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Adaptive confidence sets for matrix completion

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In the present paper, we study the problem of existence of honest and adaptive confidence sets for matrix completion. We consider two statistical models: the trace regression model and the Bernoulli model. In the trace regression model, we show that honest confidence sets that adapt to the unknown rank of the matrix exist even when the error variance is unknown. Contrary to this, we prove that in the Bernoulli model, honest and adaptive confidence sets exist only when the error variance is known a priori. In the course of our proofs, we obtain bounds for the minimax rates of certain composite hypothesis testing problems arising in low rank inference.

Keywords: adaptivity; confidence sets; low rank recovery; matrix completion; minimax hypothesis testing; unknown variance

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Evolution of the Wasserstein distance between the marginals of two Markov processes

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In this paper, we are interested in the time derivative of the Wasserstein distance between the marginals of two Markov processes. As recalled in the introduction, the Kantorovich duality leads to a natural candidate for this derivative. Up to the sign, it is the sum of the integrals with respect to each of the two marginals of the corresponding generator applied to the corresponding Kantorovich potential. For pure jump processes with bounded intensity of jumps, we prove that the evolution of the Wasserstein distance is actually given by this candidate. In dimension one, we show that this remains true for Piecewise Deterministic Markov Processes. We apply the formula to estimate the exponential decrease rate of the Wasserstein distance between the marginals of two birth and death processes with the same generator in terms of the Wasserstein curvature.

Keywords: birth and death processes; optimal transport; piecewise deterministic Markov processes; pure jump Markov processes; Wasserstein distance

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The sharp constant for the Burkholder–Davis–Gundy inequality and non-smooth pasting

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We revisit the celebrated family of BDG-inequalities introduced by Burkholder, Gundy (*Acta Math.* **124** (1970) 249–304) and Davis (*Israel J. Math.* **8** (1970) 187–190) for continuous martingales. For the inequalities $\mathbb{E}[\tau^{\frac{p}{2}}] \le C_p \mathbb{E}[(B^*(\tau))^p]$ with $0 < p < 2$ we propose a connection of the optimal constant C_p with an ordinary integro-differential equation which gives rise to a numerical method of finding this constant. Based on numerical evidence, we are able to calculate, for $p = 1$, the explicit value of the optimal constant C_1, namely $C_1 = 1.27267\dots$. In the course of our analysis, we find a remarkable appearance of "non-smooth pasting" for a solution of a related ordinary integro-differential equation.

Keywords: BDG inequality; non-smooth pasting; optimal stopping; ordinary integro-differential equations

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The $M/G/\infty$ estimation problem revisited

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The subject of this paper is *the $M/G/\infty$ estimation problem*: the goal is to estimate the service time distribution G of the $M/G/\infty$ queue from the arrival–departure observations without identification of customers. We develop estimators of G and derive exact non-asymptotic expressions for their mean squared errors. The problem of estimating the service time expectation is addressed as well. We present some numerical results on comparison of different estimators of the service time distribution.

Keywords: $M/G/\infty$ queue; nonparametric estimation; Poisson point process; rates of convergence

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Special weak Dirichlet processes and BSDEs driven by a random measure

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This paper considers a forward BSDE driven by a random measure, when the underlying forward process X is a special semimartingale, or even more generally, a special weak Dirichlet process. Given a solution (Y, Z, U), generally Y appears to be of the type $u(t, X_t)$ where u is a deterministic function. In this paper, we identify Z and U in terms of u applying stochastic calculus with respect to weak Dirichlet processes.

Keywords: backward stochastic differential equations; random measure; stochastic integrals for jump processes; weak Dirichlet processes

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Perturbation theory for Markov chains via Wasserstein distance

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Perturbation theory for Markov chains addresses the question of how small differences in the transition probabilities of Markov chains are reflected in differences between their distributions. We prove powerful and flexible bounds on the distance of the nth step distributions of two Markov chains when one of them satisfies a Wasserstein ergodicity condition. Our work is motivated by the recent interest in approximate Markov chain Monte Carlo (MCMC) methods in the analysis of big data sets. By using an approach based on Lyapunov functions, we provide estimates for geometrically ergodic Markov chains under weak assumptions. In an autoregressive model, our bounds cannot be improved in general. We illustrate our theory by showing quantitative estimates for approximate versions of two prominent MCMC algorithms, the Metropolis–Hastings and stochastic Langevin algorithms.

Keywords: big data; Markov chains; MCMC; perturbations; Wasserstein distance

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Gaussian approximation for high dimensional vector under physical dependence

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We develop a Gaussian approximation result for the maximum of a sum of weakly dependent vectors, where the data dimension is allowed to be exponentially larger than sample size. Our result is established under the physical/functional dependence framework. This work can be viewed as a substantive extension of Chernozhukov et al. (*Ann. Statist.* **41** (2013) 2786–2819) to time series based on a variant of Stein's method developed therein.

