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Aims and Scope

BERNOULLI is the journal of the Bernoulli Society for Mathematical Statistics and Probability, issued four times per year. The journal provides a comprehensive account of important developments in the fields of statistics and probability, offering an international forum for both theoretical and applied work.

Bernoulli Society for Mathematical Statistics and Probability

The Bernoulli Society was founded in 1973. It is an autonomous Association of the International Statistical Institute, ISI. According to its statutes, the object of the Bernoulli Society is the advancement, through international contacts, of the sciences of probability (including the theory of stochastic processes) and mathematical statistics and of their applications to all those aspects of human endeavour which are directed towards the increase of natural knowledge and the welfare of mankind.

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Posteriors, conjugacy, and exponential families for completely random measures

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We demonstrate how to calculate posteriors for general Bayesian nonparametric priors and likelihoods based on completely random measures (CRMs). We further show how to represent Bayesian nonparametric priors as a sequence of finite draws using a size-biasing approach – and how to represent full Bayesian nonparametric models via finite marginals. Motivated by conjugate priors based on exponential family representations of likelihoods, we introduce a notion of exponential families for CRMs, which we call exponential CRMs. This construction allows us to specify automatic Bayesian nonparametric conjugate priors for exponential CRM likelihoods. We demonstrate that our exponential CRMs allow particularly straightforward recipes for size-biased and marginal representations of Bayesian nonparametric models. Along the way, we prove that the gamma process is a conjugate prior for the Poisson likelihood process and the beta prime process is a conjugate prior for a process we call the odds Bernoulli process. We deliver a size-biased representation of the gamma process and a marginal representation of the gamma process coupled with a Poisson likelihood process.

Keywords: Bayesian nonparametrics; beta process; completely random measure; conjugacy; exponential family; Indian buffet process; posterior; size-biased

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Applications of pathwise Burkholder–Davis–Gundy inequalities

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In this paper, after generalizing the *pathwise* Burkholder–Davis–Gundy (BDG) inequalities from discrete time to cadlag semimartingales, we present several applications of the pathwise inequalities. In particular we show that they allow to extend the classical BDG inequalities

1. to the Bessel process of order $\alpha \geq 1$
2. to the case of a *random* exponent p
3. to martingales stopped at a time τ which belongs to a well studied class of *random* times

Keywords: Bessel process; Burkholder–Davis–Gundy; pathwise martingale inequalities; pseudo stopping time; semimartingale; variable exponent

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Entropy production in nonlinear recombination models

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We study the convergence to equilibrium of a class of nonlinear recombination models. In analogy with Boltzmann's H-theorem from kinetic theory, and in contrast with previous analysis of these models, convergence is measured in terms of relative entropy. The problem is formulated within a general framework that we refer to as Reversible Quadratic Systems. Our main result is a tight quantitative estimate for the entropy production functional. Along the way, we establish some new entropy inequalities generalizing Shearer's and related inequalities.

Keywords: Boltzmann equation; entropy; functional inequalities; nonlinear equations; population dynamics

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Bounded size biased couplings, log concave distributions and concentration of measure for occupancy models

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Threshold-type counts based on multivariate occupancy models with log concave marginals admit bounded size biased couplings under weak conditions, leading to new concentration of measure results for random graphs, germ-grain models in stochastic geometry and multinomial allocation models. The results obtained compare favorably with classical methods, including the use of McDiarmid's inequality, negative association, and self bounding functions.

Keywords: concentration; coupling; log concave; occupancy; Poisson Binomial distribution

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Parametric inference for nonsynchronously observed diffusion processes in the presence of market microstructure noise

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We study parametric inference for diffusion processes when observations occur nonsynchronously and are contaminated by market microstructure noise. We construct a quasi-likelihood function and study asymptotic mixed normality of maximum-likelihood- and Bayes-type estimators based on it. We also prove the local asymptotic normality of the model and asymptotic efficiency of our estimator when the diffusion coefficients are deterministic and noise follows a normal distribution. We conjecture that our estimator is asymptotically efficient even when the latent process is a general diffusion process. An estimator for the quadratic covariation of the latent process is also constructed. Some numerical examples show that this estimator performs better compared to existing estimators of the quadratic covariation.

