BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Volume Twenty Five Number Four Part B November 2019 ISSN: 1350-7265

CONTENTS

(continued)

The papers published in Bernoulli are indexed or abstracted in *Current Index to Statistics*, *Mathematical Reviews*, *Statistical Theory and Method Abstracts-Zentralblatt (STMA-Z)*, and *Zentralblatt für Mathematik* (also avalaible on the *MATH via STN* database and *Compact MATH CD-ROM*). A list of forthcoming papers can be found online at http://www.bernoulli-society.org/index.php/publications/bernoulli-journal/bernoulli-journal-papers

BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Volume Twenty Five Number Four Part B November 2019 ISSN: 1350-7265

CONTENTS

(continued)

BERNOULLI Volume 25 Number 4B November 2019 Pages 3203–4006 ISI/BS

Volume 25 Number 4B November 2019 ISSN 1350-7265

BERNOULLI

Official Journal of the Bernoulli Society for Mathematical Statistics and Probability

Aims and Scope

BERNOULLI is the journal of the Bernoulli Society for Mathematical Statistics and Probability, issued four times per year. The journal provides a comprehensive account of important developments in the fields of statistics and probability, offering an international forum for both theoretical and applied work.

Bernoulli Society for Mathematical Statistics and Probability

The Bernoulli Society was founded in 1973. It is an autonomous Association of the International Statistical Institute, ISI. According to its statutes, the object of the Bernoulli Society is the advancement, through international contacts, of the sciences of probability (including the theory of stochastic processes) and mathematical statistics and of their applications to all those aspects of human endeavour which are directed towards the increase of natural knowledge and the welfare of mankind.

Meetings: http://www.bernoulli-society.org/index.php/meetings

The Society holds a World Congress every four years; more frequent meetings, coordinated by the Society's standing committees and often organised in collaboration with other organisations, are the European Meeting of Statisticians, the Conference on Stochastic Processes and their Applications, the CLAPEM meeting (Latin-American Congress on Probability and Mathematical Statistics), the European Young Statisticians Meeting, and various meetings on special topics – in the physical sciences in particular. The Society, as an association of the ISI, also collaborates with other ISI associations in the organization of the biennial ISI World Statistics Congresses (formerly ISI Sessions).

Executive Committee

The papers published in Bernoulli are indexed or abstracted in *Current Index to Statistics, Mathematical Reviews, Statistical Theory and Method Abstracts-Zentralblatt (STMA-Z), Thomson Scientific* and *Zentralblatt für Mathematik* (also available on the *MATH via STN* database and *Compact MATH CD-ROM*). A list of forthcoming papers can be found online at http://www.bernoulli-society.org/index.php/publications/bernoulli-journal/bernoulli-journal-papers

In 2019 Bernoulli consists of 4 issues published in February, May, August and November.

RESURGO MUTATA EADEM

Bernoulli Society
for Mathematical Statistics
and Probability

Bernoulli **25**(4B), 2019, 3203–3233
https://doi.org/10.3150/18-BEJ1088

Functional CLT for martingale-like nonstationary dependent structures

FLORENCE MERLEVÈDE[1], MAGDA PELIGRAD[2] and SERGEY UTEV[3]

[1]*Université Paris-Est, LAMA (UMR 8050), UPEM, CNRS, UPEC, France. E-mail: florence.merlevede@u-pem.fr*
[2]*University of Cincinnati, USA. E-mail: peligrm@ucmail.uc.edu*
[3]*University of Leicester, United Kingdom. E-mail: su35@leicester.ac.uk*

In this paper, we develop non-stationary martingale techniques for dependent data. We shall stress the non-stationary version of the projective Maxwell–Woodroofe condition, which will be essential for obtaining maximal inequalities and functional central limit theorem for the following examples: nonstationary ρ-mixing sequences, functions of linear processes with non-stationary innovations, locally stationary processes, quenched version of the functional central limit theorem for a stationary sequence, evolutions in random media such as a process sampled by a shifted Markov chain.

Keywords: ρ-mixing arrays; dependent structures; functional central limit theorem; non-stationary triangular arrays; projective criteria

References

[1] Aldous, D. (1978). Stopping times and tightness. *Ann. Probab.* **6** 335–340. MR0474446
[2] Billingsley, P. (1968). *Convergence of Probability Measures*. New York: Wiley. MR0233396
[3] Billingsley, P. (1995). *Probability and Measure*, 3rd ed. *Wiley Series in Probability and Mathematical Statistics*. New York: Wiley. A Wiley-Interscience Publication. MR1324786
[4] Bradley, R.C. (2007). *Introduction to Strong Mixing Conditions*. *Vol*. 2. Heber City, UT: Kendrick Press. MR2325295
[5] Cuny, C., Dedecker, J. and Merlevède, F. (2017). Large and moderate deviations for the left random walk on $GL_d(\mathbb{R})$. *ALEA Lat. Am. J. Probab. Math. Stat.* **14** 503–527. MR3659265
[6] Cuny, C. and Merlevède, F. (2014). On martingale approximations and the quenched weak invariance principle. *Ann. Probab.* **42** 760–793. MR3178473
[7] Dahlhaus, R., Richter, S. and Wu, W.B. (2018). Towards a general theory for non-linear locally stationary processes. *Bernoulli*. To appear.
[8] Dedecker, J. (2008). Inégalités de Hoeffding et théorème limite central pour des fonctions peu régulières de chaînes de Markov non irréductibles. *Ann. I.S.U.P.* **52** 39–46. MR2435039
[9] Dedecker, J., Merlevède, F. and Peligrad, M. (2014). A quenched weak invariance principle. *Ann. Inst. Henri Poincaré Probab. Stat.* **50** 872–898. MR3224292
[10] Dobrushin, R. (1956). Central limit theorems for non-stationary Markov chains I, II. *Theory Probab. Appl.* **1** 65–80, 329–383.
[11] Durieu, O. (2009). Independence of four projective criteria for the weak invariance principle. *ALEA Lat. Am. J. Probab. Math. Stat.* **5** 21–26. MR2475604
[12] Gänssler, P. and Häusler, E. (1979). Remarks on the functional central limit theorem for martingales. *Z. Wahrsch. Verw. Gebiete* **50** 237–243. MR0554543

[13] Gordin, M. and Peligrad, M. (2011). On the functional central limit theorem via martingale approximation. *Bernoulli* **17** 424–440. MR2797997

[14] Gordin, M.I. (1969). The central limit theorem for stationary processes. *Sov. Math., Dokl.* **10** 1174–1176.

[15] Hall, P. and Heyde, C.C. (1980). *Martingale Limit Theory and Its Application. Probability and Mathematical Statistics*. New York–London: Academic Press [Harcourt Brace Jovanovich, Publishers]. MR0624435

[16] Helland, I.S. (1982). Central limit theorems for martingales with discrete or continuous time. *Scand. J. Stat.* **9** 79–94. MR0668684

[17] Heyde, C.C. (1974). On the central limit theorem for stationary processes. *Z. Wahrsch. Verw. Gebiete* **30** 315–320. MR0372955

[18] Krengel, U. (1985). *Ergodic Theorems. De Gruyter Studies in Mathematics* **6**. Berlin: de Gruyter. With a supplement by Antoine Brunel. MR0797411

[19] Maxwell, M. and Woodroofe, M. (2000). Central limit theorems for additive functionals of Markov chains. *Ann. Probab.* **28** 713–724. MR1782272

[20] McLeish, D.L. (1975). Invariance principles for dependent variables. *Z. Wahrsch. Verw. Gebiete* **32** 165–178. MR0388483

[21] McLeish, D.L. (1977). On the invariance principle for nonstationary mixingales. *Ann. Probab.* **5** 616–621. MR0445583

[22] Merlevède, F., Peligrad, M. and Utev, S. (2019). Supplement to "Functional CLT for martingale-like nonstationary dependent structures." DOI:10.3150/18-BEJ1088SUPP.