Keywords: Gaussian approximation; high dimensionality; physical dependence measure; Slepian interpolation; Stein's method; time series

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Equivalence classes of staged trees

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In this paper, we give a complete characterization of the statistical equivalence classes of CEGs and of staged trees. We are able to show that all graphical representations of the same model share a common polynomial description. Then, simple transformations on that polynomial enable us to traverse the corresponding class of graphs. We illustrate our results with a real analysis of the implicit dependence relationships within a previously studied dataset.

Keywords: algebraic statistics; Chain Event Graphs; probability trees; staged trees

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Max-linear models on directed acyclic graphs

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We consider a new recursive structural equation model where all variables can be written as max-linear function of their parental node variables and independent noise variables. The model is max-linear in terms of the noise variables, and its causal structure is represented by a directed acyclic graph. We detail the relation between the weights of the recursive structural equation model and the coefficients in its max-linear representation. In particular, we characterize all max-linear models which are generated by a recursive structural equation model, and show that its max-linear coefficient matrix is the solution of a fixed point equation. We also find the minimum directed acyclic graph representing the recursive structural equations of the variables. The model structure introduces a natural order between the node variables and the max-linear coefficients. This yields representations of the vector components, which are based on the minimum number of node and noise variables.

Keywords: directed acyclic graph; graphical model; max-linear model; minimal representation; path analysis; structural equation model

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Coalescence of Euclidean geodesics on the Poisson–Delaunay triangulation

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Let us consider Euclidean first-passage percolation on the Poisson–Delaunay triangulation. We prove almost sure coalescence of any two semi-infinite geodesics with the same asymptotic direction. The proof is based on an argument of Burton–Keane type and makes use of the concentration property for shortest-path lengths in the considered graphs. Moreover, by considering the specific example of the relative neighborhood graph, we illustrate that our approach extends to further well-known graphs in computational geometry. As an application, we show that the expected number of semi-infinite geodesics starting at a given vertex and leaving a disk of a certain radius grows at most sublinearly in the radius.

Keywords: Burton–Keane argument; coalescence; Delaunay triangulation; first-passage percolation; Poisson point process; relative neighborhood graph; sublinearity

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Sticky processes, local and true martingales

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We prove that for a so-called sticky process S there exists an equivalent probability Q and a Q-martingale $\tilde{S}$ that is arbitrarily close to S in $L^p(Q)$ norm. For continuous S, $\tilde{S}$ can be chosen arbitrarily close to S in supremum norm. In the case where S is a local martingale we may choose Q arbitrarily close to the original probability in the total variation norm. We provide examples to illustrate the power of our results and present an application in mathematical finance.

Keywords: consistent price systems; illiquid markets; martingales; processes with jumps; sticky processes

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A new approach to estimator selection

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In the framework of an abstract statistical model, we discuss how to use the solution of one estimation problem (*Problem A*) in order to construct an estimator in another, completely different, *Problem B*. As a solution of *Problem A* we understand a data-driven selection from a given family of estimators $\mathbf{A}(\mathfrak{H}) = \{\widehat{A}_{\mathfrak{h}}, \mathfrak{h} \in \mathfrak{H}\}$ and establishing for the selected estimator so-called oracle inequality. If $\hat{\mathfrak{h}} \in \mathfrak{H}$ is the selected parameter and $\mathbf{B}(\mathfrak{H}) = \{\widehat{B}_{\mathfrak{h}}, \mathfrak{h} \in \mathfrak{H}\}$ is an estimator's collection built in *Problem B*, we suggest to use the estimator $\widehat{B}_{\hat{\mathfrak{h}}}$. We present very general selection rule led to selector $\hat{\mathfrak{h}}$ and find conditions under which the estimator $\widehat{B}_{\hat{\mathfrak{h}}}$ is reasonable. Our approach is illustrated by several examples related to adaptive estimation.

Keywords: adaptive estimation; density model; generalized deconvolution model; oracle approach; upper function

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Concentration and moderate deviations for Poisson polytopes and polyhedra

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The convex hull generated by the restriction to the unit ball of a stationary Poisson point process in the d-dimensional Euclidean space is considered. By establishing sharp bounds on cumulants, exponential estimates for large deviation probabilities are derived and the relative error in the central limit theorem on a logarithmic scale is investigated for a large class of key geometric characteristics. This includes the number of lower-dimensional faces and the intrinsic volumes of the random polytopes. Furthermore, moderate deviation principles for the spatial empirical measures induced by these functionals are also established using the method of cumulants. The results are applied to a class of zero cells associated with Poisson hyperplane mosaics. As a special case, this comprises the typical Poisson–Voronoi cell conditioned on having large inradius.