Keywords: asymptotic efficiency; Bayes-type estimation; diffusion processes; local asymptotic normality; market microstructure noise; maximum-likelihood-type estimation; nonsynchronous observations; parametric estimation

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The Gamma Stein equation and noncentral de Jong theorems

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Dedicated to the memory of Charles M. Stein

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We study the Stein equation associated with the one-dimensional Gamma distribution, and provide novel bounds, allowing one to effectively deal with test functions supported by the whole real line. We apply our estimates to derive new quantitative results involving random variables that are non-linear functionals of random fields, namely: (i) a non-central quantitative de Jong theorem for sequences of degenerate U-statistics satisfying minimal uniform integrability conditions, significantly extending previous findings by de Jong (*J. Multivariate Anal.* **34** (1990) 275–289), Nourdin, Peccati and Reinert (*Ann. Probab.* **38** (2010) 1947–1985) and Döbler and Peccati (*Electron. J. Probab.* **22** (2017) no. 2), (ii) a new Gamma approximation bound on the Poisson space, refining previous estimates by Peccati and Thäle (*ALEA Lat. Am. J. Probab. Math. Stat.* **10** (2013) 525–560) and (iii) new Gamma bounds on a Gaussian space, strengthening estimates by Nourdin and Peccati (*Probab. Theory Related Fields* **145** (2009) 75–118). As a by-product of our analysis, we also deduce a new inequality for Gamma approximations *via* exchangeable pairs, that is of independent interest.

Keywords: de Jong theorem; degenerate U-statistics; exchangeable pairs; Gamma approximation; Hoeffding decomposition; multiple stochastic integrals; Stein equation; Stein's method

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Expected number and height distribution of critical points of smooth isotropic Gaussian random fields

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We obtain formulae for the expected number and height distribution of critical points of smooth isotropic Gaussian random fields parameterized on Euclidean space or spheres of arbitrary dimension. The results hold in general in the sense that there are no restrictions on the covariance function of the field except for smoothness and isotropy. The results are based on a characterization of the distribution of the Hessian of the Gaussian field by means of the family of Gaussian orthogonally invariant (GOI) matrices, of which the Gaussian orthogonal ensemble (GOE) is a special case. The obtained formulae depend on the covariance function only through a single parameter (Euclidean space) or two parameters (spheres), and include the special boundary case of random Laplacian eigenfunctions.

Keywords: boundary; critical points; Gaussian random fields; GOE; GOI; height density; isotropic; Kac–Rice formula; random matrices; sphere

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A unified matrix model including both CCA and F matrices in multivariate analysis: The largest eigenvalue and its applications

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Let $\mathbf{Z}_{M_1\times N} = \mathbf{T}^{\frac{1}{2}}\mathbf{X}$ where $(\mathbf{T}^{\frac{1}{2}})^2 = \mathbf{T}$ is a positive definite matrix and $\mathbf{X}$ consists of independent random variables with mean zero and variance one. This paper proposes a unified matrix model

$$\Omega = \left(\mathbf{Z}\mathbf{U}_2\mathbf{U}_2^T\mathbf{Z}^T\right)^{-1}\mathbf{Z}\mathbf{U}_1\mathbf{U}_1^T\mathbf{Z}^T,$$

where $\mathbf{U}_1$ and $\mathbf{U}_2$ are isometric with dimensions $N\times N_1$ and $N\times(N-N_2)$ respectively such that $\mathbf{U}_1^T\mathbf{U}_1 = \mathbf{I}_{N_1}$, $\mathbf{U}_2^T\mathbf{U}_2 = \mathbf{I}_{N-N_2}$ and $\mathbf{U}_1^T\mathbf{U}_2 = 0$. Moreover, $\mathbf{U}_1$ and $\mathbf{U}_2$ (random or non-random) are independent of $\mathbf{Z}_{M_1\times N}$ and with probability tending to one, $\operatorname{rank}(\mathbf{U}_1) = N_1$ and $\operatorname{rank}(\mathbf{U}_2) = N - N_2$. We establish the asymptotic Tracy–Widom distribution for its largest eigenvalue under moment assumptions on $\mathbf{X}$ when N_1, N_2 and M_1 are comparable.