[23] Peligrad, M. (2012). Central limit theorem for triangular arrays of non-homogeneous Markov chains. *Probab. Theory Related Fields* **154** 409–428. MR3000551

[24] Peligrad, M. and Utev, S. (2005). A new maximal inequality and invariance principle for stationary sequences. *Ann. Probab.* **33** 798–815. MR2123210

[25] Sethuraman, S. and Varadhan, S.R.S. (2005). A martingale proof of Dobrushin's theorem for non-homogeneous Markov chains. *Electron. J. Probab.* **10** 1221–1235. MR2164043

[26] Shao, Q.M. (1989). On the invariance principle for ρ-mixing sequences of random variables. *Chin. Ann. Math. Ser. B* **10** 427–433. A Chinese summary appears in *Chin. Ann. Math. Ser. A* **10** (1989), no. 5, 640. MR1038376

[27] Utev, S.A. (1989). Sums of φ-mixing random variables. *Siberian Adv. Math.* **1** 24–155. Translation of *Tr. Inst. Mat. (Novosib.)* **13** (1989), Asimptot. Analiz Raspred. Sluch. Protsess., 78–100.

[28] Utev, S.A. (1990). Central limit theorem for dependent random variables. In *Probability Theory and Mathematical Statistics, Vol. II* (*Vilnius*, 1989) 519–528. Vilnius: "Mokslas". MR1153906

[29] Vogt, M. (2012). Nonparametric regression for locally stationary time series. *Ann. Statist.* **40** 2601–2633. MR3097614

[30] Wu, W.B. and Zhao, Z. (2008). Moderate deviations for stationary processes. *Statist. Sinica* **18** 769–782. MR2411619

[31] Wu, W.B. and Zhou, Z. (2011). Gaussian approximations for non-stationary multiple time series. *Statist. Sinica* **21** 1397–1413. MR2827528

[32] Zhao, O. and Woodroofe, M. (2008). On martingale approximations. *Ann. Appl. Probab.* **18** 1831–1847. MR2462550

Bernoulli **25**(4B), 2019, 3234–3275
https://doi.org/10.3150/18-BEJ1089

Rate of convergence to equilibrium for discrete-time stochastic dynamics with memory

MAYLIS VARVENNE

Institut de Mathématiques de Toulouse, 118 route de Narbonne, 31400 Toulouse, France.
E-mail: maylis.varvenne@math.univ-toulouse.fr

The main objective of the paper is to study the long-time behavior of general discrete dynamics driven by an ergodic stationary Gaussian noise. In our main result, we prove existence and uniqueness of the invariant distribution and exhibit some upper-bounds on the rate of convergence to equilibrium in terms of the asymptotic behavior of the covariance function of the Gaussian noise (or equivalently to its moving average representation). Then, we apply our general results to fractional dynamics (including the Euler Scheme associated to fractional driven Stochastic Differential Equations). When the Hurst parameter H belongs to $(0, 1/2)$ we retrieve, with a slightly more explicit approach due to the discrete-time setting, the rate exhibited by Hairer in a continuous time setting (*Ann. Probab.* **33** (2005) 703–758). In this fractional setting, we also emphasize the significant dependence of the rate of convergence to equilibrium on the local behaviour of the covariance function of the Gaussian noise.

Keywords: discrete stochastic dynamics; Lyapunov function; rate of convergence to equilibrium; stationary Gaussian noise; Toeplitz operator; total variation distance

References

[1] Bradley, R.C. (2005). Basic properties of strong mixing conditions. A survey and some open questions. *Probab. Surv.* **2** 107–144. Update of, and a supplement to, the 1986 original. MR2178042
[2] Brockwell, P.J. and Davis, R.A. (2013). *Time Series*: *Theory and Methods*. Springer Science & Business Media.
[3] Brouste, A., Cai, C. and Kleptsyna, M. (2014). Asymptotic properties of the MLE for the autoregressive process coefficients under stationary Gaussian noise. *Math. Methods Statist.* **23** 103–115. MR3224634
[4] Brouste, A. and Kleptsyna, M. (2012). Kalman type filter under stationary noises. *Systems Control Lett.* **61** 1229–1234. MR2998208
[5] Cohen, S. and Panloup, F. (2011). Approximation of stationary solutions of Gaussian driven stochastic differential equations. *Stochastic Process. Appl.* **121** 2776–2801. MR2844540
[6] Cohen, S., Panloup, F. and Tindel, S. (2014). Approximation of stationary solutions to SDEs driven by multiplicative fractional noise. *Stochastic Process. Appl.* **124** 1197–1225. MR3148010
[7] Deya, A., Panloup, F. and Tindel, S. (2016). Rate of convergence to equilibrium of fractional driven stochastic differential equations with rough multiplicative noise. arXiv preprint, arXiv:1605.00880.
[8] Down, D., Meyn, S.P. and Tweedie, R.L. (1995). Exponential and uniform ergodicity of Markov processes. *Ann. Probab.* **23** 1671–1691. MR1379163

[9] Fontbona, J. and Panloup, F. (2017). Rate of convergence to equilibrium of fractional driven stochastic differential equations with some multiplicative noise. *Ann. Inst. Henri Poincaré Probab. Stat.* **53** 503–538. MR3634264
[10] Ford, N.J., Savostyanov, D.V. and Zamarashkin, N.L. (2014). On the decay of the elements of inverse triangular Toeplitz matrices. *SIAM J. Matrix Anal. Appl.* **35** 1288–1302. MR3272624
[11] Guasoni, P. (2006). No arbitrage under transaction costs, with fractional Brownian motion and beyond. *Math. Finance* **16** 569–582. MR2239592
[12] Hairer, M. (2005). Ergodicity of stochastic differential equations driven by fractional Brownian motion. *Ann. Probab.* **33** 703–758. MR2123208
[13] Hairer, M. (2008). Ergodic properties of a class of non-Markovian processes. *Lecture Notes*.
[14] Hairer, M. and Ohashi, A. (2007). Ergodic theory for SDEs with extrinsic memory. *Ann. Probab.* **35** 1950–1977. MR2349580
[15] Hairer, M. and Pillai, N.S. (2013). Regularity of laws and ergodicity of hypoelliptic SDEs driven by rough paths. *Ann. Probab.* **41** 2544–2598. MR3112925
[16] Jeon, J.-H., Tejedor, V., Burov, S., Barkai, E., Selhuber-Unkel, C., Berg-Sørensen, K., Oddershede, L. and Metzler, R. (2011). In vivo anomalous diffusion and weak ergodicity breaking of lipid granules. *Phys. Rev. Lett.* **106** 048103.
[17] Kolmogorov, A.N. and Rozanov, Ju.A. (1960). On a strong mixing condition for stationary Gaussian processes. *Theory Probab. Appl.* **5** 204–208. MR0133175
[18] Kou, S.C. (2008). Stochastic modeling in nanoscale biophysics: Subdiffusion within proteins. *Ann. Appl. Stat.* **2** 501–535. MR2524344
[19] Odde, D.J., Tanaka, E.M., Hawkins, S.S. and Buettner, H.M. (1996). Stochastic dynamics of the nerve growth cone and its microtubules during neurite outgrowth. *Biotechnol. Bioeng.* **50** 452–461.
[20] Ostry, D.I. (2006). Synthesis of accurate fractional Gaussian noise by filtering. *IEEE Trans. Inform. Theory* **52** 1609–1623. MR2241208
[21] Panloup, F. and Richard, A. (2018). Sub-exponential convergence to equilibrium for gaussian driven stochastic differential equations with semi-contractive drift. arXiv preprint, arXiv:1804.01348.
[22] Rosenblatt, M. (1972). Central limit theorem for stationary processes. In *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability, Volume 2: Probability Theory* 551–561. Berkeley: Univ. California Press. MR0402869
[23] Varvenne, M. (2019). Supplement to "Rate of convergence to equilibrium for discrete-time stochastic dynamics with memory." DOI:10.3150/18-BEJ1089SUPP.