Keywords: concentration inequalities; convex hulls; cumulants; deviation probabilities; moderate deviation principles; Poisson hyperplanes; Poisson–Voronoi mosaics; random polytopes; zero cells

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Large deviations for locally monotone stochastic partial differential equations driven by Lévy noise

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We establish a large deviation principle for a type of stochastic partial differential equations (SPDEs) with locally monotone coefficients driven by Lévy noise. The weak convergence method plays an important role.

Keywords: Freidlin–Wentzell type large deviation principle; Levy processes; locally monotone coefficients; stochastic partial differential equations

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Large deviations and applications for Markovian Hawkes processes with a large initial intensity

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Hawkes process is a class of simple point processes that is self-exciting and has clustering effect. The intensity of this point process depends on its entire past history. It has wide applications in finance, insurance, neuroscience, social networks, criminology, seismology, and many other fields. In this paper, we study linear Hawkes process with an exponential kernel in the asymptotic regime where the initial intensity of the Hawkes process is large. We establish large deviations for Hawkes processes in this regime as well as the regime when both the initial intensity and the time are large. We illustrate the strength of our results by discussing the applications to insurance and queueing systems.

Keywords: Hawkes processes; insurance; large deviations; large initial intensity; queueing systems

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Concentration inequalities for separately convex functions

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We provide new comparison inequalities for separately convex functions of independent random variables. Our method is based on the decomposition in Doob martingale. However, we only impose that the martingale increments are stochastically bounded. For this purpose, building on the results of Bentkus (*Lith. Math. J.* **48** (2008) 237–255; *Lith. Math. J.* **48** (2008) 137–157; Bounds for the stop loss premium for unbounded risks under the variance constraints (2010) Preprint), we establish comparison inequalities for random variables stochastically dominated from below and from above. We illustrate our main results by showing how they can be used to derive deviation or moment inequalities for functions which are both separately convex and separately Lipschitz, for weighted empirical distribution functions, for suprema of randomized empirical processes and for chaos of order two.

Keywords: concentration inequalities; deviation inequalities; generalized moments; martingale method; suprema of empirical processes

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Nonparametric volatility estimation in scalar diffusions: Optimality across observation frequencies

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The nonparametric volatility estimation problem of a scalar diffusion process observed at equidistant time points is addressed. Using the spectral representation of the volatility in terms of the invariant density and an eigenpair of the infinitesimal generator the first known estimator that attains the minimax optimal convergence rates for both high and low-frequency observations is constructed. The proofs are based on a posteriori error bounds for generalized eigenvalue problems as well as the path properties of scalar diffusions and stochastic analysis. The finite sample performance is illustrated by a numerical example.

Keywords: diffusion processes; nonparametric estimation; sampling frequency; spectral approximation

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Simultaneous quantile inference for non-stationary long-memory time series

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We consider the simultaneous or functional inference of time-varying quantile curves for a class of non-stationary long-memory time series. New uniform Bahadur representations and Gaussian approximation schemes are established for a broad class of non-stationary long-memory linear processes. Furthermore, an asymptotic distribution theory is developed for the maxima of a class of non-stationary long-memory Gaussian processes. Using the latter theoretical results, simultaneous confidence bands for the aforementioned quantile curves with asymptotically correct coverage probabilities are constructed.

Keywords: heterogeneity; local linear quantile estimation; long memory; simultaneous confidence bands

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Simultaneous nonparametric regression analysis of sparse longitudinal data

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Longitudinal data arise frequently in many scientific inquiries. To capture the dynamic relationship between longitudinal covariates and response, varying coefficient models have been proposed with point-wise inference procedures. This paper considers the challenging problem of asymptotically accurate simultaneous inference of varying coefficient models for sparse and irregularly observed longitudinal data via the local linear kernel method. The error and covariate processes are modeled as very general classes of non-Gaussian and non-stationary processes and are allowed to be statistically dependent. Simultaneous confidence bands (SCBs) with asymptotically correct coverage probabilities are constructed to assess the overall pattern and magnitude of the dynamic association between the response and covariates. A simulation based method is proposed to overcome the problem of slow convergence of the asymptotic results. Simulation studies demonstrate that the proposed inference procedure performs well in realistic settings and is favored over the existing point-wise and Bonferroni methods. A longitudinal dataset from the Chicago Health and Aging Project is used to illustrate our methodology.

Keywords: local polynomial estimation; maximum deviation; nonparametric regression; simultaneous confidence band; sparse longitudinal data

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Nested particle filters for online parameter estimation in discrete-time state-space Markov models

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We address the problem of approximating the posterior probability distribution of the fixed parameters of a state-space dynamical system using a sequential Monte Carlo method. The proposed approach relies on a nested structure that employs two layers of particle filters to approximate the posterior probability measure of the static parameters and the dynamic state variables of the system of interest, in a vein similar to the recent "sequential Monte Carlo square" (SMC^2) algorithm. However, unlike the SMC^2 scheme, the proposed technique operates in a purely recursive manner. In particular, the computational complexity of the recursive steps of the method introduced herein is constant over time. We analyse the approximation of integrals of real bounded functions with respect to the posterior distribution of the system parameters computed via the proposed scheme. As a result, we prove, under regularity assumptions, that the approximation errors vanish asymptotically in L_p ($p \geq 1$) with convergence rate proportional to $\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{M}}$, where N is the number of Monte Carlo samples in the parameter space and $N \times M$ is the number of samples in the state space. This result also holds for the approximation of the joint posterior distribution of the parameters and the state variables. We discuss the relationship between the SMC^2 algorithm and the new recursive method and present a simple example in order to illustrate some of the theoretical findings with computer simulations.

Keywords: error bounds; model inference; Monte Carlo; parameter estimation; particle filtering; recursive algorithms; state space models

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Applications of distance correlation to time series

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The use of empirical characteristic functions for inference problems, including estimation in some special parametric settings and testing for goodness of fit, has a long history dating back to the 70s. More recently, there has been renewed interest in using empirical characteristic functions in other inference settings. The distance covariance and correlation, developed by Székely et al. (*Ann. Statist.* **35** (2007) 2769–2794) and Székely and Rizzo (*Ann. Appl. Stat.* **3** (2009) 1236–1265) for measuring dependence and testing independence between two random vectors, are perhaps the best known illustrations of this. We apply these ideas to stationary univariate and multivariate time series to measure lagged auto- and cross-dependence in a time series. Assuming strong mixing, we establish the relevant asymptotic theory for the sample auto- and cross-distance correlation functions. We also apply the auto-distance correlation function (ADCF) to the residuals of an autoregressive processes as a test of goodness of fit. Under the null that an autoregressive model is true, the limit distribution of the empirical ADCF can differ markedly from the corresponding one based on an i.i.d. sequence. We illustrate the use of the empirical auto- and cross-distance correlation functions for testing dependence and cross-dependence of time series in a variety of contexts.

Keywords: U-statistics; AR process; auto- and cross-distance correlation function; ergodicity; Fourier analysis; residuals; strong mixing; testing independence; time series

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On limit theory for Lévy semi-stationary processes

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In this paper, we present some limit theorems for power variation of Lévy semi-stationary processes in the setting of infill asymptotics. Lévy semi-stationary processes, which are a one-dimensional analogue of ambit fields, are moving average type processes with a multiplicative random component, which is usually referred to as volatility or intermittency. From the mathematical point of view this work extends the asymptotic theory investigated in (*Power variation for a class of stationary increments Lévy driven moving averages*. Preprint), where the authors derived the limit theory for kth order increments of stationary increments Lévy driven moving averages. The asymptotic results turn out to heavily depend on the interplay between the given order of the increments, the considered power $p > 0$, the Blumenthal–Getoor index $\beta \in (0, 2)$ of the driving pure jump Lévy process L and the behaviour of the kernel function g at 0 determined by the power α. In this paper, we will study the first order asymptotic theory for Lévy semi-stationary processes with a random volatility/intermittency component and present some statistical applications of the probabilistic results.

Keywords: high frequency data; Lévy semi-stationary processes; limit theorems; power variation; stable convergence

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Wide consensus aggregation in the Wasserstein space. Application to location-scatter families

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We introduce a general theory for a consensus-based combination of estimations of probability measures. Potential applications include parallelized or distributed sampling schemes as well as variations on aggregation from resampling techniques like boosting or bagging. Taking into account the possibility of very discrepant estimations, instead of a full consensus we consider a "wide consensus" procedure. The approach is based on the consideration of trimmed barycenters in the Wasserstein space of probability measures. We provide general existence and consistency results as well as suitable properties of these robustified Fréchet means. In order to get quick applicability, we also include characterizations of barycenters of probabilities that belong to (non necessarily elliptical) location and scatter families. For these families, we provide an iterative algorithm for the effective computation of trimmed barycenters, based on a consistent algorithm for computing barycenters, guarantying applicability in a wide setting of statistical problems.

Keywords: impartial trimming; parallelized inference; robust aggregation; trimmed barycenter; trimmed distributions; Wasserstein distance; wide consensus

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