The asymptotic distributions of the maximum eigenvalues of the matrices used in Canonical Correlation Analysis (CCA) and of F matrices (including centered and non-centered versions) can be both obtained from that of Ω by selecting appropriate matrices $\mathbf{U}_1$ and $\mathbf{U}_2$. Moreover, via appropriate matrices $\mathbf{U}_1$ and $\mathbf{U}_2$, this matrix Ω can be applied to some multivariate testing problems that cannot be done by both types of matrices. To see this, we explore two more applications. One is in the MANOVA approach for testing the equivalence of several high-dimensional mean vectors, where $\mathbf{U}_1$ and $\mathbf{U}_2$ are chosen to be two nonrandom matrices. The other one is in the multivariate linear model for testing the unknown parameter matrix, where $\mathbf{U}_1$ and $\mathbf{U}_2$ are random. For each application, theoretical results are developed and various numerical studies are conducted to investigate the empirical performance.

Keywords: canonical correlation analysis; F matrix; largest eigenvalue; MANOVA; multivariate linear model; random matrix theory; Tracy–Widom distribution

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Statistical inference for the doubly stochastic self-exciting process

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We introduce and show the existence of a Hawkes self-exciting point process with exponentially-decreasing kernel and where parameters are time-varying. The quantity of interest is defined as the integrated parameter $T^{-1}\int_0^T \theta_t^* \, dt$, where θ_t^* is the time-varying parameter, and we consider the high-frequency asymptotics. To estimate it naïvely, we chop the data into several blocks, compute the maximum likelihood estimator (MLE) on each block, and take the average of the local estimates. The asymptotic bias explodes asymptotically, thus we provide a non-naïve estimator which is constructed as the naïve one when applying a first-order bias reduction to the local MLE. We show the associated central limit theorem. Monte Carlo simulations show the importance of the bias correction and that the method performs well in finite sample, whereas the empirical study discusses the implementation in practice and documents the stochastic behavior of the parameters.

Keywords: Hawkes process; high-frequency data; integrated parameter; self-exciting process; stochastic; time-varying parameter

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Small deviations of a Galton–Watson process with immigration

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We consider a Galton–Watson process with immigration $(\mathcal{Z}_n)$, with offspring probabilities (p_i) and immigration probabilities (q_i). In the case when $p_0 = 0$, $p_1 \neq 0$, $q_0 = 0$ (that is, when $\operatorname{essinf}(\mathcal{Z}_n)$ grows linearly in n), we establish the asymptotics of the left tail $\mathbb{P}\{\mathcal{W} < \varepsilon\}$, as $\varepsilon \downarrow 0$, of the martingale limit $\mathcal{W}$ of the process $(\mathcal{Z}_n)$. Further, we consider the first generation $\mathcal{K}$ such that $\mathcal{Z}_{\mathcal{K}} > \operatorname{essinf}(\mathcal{Z}_{\mathcal{K}})$ and study the asymptotic behaviour of $\mathcal{K}$ conditionally on $\{\mathcal{W} < \varepsilon\}$, as $\varepsilon \downarrow 0$. We find the growth scale and the fluctuations of $\mathcal{K}$ and compare the results with those for standard Galton–Watson processes.

Keywords: conditioning; Galton–Watson processes; Galton–Watson trees; immigration; large deviations; lower tail; martingale limit; small value probabilities

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Testing for simultaneous jumps in case of asynchronous observations

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This paper proposes a novel test for simultaneous jumps in a bivariate Itô semimartingale when observation times are asynchronous and irregular. Inference is built on a realized correlation coefficient for the squared jumps of the two processes which is estimated using bivariate power variations of Hayashi–Yoshida type without an additional synchronization step. An associated central limit theorem is shown whose asymptotic distribution is assessed using a bootstrap procedure. Simulations show that the test works remarkably well in comparison with the much simpler case of regular observations.

Keywords: asynchronous observations; common jumps; high-frequency statistics; Itô semimartingale; stable convergence

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Statistical estimation of the Oscillating Brownian Motion

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We study the asymptotic behavior of estimators of a two-valued, discontinuous diffusion coefficient in a Stochastic Differential Equation, called an Oscillating Brownian Motion. Using the relation of the latter process with the Skew Brownian Motion, we propose two natural consistent estimators, which are variants of the integrated volatility estimator and take the occupation times into account. We show the stable convergence of the renormalized errors' estimations toward some Gaussian mixture, possibly corrected by a term that depends on the local time. These limits stem from the lack of ergodicity as well as the behavior of the local time at zero of the process. We test both estimators on simulated processes, finding a complete agreement with the theoretical predictions.