Bernoulli **25**(4B), 2019, 3276–3310
https://doi.org/10.3150/18-BEJ1090

Least squares estimation in the monotone single index model

FADOUA BALABDAOUI[1,2], CÉCILE DUROT[3] and HANNA JANKOWSKI[4]

[1]*Seminar für Statistik, ETH Zürich, 8092, Zürich, Schweiz. E-mail: fadoua.balabdaoui@stat.math.ethz.ch*
[2]*Université Paris-Dauphine, PSL Research University, CNRS, CEREMADE, 75016 Paris, France*
[3]*Modal'x, UPL, Univ Paris Nanterre, F92000 Nanterre, France. E-mail: cecile.durot@gmail.com*
[4]*Department of Mathematics and Statistics, York University, 4700 Keele Street, Toronto ON, M3J 1P3, Canada. E-mail: hkj@yorku.ca*

We study the monotone single index model where a real response variable Y is linked to a d-dimensional covariate X through the relationship $E[Y|X] = \Psi_0(\alpha_0^T X)$, almost surely. Both the ridge function, Ψ_0, and the index parameter, α_0, are unknown and the ridge function is assumed to be monotone. Under some appropriate conditions, we show that the rate of convergence in the L_2-norm for the least squares estimator of the bundled function $\Psi_0(\alpha_0^T \cdot)$ is $n^{1/3}$. A similar result is established for the isolated ridge function, and the index is shown to converge at least at the rate $n^{1/3}$. Since the least squares estimator of the index is computationally intensive, we also consider alternative estimators of the index α_0 from earlier literature. Moreover, we show that if the rate of convergence of such an alternative estimator is at least $n^{1/3}$, then the corresponding least-squares type estimators (obtained via a "plug-in" approach) of both the bundled and isolated ridge functions still converge at the rate $n^{1/3}$.

Keywords: least squares; maximum likelihood; monotone; semi-parametric; shape-constraints; single-index model

References

[1] Balabdaoui, F., Durot, C. and Jankowski, H. (2019). Supplement to "Least squares estimation in the monotone single index model." DOI:10.3150/18-BEJ1090SUPP.

[2] Barlow, R.E., Bartholomew, D.J., Bremner, J.M. and Brunk, H.D. (1972). *Statistical Inference Under Order Restrictions. The Theory and Application of Isotonic Regression*. London–Sydney: Wiley. Wiley Series in Probability and Mathematical Statistics. MR0326887

[3] Brillinger, D.R. (1983). A generalized linear model with "Gaussian" regressor variables. In *A Festschrift for Erich L. Lehmann*. *Wadsworth Statist./Probab. Ser.* 97–114. Belmont, CA: Wadsworth. MR0689741

[4] Chen, Y. and Samworth, R.J. (2016). Generalized additive and index models with shape constraints. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **78** 729–754. MR3534348

[5] Chiou, J.-M. and Müller, H.-G. (2004). Quasi-likelihood regression with multiple indices and smooth link and variance functions. *Scand. J. Stat.* **31** 367–386. MR2087831

[6] Chmielewski, M.A. (1981). Elliptically symmetric distributions: A review and bibliography. *Int. Stat. Rev.* **49** 67–74. MR0623010

[7] Cosslett, S.R. (1983). Distribution-free maximum likelihood estimator of the binary choice model. *Econometrica* **51** 765–782. MR0712369

1350-7265

[8] Cover, T.M. (1967). The number of linearly inducible orderings of points in d-space. *SIAM J. Appl. Math.* **15** 434–439. MR0211892

[9] Dobson, A.J. and Barnett, A.G. (2008). *An Introduction to Generalized Linear Models*, 3rd ed. *Texts in Statistical Science Series*. Boca Raton, FL: CRC Press. MR2459739

[10] Feige, U. and Schechtman, G. (2002). On the optimality of the random hyperplane rounding technique for MAX CUT. *Random Structures Algorithms* **20** 403–440. Probabilistic methods in combinatorial optimization. MR1900615

[11] Foster, J.C., Taylor, J.M.G. and Nan, B. (2013). Variable selection in monotone single-index models via the adaptive LASSO. *Stat. Med.* **32** 3944–3954. MR3102450

[12] Ganti, R., Rao, N., Willett, R.M. and Nowak, R. (2015). Learning single index models in high dimensions. Preprint. Available at arXiv:1506.08910.

[13] Goldstein, L., Minsker, S. and Wei, X. (2018). Structured signal recovery from non-linear and heavy-tailed measurements. *IEEE Trans. Inform. Theory* **64** 5513–5530. MR3832320

[14] Groeneboom, P. and Hendrickx, K. (2018). Current status linear regression. *Ann. Statist.* **46** 1415–1444. MR3819105

[15] Han, A.K. (1987). Nonparametric analysis of a generalized regression model. The maximum rank correlation estimator. *J. Econometrics* **35** 303–316. MR0903188

[16] Härdle, W., Hall, P. and Ichimura, H. (1993). Optimal smoothing in single-index models. *Ann. Statist.* **21** 157–178. MR1212171

[17] Hristache, M., Juditsky, A. and Spokoiny, V. (2001). Direct estimation of the index coefficient in a single-index model. *Ann. Statist.* **29** 595–623. MR1865333

[18] Kakade, S.M., Kanade, V., Shamir, O. and Kalai, A. (2011). Efficient learning of generalized linear and single index models with isotonic regression. In *Advances in Neural Information Processing Systems* 927–935.

[19] Kalai, A. and Sastry, R. (2009). The isotron algorithm: High-dimensional isotonic regression. In *Proceedings of the* 22*nd Annual Conference on Learning Theory* (*COLT*).

[20] Li, K.-C. and Duan, N. (1989). Regression analysis under link violation. *Ann. Statist.* **17** 1009–1052. MR1015136

[21] Li, Q. and Racine, J.S. (2007). *Nonparametric Econometrics*: *Theory and Practice*. Princeton, NJ: Princeton Univ. Press. MR2283034

[22] Lin, W. and Kulasekera, K.B. (2007). Identifiability of single-index models and additive-index models. *Biometrika* **94** 496–501. MR2380574

[23] Murphy, S.A., van der Vaart, A.W. and Wellner, J.A. (1999). Current status regression. *Math. Methods Statist.* **8** 407–425. MR1735473

[24] Plan, Y. and Vershynin, R. (2013). Robust 1-bit compressed sensing and sparse logistic regression: A convex programming approach. *IEEE Trans. Inform. Theory* **59** 482–494. MR3008160

[25] Plan, Y., Vershynin, R. and Yudovina, E. (2017). High-dimensional estimation with geometric constraints. *Inf. Inference* **6** 1–40. MR3636866

[26] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes*: *With Applications to Statistics*. *Springer Series in Statistics*. New York: Springer. MR1385671

Bernoulli **25**(4B), 2019, 3311–3338
https://doi.org/10.3150/18-BEJ1091

Adaptively weighted group Lasso for semiparametric quantile regression models

TOSHIO HONDA[1], CHING-KANG ING[2] and WEI-YING WU[3]

[1]*Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. E-mail: t.honda@r.hit-u.ac.jp*

[2]*Institute of Statistics, National Tsing Hua University, 101, Section 2, Kuang-Fu Rd., Hsinchu 30013, Taiwan. E-mail: cking@stat.nthu.edu.tw*

[3]*Department of Applied Mathematics, National Dong Hwa University, Taiwan. E-mail: wuweiying1011@gmail.com*

We propose an adaptively weighted group Lasso procedure for simultaneous variable selection and structure identification for varying coefficient quantile regression models and additive quantile regression models with ultra-high dimensional covariates. Under a strong sparsity condition, we establish selection consistency of the proposed Lasso procedure when the weights therein satisfy a set of general conditions. This consistency result, however, is reliant on a suitable choice of the tuning parameter for the Lasso penalty, which can be hard to make in practice. To alleviate this difficulty, we suggest a BIC-type criterion, which we call high-dimensional information criterion (HDIC), and show that the proposed Lasso procedure with the tuning parameter determined by HDIC still achieves selection consistency. Our simulation studies support strongly our theoretical findings.

Keywords: additive models; B-spline; high-dimensional information criteria; Lasso; structure identification; varying coefficient models

References

[1] Belloni, A. and Chernozhukov, V. (2011). ℓ_1-penalized quantile regression in high-dimensional sparse models. *Ann. Statist.* **39** 82–130. MR2797841

[2] Bickel, P.J., Ritov, Y. and Tsybakov, A.B. (2009). Simultaneous analysis of lasso and Dantzig selector. *Ann. Statist.* **37** 1705–1732. MR2533469

[3] Bühlmann, P. and van de Geer, S. (2011). *Statistics for High-Dimensional Data: Methods, Theory and Applications. Springer Series in Statistics*. Heidelberg: Springer. MR2807761

[4] Cai, Z. and Xiao, Z. (2012). Semiparametric quantile regression estimation in dynamic models with partially varying coefficients. *J. Econometrics* **167** 413–425. MR2892085

[5] Chen, J. and Chen, Z. (2008). Extended Bayesian information criteria for model selection with large model spaces. *Biometrika* **95** 759–771. MR2443189

[6] Cheng, M.-Y., Honda, T., Li, J. and Peng, H. (2014). Nonparametric independence screening and structure identification for ultra-high dimensional longitudinal data. *Ann. Statist.* **42** 1819–1849. MR3262469

[7] Fan, J., Fan, Y. and Barut, E. (2014). Adaptive robust variable selection. *Ann. Statist.* **42** 324–351. MR3189488

[8] Fan, J., Feng, Y. and Song, R. (2011). Nonparametric independence screening in sparse ultra-high-dimensional additive models. *J. Amer. Statist. Assoc.* **106** 544–557. MR2847969

[9] Fan, J. and Li, R. (2001). Variable selection via nonconcave penalized likelihood and its oracle properties. *J. Amer. Statist. Assoc.* **96** 1348–1360. MR1946581
[10] Fan, J., Ma, Y. and Dai, W. (2014). Nonparametric independence screening in sparse ultra-high-dimensional varying coefficient models. *J. Amer. Statist. Assoc.* **109** 1270–1284. MR3265696
[11] Fan, J. and Song, R. (2010). Sure independence screening in generalized linear models with NP-dimensionality. *Ann. Statist.* **38** 3567–3604. MR2766861
[12] Fan, J., Xue, L. and Zou, H. (2014). Strong oracle optimality of folded concave penalized estimation. *Ann. Statist.* **42** 819–849. MR3210988
[13] Fan, J., Xue, L. and Zou, H. (2016). Multitask quantile regression under the transnormal model. *J. Amer. Statist. Assoc.* **111** 1726–1735. MR3601731
[14] Hastie, T., Tibshirani, R. and Wainwright, M. (2015). *Statistical Learning with Sparsity: The Lasso and Generalizations. Monographs on Statistics and Applied Probability* **143**. Boca Raton, FL: CRC Press. MR3616141
[15] He, X., Wang, L. and Hong, H.G. (2013). Quantile-adaptive model-free variable screening for high-dimensional heterogeneous data. *Ann. Statist.* **41** 342–369. MR3059421
[16] Honda, T., Ing, C.-K. and Wu, W.-Y. (2019). Supplement to "Adaptively weighted group Lasso for semiparametric quantile regression models." DOI:10.3150/18-BEJ1091SUPP.
[17] Honda, T. and Yabe, R. (2017). Variable selection and structure identification for varying coefficient Cox models. *J. Multivariate Anal.* **161** 103–122. MR3698118
[18] Ing, C.-K. and Lai, T.L. (2011). A stepwise regression method and consistent model selection for high-dimensional sparse linear models. *Statist. Sinica* **21** 1473–1513. MR2895106
[19] Kato, K. (2011). Group lasso for high dimensional sparse quantile regression models. ArXiv preprint. Available at arXiv:1103.1458.
[20] Kim, Y. and Jeon, J.-J. (2016). Consistent model selection criteria for quadratically supported risks. *Ann. Statist.* **44** 2467–2496. MR3576551
[21] Lee, E.R. and Mammen, E. (2016). Local linear smoothing for sparse high dimensional varying coefficient models. *Electron. J. Stat.* **10** 855–894. MR3486419
[22] Lee, E.R., Noh, H. and Park, B.U. (2014). Model selection via Bayesian information criterion for quantile regression models. *J. Amer. Statist. Assoc.* **109** 216–229. MR3180558
[23] Lian, H. (2012). Semiparametric estimation of additive quantile regression models by two-fold penalty. *J. Bus. Econom. Statist.* **30** 337–350. MR2969217
[24] Lian, H., Liang, H. and Ruppert, D. (2015). Separation of covariates into nonparametric and parametric parts in high-dimensional partially linear additive models. *Statist. Sinica* **25** 591–607. MR3379090
[25] Lv, S., Lin, H., Lian, H. and Huang, J. (2018). Oracle inequalities for sparse additive quantile regression in reproducing kernel Hilbert space. *Ann. Statist.* **46** 781–813. MR3782384
[26] Ma, S. and He, X. (2016). Inference for single-index quantile regression models with profile optimization. *Ann. Statist.* **44** 1234–1268. MR3485959
[27] Meier, L., van de Geer, S. and Bühlmann, P. (2008). The group Lasso for logistic regression. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **70** 53–71. MR2412631
[28] Sherwood, B. and Wang, L. (2016). Partially linear additive quantile regression in ultra-high dimension. *Ann. Statist.* **44** 288–317. MR3449769
[29] Tang, Y., Song, X., Wang, H.J. and Zhu, Z. (2013). Variable selection in high-dimensional quantile varying coefficient models. *J. Multivariate Anal.* **122** 115–132. MR3189311
[30] Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. *J. Roy. Statist. Soc. Ser. B* **58** 267–288. MR1379242
[31] van de Geer, S. (2016). *Estimation and Testing Under Sparsity. Lecture Notes in Math.* **2159**. Cham: Springer. Lecture notes from the 45th Probability Summer School held in Saint-Four, 2015, École d'Été de Probabilités de Saint-Flour. [Saint-Flour Probability Summer School]. MR3526202

[32] Wang, H. (2009). Forward regression for ultra-high dimensional variable screening. *J. Amer. Statist. Assoc.* **104** 1512–1524. MR2750576

[33] Wang, H.J., Zhu, Z. and Zhou, J. (2009). Quantile regression in partially linear varying coefficient models. *Ann. Statist.* **37** 3841–3866. MR2572445

[34] Yan, J. and Huang, J. (2012). Model selection for Cox models with time-varying coefficients. *Biometrics* **68** 419–428. MR2959608

[35] Yuan, M. and Lin, Y. (2006). Model selection and estimation in regression with grouped variables. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **68** 49–67. MR2212574

[36] Zhang, H.H., Cheng, G. and Liu, Y. (2011). Linear or nonlinear? Automatic structure discovery for partially linear models. *J. Amer. Statist. Assoc.* **106** 1099–1112. MR2894767

[37] Zheng, Q., Peng, L. and He, X. (2015). Globally adaptive quantile regression with ultra-high dimensional data. *Ann. Statist.* **43** 2225–2258. MR3396984

[38] Zhu, L., Huang, M. and Li, R. (2012). Semiparametric quantile regression with high-dimensional covariates. *Statist. Sinica* **22** 1379–1401. MR3027092

[39] Zou, H. (2006). The adaptive lasso and its oracle properties. *J. Amer. Statist. Assoc.* **101** 1418–1429. MR2279469

Bernoulli **25**(4B), 2019, 3339–3378
https://doi.org/10.3150/18-BEJ1092

Networks of reinforced stochastic processes: Asymptotics for the empirical means

GIACOMO ALETTI[1,*] IRENE CRIMALDI[2] and ANDREA GHIGLIETTI[1,**]

[1]*ADAMSS Center, Università degli Studi di Milano, Milan, Italy.*
E-mail: [*]*giacomo.aletti@unimi.it;* [**]*andrea.ghiglietti@unimi.it*
[2]*IMT School for Advanced Studies, Lucca, Italy. E-mail: irene.crimaldi@imtlucca.it*

This work deals with systems of *interacting reinforced stochastic processes*, where each process $X^j = (X_{n,j})_n$ is located at a vertex j of a finite *weighted direct graph*, and it can be interpreted as the sequence of "actions" adopted by an agent j of the network. The interaction among the evolving dynamics of these processes depends on the weighted adjacency matrix W associated to the underlying graph: indeed, the probability that an agent j chooses a certain action depends on its personal "inclination" $Z_{n,j}$ and on the inclinations $Z_{n,h}$, with $h \neq j$, of the other agents according to the elements of W.

Asymptotic results for the stochastic processes of the personal inclinations $Z^j = (Z_{n,j})_n$ have been subject of studies in recent papers (e.g., Aletti, Crimaldi and Ghiglietti [*Ann. Appl. Probab.* **27** (2017) 3787–3844], Crimaldi et al. [Synchronization and functional central limit theorems for interacting reinforced random walks (2019)]); while the asymptotic behavior of quantities based on the stochastic processes X^j of the actions has never been studied yet. In this paper, we fill this gap by characterizing the asymptotic behavior of the *empirical means* $N_{n,j} = \sum_{k=1}^{n} X_{k,j}/n$, proving their almost sure synchronization and some central limit theorems in the sense of stable convergence. Moreover, we discuss some statistical applications of these convergence results concerning confidence intervals for the random limit toward which all the processes of the system almost surely converge and tools to make inference on the matrix W.

Keywords: asymptotic normality; complex networks; interacting systems; reinforced stochastic processes; synchronization; urn models

References

[1] Albert, R. and Barabási, A.-L. (2002). Statistical mechanics of complex networks. *Rev. Modern Phys.* **74** 47–97. MR1895096

[2] Aletti, G., Crimaldi, I. and Ghiglietti, A. (2019). Supplement to "Networks of reinforced stochastic processes: asymptotics for the empirical means." DOI:10.3150/18-BEJ1092SUPP.

[3] Aletti, G., Crimaldi, I. and Ghiglietti, A. (2017). Synchronization of reinforced stochastic processes with a network-based interaction. *Ann. Appl. Probab.* **27** 3787–3844. MR3737938

[4] Aletti, G. and Ghiglietti, A. (2017). Interacting generalized Friedman's urn systems. *Stochastic Process. Appl.* **127** 2650–2678. MR3660886

[5] Aletti, G., Ghiglietti, A. and Vidyashankar, A.N. (2018). Dynamics of an adaptive randomly reinforced urn. *Bernoulli* **24** 2204–2255. MR3757528

[6] Aletti, G., May, C. and Secchi, P. (2009). A central limit theorem, and related results, for a two-color randomly reinforced urn. *Adv. in Appl. Probab.* **41** 829–844. MR2571318

[7] Arenas, A., Díaz-Guilera, A., Kurths, J., Moreno, Y. and Zhou, C. (2008). Synchronization in complex networks. *Phys. Rep.* **469** 93–153. MR2477097

1350-7265

[8] Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. *Science* **286** 509–512. MR2091634

[9] Benaïm, M., Benjamini, I., Chen, J. and Lima, Y. (2015). A generalized Pólya's urn with graph based interactions. *Random Structures Algorithms* **46** 614–634. MR3346459

[10] Berti, P., Crimaldi, I., Pratelli, L. and Rigo, P. (2011). A central limit theorem and its applications to multicolor randomly reinforced urns. *J. Appl. Probab.* **48** 527–546. MR2840314

[11] Berti, P., Crimaldi, I., Pratelli, L. and Rigo, P. (2016). Asymptotics for randomly reinforced urns with random barriers. *J. Appl. Probab.* **53** 1206–1220. MR3581252

[12] Chen, J. and Lucas, C. (2014). A generalized Pólya's urn with graph based interactions: Convergence at linearity. *Electron. Commun. Probab.* **19** no. 67, 13. MR3269167

[13] Chen, M.-R. and Kuba, M. (2013). On generalized Pólya urn models. *J. Appl. Probab.* **50** 1169–1186. MR3161380

[14] Cirillo, P., Gallegati, M. and Hüsler, J. (2012). A Pólya lattice model to study leverage dynamics and contagious financial fragility. *Adv. Complex Syst.* **15** 1250069, 26. MR2972682

[15] Collevecchio, A., Cotar, C. and LiCalzi, M. (2013). On a preferential attachment and generalized Pólya's urn model. *Ann. Appl. Probab.* **23** 1219–1253. MR3076683

[16] Crimaldi, I. (2009). An almost sure conditional convergence result and an application to a generalized Pólya urn. *Int. Math. Forum* **4** 1139–1156. MR2524635

[17] Crimaldi, I. (2016). Central limit theorems for a hypergeometric randomly reinforced urn. *J. Appl. Probab.* **53** 899–913. MR3570102

[18] Crimaldi, I. (2016). *Introduzione Alla Nozione di Convergenza Stabile e sue Varianti* (*Introduction to the Notion of Stable Convergence and Its Variants*) **57**. Bologna, Italy: Unione Matematica Italiana, Monograf s.r.l. Book written in Italian.

[19] Crimaldi, I., Dai Pra, P. and Minelli, I.G. (2016). Fluctuation theorems for synchronization of interacting Pólya's urns. *Stochastic Process. Appl.* **126** 930–947. MR3452818

[20] Crimaldi, I., Dai Pra, P., Louis, P.Y. and Minelli, I.G. (2019). Synchronization and functional central limit theorems for interacting reinforced random walks. *Stochastic Process. Appl.* **129** 70–101.

[21] Crimaldi, I., Letta, G. and Pratelli, L. (2007). A strong form of stable convergence. In *Séminaire de Probabilités XL. Lecture Notes in Math.* **1899** 203–225. Berlin: Springer. MR2409006

[22] Crimaldi, I. and Pratelli, L. (2005). Convergence results for multivariate martingales. *Stochastic Process. Appl.* **115** 571–577. MR2128630

[23] Dai Pra, P., Louis, P.-Y. and Minelli, I.G. (2014). Synchronization via interacting reinforcement. *J. Appl. Probab.* **51** 556–568. MR3217785

[24] Fortini, S., Petrone, S. and Sporysheva, P. (2018). On a notion of partially conditionally identically distributed sequences. *Stochastic Process. Appl.* **128** 819–846. MR3758339

[25] Ghiglietti, A. and Paganoni, A.M. (2014). Statistical properties of two-color randomly reinforced urn design targeting fixed allocations. *Electron. J. Stat.* **8** 708–737. MR3211029

[26] Ghiglietti, A., Vidyashankar, A.N. and Rosenberger, W.F. (2017). Central limit theorem for an adaptive randomly reinforced urn model. *Ann. Appl. Probab.* **27** 2956–3003. MR3719951

[27] Hall, P. and Heyde, C.C. (1980). *Martingale Limit Theory and Its Application*: *Probability and Mathematical Statistics*. New York-London: Academic Press. MR0624435

[28] Konda, V.R. and Tsitsiklis, J.N. (2004). Convergence rate of linear two-time-scale stochastic approximation. *Ann. Appl. Probab.* **14** 796–819. MR2052903

[29] Laruelle, S. and Pagès, G. (2013). Randomized urn models revisited using stochastic approximation. *Ann. Appl. Probab.* **23** 1409–1436. MR3098437

[30] Lima, Y. (2016). Graph-based Pólya's urn: Completion of the linear case. *Stoch. Dyn.* **16** 1660007, 13. MR3470556

[31] Mokkadem, A. and Pelletier, M. (2006). Convergence rate and averaging of nonlinear two-time-scale stochastic approximation algorithms. *Ann*. *Appl*. *Probab*. **16** 1671–1702. MR2260078
[32] Newman, M.E.J. (2010). *Networks*: *An Introduction*. Oxford: Oxford Univ. Press. MR2676073
[33] Paganoni, A.M. and Secchi, P. (2004). Interacting reinforced-urn systems. *Adv*. *in Appl*. *Probab*. **36** 791–804. MR2079914
[34] Pelletier, M. (1998). Weak convergence rates for stochastic approximation with application to multiple targets and simulated annealing. *Ann*. *Appl*. *Probab*. **8** 10–44. MR1620405
[35] Pemantle, R. (2007). A survey of random processes with reinforcement. *Probab*. *Surv*. **4** 1–79. MR2282181
[36] van der Hofstad, R. (2016). Random Graphs and Complex Networks. https://www.win.tue.nl/~rhofstad/NotesRGCN.html.
[37] Zhang, L.-X. (2014). A Gaussian process approximation for two-color randomly reinforced urns. *Electron*. *J*. *Probab*. **19** no. 86, 19. MR3263643

Bernoulli **25**(4B), 2019, 3379–3399
https://doi.org/10.3150/18-BEJ1093

Limiting saddlepoint relative errors in large deviation regions under purely Tauberian conditions

RONALD W. BUTLER[1] and ANDREW T.A. WOOD[2]

[1]*Department of Statstical Science, Southern Methodist University, Dallas, TX 75275, USA.*
E-mail: rbutler@smu.edu
[2]*School of Mathematical Sciences, University of Nottingham, Nottingham, NG7 2RD, UK.*
E-mail: Andrew.Wood@nottingham.ac.uk

Most theoretical results on the relative errors of saddlepoint approximations in the extreme tails have involved placing conditions on the density/mass function. Checking the validity of such conditions is problematic when density/mass functions are intractable, as is typically the case in important practical applications involving convolved, compound, and first-passage distributions as well as for moment generating functions MGFs that are regularly varying. In this paper, we present novel conditions which ensure the existence of positive finite limiting relative errors for saddlepoint density/mass function and survival function approximations. These conditions, which are rather weak, are expressed entirely in terms of the MGF, hence the description *purely Tauberian*. We focus mainly on the cases in which there are positive and negative gamma distributional limits (the only other non-degenerate possibility being a Gaussian limit) and we show how to check the new conditions in important classes of models in these two settings.

Keywords: compound distribution; first-passage distribution; regular variation; saddlepoint approximation; Tauberian arguments

References

[1] Abramowitz, M. and Stegun, I.A. (1972). *Handbook of Mathematical Functions*. 9th ed. New York: Dover.
[2] Agresti, A. (1992). A survey of exact inference for contingency tables. *Statist. Sci.* **7** 131–177. MR1173420
[3] Balkema, A.A., Klüppelberg, C. and Resnick, S.I. (1999). Limit laws for exponential families. *Bernoulli* **5** 951–968. MR1735779
[4] Balkema, A.A., Klüppelberg, C. and Resnick, S.I. (1999). Domains of attraction for exponential families and asymptotic behaviour of Laplace transforms. Unpublished Research Report.
[5] Balkema, A.A., Klüppelberg, C. and Resnick, S.I. (2003). Domains of attraction for exponential families. *Stochastic Process. Appl.* **107** 83–103. MR1995922
[6] Barndorff-Nielsen, O. (1978). *Information and Exponential Families in Statistical Theory*. *Wiley Series in Probability and Mathematical Statistics*. Chichester: Wiley. MR0489333
[7] Barndorff-Nielsen, O.E. (1990). Approximate interval probabilities. *J. Roy. Statist. Soc. Ser. B* **52** 485–496. MR1086800
[8] Barndorff-Nielsen, O.E. and Klüppelberg, C. (1992). A note on the tail accuracy of the univariate saddlepoint approximation. *Ann. Fac. Sci. Toulouse Math.* (6) **1** 5–14. MR1191724

[9] Barndorff-Nielsen, O.E. and Klüppelberg, C. (1999). Tail exactness of multivariate saddlepoint approximations. *Scand. J. Stat.* **26** 253–264. MR1707583
[10] Bingham, N.H., Goldie, C.M. and Teugels, J.L. (1987). *Regular Variation. Encyclopedia of Mathematics and Its Applications* **27**. Cambridge: Cambridge Univ. Press. MR0898871
[11] Booth, J.G. (1994). A note on the accuracy of two saddlepoint tail approximations. *Proc. Amer. Statist. Assoc. Phys. Engr. Sci.*
[12] Butler, R.W. (2000). Reliabilities for feedback systems and their saddlepoint approximation. *Statist. Sci.* **15** 279–298. MR1821330
[13] Butler, R.W. (2007). *Saddlepoint Approximations with Applications. Cambridge Series in Statistical and Probabilistic Mathematics* **22**. Cambridge: Cambridge Univ. Press. MR2357347
[14] Butler, R.W. (2017). Asymptotic expansions and hazard rates for compound and first-passage distributions. *Bernoulli* **23** 3508–3536. MR3654814
[15] Butler, R.W. (2019). Asymptotic expansions and saddlepoint approximations for first-passage distributions in semi-Markov processes. Unpublished manuscript.
[16] Butler, R.W. and Wood, A.T.A. (2019). Supplement to "Limiting saddlepoint relative errors in large deviation regions under purely Tauberian conditions." DOI:10.3150/18-BEJ1093SUPP.
[17] Copson, E.T. (1965). *Asymptotic Expansions. Cambridge Tracts in Mathematics and Mathematical Physics, No.* 55. New York: Cambridge Univ. Press. MR0168979
[18] Daniels, H.E. (1954). Saddlepoint approximations in statistics. *Ann. Math. Stat.* **25** 631–650. MR0066602
[19] Daniels, H.E. (1980). Exact saddlepoint approximations. *Biometrika* **67** 59–63. MR0570505
[20] Daniels, H.E. (1987). Tail probability approximations. *Int. Stat. Rev.* **55** 37–48. MR0962940
[21] Daniels, H.E. (1991). *A Class of Nearly Exact Saddlepoint Approximations* (K. Mardia, ed.). *The Art of Statistical Science*. New York: Wiley.
[22] Doetsch, G. (1974). *Introduction to the Theory and Application of the Laplace Transformation*. New York: Springer. MR0344810
[23] Feller, W. (1971). *An Introduction to Probability Theory and Its Applications. Vol. II*, 2nd ed. New York: Wiley. MR0270403
[24] Flajolet, P. and Odlyzko, A. (1990). Singularity analysis of generating functions. *SIAM J. Discrete Math.* **3** 216–240. MR1039294
[25] Henrici, P. (1977). *Applied and Computational Complex Analysis. Vol.* 2. New York–London–Sydney: Wiley. MR0453984
[26] Huang, X.C. (1988). A discrete L'Hôpital's rule. *College Math. J.* **19** 321–329. MR0962484
[27] Jensen, J.L. (1988). Uniform saddlepoint approximations. *Adv. in Appl. Probab.* **20** 622–634. MR0955507
[28] Jensen, J.L. (1991). Uniform saddlepoint approximations and log-concave densities. *J. Roy. Statist. Soc. Ser. B* **53** 157–172. MR1094278
[29] Jensen, J.L. (1991). Saddlepoint approximations to the distribution of the total claim amount in some recent risk models. *Scand. Actuar. J.* **1991** 154–168. MR1183449
[30] Jensen, J.L. (1995). *Saddlepoint Approximations. Oxford Statistical Science Series* **16**. New York: The Clarendon Press. MR1354837
[31] Jørgensen, B. (1982). *Statistical Properties of the Generalized Inverse Gaussian Distribution. Lecture Notes in Statistics* **9**. New York-Berlin: Springer. MR0648107
[32] Korevaar, J. (2004). *Tauberian Theory. A Century of Developments. Grundlehren der Mathematischen Wissenschaften* [*Fundamental Principles of Mathematical Sciences*] **329**. Berlin: Springer. MR2073637
[33] Lugannani, R. and Rice, S. (1980). Saddle point approximation for the distribution of the sum of independent random variables. *Adv. in Appl. Probab.* **12** 475–490. MR0569438

[34] Lukacs, E. (1964). Inversion formulae for characteristic functions of absolutely continuous distributions. *Amer. Math. Monthly* **71** 44–47. MR0174072

[35] McCullagh, P. (1994). Does the moment-generating function characterize a distribution? *Amer. Statist.* **48** 208. MR1304977

[36] NIST Digital Library of Mathematical Functions. http://dlmf.nist.gov/, Release 1.0.8 of 2014-04-25.

[37] Paige, R.L., Chapman, P.L. and Butler, R.W. (2011). Small sample $LD50$ confidence intervals using saddlepoint approximations. *J. Amer. Statist. Assoc.* **106** 334–344. MR2816725

[38] Vuilleumier, M. (1976). Slowly varying functions in the complex plane. *Trans. Amer. Math. Soc.* **218** 343–348. MR0399479

[39] Widder, D.V. (2010). *The Laplace Transform*. Republication of 1941 Edition. New York: Dover.

Bernoulli **25**(4B), 2019, 3400–3420
https://doi.org/10.3150/18-BEJ1094

Rate of divergence of the nonparametric likelihood ratio test for Gaussian mixtures

WENHUA JIANG[1] and CUN-HUI ZHANG[2]

[1]*School of Mathematical Sciences, Fudan University and Shanghai Center for Mathematical Sciences, 220 Handan Road, Shanghai 200433, China. E-mail: wenhuajiang@yahoo.com*
[2]*Department of Statistics and Biostatistics, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854, U.S.A. E-mail: czhang@stat.rutgers.edu*

We study a nonparametric likelihood ratio test (NPLRT) for Gaussian mixtures. It is based on the nonparametric maximum likelihood estimator in the context of demixing. The test concerns if a random sample is from the standard normal distribution. We consider mixing distributions of unbounded support for alternative hypothesis. We prove that the divergence rate of the NPLRT under the null is bounded by $\log n$, provided that the support range of the mixing distribution increases no faster than $(\log n/\log 9)^{1/2}$. We prove that the rate of $\sqrt{\log n}$ is a lower bound for the divergence rate if the support range increases no slower than the order of $\sqrt{\log n}$. Implications of the upper bound for the rate of divergence are discussed.

Keywords: Gaussian mixtures; Hermite polynomials; likelihood ratio test; rate of divergence; two-component mixtures

References

[1] Azaïs, J.-M., Gassiat, É. and Mercadier, C. (2006). Asymptotic distribution and local power of the log-likelihood ratio test for mixtures: Bounded and unbounded cases. *Bernoulli* **12** 775–799. MR2265342
[2] Azaïs, J.-M., Gassiat, É. and Mercadier, C. (2009). The likelihood ratio test for general mixture models with or without structural parameter. *ESAIM Probab. Stat.* **13** 301–327. MR2528086
[3] Bickel, P. and Chernoff, H. (1993). Asymptotic distribution of the likelihood ratio statistic in a prototypical non regular problem. In *Statistics and Probability: A Raghu Raj Bahadur Festschrift* (J.K. Ghosh, S.K. Mitra, K.R. Parthasarathy and B.L.S. Prakasa Rao, eds.) 83–96. New Delhi: Wiley Eastern.
[4] DasGupta, A. (2008). *Asymptotic Theory of Statistics and Probability*. *Springer Texts in Statistics*. New York: Springer. MR2664452
[5] Donoho, D. and Jin, J. (2004). Higher criticism for detecting sparse heterogeneous mixtures. *Ann. Statist.* **32** 962–994. MR2065195
[6] Genovese, C.R. and Wasserman, L. (2000). Rates of convergence for the Gaussian mixture sieve. *Ann. Statist.* **28** 1105–1127. MR1810921
[7] Ghosal, S. and van der Vaart, A.W. (2001). Entropies and rates of convergence for maximum likelihood and Bayes estimation for mixtures of normal densities. *Ann. Statist.* **29** 1233–1263. MR1873329
[8] Gu, J., Koenker, R. and Volgushev, S. (2018). Testing for homogeneity in mixture models. *Econometric Theory* **34** 850–895. MR3815469
[9] Hartigan, J.A. (1985). A failure of likelihood asymptotics for normal mixtures. In *Proceedings of the Berkeley Conference in Honor of Jerzy Neyman and Jack Kiefer, Vol. II* (*Berkeley, Calif.*, 1983). *Wadsworth Statist./Probab. Ser.* 807–810. Belmont, CA: Wadsworth. MR0822066

[10] Ingster, Yu.I. (1999). Minimax detection of a signal for l^n-balls. *Math*. *Methods Statist*. **7** 401–428. MR1680087

[11] Jiang, W. and Zhang, C.-H. (2009). General maximum likelihood empirical Bayes estimation of normal means. *Ann*. *Statist*. **37** 1647–1684. MR2533467

[12] Jiang, W. and Zhang, C.-H. (2016). Generalized likelihood ratio test for normal mixtures. *Statist*. *Sinica* **26** 955–978. MR3559938

[13] Kiefer, J. and Wolfowitz, J. (1956). Consistency of the maximum likelihood estimator in the presence of infinitely many incidental parameters. *Ann*. *Math*. *Stat*. **27** 887–906. MR0086464

[14] Lindsay, B.G. (1995). *Mixture Models*: *Theory*, *Geometry*, *and Applications*. Hayward, CA: IMS.

[15] Liu, X. and Shao, Y. (2003). Asymptotics for likelihood ratio tests under loss of identifiability. *Ann*. *Statist*. **31** 807–832. MR1994731

[16] Liu, X. and Shao, Y. (2004). Asymptotics for the likelihood ratio test in a two-component normal mixture model. *J*. *Statist*. *Plann*. *Inference* **123** 61–81. MR2058122

[17] Robbins, H. (1950). A generalization of the method of maximum likelihood: Estimating a mixing distribution (abstract). *Ann*. *Math*. *Stat*. **21** 314–315.

[18] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes*: *With Applications to Statistics*. *Springer Series in Statistics*. New York: Springer. MR1385671

[19] Zhang, C.-H. (2009). Generalized maximum likelihood estimation of normal mixture densities. *Statist*. *Sinica* **19** 1297–1318. MR2536157

Bernoulli **25**(4B), 2019, 3421–3458
https://doi.org/10.3150/18-BEJ1095

Concentration of weakly dependent Banach-valued sums and applications to statistical learning methods

GILLES BLANCHARD* and OLEKSANDR ZADOROZHNYI**

*University of Potsdam, Institute of Mathematics, Karl-Liebknecht-Str. 24/25, Potsdam, 14476, Germany. E-mail: *blanchard@uni-potsdam.de; **zadorozh@uni-potsdam.de*

We obtain a Bernstein-type inequality for sums of Banach-valued random variables satisfying a weak dependence assumption of general type and under certain smoothness assumptions of the underlying Banach norm. We use this inequality in order to investigate in the asymptotical regime the error upper bounds for the broad family of spectral regularization methods for reproducing kernel decision rules, when trained on a sample coming from a τ-mixing process.

Keywords: Banach-valued process; Bernstein inequality; concentration; spectral regularization; weak dependence

References

[1] Andrews, D.W.K. (1984). Nonstrong mixing autoregressive processes. *J. Appl. Probab.* **21** 930–934. MR0766830
[2] Andrews, D.W.K. (1988). Laws of large numbers for dependent nonidentically distributed random variables. *Econometric Theory* **4** 458–467. MR0985156
[3] Argyriou, A. and Dinuzzo, F. (2014). A unifying view of representer theorems. In *International Conference on Machine Learning* 31 (*ICML* 2014) (E.P. Xing and T. Jebara, eds.). *Proceedings of Machine Learning Research* **32** 748–756.
[4] Bauer, F., Pereverzev, S. and Rosasco, L. (2007). On regularization algorithms in learning theory. *J. Complexity* **23** 52–72. MR2297015
[5] Benett, K. and Bredensteiner, J. (2000). Duality and geometry in support vector machine classifiers. In *International Conference on Machine Learning* 17 (*ICML* 2000) (P. Langley, ed.) 57–64.
[6] Bernstein, S. (1924). On a modification of Chebyschev's inequality and of the error formula of Laplace. *Ann. Sci. Inst. Sav. Ukraine*, *Sect. Math* **4**.
[7] Bhatia, R. (1997). *Matrix Analysis*. *Graduate Texts in Mathematics* **169**. New York: Springer. MR1477662
[8] Bickel, P.J. and Bühlmann, P. (1999). A new mixing notion and functional central limit theorems for a sieve bootstrap in time series. *Bernoulli* **5** 413–446. MR1693612
[9] Blanchard, G., Lee, G. and Scott, C. (2011). Generalizing from several related classification tasks to a new unlabeled sample. In *Advances in Neural Inf. Proc. Systems* 24 (*NIPS* 2011) (J. Shawe-Taylor, R.S. Zemel, P.L. Bartlett, F. Pereira and K.Q. Weinberger, eds.) 2438–2446.
[10] Blanchard, G. and Mücke, N. (2018). Optimal rates for regularization of statistical inverse learning problems. *Found. Comput. Math.* **18** 971–1013. MR3833647

[11] Bosq, D. (1993). Bernstein-type large deviations inequalities for partial sums of strong mixing processes. *Statistics* **24** 59–70. MR1238263
[12] Bosq, D. (2000). *Linear Processes in Function Spaces*: *Theory and Applications*. *Lecture Notes in Statistics* **149**. New York: Springer. MR1783138
[13] Bradley, R.C. (2005). Basic properties of strong mixing conditions. A survey and some open questions. *Probab. Surv.* **2** 107–144. MR2178042
[14] Canu, S., Mary, X. and Rakotomamonjy, A. (2003). Functional learning through kernel. **5** 89–110. IOS Press.
[15] Caponnetto, A. and De Vito, E. (2007). Optimal rates for the regularized least-squares algorithm. *Found*. *Comput*. *Math*. **7** 331–368. MR2335249
[16] Combettes, P.L., Salzo, S. and Villa, S. (2018). Regularized learning schemes in feature Banach spaces. *Anal*. *Appl*. (*Singap*.) **16** 1–54. MR3716734
[17] De Vito, E., Rosasco, L. and Caponnetto, A. (2006). Discretization error analysis for Tikhonov regularization. *Anal*. *Appl*. (*Singap*.) **4** 81–99. MR2199794
[18] Dedecker, J., Doukhan, P., Lang, G., León R., J.R., Louhichi, S. and Prieur, C. (2007). *Weak Dependence*: *With Examples and Applications*. *Lecture Notes in Statistics* **190**. New York: Springer. MR2338725
[19] Dedecker, J. and Merlevède, F. (2015). Moment bounds for dependent sequences in smooth Banach spaces. *Stochastic Process*. *Appl*. **125** 3401–3429. MR3357614
[20] Doukhan, P. and Louhichi, S. (1999). A new weak dependence condition and applications to moment inequalities. *Stochastic Process*. *Appl*. **84** 313–342. MR1719345
[21] Engl, H.W., Hanke, M. and Neubauer, A. (1996). *Regularization of Inverse Problems*. *Mathematics and Its Applications* **375**. Dordrecht: Kluwer Academic. MR1408680
[22] Esary, J.D., Proschan, F. and Walkup, D.W. (1967). Association of random variables, with applications. *Ann*. *Math*. *Stat*. **38** 1466–1474. MR0217826
[23] Fan, X., Grama, I. and Liu, Q. (2015). Exponential inequalities for martingales with applications. *Electron*. *J*. *Probab.* **20** 1–22. MR3311214
[24] Fortuin, C.M., Kasteleyn, P.W. and Ginibre, J. (1971). Correlation inequalities on some partially ordered sets. *Comm*. *Math*. *Phys*. **22** 89–103. MR0309498
[25] Freedman, D.A. (1975). On tail probabilities for martingales. *Ann*. *Probab.* **3** 100–118. MR0380971
[26] Hang, H. and Steinwart, I. (2017). A Bernstein-type inequality for some mixing processes and dynamical systems with an application to learning. *Ann*. *Statist*. **45** 708–743. MR3650398
[27] Hein, M., Bousquet, O. and Schölkopf, B. (2005). Maximal margin classification for metric spaces. *J*. *Comput*. *System Sci*. **71** 333–359. MR2168357
[28] Hoeffding, W. (1963). Probability inequalities for sums of bounded random variables. *J*. *Amer. Statist*. *Assoc*. **58** 13–30. MR0144363
[29] Horváth, L. and Kokoszka, P. (2012). *Inference for Functional Data with Applications*. *Springer Series in Statistics*. New York: Springer. MR2920735
[30] Ibragimov, I.A. (1959). Some limit theorems for stochastic processes stationary in the strict sense. *Dokl*. *Akad*. *Nauk SSSR* **125** 711–714. MR0105180
[31] Jirak, M. (2018). