stat. Methodol*. **73** 629–662. MR2867452

[15] Cai, T.T. and Wu, Y. (2014). Optimal detection of sparse mixtures against a given null distribution. *IEEE Trans. Inform. Theory* **60** 2217–2232. MR3181520

[16] Carlstein, E., Müller, H.-G. and Siegmund, D., eds. (1994). *Change-Point Problems*. *Institute of Mathematical Statistics Lecture Notes—Monograph Series* **23**. Hayward, CA: IMS. MR1477909

[17] Castro, R.M., Haupt, J., Nowak, R. and Raz, G.M. (2008). Finding needles in noisy haystacks. In *IEEE International Conference on Acoustics*, *Speech and Signal Processing*, 2008. *ICASSP* 2008 5133–5136. New York: IEEE.

[18] Chan, H.P. and Walther, G. (2013). Detection with the scan and the average likelihood ratio. *Statist. Sinica* **23** 409–428. MR3076173

[19] Csörgő, M. and Horváth, L. (1997). *Limit Theorems in Change-Point Analysis*. Chichester: Wiley. MR2743035

[20] Donoho, D. and Jin, J. (2004). Higher criticism for detecting sparse heterogeneous mixtures. *Ann. Statist*. **32** 962–994. MR2065195

[21] Du, C., Kao, C.-L.M. and Kou, S.C. (2016). Stepwise signal extraction via marginal likelihood. *J. Amer. Statist. Assoc*. **111** 314–330. MR3494662

[22] Dümbgen, L. and Spokoiny, V.G. (2001). Multiscale testing of qualitative hypotheses. *Ann. Statist*. **29** 124–152. MR1833961

[23] Dümbgen, L. and Walther, G. (2008). Multiscale inference about a density. *Ann. Statist*. **36** 1758–1785. MR2435455

[24] Frick, K., Munk, A. and Sieling, H. (2014). Multiscale change point inference. *J. R. Stat. Soc. Ser. B. Stat. Methodol*. **76** 495–580. MR3210728

[25] Goldenshluger, A., Juditsky, A. and Nemirovski, A. (2015). Hypothesis testing by convex optimization. *Electron. J. Stat*. **9** 1645–1712. MR3379005

[26] Harchaoui, Z. and Lévy-Leduc, C. (2010). Multiple change-point estimation with a total variation penalty. *J. Amer. Statist. Assoc*. **105** 1480–1493. MR2796565

[27] Hsu, D., Kakade, S.M. and Zhang, T. (2012). A tail inequality for quadratic forms of subgaussian random vectors. *Electron. Commun. Probab*. **17** no. 52, 6. MR2994877

[28] Huang, W.T. and Chang, Y.P. (1993). Nonparametric estimation in change-point models. *J. Statist. Plann. Inference* **35** 335–347. MR1229248

[29] Ingster, Yu.I. (1993). Asymptotically minimax hypothesis testing for nonparametric alternatives. I. *Math. Methods Statist*. **2** 85–114. MR1257978

[30] Ingster, Y.I. and Suslina, I.A. (2002). On detection of a signal of known shape in multi-channel system. *Zap. Nauchn. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov.* (*POMI*) **294** 88–112.
[31] Jacod, J. and Todorov, V. (2010). Do price and volatility jump together? *Ann. Appl. Probab.* **20** 1425–1469. MR2676944
[32] Jeng, X.J., Cai, T.T. and Li, H. (2010). Optimal sparse segment identification with application in copy number variation analysis. *J. Amer. Statist. Assoc.* **105** 1156–1166. MR2752611
[33] Killick, R., Fearnhead, P. and Eckley, I.A. (2012). Optimal detection of changepoints with a linear computational cost. *J. Amer. Statist. Assoc.* **107** 1590–1598. MR3036418
[34] Korostelev, A. and Korosteleva, O. (2011). *Mathematical Statistics*: *Asymptotic Minimax Theory*. *Graduate Studies in Mathematics* **119**. Providence, RI: Amer. Math. Soc. MR2767163
[35] Laurent, B., Loubes, J.-M. and Marteau, C. (2012). Non asymptotic minimax rates of testing in signal detection with heterogeneous variances. *Electron. J. Stat.* **6** 91–122. MR2879673
[36] Lavielle, M. (2005). Using penalized contrasts for the change-point problem. *Signal Process.* **85** 1501–1510.
[37] Lehmann, E.L. and Romano, J.P. (2005). *Testing Statistical Hypotheses*, 3rd ed. *Springer Texts in Statistics*. New York: Springer. MR2135927
[38] Muggeo, V. and Adelfio, G. (2010). Efficient change point detection for genomic sequences of continuous measurements. *Bioinformatics* **27** 161–166.
[39] Munk, A. and Werner, F. (2015). Discussion of "Hypotheses testing by convex optimization" [MR3379005]. *Electron. J. Stat.* **9** 1720–1722. MR3379007
[40] Neher, E. and Sakmann, B. (1995). *Single-Channel Recording*. New York: Plenum Press.
[41] Rivera, C. and Walther, G. (2013). Optimal detection of a jump in the intensity of a Poisson process or in a density with likelihood ratio statistics. *Scand. J. Stat.* **40** 752–769. MR3145116
[42] Rohde, A. and Dümbgen, L. (2013). Statistical inference for the optimal approximating model. *Probab. Theory Related Fields* **155** 839–865. MR3034794
[43] Schirmer, T. (1998). General and specific porins from bacterial outer membranes. *J. Struct. Biol.* **121** 101–109.
[44] Shorack, G.R. (2000). *Probability for Statisticians*. New York: Springer. MR1762415
[45] Siegmund, D. (2013). Change-points: From sequential detection to biology and back. *Sequential Anal.* **32** 2–14. MR3023983
[46] Siegmund, D. and Venkatraman, E.S. (1995). Using the generalized likelihood ratio statistic for sequential detection of a change-point. *Ann. Statist.* **23** 255–271. MR1331667
[47] Siegmund, D., Yakir, B. and Zhang, N.R. (2011). Detecting simultaneous variant intervals in aligned sequences. *Ann. Appl. Stat.* **5** 645–668. MR2840169
[48] Sigworth, F.J. (1985). Open channel noise. I. Noise in acetylcholine receptor currents suggests conformational fluctuations. *Biophys. J.* **47** 709–720.
[49] Spokoiny, V. and Zhilova, M. (2013). Sharp deviation bounds for quadratic forms. *Math. Methods Statist.* **22** 100–113. MR3071957
[50] Tsybakov, A.B. (2009). *Introduction to Nonparametric Estimation*. New York: Springer. MR2724359
[51] Yao, Y.-C. (1988). Estimating the number of change-points via Schwarz' criterion. *Statist. Probab. Lett.* **6** 181–189. MR0919373

Bernoulli **24**(2), 2018, 1307–1350
https://doi.org/10.3150/16-BEJ900

Quenched invariance principles for the discrete Fourier transforms of a stationary process

DAVID BARRERA[1,2]

[1]*Department of Mathematical Sciences, University of Cincinnati, PO Box 210025, Cincinnati, OH 45221-0025, USA. E-mail: barrerjd@mail.uc.edu*
[2]*Centre de Mathématiques Appliquées, École Polytechnique, Route de Saclay 91128, Palaiseau Cedex, France. E-mail: david.barrera@polytechnique.edu*

In this paper, we study the asymptotic behavior of the normalized cadlag functions generated by the discrete Fourier transforms of a stationary centered square-integrable process, started at a point.

We prove that the quenched invariance principle holds for averaged frequencies under no assumption other than ergodicity, and that this result holds also for almost every fixed frequency under a certain generalization of the Hannan condition and a certain rotated form of the Maxwell and Woodroofe condition which, under a condition of weak dependence that we specify, is guaranteed for a.e. frequency. If the process is in particular weakly mixing, our results describe the asymptotic distributions of the normalized discrete Fourier transforms at every frequency other than 0 and π under the generalized Hannan condition.

We prove also that under a certain regularity hypothesis the conditional centering is irrelevant for averaged frequencies, and that the same holds for a given fixed frequency under the rotated Maxwell and Woodroofe condition but not necessarily under the generalized Hannan condition. In particular, this implies that the hypothesis of regularity is not sufficient for functional convergence without random centering at a.e. fixed frequency.

The proofs are based on martingale approximations and combine results from Ergodic theory of recent and classical origin with approximation results by contemporary authors and with some facts from Harmonic Analysis and Functional Analysis.

Keywords: central limit theorem; discrete Fourier transform; invariance principle; martingale approximation; quenched convergence

References

[1] Akcoglu, M.A. and Sucheston, L. (1978). A ratio ergodic theorem for superadditive processes. *Z. Wahrsch. Verw. Gebiete* **44** 269–278. MR0509202
[2] Assani, I. (1990). Minimal convergence on L^p spaces. *Ergodic Theory Dynam. Systems* **10** 411–420. MR1074311
[3] Barrera, D. (2015). An example of non-quenched convergence in the conditional CLT for discrete Fourier transforms. *ALEA Lat. Am. J. Probab. Math. Stat.* **12** 685–697. MR3430503
[4] Barrera, D. (2016). Quenched Asymptotics for the Discrete Fourier Transforms of a Stationary Process Doctoral Dissertation. Version May 2016. Available at arXiv:1604.00428v2.

1350-7265

[5] Barrera, D. and Peligrad, M. (2016). Quenched limit theorems for Fourier transforms and periodogram. *Bernoulli* **22** 275–301. MR3449783
[6] Billingsley, P. (1995). *Probability and Measure*, 3rd ed. *Wiley Series in Probability and Mathematical Statistics*. New York: Wiley. MR1324786
[7] Billingsley, P. (1999). *Convergence of Probability Measures*, 2nd ed. *Wiley Series in Probability and Statistics*: *Probability and Statistics*. New York: Wiley. MR1700749
[8] Cuny, C. and Merlevède, F. (2014). On martingale approximations and the quenched weak invariance principle. *Ann*. *Probab*. **42** 760–793. MR3178473
[9] Cuny, C., Merlevède, F. and Peligrad, M. (2013). Law of the iterated logarithm for the periodogram. *Stochastic Process*. *Appl*. **123** 4065–4089. MR3091099
[10] Cuny, C. and Volný, D. (2013). A quenched invariance principle for stationary processes. *ALEA Lat*. *Am*. *J*. *Probab*. *Math*. *Stat*. **10** 107–115. MR3083921
[11] Dedecker, J., Merlevède, F. and Peligrad, M. (2014). A quenched weak invariance principle. *Ann*. *I.H.P. Probab*. *Stat*. **50** 872–898. MR3224292
[12] Dehling, H., Durieu, O. and Volny, D. (2009). New techniques for empirical processes of dependent data. *Stochastic Process*. *Appl*. **119** 3699–3718. MR2568292
[13] Durieu, O. (2009). Independence of four projective criteria for the weak invariance principle. *ALEA Lat*. *Am*. *J*. *Probab*. *Math*. *Stat*. **5** 21–26. MR2475604
[14] Durrett, R. (2010). *Probability*: *Theory and Examples*, 4th ed. *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge: Cambridge Univ. Press. MR2722836
[15] Einsiedler, M. and Ward, T. (2011). *Ergodic Theory with a View Towards Number Theory*. *Graduate Texts in Mathematics* **259**. London: Springer. MR2723325
[16] Eisner, T., Farkas, B., Haase, M. and Nagel, R. (2015). *Operator Theoretic Aspects of Ergodic Theory*. *Graduate Texts in Mathematics* **272**. New York: Springer. MR3410920
[17] Hunt, R.A. and Young, W.S. (1974). A weighted norm inequality for Fourier series. *Bull*. *Amer*. *Math*. *Soc*. **80** 274–277. MR0338655
[18] Krengel, U. (1985). *Ergodic Theorems*. *de Gruyter Studies in Mathematics* **6**. Berlin: de Gruyter. MR0797411
[19] Lifshits, M.A. and Peligrad, M. (2015). On the spectral density of stationary processes and random fields. *Zap*. *Nauchn*. *Sem*. *S*.-*Peterburg*. *Otdel*. *Mat*. *Inst*. *Steklov*. (*POMI*) **441** 274–285. MR3504510
[20] Peligrad, M. and Wu, W.B. (2010). Central limit theorem for Fourier transforms of stationary processes. *Ann*. *Probab*. **38** 2009–2022. MR2722793
[21] Quas, A. (2009). Ergodicity and mixing properties. In *Encyclopedia of Complexity and Systems Science* 2918–2933.
[22] Revu, D. and Yor, M. (2005). *Continuous Martingales and Brownian Motion*, *Grundlehren Math*. *Wiss*. New York: Springer. Corrected 3rd printing.
[23] Wu, W.B. (2005). Fourier transforms of stationary processes. *Proc*. *Amer*. *Math*. *Soc*. **133** 285–293 (electronic). MR2086221

Bernoulli **24**(2), 2018, 1351–1393
https://doi.org/10.3150/16-BEJ901

The eigenvalues of the sample covariance matrix of a multivariate heavy-tailed stochastic volatility model

ANJA JANSSEN[1,*], THOMAS MIKOSCH[1,**,†], MOHSEN REZAPOUR[2] and XIAOLEI XIE[1,‡]

[1] *Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark. E-mail:* [*] *anja@math.ku.dk,* [**] *mikosch@math.ku.dk, url:* [†] *www.math.ku.dk/~mikosch,* [‡] *xie@math.ku.dk*

[2] *Department of Statistics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran. E-mail: mohsenrzp@gmail.com*

We consider a multivariate heavy-tailed stochastic volatility model and analyze the large-sample behavior of its sample covariance matrix. We study the limiting behavior of its entries in the infinite-variance case and derive results for the ordered eigenvalues and corresponding eigenvectors. Essentially, we consider two different cases where the tail behavior either stems from the i.i.d. innovations of the process or from its volatility sequence. In both cases, we make use of a large deviations technique for regularly varying time series to derive multivariate α-stable limit distributions of the sample covariance matrix. For the case of heavy-tailed innovations, we show that the limiting behavior resembles that of completely independent observations. In contrast to this, for a heavy-tailed volatility sequence the possible limiting behavior is more diverse and allows for dependencies in the limiting distributions which are determined by the structure of the underlying volatility sequence.

Keywords: dependent entries; eigenvectors; largest eigenvalues; regular variation; sample covariance matrix; stochastic volatility

References

[1] Andersen, T.G., Davis, R.A., Kreiss, J.-P. and Mikosch, T., eds. (2009). *Handbook of Financial Time Series*. Berlin: Springer.

[2] Auffinger, A., Ben Arous, G. and Péché, S. (2009). Poisson convergence for the largest eigenvalues of heavy tailed random matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **45** 589–610. MR2548495

[3] Bai, Z.D. and Yin, Y.Q. (1993). Limit of the smallest eigenvalue of a large-dimensional sample covariance matrix. *Ann. Probab.* **21** 1275–1294. MR1235416

[4] Basrak, B., Davis, R.A. and Mikosch, T. (2002). A characterization of multivariate regular variation. *Ann. Appl. Probab.* **12** 908–920. MR1925445

[5] Basrak, B. and Segers, J. (2009). Regularly varying multivariate time series. *Stochastic Process. Appl.* **119** 1055–1080. MR2508565

[6] Belinschi, S., Dembo, A. and Guionnet, A. (2009). Spectral measure of heavy tailed band and covariance random matrices. *Comm. Math. Phys.* **289** 1023–1055. MR2511659

[7] Benaych-Georges, F. and Péché, S. (2014). Localization and delocalization for heavy tailed band matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **50** 1385–1403. MR3269999

1350-7265

[8] Ben Arous, G. and Guionnet, A. (2008). The spectrum of heavy tailed random matrices. *Comm. Math. Phys.* **278** 715–751. MR2373441

[9] Bhatia, R. (1997). *Matrix Analysis. Graduate Texts in Mathematics* **169**. New York: Springer. MR1477662

[10] Bordenave, C. and Guionnet, A. (2013). Localization and delocalization of eigenvectors for heavy-tailed random matrices. *Probab. Theory Related Fields* **157** 885–953. MR3129806

[11] Breiman, L. (1965). On some limit theorems similar to the arc-sin law. *Theory Probab. Appl.* **10** 323–331.

[12] Campbell, J., Lo, A.W. and MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. Princeton: Princeton Univ. Press.

[13] Chover, J., Ney, P. and Wainger, S. (1973). Functions of probability measures. *J. Anal. Math.* **26** 255–302. MR0348393

[14] Cline, D.B.H. (1986). Convolution tails, product tails and domains of attraction. *Probab. Theory Related Fields* **72** 529–557. MR0847385

[15] Davis, R.A. and Resnick, S. (1985). More limit theory for the sample correlation function of moving averages. *Stochastic Process. Appl.* **20** 257–279. MR0808161

[16] Davis, R.A. and Resnick, S. (1986). Limit theory for the sample covariance and correlation functions of moving averages. *Ann. Statist.* **14** 533–558. MR0840513

[17] Davis, R.A., Heiny, J., Mikosch, T. and Xie, X. (2016). Extreme value analysis for the sample autocovariance matrices of heavy-tailed multivariate time series. *Extremes* **19** 517–547. MR3535965

[18] Davis, R.A. and Hsing, T. (1995). Point process and partial sum convergence for weakly dependent random variables with infinite variance. *Ann. Probab.* **23** 879–917. MR1334176

[19] Davis, R.A. and Mikosch, T. (2000). The sample autocorrelations of financial time series models. In *Nonlinear and Nonstationary Signal Processing* (*Cambridge*, 1998) 247–274. Cambridge: Cambridge Univ. Press. MR1831322

[20] Davis, R.A. and Mikosch, T. (2001). Point process convergence of stochastic volatility processes with application to sample autocorrelation. *J. Appl. Probab.* **38A** 93–104. MR1915537

[21] Davis, R.A. and Mikosch, T. (2009). Probabilistic properties of stochastic volatility models. In *Handbook of Financial Time Series* (T.G. Andersen, R.A. Davis, J.-P. Kreiss and T. Mikosch, eds.) 255–268. Berlin: Springer. 2009.

[22] Davis, R.A. and Mikosch, T. (2009). The extremogram: A correlogram for extreme events. *Bernoulli* **15** 977–1009. MR2597580

[23] Davis, R.A., Mikosch, T. and Pfaffel, O. (2016). Asymptotic theory for the sample covariance matrix of a heavy-tailed multivariate time series. *Stochastic Process. Appl.* **126** 767–799. MR3452812

[24] Davis, R.A., Pfaffel, O. and Stelzer, R. (2014). Limit theory for the largest eigenvalues of sample covariance matrices with heavy-tails. *Stochastic Process. Appl.* **124** 18–50. MR3131285

[25] Ding, X. and Yang, F. (2016). A necessary and sufficient condition for edge universality at the largest singular values of covariance matrices. Preprint. Available at arXiv:1607.06873.

[26] Doukhan, P. (1994). *Mixing: Properties and Examples. Lecture Notes in Statistics* **85**. Springer: Berlin. MR1312160

[27] Drees, H. (2003). Extreme quantile estimation for dependent data, with applications to finance. *Bernoulli* **9** 617–657. MR1996273

[28] Embrechts, P. and Goldie, C.M. (1980). On closure and factorization properties of subexponential and related distributions. *J. Aust. Math. Soc. A* **29** 243–256. MR0566289

[29] Embrechts, P. and Goldie, C.M. (1982). On convolution tails. *Stochastic Process. Appl.* **13** 263–278. MR0671036

[30] Feller, W. (1971). *An Introduction to Probability Theory and Its Applications. Vol. II.*, 2nd ed. New York: Wiley. MR0270403

[31] Heiny, J. and Mikosch, T. (2016). Eigenvalues and eigenvectors of heavy-tailed sample covariance matrices with general growth rates: The iid case. Technical report.
[32] Hult, H. and Lindskog, F. (2005). Extremal behavior of regularly varying stochastic processes. *Stochastic Process*. *Appl*. **115** 249–274. MR2111194
[33] Hult, H. and Lindskog, F. (2006). Regular variation for measures on metric spaces. *Publ*. *Inst*. *Math*. (*Beograd*) (*N*.*S*.) **80(94)** 121–140. MR2281910
[34] Janssen, A. and Drees, H. (2016). A stochastic volatility model with flexible extremal dependence structure. *Bernoulli* **22** 1448–1490. MR3474822
[35] Jessen, A.H. and Mikosch, T. (2006). Regularly varying functions. *Publ*. *Inst*. *Math*. (*Beograd*) (*N*.*S*.) **80(94)** 171–192. MR2281913
[36] Kulik, R. and Soulier, P. (2015). Heavy tailed time series with extremal independence. *Extremes* **18** 273–299. MR3351817
[37] Lam, C. and Yao, Q. (2012). Factor modeling for high-dimensional time series: Inference for the number of factors. *Ann*. *Statist*. **40** 694–726. MR2933663
[38] Mikosch, T. and Rezapour, M. (2013). Stochastic volatility models with possible extremal clustering. *Bernoulli* **19** 1688–1713. MR3129030
[39] Mikosch, T. and Wintenberger, O. (2016). A large deviations approach to limit theory for heavy-tailed time series. *Probab*. *Theory Related Fields* **166** 233–269. MR3547739
[40] Petrov, V.V. (1995). *Limit Theorems of Probability Theory*. *Oxford Studies in Probability* **4**. New York: Oxford Univ. Press. MR1353441
[41] Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L.A.N., Guhr, T. and Stanley, H.E. (2002). Random matrix approach to cross correlations in financial data. *Phys*. *Rev*. *E* **65** 066126.
[42] Resnick, S.I. (1987). *Extreme Values*, *Regular Variation*, *and Point Processes*. New York: Springer. MR0900810
[43] Resnick, S.I. (2007). *Heavy-Tail Phenomena*: *Probabilistic and Statistical Modeling*. *Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2271424
[44] Rootzén, H. (1986). Extreme value theory for moving average processes. *Ann*. *Probab*. **14** 612–652. MR0832027
[45] Soshnikov, A. (2004). Poisson statistics for the largest eigenvalues of Wigner random matrices with heavy tails. *Electron*. *Commun*. *Probab*. **9** 82–91 (electronic). MR2081462
[46] Soshnikov, A. (2006). Poisson statistics for the largest eigenvalues in random matrix ensembles. In *Mathematical Physics of Quantum Mechanics*. *Lecture Notes in Physics* **690** 351–364. Berlin: Springer. MR2234922
[47] Tao, T. and Vu, V. (2012). Random covariance matrices: Universality of local statistics of eigenvalues. *Ann*. *Probab*. **40** 1285–1315. MR2962092
[48] Taylor, S.J. (1986). *Modelling Financial Time Series*. Chichester: Wiley.
[49] von Bahr, B. and Esseen, C.-G. (1965). Inequalities for the rth absolute moment of a sum of random variables, $1 \le r \le 2$. *Ann*. *Math*. *Stat*. **36** 299–303. MR0170407

Bernoulli **24**(2), 2018, 1394–1426
https://doi.org/10.3150/16-BEJ902

American options with asymmetric information and reflected BSDE

NEDA ESMAEELI[1] and PETER IMKELLER[2]

[1]*Department of Mathematical Sciences, Sharif University of Technology, Azadi Avenue, Tehran 14588-89694, Iran. E-mail: n_esmaili@mehr.sharif.ir*
[2]*Institut für Mathematik, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany. E-mail: imkeller@math.hu-berlin.de*

We consider an American contingent claim on a financial market where the buyer has additional information. Both agents (seller and buyer) observe the same prices, while the information available to them may differ due to some extra exogenous knowledge the buyer has. The buyer's information flow is modeled by an initial enlargement of the reference filtration. It seems natural to investigate the value of the American contingent claim with asymmetric information. We provide a representation for the cost of the additional information relying on some results on reflected backward stochastic differential equations (RBSDE). This is done by using an interpretation of prices of American contingent claims with extra information for the buyer by solutions of appropriate RBSDE.

Keywords: American contingent claims; asymmetric information; cost of information; initial enlargement of filtrations; reflected BSDE

References

[1] Amendinger, J. (1999). Initial enlargement of filtrations and additional information in financial markets. Ph.D. thesis, Technischen Universität zu Berlin.
[2] Amendinger, J., Imkeller, P. and Schweizer, M. (1998). Additional logarithmic utility of an insider. *Stochastic Process*. *Appl*. **75** 263–286. MR1632213
[3] Ankirchner, S., Dereich, S. and Imkeller, P. (2006). The Shannon information of filtrations and the additional logarithmic utility of insiders. *Ann*. *Probab*. **34** 743–778.
[4] Bismut, J.-M. (1973). Conjugate convex functions in optimal stochastic control. *J*. *Math*. *Anal*. *Appl*. **44** 384–404. MR0329726
[5] Black, F. and Scholes, M. (1973). The pricing of options and corporate liabilities. *J*. *Polit*. *Econ*. **81** 637–654. MR3363443
[6] Bouchaud, J.-P. and Sornette, D. (1994). The Black–Scholes option pricing problem in mathematical finance: Generalization and extensions for a large class of stochastic processes. *J*. *Phys*., *I* **4** 863–881.
[7] Callegaro, G., Jeanblanc, M. and Zargari, B. (2013). Carthaginian enlargement of filtrations. *ESAIM Probab*. *Stat*. **17** 550–566. MR3085632
[8] Duffie, D. (1988). *Security Markets*: *Stochastic Models*. *Economic Theory*, *Econometrics*, *and Mathematical Economics*. Boston, MA: Academic Press, Inc. MR0955269
[9] El Karoui, N., Kapoudjian, C., Pardoux, E., Peng, S. and Quenez, M.C. (1997). Reflected solutions of backward SDE's, and related obstacle problems for PDE's. *Ann*. *Probab*. **25** 702–737. MR1434123
[10] El Karoui, N. and Quenez, M.C. (1995). Dynamic programming and pricing of contingent claims in an incomplete market. *SIAM J*. *Control Optim*. **33** 29–66.

[11] Elliott, R.J., Geman, H. and Korkie, B.M. (1997). Portfolio optimization and contingent claim pricing with differential information. *Stoch. Stoch. Rep.* **60** 185–203. MR1467716

[12] Eyraud-Loisel, A. (2005). Backward stochastic differential equations with enlarged filtration: Option hedging of an insider trader in a financial market with jumps. *Stochastic Process. Appl.* **115** 1745–1763. MR2172884

[13] Föllmer, H. and Schweizer, M. (1990). Hedging of contingent claims under incomplete information. In *Applied Stochastic Analysis* (M.H.A. Davis and R.J. Elliott, eds.). London: Gordon and Breach.

[14] Föllmer, H. and Sondermann, D. (1986). Hedging of non-redundant contingent claims. In *Contributions to Mathematical Economics* (W. Hildenbrand and A. Mas-Collel, eds.) 205–223. Amsterdam: North-Holland.

[15] Grigorova, M., Imkeller, P., Offen, E. and Ouknine, Y. (2016). Reflected BSDEs when the obstacle is not right-continuous and optimal stopping. Available at arXiv:1504.06094.

[16] Grorud, A. and Pontier, M. (1998). Asymmetric information and incomplete market. *Int. J. Theor. Appl. Finance* **4** 285–302.

[17] Grorud, A. and Pontier, M. (1998). Insider trading in a continuous time market model. *Int. J. Theor. Appl. Finance* **1** 331–347.

[18] Harrison, J.M. and Kreps, D.M. (1979). Martingale and arbitrage in multiperiod securiries markets. *Int. J. Econ. Theory* **20** 381–408.

[19] Harrison, J.M. and Pliska, S.R. (1981). Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Process. Appl.* **11** 215–260. MR0622165

[20] Imkeller, P. (2003). Malliavin's calculus in insider models: Additional utility and free lunches. *Math. Finance* **13** 153–169. *Conference on Applications of Malliavin Calculus in Finance (Rocquencourt, 2001)*. MR1968102

[21] Imkeller, P. and Perkowski, N. (2015). The existence of dominating local martingale measures. *Finance Stoch.* **19** 685–717. MR3413932

[22] Jacod, J. (1979). *Calcul Stochastique et Problémes de Martingales*. *Lecture Notes in Math.* **714**. Berlin: Springer. MR0542115

[23] Jacod, J. (1985). Grossissement initial, hypothése (H′) et thèoréme de Girsanov. In *Grossissements de filtrations: exemples et applications*. *Lecture Notes in Math.* **1118**. Berlin: Springer.

[24] Karatzas, I. (1989). Optimization problems in the theory of continuous trading. *SIAM J. Control Optim.* **27** 1221–1259. MR1022426

[25] Kharroubi, I. and Lim, T. (2014). Progressive enlargement of filtrations and backward stochastic differential equations with jumps. *J. Theoret. Probab.* **27** 683–724. MR3245982

[26] Kobylanski, M. and Quenez, M.C. (2012). Optimal stopping time problem in a general framework. *Electron. J. Probab.* **17** 1–28.

[27] Merton, R. (1973). Theory of rational option pricing. *Bell J. Econ. Manag. Sci.* **4** 141–183.

[28] Merton, R. (1991). *Continuous Time Finance*. Oxford: Basil Blackwell.

[29] Müller, S. (1985). *Arbitrage Pricing of Contingent Claims*. *Lecture Notes in Economics and Mathematical Systems* **254**. Berlin: Springer. MR0812055

[30] Neveu, J. (1975). *Discrete-Parameter Martingales*, revised ed. *North-Holland Mathematical Library* **10**. Amsterdam: North-Holland. Translated from the French by T.P. Speed. MR0402915

[31] Oksendal, B. (2003). *Stochastic Differential Equations*: *An Introduction with Applications*. Berlin: Springer.

[32] Pardoux, É. and Peng, S.G. (1990). Adapted solution of a backward stochastic differential equation. *Systems Control Lett.* **14** 55–61. MR1037747

[33] Pikovsky, I. and Karatzas, I. (1996). Anticipative portfolio optimization. *Adv. in Appl. Probab.* **28** 1095–1122. MR1418248

[34] Schäl, M. (1994). On quadratic cost criteria for option hedging. *Math. Oper. Res.* **19** 121–131.

[35] Schweizer, M. (1995). Variance-optimal hedging in discrete time. *Math. Oper. Res.* **20** 1–32.
[36] Stricker, C. and Yor, M. (1978). Calcul stochastique dépendant d'un paramêtre. *Z. Wahrsch. Verw. Gebiete* **45** 109–133.

Bernoulli **24**(2), 2018, 1427–1462
https://doi.org/10.3150/16-BEJ903

Maximum likelihood estimation for the Fréchet distribution based on block maxima extracted from a time series

AXEL BÜCHER[1] and JOHAN SEGERS[2]

[1]*Fakultät für Mathematik, Ruhr-Universität Bochum, Universitätsstr. 150, 44780 Bochum, Germany. E-mail: axel.buecher@ruhr-uni-bochum.de*
[2]*Université catholique de Louvain, Institut de Statistique, Biostatistique et Sciences Actuarielles, Voie du Roman Pays 20, B-1348 Louvain-la-Neuve, Belgium. E-mail: johan.segers@uclouvain.be*

The block maxima method in extreme-value analysis proceeds by fitting an extreme-value distribution to a sample of block maxima extracted from an observed stretch of a time series. The method is usually validated under two simplifying assumptions: the block maxima should be distributed exactly according to an extreme-value distribution and the sample of block maxima should be independent. Both assumptions are only approximately true. The present paper validates that the simplifying assumptions can in fact be safely made.

For general triangular arrays of block maxima attracted to the Fréchet distribution, consistency and asymptotic normality is established for the maximum likelihood estimator of the parameters of the limiting Fréchet distribution. The results are specialized to the common setting of block maxima extracted from a strictly stationary time series. The case where the underlying random variables are independent and identically distributed is further worked out in detail. The results are illustrated by theoretical examples and Monte Carlo simulations.

Keywords: asymptotic normality; block maxima method; heavy tails; maximum likelihood estimation; stationary time series; triangular arrays

References

[1] Balakrishnan, N. and Kateri, M. (2008). On the maximum likelihood estimation of parameters of Weibull distribution based on complete and censored data. *Statist. Probab. Lett.* **78** 2971–2975. MR2516812

[2] Bingham, N.H., Goldie, C.M. and Teugels, J.L. (1987). *Regular Variation. Encyclopedia of Mathematics and Its Applications* **27**. Cambridge: Cambridge Univ. Press. MR0898871

[3] Bücher, A. and Segers, J. (2014). Extreme value copula estimation based on block maxima of a multivariate stationary time series. *Extremes* **17** 495–528. MR3252823

[4] Bücher, A. and Segers, J. (2016). On the maximum likelihood estimator for the Generalized Extreme-Value distribution. Available at arXiv:1601.05702.

[5] Bücher, A. and Segers, J. (2016). Supplement to "Maximum likelihood estimation for the Fréchet distribution based on block maxima extracted from a time series". DOI:10.3150/16-BEJ903SUPP.

[6] Cai, J.-J., de Haan, L. and Zhou, C. (2013). Bias correction in extreme value statistics with index around zero. *Extremes* **16** 173–201. MR3057195

[7] de Haan, L. and Ferreira, A. (2006). *Extreme Value Theory*: *An Introduction*. *Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2234156
[8] Dehling, H. and Philipp, W. (2002). Empirical process techniques for dependent data. In *Empirical Process Techniques for Dependent Data* 3–113. Boston, MA: Birkhäuser Boston.
[9] Dombry, C. (2015). Existence and consistency of the maximum likelihood estimators for the extreme value index within the block maxima framework. *Bernoulli* **21** 420–436. MR3322325
[10] Drees, H. (1998). On smooth statistical tail functionals. *Scand*. *J*. *Stat*. **25** 187–210. MR1614276
[11] Drees, H. (2000). Weighted approximations of tail processes for β-mixing random variables. *Ann*. *Appl*. *Probab*. **10** 1274–1301. MR1810875
[12] Ferreira, A. and de Haan, L. (2015). On the block maxima method in extreme value theory: PWM estimators. *Ann*. *Statist*. **43** 276–298. MR3285607
[13] Gnedenko, B. (1943). Sur la distribution limite du terme maximum d'une série aléatoire. *Ann*. *of Math*. (2) **44** 423–453. MR0008655
[14] Gumbel, E.J. (1958). *Statistics of Extremes*. New York: Columbia Univ. Press. MR0096342
[15] Hosking, J.R.M., Wallis, J.R. and Wood, E.F. (1985). Estimation of the generalized extreme-value distribution by the method of probability-weighted moments. *Technometrics* **27** 251–261. MR0797563
[16] Hsing, T. (1991). On tail index estimation using dependent data. *Ann*. *Statist*. **19** 1547–1569. MR1126337
[17] Leadbetter, M.R. (1983). Extremes and local dependence in stationary sequences. *Z*. *Wahrsch*. *Verw*. *Gebiete* **65** 291–306. MR0722133
[18] Marohn, F. (1994). On testing the Exponential and Gumbel distribution. In *Extreme Value Theory and Applications* 159–174. Kluwer Academic.
[19] Mikosch, T. and Stărică, C. (2000). Limit theory for the sample autocorrelations and extremes of a GARCH(1, 1) process. *Ann*. *Statist*. **28** 1427–1451. MR1805791
[20] Peng, L. (1998). Asymptotically unbiased estimators for the extreme-value index. *Statist*. *Probab*. *Lett*. **38** 107–115. MR1627906
[21] Pickands, J. III (1975). Statistical inference using extreme order statistics. *Ann*. *Statist*. **3** 119–131. MR0423667
[22] Prescott, P. and Walden, A.T. (1980). Maximum likelihood estimation of the parameters of the generalized extreme-value distribution. *Biometrika* **67** 723–724. MR0601119
[23] Resnick, S.I. (1987). *Extreme Values*, *Regular Variation*, *and Point Processes*. *Applied Probability*. *A Series of the Applied Probability Trust* **4**. New York: Springer. MR0900810
[24] Rootzén, H. (2009). Weak convergence of the tail empirical process for dependent sequences. *Stochastic Process*. *Appl*. **119** 468–490. MR2494000
[25] Smith, R.L. (1985). Maximum likelihood estimation in a class of nonregular cases. *Biometrika* **72** 67–90. MR0790201
[26] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247

Bernoulli **24**(2), 2018, 1463–1496
https://doi.org/10.3150/16-BEJ904

Characterization of the convergence in total variation and extension of the Fourth Moment Theorem to invariant measures of diffusions

SEIICHIRO KUSUOKA[1] and CIPRIAN A. TUDOR[2,3]

[1]*Research Institute for Interdisciplinary Science, Okayama University, 3-1-1 Tsushima-naka, Kita-ku, Okayama 700-8530, Japan. E-mail: kusuoka@okayama-u.ac.jp*
[2]*Laboratoire Paul Painlevé, Université de Lille 1, F-59655 Villeneuve d'Ascq, France*
[3]*Department of Mathematics, Academy of Economical Studies, Bucharest, Romania. E-mail: tudor@math.univ-lille1.fr*

We give necessary and sufficient conditions to characterize the convergence in distribution of a sequence of arbitrary random variables to a probability distribution which is the invariant measure of a diffusion process. This class of target distributions includes the most known continuous probability distributions. Precisely speaking, we characterize the convergence in total variation to target distributions which are not Gaussian or Gamma distributed, in terms of the Malliavin calculus and of the coefficients of the associated diffusion process. We also prove that, among the distributions whose associated squared diffusion coefficient is a polynomial of second degree (with some restrictions on its coefficients), the only possible limits of sequences of multiple integrals are the Gaussian and the Gamma laws.

Keywords: convergence in total variation; diffusions; Fourth Moment Theorem; invariant measure; Malliavin calculus; multiple stochastic integrals; Stein's method; weak convergence

References

[1] Azmoodeh, E., Peccati, G. and Poly, G. (2014). Convergence towards linear combinations of chi-squared random variables. Preprint.
[2] Bibby, B.M., Skovgaard, I.M. and Sørensen, M. (2005). Diffusion-type models with given marginal distribution and autocorrelation function. *Bernoulli* **11** 191–220. MR2132002
[3] Bourguin, S. and Tudor, C.A. (2011). Cramér theorem for gamma random variables. *Electron. Commun. Probab.* **16** 365–378. MR2819659
[4] Eden, R. and Viens, F. (2013). General upper and lower tail estimates using Malliavin calculus and Stein's equations. In *Seminar on Stochastic Analysis, Random Fields and Applications VII. Progress in Probability* **67** 55–84. Basel: Birkhäuser/Springer. MR3380095
[5] Kusuoka, S. and Tudor, C.A. (2012). Stein's method for invariant measures of diffusions via Malliavin calculus. *Stochastic Process. Appl.* **122** 1627–1651. MR2914766
[6] Nourdin, I. and Peccati, G. (2009). Stein's method on Wiener chaos. *Probab. Theory Related Fields* **145** 75–118. MR2520122
[7] Nourdin, I. and Peccati, G. (2009). Noncentral convergence of multiple integrals. *Ann. Probab.* **37** 1412–1426. MR2546749

1350-7265

[8] Nourdin, I. and Peccati, G. (2012). *Normal Approximations with Malliavin Calculus*: *From Stein's Method to Universality*. *Cambridge Tracts in Mathematics* **192**. Cambridge: Cambridge Univ. Press. MR2962301

[9] Nourdin, I., Peccati, G. and Swan, Y. (2014). Entropy and the fourth moment phenomenon. *J. Funct. Anal*. **266** 3170–3207. MR3158721

[10] Nourdin, I. and Poly, G. (2013). Convergence in total variation on Wiener chaos. *Stochastic Process. Appl*. **123** 651–674. MR3003367

[11] Nourdin, I. and Viens, F.G. (2009). Density formula and concentration inequalities with Malliavin calculus. *Electron. J. Probab.* **14** 2287–2309. MR2556018

[12] Nualart, D. (2006). *The Malliavin Calculus and Related Topics*, 2nd ed. *Probability and Its Applications* (*New York*). Berlin: Springer. MR2200233

[13] Nualart, D. and Ortiz-Latorre, S. (2008). Central limit theorems for multiple stochastic integrals and Malliavin calculus. *Stochastic Process. Appl*. **118** 614–628. MR2394845

[14] Nualart, D. and Peccati, G. (2005). Central limit theorems for sequences of multiple stochastic integrals. *Ann. Probab.* **33** 177–193. MR2118863

[15] Peccati, G. and Tudor, C.A. (2005). Gaussian limits for vector-valued multiple stochastic integrals. In *Séminaire de Probabilités XXXVIII*. *Lecture Notes in Math*. **1857** 247–262. Berlin: Springer. MR2126978

[16] Tudor, C.A. (2013). *Analysis of Variations for Self-Similar Processes*: *A Stochastic Calculus Approach*. *Probability and Its Applications* (*New York*). Cham: Springer. MR3112799

[17] Tudor, C.A. (2014). Chaos expansion and asymptotic behavior of the Pareto distribution. *Statist. Probab. Lett*. **91** 62–68. MR3208117

[18] Üstünel, A.S. and Zakai, M. (1989). On independence and conditioning on Wiener space. *Ann. Probab.* **17** 1441–1453. MR1048936

Bernoulli **24**(2), 2018, 1497–1530
https://doi.org/10.3150/16-BEJ905

Exact and fast simulation of max-stable processes on a compact set using the normalized spectral representation

MARCO OESTING[1], MARTIN SCHLATHER[2] and CHEN ZHOU[3,4]

[1]*Department Mathematik, Universität Siegen, Walter-Flex-Str. 3, 57072 Siegen, Germany. E-mail: oesting@mathematik.uni-siegen.de*
[2]*Institut für Mathematik, Universität Mannheim, A 5, 6, 68161 Mannheim, Germany. E-mail: schlather@math.uni-mannheim.de*
[3]*Erasmus School of Economics, Erasmus University Rotterdam, P.O. Box 1738, 3000DR Rotterdam, The Netherlands. E-mail: zhou@ese.eur.nl*
[4]*Economics and Research Division, De Nederlandsche Bank, P.O. Box 98, 1000AB Amsterdam, The Netherlands. E-mail: c.zhou@dnb.nl*

The efficiency of simulation algorithms for max-stable processes relies on the choice of the spectral representation: different choices result in different sequences of finite approximations to the process. We propose a constructive approach yielding a normalized spectral representation that solves an optimization problem related to the efficiency of simulating max-stable processes. The simulation algorithm based on the normalized spectral representation can be regarded as max-importance sampling. Compared to other simulation algorithms hitherto, our approach has at least two advantages. First, it allows the exact simulation of a comprising class of max-stable processes. Second, the algorithm has a stopping time with finite expectation. In practice, our approach has the potential of considerably reducing the simulation time of max-stable processes.

Keywords: importance sampling; mixed moving maxima; optimal simulation

References

[1] Blanchet, J. and Davison, A.C. (2011). Spatial modeling of extreme snow depth. *Ann. Appl. Stat.* **5** 1699–1725. MR2884920
[2] Buishand, T.A., de Haan, L. and Zhou, C. (2008). On spatial extremes: With application to a rainfall problem. *Ann. Appl. Stat.* **2** 624–642. MR2524349
[3] Coles, S.G. (1993). Regional modelling of extreme storms via max-stable processes. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **55** 797–816. MR1229882
[4] Coles, S.G. and Tawn, J.A. (1996). Modelling extremes of the areal rainfall process. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **58** 329–347. MR1377836
[5] Cooley, D., Naveau, P. and Poncet, P. (2006). Variograms for spatial max-stable random fields. In *Dependence in Probability and Statistics* 373–390. New York: Springer.
[6] de Haan, L. (1984). A spectral representation for max-stable processes. *Ann. Probab.* **12** 1194–1204. MR0757776
[7] de Haan, L. and Ferreira, A. (2006). *Extreme Value Theory: An Introduction. Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2234156

[8] de Haan, L. and Lin, T. (2001). On convergence toward an extreme value distribution in $C[0,1]$. *Ann. Probab.* **29** 467–483.
[9] Dieker, A.B. and Mikosch, T. (2015). Exact simulation of Brown–Resnick random fields at a finite number of locations. *Extremes* **18** 301–314.
[10] Dombry, C., Engelke, S. and Oesting, M. (2016). Exact simulation of max-stable processes. *Biometrika* **103** 303–317.
[11] Dombry, C. and Eyi-Minko, F. (2012). Strong mixing properties of max-infinitely divisible random fields. *Stochastic Process. Appl.* **122** 3790–3811. MR2965926
[12] Dombry, C. and Eyi-Minko, F. (2013). Regular conditional distributions of continuous max-infinitely divisible random fields. *Electron. J. Probab.* **18** no. 7. MR3024101
[13] Dombry, C., Éyi-Minko, F. and Ribatet, M. (2013). Conditional simulation of max-stable processes. *Biometrika* **100** 111–124. MR3034327
[14] Giné, E., Hahn, M.G. and Vatan, P. (1990). Max-infinitely divisible and max-stable sample continuous processes. *Probab. Theory Related Fields* **87** 139–165.
[15] Hastings, W.K. (1970). Monte Carlo sampling methods using Markov chains and their applications. *Biometrika* **57** 97–109.
[16] Kabluchko, Z. (2009). Spectral representations of sum- and max-stable processes. *Extremes* **12** 401–424. MR2562988
[17] Kabluchko, Z., Schlather, M. and de Haan, L. (2009). Stationary max-stable fields associated to negative definite functions. *Ann. Probab.* **37** 2042–2065. MR2561440
[18] Oesting, M., Kabluchko, Z. and Schlather, M. (2012). Simulation of Brown–Resnick processes. *Extremes* **15** 89–107. MR2891311
[19] Oesting, M. and Schlather, M. (2014). Conditional sampling for max-stable processes with a mixed moving maxima representation.
[20] Oesting, M., Schlather, M. and Zhou, C. (2013). On the normalized spectral representation of max-stable processes on a compact set. Available at arXiv:1310.1813v1.
[21] Opitz, T. (2013). Extremal t processes: Elliptical domain of attraction and a spectral representation. *J. Multivariate Anal.* **122** 409–413. MR3189331
[22] Padoan, S.A., Ribatet, M. and Sisson, S.A. (2010). Likelihood-based inference for max-stable processes. *J. Amer. Statist. Assoc.* **105** 263–277.
[23] R Core Team (2015). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing.
[24] Resnick, S.I. and Roy, R. (1991). Random usc functions, max-stable processes and continuous choice. *Ann. Appl. Probab.* 267–292.
[25] Ribatet, M. (2013). Spatial extremes: Max-stable processes at work. *J. Soc. Fr. Stat.* **154** 156–177.
[26] Schlather, M. (2002). Models for stationary max-stable random fields. *Extremes* **5** 33–44.
[27] Schlather, M., Malinowski, A., Oesting, M., Boecker, D., Strokorb, K., Engelke, S., Martini, J., Ballani, F., Moreva, O., Menck, P.J., Gross, S., Ober, U., Berreth, C., Burmeister, K., Manitz, J., Ribeiro, P., Singleton, R., Pfaff, B. and R Core Team (2015). RandomFields: Simulation and analysis of random fields. R package version 3.1.3.
[28] Schlather, M. and Tawn, J.A. (2003). A dependence measure for multivariate and spatial extreme values: Properties and inference. *Biometrika* **90** 139–156.
[29] Smith, R.L. (1990). Max-stable processes and spatial extremes. Unpublished manuscript.
[30] Stoev, S.A. and Taqqu, M.S. (2005). Extremal stochastic integrals: A parallel between max-stable processes and α-stable processes. *Extremes* **8** 237–266. MR2324891
[31] Strokorb, K., Ballani, F. and Schlather, M. (2015). Tail correlation functions of max-stable processes. *Extremes* **18** 241–271.
[32] Wang, Y. and Stoev, S.A. (2010). On the structure and representations of max-stable processes. *Adv. in Appl. Probab.* **42** 855–877.

Bernoulli **24**(2), 2018, 1531–1575
https://doi.org/10.3150/16-BEJ906

Asymptotic analysis of covariance parameter estimation for Gaussian processes in the misspecified case

FRANÇOIS BACHOC

Institut de Mathématiques de Toulouse, Université Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse. E-mail: francois.bachoc@math.univ-toulouse.fr

In parametric estimation of covariance function of Gaussian processes, it is often the case that the true covariance function does not belong to the parametric set used for estimation. This situation is called the misspecified case. In this case, it has been shown that, for irregular spatial sampling of observation points, Cross Validation can yield smaller prediction errors than Maximum Likelihood. Motivated by this observation, we provide a general asymptotic analysis of the misspecified case, for independent and uniformly distributed observation points. We prove that the Maximum Likelihood estimator asymptotically minimizes a Kullback–Leibler divergence, within the misspecified parametric set, while Cross Validation asymptotically minimizes the integrated square prediction error. In Monte Carlo simulations, we show that the covariance parameters estimated by Maximum Likelihood and Cross Validation, and the corresponding Kullback–Leibler divergences and integrated square prediction errors, can be strongly contrasting. On a more technical level, we provide new increasing-domain asymptotic results for independent and uniformly distributed observation points.

Keywords: covariance parameter estimation; cross validation; Gaussian processes; increasing-domain asymptotics; integrated square prediction error; Kullback–Leibler divergence; maximum likelihood

References

[1] Abrahamsen, P. (1997). A review of Gaussian random fields and correlation functions. Technical report, Norwegian computing center.
[2] Anderes, E. (2010). On the consistent separation of scale and variance for Gaussian random fields. *Ann. Statist.* **38** 870–893. MR2604700
[3] Andrianakis, I. and Challenor, P.G. (2012). The effect of the nugget on Gaussian process emulators of computer models. *Comput. Statist. Data Anal.* **56** 4215–4228. MR2957866
[4] Azencott, R. and Dacunha-Castelle, D. (1986). *Series of Irregular Observations: Forecasting and Model Building. Applied Probability. A Series of the Applied Probability Trust*. New York: Springer. MR0848355
[5] Bachoc, F. (2013). Cross validation and maximum likelihood estimations of hyper-parameters of Gaussian processes with model misspecification. *Comput. Statist. Data Anal.* **66** 55–69. MR3064023
[6] Bachoc, F. (2013). Parametric estimation of covariance function in Gaussian-process based Kriging models. Application to uncertainty quantification for computer experiments. Ph.D. thesis, Université Paris-Diderot – Paris VII. Available at https://tel.archives-ouvertes.fr/tel-00881002/document.
[7] Bachoc, F. (2014). Asymptotic analysis of the role of spatial sampling for covariance parameter estimation of Gaussian processes. *J. Multivariate Anal.* **125** 1–35. MR3163828

1350-7265

[8] Bachoc, F. (2016). Supplement to "Asymptotic analysis of covariance parameter estimation for Gaussian processes in the misspecified case." DOI:10.3150/16-BEJ906SUPP.

[9] Bachoc, F., Bois, G., Garnier, J. and Martinez, J.M. (2014). Calibration and improved prediction of computer models by universal Kriging. *Nucl. Sci. Eng.* **176** 81–97.

[10] Chevalier, C., Ginsbourger, D., Bect, J., Vazquez, E., Picheny, V. and Richet, Y. (2014). Fast parallel Kriging-based stepwise uncertainty reduction with application to the identification of an excursion set. *Technometrics* **56** 455–465. MR3290615

[11] Conti, S. and O'Hagan, A. (2010). Bayesian emulation of complex multi-output and dynamic computer models. *J. Statist. Plann. Inference* **140** 640–651. MR2558393

[12] Cressie, N. and Lahiri, S.N. (1993). The asymptotic distribution of REML estimators. *J. Multivariate Anal.* **45** 217–233. MR1221918

[13] Cressie, N. and Lahiri, S.N. (1996). Asymptotics for REML estimation of spatial covariance parameters. *J. Statist. Plann. Inference* **50** 327–341. MR1394135

[14] Dubrule, O. (1983). Cross validation of Kriging in a unique neighborhood. *J. Int. Assoc. Math. Geol.* **15** 687–699. MR0720633

[15] Furrer, R., Bachoc, F. and Du, J. (2016). Asymptotic properties of multivariate tapering for estimation and prediction. *J. Multivariate Anal.* **149** 177–191. MR3507322

[16] Furrer, R., Genton, M.G. and Nychka, D. (2006). Covariance tapering for interpolation of large spatial datasets. *J. Comput. Graph. Statist.* **15** 502–523. MR2291261

[17] Gneiting, T. (2011). Making and evaluating point forecasts. *J. Amer. Statist. Assoc.* **106** 746–762. MR2847988

[18] Gneiting, T. and Raftery, A.E. (2007). Strictly proper scoring rules, prediction, and estimation. *J. Amer. Statist. Assoc.* **102** 359–378. MR2345548

[19] Gramacy, R.B. and Apley, D.W. (2015). Local Gaussian process approximation for large computer experiments. *J. Comput. Graph. Statist.* **24** 561–578. MR3357395

[20] Gray, R.M. (2006). Toeplitz and circulant matrices: A review. *Found. Trends Commun. Inf. Theory* **2** 155–239.

[21] Hallin, M., Lu, Z. and Yu, K. (2009). Local linear spatial quantile regression. *Bernoulli* **15** 659–686. MR2555194

[22] Handcock, M.S. and Wallis, J.R. (1994). An approach to statistical spatial-temporal modeling of meteorological fields. *J. Amer. Statist. Assoc.* **89** 368–390. MR1294070

[23] Iooss, B., Boussouf, L., Feuillard, V. and Marrel, A. (2010). Numerical studies of the metamodel fitting and validation processes. *International Journal of Advances in Systems and Measurements* **3** 11–21.

[24] Jones, D.R., Schonlau, M. and Welch, W.J. (1998). Efficient global optimization of expensive black-box functions. *J. Global Optim.* **13** 455–492. MR1673460

[25] Kou, S.C. (2003). On the efficiency of selection criteria in spline regression. *Probab. Theory Related Fields* **127** 153–176. MR2013979

[26] Lahiri, S.N. (2003). Central limit theorems for weighted sums of a spatial process under a class of stochastic and fixed designs. *Sankhyā* **65** 356–388. MR2028905

[27] Lahiri, S.N. and Mukherjee, K. (2004). Asymptotic distributions of M-estimators in a spatial regression model under some fixed and stochastic spatial sampling designs. *Ann. Inst. Statist. Math.* **56** 225–250. MR2067154

[28] Lahiri, S.N. and Robinson, P.M. (2016). Central limit theorems for long range dependent spatial linear processes. *Bernoulli* **22** 345–375. MR3449786

[29] Lahiri, S.N. and Zhu, J. (2006). Resampling methods for spatial regression models under a class of stochastic designs. *Ann. Statist.* **34** 1774–1813. MR2283717

[30] Le Gratiet, L. and Garnier, J. (2014). Asymptotic analysis of the learning curve for Gaussian process regression. *Mach*. *Learn*. 1–27.
[31] Loh, W.-L. (2005). Fixed-domain asymptotics for a subclass of Matérn-type Gaussian random fields. *Ann*. *Statist*. **33** 2344–2394. MR2211089
[32] Mardia, K.V. and Marshall, R.J. (1984). Maximum likelihood estimation of models for residual covariance in spatial regression. *Biometrika* **71** 135–146. MR0738334
[33] Marrel, A., Iooss, B., Van Dorpe, F. and Volkova, E. (2008). An efficient methodology for modeling complex computer codes with Gaussian processes. *Comput*. *Statist*. *Data Anal*. **52** 4731–4744. MR2521618
[34] Martin, J.D. and Simpson, T.W. (2004). On the use of Kriging models to approximate deterministic computer models. In *DETC*'04 *ASME* 2004 *International Design Engineering Technical Conferences and Computers and Information in Engineering Conference Salt Lake City*, *Utah USA*, *September* 28 – *October* 2, 2004.
[35] Paulo, R., García-Donato, G. and Palomo, J. (2012). Calibration of computer models with multivariate output. *Comput*. *Statist*. *Data Anal*. **56** 3959–3974. MR2957846
[36] Putter, H. and Young, G.A. (2001). On the effect of covariance function estimation on the accuracy of Kriging predictors. *Bernoulli* **7** 421–438. MR1836738
[37] Rasmussen, C.E. and Williams, C.K.I. (2006). *Gaussian Processes for Machine Learning*. *Adaptive Computation and Machine Learning*. Cambridge, MA: MIT Press. MR2514435
[38] Ripley, B.D. (1981). *Spatial Statistics*. New York: Wiley. MR0624436
[39] Sacks, J., Welch, W.J., Mitchell, T.J. and Wynn, H.P. (1989). Design and analysis of computer experiments. *Statist*. *Sci*. **4** 409–423. MR1041765
[40] Santner, T.J., Williams, B.J. and Notz, W.I. (2003). *The Design and Analysis of Computer Experiments*. *Springer Series in Statistics*. New York: Springer. MR2160708
[41] Scheuerer, M. (2010). Regularity of the sample paths of a general second order random field. *Stochastic Process*. *Appl*. **120** 1879–1897. MR2673978
[42] Shi, T., Belkin, M. and Yu, B. (2009). Data spectroscopy: Eigenspaces of convolution operators and clustering. *Ann*. *Statist*. **37** 3960–3984. MR2572449
[43] Stein, M. (1990). Uniform asymptotic optimality of linear predictions of a random field using an incorrect second-order structure. *Ann*. *Statist*. **18** 850–872. MR1056340
[44] Stein, M.L. (1988). Asymptotically efficient prediction of a random field with a misspecified covariance function. *Ann*. *Statist*. **16** 55–63. MR0924856
[45] Stein, M.L. (1990). Bounds on the efficiency of linear predictions using an incorrect covariance function. *Ann*. *Statist*. **18** 1116–1138. MR1062701
[46] Stein, M.L. (1990). A comparison of generalized cross validation and modified maximum likelihood for estimating the parameters of a stochastic process. *Ann*. *Statist*. **18** 1139–1157. MR1062702
[47] Stein, M.L. (1993). Spline smoothing with an estimated order parameter. *Ann*. *Statist*. **21** 1522–1544. MR1241277
[48] Stein, M.L. (1999). *Interpolation of Spatial Data*: *Some Theory for Kriging*. *Springer Series in Statistics*. New York: Springer. MR1697409
[49] Sundararajan, S. and Keerthi, S.S. (2001). Predictive approaches for choosing hyperparameters in Gaussian processes. *Neural Comput*. **13** 1103–1118.
[50] Wendland, H. (1995). Piecewise polynomial, positive definite and compactly supported radial functions of minimal degree. *Adv*. *Comput*. *Math*. **4** 389–396. MR1366510
[51] White, H. (1982). Maximum likelihood estimation of misspecified models. *Econometrica* **50** 1–25. MR0640163
[52] Ying, Z. (1991). Asymptotic properties of a maximum likelihood estimator with data from a Gaussian process. *J*. *Multivariate Anal*. **36** 280–296. MR1096671

[53] Ying, Z. (1993). Maximum likelihood estimation of parameters under a spatial sampling scheme. *Ann. Statist.* **21** 1567–1590. MR1241279

[54] Zhang, H. (2004). Inconsistent estimation and asymptotically equal interpolations in model-based geostatistics. *J. Amer. Statist. Assoc.* **99** 250–261. MR2054303

[55] Zhang, H. and Wang, Y. (2010). Kriging and cross-validation for massive spatial data. *Environmetrics* **21** 290–304. MR2842244

[56] Zhang, H. and Zimmerman, D.L. (2005). Towards reconciling two asymptotic frameworks in spatial statistics. *Biometrika* **92** 921–936. MR2234195

Bernoulli **24**(2), 2018, 1576–1612
https://doi.org/10.3150/16-BEJ907

On branching process with rare neutral mutation

AIRAM BLANCAS* and VÍCTOR RIVERO**

Centro de Investigación en Matemáticas (CIMAT A.C.). Calle Jalisco s/n, Col. Valenciana, 36240 Guanajuato, Gto. México. E-mail: *airam@cimat.mx; **rivero@cimat.mx

In this paper, we study the genealogical structure of a Galton–Watson process with neutral mutations. Namely, we extend in two directions the asymptotic results obtained in Bertoin [*Stochastic Process. Appl.* **120** (2010) 678–697]. In the critical case, we construct the version of the model in Bertoin [*Stochastic Process. Appl.* **120** (2010) 678–697], conditioned not to be extinct. We establish a version of the limit theorems in Bertoin [*Stochastic Process. Appl.* **120** (2010) 678–697], when the reproduction law has an infinite variance and it is in the domain of attraction of an α-stable distribution, both for the unconditioned process and for the process conditioned to nonextinction. In the latter case, we obtain the convergence (after re-normalization) of the allelic sub-populations towards a tree indexed CSBP with immigration.

Keywords: branching process; domain of attraction of α-stable laws; neutral mutations; Q-processes; regular variation

References

[1] Athreya, K.B. and Ney, P.E. (1972). *Branching Processes*. *Die Grundlehren der Mathematischen Wissenschaften* **196**. New York: Springer.
[2] Bertoin, J. (1996). *Lévy Processes*. *Cambridge Tracts in Mathematics* **121**. Cambridge: Cambridge Univ. Press.
[3] Bertoin, J. (2010). A limit theorem for trees of alleles in branching processes with rare neutral mutations. *Stochastic Process*. *Appl*. **120** 678–697. MR2603059
[4] Bingham, N.H., Goldie, C.M. and Teugels, J.L. (1989). *Regular Variation*. *Encyclopedia of Mathematics and Its Applications* **27**. Cambridge: Cambridge Univ. Press. MR1015093
[5] Chauvin, B. (1986). Arbres et processus de Bellman–Harris. *Ann*. *Inst*. *Henri Poincaré B*, *Probab*. *Stat*. **22** 209–232.
[6] Chung, K.L. and Walsh, J.B. (2005). *Markov Processes*, *Brownian Motion*, *and Time Symmetry*, 2nd ed. *Grundlehren der Mathematischen Wissenschaften* **249**. New York: Springer.
[7] Feller, W. (1971). *An Introduction to Probability Theory and Its Applications*. *Vol*. *II*, 2nd ed. New York: Wiley. MR0270403
[8] Jacod, J. and Shiryaev, A.N. (1987). *Limit Theorems for Stochastic Processes*. *Grundlehren der Mathematischen Wissenschaften* **288**. Berlin: Springer. MR0959133
[9] Joffe, A. (1967). On the Galton–Watson branching process with mean less than one. *Ann*. *Math*. *Stat*. **38** 264–266. MR0205337
[10] Kallenberg, O. (2002). *Foundations of Modern Probability*, 2nd ed. *Probability and Its Applications* (*New York*). New York: Springer.
[11] Kawazu, K. and Watanabe, S. (1971). Branching processes with immigration and related limit theorems. *Teor*. *Verojatnost*. *i Primenen*. **16** 34–51.

1350-7265

[12] Lambert, A. (2007). Quasi-stationary distributions and the continuous-state branching process conditioned to be never extinct. *Electron. J. Probab.* **12** 420–446.
[13] Lambert, A. (2008). Population dynamics and random genealogies. *Stoch. Models* **24** 45–163. MR2466449
[14] Lamperti, J. and Ney, P. (1968). Conditioned branching processes and their limiting diffusions. *Teor. Verojatnost. i Primenen.* **13** 126–137.
[15] Li, Z. (2011). *Measure-Valued Branching Markov Processes*. *Probability and Its Applications* (*New York*). Heidelberg: Springer. MR2760602
[16] Seneta, E. and Vere-Jones, D. (1966). On quasi-stationary distributions in discrete-time Markov chains with a denumerable infinity of states. *J. Appl. Probab.* **3** 403–434. MR0207047
[17] Whitt, W. (2002). *Stochastic-Process Limits*: *An Introduction to Stochastic-Process Limits and Their Application to Queues*. *Springer Series in Operations Research*. New York: Springer. MR1876437
[18] Yaglom, A.M. (1947). Certain limit theorems of the theory of branching random processes. *Dokl. Akad. Nauk SSSR* **56** 795–798.

Bernoulli Forthcoming Papers

BOUCHARD, B., ELIE, R. and MOREAU, L.
Regularity of BSDEs with a convex constraint on the gains-process

BRODERICK, T., WILSON, A.C. and JORDAN, M.I.
Posteriors, conjugacy, and exponential families for completely random measures

THAI, M.-N.
Birth and death process in mean field type interaction

LEE, C. and NEAL, P.
Optimal scaling of the independence sampler: Theory and practice

FRIKHA, N.
On the weak approximation of a skew diffusion by an Euler-type scheme

AY, N., JOST, J., LÊ, H.V. and SCHWACHHÖFER, L.
Parametrized measure models

AGAPIOU, S., ROBERTS, G.O. and VOLLMER, S.J.
Unbiased Monte Carlo: Posterior estimation for intractable/infinite-dimensional models

HAN, F., XU, S. and ZHOU, W.-X.
On Gaussian comparison inequality and its application to spectral analysis of large random matrices

QIAO, W. and POLONIK, W.
Extrema of rescaled locally stationary Gaussian fields on manifolds

VATS, D., FLEGAL, J.M. and JONES, G.L.
Strong consistency of multivariate spectral variance estimators in Markov chain Monte Carlo

DECROUEZ, G., GRABCHAK, M. and PARIS, Q.
Finite sample properties of the mean occupancy counts and probabilities

PITMAN, J. and TANG, W.
Tree formulas, mean first passage times and Kemeny's constant of a Markov chain

KEELER, H.P., ROSS, N. and XIA, A.
When do wireless network signals appear Poisson?

BOSSY, M. and OLIVERO, H.
Strong convergence of the symmetrized Milstein scheme for some CEV-like SDEs

ARUTYUNYAN, L.M. and KOSOV, E.D.
Deviation of polynomials from their expectations and isoperimetry

KHASMINSKII, R.Z. and KUTOYANTS, Y.A.
On parameter estimation of hidden telegraph process

KNAPIK, B. and SALOMOND, J.-B.
A general approach to posterior contraction in nonparametric inverse problems

Continues

Bernoulli Forthcoming Papers—*Continued*

CHATTERJEE, S. and LAFFERTY, J.
Adaptive risk bounds in unimodal regression

KOSKELA, J., JENKINS, P.A. and SPANÒ, D.
Bayesian non-parametric inference for Λ-coalescents: Posterior consistency and a parametric method

BUCHSTEINER, J.
The function-indexed sequential empirical process under long-range dependence

SAUMARD, A.
On optimality of empirical risk minimization in linear aggregation

ALETTI, G., GHIGLIETTI, A. and VIDYASHANKAR, A.N.
Dynamics of an adaptive randomly reinforced urn

ANDJEL, E., MOUNTFORD, T. and VALESIN, D.
Equilibrium of the interface of the grass-bushes-trees process

EGUCHI, S. and MASUDA, H.
Schwarz type model comparison for LAQ models

SINOVA, B., GONZÁLEZ-RODRÍGUEZ, G. and VAN AELST, S.
M-estimators of location for functional data

EVANS, S.N., RIVEST, R.L. and STARK, P.B.
Leading the field: Fortune favors the bold in Thurstonian choice models

GÖTZE, F., NAUMOV, A., TIKHOMIROV, A. and TIMUSHEV, D.
On the local semicircular law for Wigner ensembles

MIJATOVIĆ, A. and VOGRINC, J.
On the Poisson equation for Metropolis–Hastings chains

CARPENTIER, A., KLOPP, O., LÖFFLER, M. and NICKL, R.
Adaptive confidence sets for matrix completion

ALFONSI, A., CORBETTA, J. and JOURDAIN, B.
Evolution of the Wasserstein distance between the marginals of two Markov processes

SCHACHERMAYER, W. and STEBEGG, F.
The sharp constant for the Burkholder–Davis–Gundy inequality and non-smooth pasting

GOLDENSHLUGER, A.
The $M/G/\infty$ estimation problem revisited

BANDINI, E. and RUSSO, F.
Special weak Dirichlet processes and BSDEs driven by a random measure

RUDOLF, D. and SCHWEIZER, N.
Perturbation theory for Markov chains via Wasserstein distance

Continues

Bernoulli Forthcoming Papers—*Continued*

ZHANG, X. and CHENG, G.
Gaussian approximation for high dimensional vector under physical dependence

GÖRGEN, C. and SMITH, J.Q.
Equivalence classes of staged trees

GISSIBL, N. and KLÜPPELBERG, C.
Max-linear models on directed acyclic graphs

BICKEL, D.R. and PATRIOTA, A.G.
Self-consistent confidence sets and tests of composite hypotheses applicable to restricted parameters

COUPIER, D. and HIRSCH, C.
Coalescence of Euclidean geodesics on the Poisson–Delaunay triangulation

RÁSONYI, M. and SAYIT, H.
Sticky processes, local and true martingales

LEPSKI, O.V.
A new approach to estimator selection

GROTE, J. and THÄLE, C.
Concentration and moderate deviations for Poisson polytopes and polyhedra

XIONG, J. and ZHAI, J.
Large deviations for locally monotone stochastic partial differential equations driven by Lévy noise

GAO, X. and ZHU, L.
Large deviations and applications for Markovian Hawkes processes with a large initial intensity

MARCHINA, A.
Concentration inequalities for separately convex functions

CHOROWSKI, J.
Nonparametric volatility estimation in scalar diffusions: Optimality across observation frequencies

WU, W. and ZHOU, Z.
Simultaneous quantile inference for non-stationary long-memory time series

CAO, H., LIU, W. and ZHOU, Z.
Simultaneous nonparametric regression analysis of sparse longitudinal data

QIU, Y.
Rigid stationary determinantal processes in non-Archimedean fields

CRISAN, D. and MÍGUEZ, J.
Nested particle filters for online parameter estimation in discrete-time state-space Markov models

Continues

Bernoulli Forthcoming Papers—*Continued*

DAVIS, R.A., MATSUI, M., MIKOSCH, T. and WAN, P.
Applications of distance correlation to time series

BASSE-O'CONNOR, A., HEINRICH, C. and PODOLSKIJ, M.
On limit theory for Lévy semi-stationary processes

ÁLVAREZ-ESTEBAN, P.C., DEL BARRIO, E., CUESTA-ALBERTOS, J.A. and MATRÁN, C.
Wide consensus aggregation in the Wasserstein space. Application to location-scatter families

CAPUTO, P. and SINCLAIR, A.
Entropy production in nonlinear recombination models

MCKEAGUE, I.W., PEKÖZ, E.A. and SWAN, Y.
Stein's method and approximating the quantum harmonic oscillator

BARTROFF, J., GOLDSTEIN, L. and IŞLAK, Ü.
Large bounded size biased couplings, log concave distributions and concentration of measure for occupancy models

OGIHARA, T.
Parametric inference for nonsynchronously observed diffusion processes in the presence of market microstructure noise

DÖBLER, C. and PECCATI, G.
The Gamma Stein equation and noncentral de Jong theorems

CHENG, D. and SCHWARTZMAN, A.
Expected number and height distribution of critical points of smooth isotropic Gaussian random fields

CLINET, S. and POTIRON, Y.
Statistical inference for the doubly stochastic self-exciting process

MARTIN, O. and VETTER, M.
Testing for simultaneous jumps in case of asynchronous observations

SIORPAES, P.
Applications of pathwise Burkholder–Davis–Gundy inequalities

HAN, X., PAN, G. and YANG, Q.
A unified matrix model including both CCA and F matrices in multivariate analysis: The largest eigenvalue and its applications

SIDOROVA, N.
Small deviations of a Galton–Watson process with immigration

LEJAY, A. and PIGATO, P.
Statistical estimation of the Oscillating Brownian Motion

CHOTARD, A. and AUGER, A.
Verifiable conditions for the irreducibility and aperiodicity of Markov chains by analyzing underlying deterministic models

Continues

Bernoulli Forthcoming Papers—*Continued*

SCHERTZER, E.
Recovering the Brownian coalescent point process from the Kingman coalescent by conditional sampling

LOENENKO, N.N., PAPIC, I., SIKORSKII, A. and SUVAK, N.
Correlated continuous time random walks and fractional Pearson diffusions

ARIAS-CASTRO, E., BUBECK, S., LUOSI, G. and VERZELEN, N.
Detecting Markov random fields hidden in white noise

KIM, D., LIU, Y. and WANG, Y.
Large volatility matrix estimation with factor-based diffusion model for high-frequency financial data

VERZELEN, N. and GASSIAT, E.
Adaptive estimation of high-dimensional signal-to-noise ratios

KAMATANI, K.
Efficient strategy for the Markov chain Monte Carlo in high-dimension with heavy-tailed target probability distribution

GENEST, C., NESLEHOVA, J.G. and RIVEST, L.-P.
The class of multivariate max-id copulas with ℓ_1-norm symmetric exponent measure

ZHOU, S.
Sparse Hanson–Wright inequalities for subgaussian quadratic forms

LEDOIT, O. and WOLF, M.
Optimal estimation of a large-dimensional covariance matrix under Stein's loss

LENG, C. and PAN, G.
Covariance estimation via sparse Kronecker structures

GIULINI, I.
Robust dimension-free Gram operator estimates

WANG, X.
Subexponential decay in kinetic Fokker–Planck equation: Weak hypocoercivity

RATH, B.
Feller property of the multiplicative coalescent with linear deletion

PEKÖZ, E., RÖLLIN, A. and ROSS, N.
Pólya urns with immigration at random times

LEUNG, D. and SHAO, Q.-S.
Asymptotic power of Rao's score test for independence in high dimensions

MAMMEN, E., VAN KEILEGOM, I. and YU, K.
Expansion for moments of regression quantiles with applications to nonparametric testing