Keywords: arcsine distribution; Gaussian mixture; local time; occupation time; Oscillating Brownian Motion; Skew Brownian Motion

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Correlated continuous time random walks and fractional Pearson diffusions

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Continuous time random walks have random waiting times between particle jumps. We define the correlated continuous time random walks (CTRWs) that converge to fractional Pearson diffusions (fPDs). The jumps in these CTRWs are obtained from Markov chains through the Bernoulli urn-scheme model and Wright–Fisher model. The jumps are correlated so that the limiting processes are not Lévy but diffusion processes with non-independent increments. The waiting times are selected from the domain of attraction of a stable law.

Keywords: continuous time random walks; fractional diffusion; Markov chains; Pearson diffusions; urn-scheme models; Wright–Fisher model

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Detecting Markov random fields hidden in white noise

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Motivated by change point problems in time series and the detection of textured objects in images, we consider the problem of detecting a piece of a Gaussian Markov random field hidden in white Gaussian noise. We derive minimax lower bounds and propose near-optimal tests.

Keywords: Markov random fields; detection; combinatorial testing; minimax test; image analysis

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Large volatility matrix estimation with factor-based diffusion model for high-frequency financial data

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Large volatility matrices are involved in many finance practices, and estimating large volatility matrices based on high-frequency financial data encounters the "curse of dimensionality". It is a common approach to impose a sparsity assumption on the large volatility matrices to produce consistent volatility matrix estimators. However, due to the existence of common factors, assets are highly correlated with each other, and it is not reasonable to assume the volatility matrices are sparse in financial applications. This paper incorporates factor influence in the asset pricing model and investigates large volatility matrix estimation under the factor price model together with some sparsity assumption. We propose to model asset prices by assuming that asset prices are governed by common factors and that the assets with similar characteristics share the same association with the factors. We then impose some reasonable sparsity condition on the part of the volatility matrices after accounting for the factor contribution. Under the proposed factor-based model and sparsity assumption, we develop an estimation scheme called "blocking and regularizing". Asymptotic properties of the proposed estimator are studied, and its finite sample performance is tested via extensive numerical studies to support theoretical results.

Keywords: adaptive threshold; diffusion; factor model; integrated volatility; kernel realized volatility; multiple-scale realized volatility; pre-averaging realized volatility; regularization; sparsity

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Adaptive estimation of high-dimensional signal-to-noise ratios

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We consider the equivalent problems of estimating the residual variance, the proportion of explained variance η and the signal strength in a high-dimensional linear regression model with Gaussian random design. Our aim is to understand the impact of not knowing the sparsity of the vector of regression coefficients and not knowing the distribution of the design on minimax estimation rates of η. Depending on the sparsity k of the vector regression coefficients, optimal estimators of η either rely on estimating the vector of regression coefficients or are based on U-type statistics. In the important situation where k is unknown, we build an adaptive procedure whose convergence rate simultaneously achieves the minimax risk over all k up to a logarithmic loss which we prove to be non avoidable. Finally, the knowledge of the design distribution is shown to play a critical role. When the distribution of the design is unknown, consistent estimation of explained variance is indeed possible in much narrower regimes than for known design distribution.

Keywords: heritability; minimax analysis; quadratic functional; signal to noise ratio

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Efficient strategy for the Markov chain Monte Carlo in high-dimension with heavy-tailed target probability distribution

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The purpose of this paper is to introduce a new Markov chain Monte Carlo method and to express its effectiveness by simulation and high-dimensional asymptotic theory. The key fact is that our algorithm has a reversible proposal kernel, which is designed to have a heavy-tailed invariant probability distribution. A high-dimensional asymptotic theory is studied for a class of heavy-tailed target probability distributions. When the number of dimensions of the state space passes to infinity, we will show that our algorithm has a much higher convergence rate than the pre-conditioned Crank–Nicolson (pCN) algorithm and the random-walk Metropolis algorithm.

Keywords: Consistency; Malliavin calculus; Markov chain; Monte Carlo; Stein's method

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The class of multivariate max-id copulas with ℓ_1-norm symmetric exponent measure

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Members of the well-known family of bivariate Galambos copulas can be expressed in a closed form in terms of the univariate Fréchet distribution. This formula extends to any dimension and can be used to define a whole new class of tractable multivariate copulas that are generated by suitable univariate distributions. This paper gives necessary and sufficient conditions on the underlying univariate distribution which ensure that the resulting copula exists. It is also shown that these new copulas are in fact dependence structures of certain max-id distributions with ℓ_1-norm symmetric exponent measure. The basic dependence properties of this new class of multivariate exchangeable copulas is investigated, and an efficient algorithm is provided for generating observations from distributions in this class.

Keywords: Clayton copula; completely monotone function; exponent measure; Galambos copula; Laplace transform; ℓ_1-norm symmetric max-id distributions

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Optimal estimation of a large-dimensional covariance matrix under Stein's loss

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This paper introduces a new method for deriving covariance matrix estimators that are decision-theoretically optimal within a class of nonlinear shrinkage estimators. The key is to employ large-dimensional asymptotics: the matrix dimension and the sample size go to infinity together, with their ratio converging to a finite, nonzero limit. As the main focus, we apply this method to Stein's loss. Compared to the estimator of Stein (Estimation of a covariance matrix (1975); *J. Math. Sci.* **34** (1986) 1373–1403), ours has five theoretical advantages: (1) it asymptotically minimizes the loss itself, instead of an estimator of the expected loss; (2) it does not necessitate post-processing via an *ad hoc* algorithm (called "isotonization") to restore the positivity or the ordering of the covariance matrix eigenvalues; (3) it does not ignore any terms in the function to be minimized; (4) it does not require normality; and (5) it is not limited to applications where the sample size exceeds the dimension. In addition to these theoretical advantages, our estimator also improves upon Stein's estimator in terms of finite-sample performance, as evidenced via extensive Monte Carlo simulations. To further demonstrate the effectiveness of our method, we show that some previously suggested estimators of the covariance matrix and its inverse are decision-theoretically optimal in the large-dimensional asymptotic limit with respect to the Frobenius loss function.

Keywords: large-dimensional asymptotics; nonlinear shrinkage estimation; random matrix theory; rotation equivariance; Stein's loss

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Covariance estimation via sparse Kronecker structures

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The problem of estimating covariance matrices is central to statistical analysis and is extensively addressed when data are vectors. This paper studies a novel Kronecker-structured approach for estimating such matrices when data are matrices and arrays. Focusing on matrix-variate data, we present simple approaches to estimate the row and the column correlation matrices, formulated separately via convex optimization. We also discuss simple thresholding estimators motivated by the recent development in the literature. Non-asymptotic results show that the proposed method greatly outperforms methods that ignore the matrix structure of the data. In particular, our framework allows the dimensionality of data to be arbitrary order even for fixed sample size, and works for flexible distributions beyond normality. Simulations and data analysis further confirm the competitiveness of the method. An extension to general array-data is also outlined.

Keywords: covariance matrix; Kronecker structure; matrix data; non-asymptotic bound

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Robust dimension-free Gram operator estimates

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In this paper, we investigate the question of estimating the Gram operator by a robust estimator from an i.i.d. sample in a separable Hilbert space and we present uniform bounds that hold under weak moment assumptions. The approach consists in first obtaining non-asymptotic dimension-free bounds in finite-dimensional spaces using some PAC-Bayesian inequalities related to Gaussian perturbations of the parameter and then in generalizing the results in a separable Hilbert space. We show both from a theoretical point of view and with the help of some simulations that such a robust estimator improves the behavior of the classical empirical one in the case of heavy tail data distributions.

Keywords: dimension-free bounds; Gram operator; PAC-Bayesian learning; robust estimation

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Uniform dimension results for a family of Markov processes

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In this paper, we prove uniform Hausdorff and packing dimension results for the images of a large family of Markov processes. The main tools are the two covering principles in Xiao (In *Fractal Geometry and Applications: A Jubilee of Benoît Mandelbrot, Part 2* (2004) 261–338 Amer. Math. Soc.). As applications, uniform Hausdorff and packing dimension results for certain classes of Lévy processes, stable jump diffusions and non-symmetric stable-type processes are obtained.

Keywords: cover principles; Markov processes; uniform Hausdorff dimension

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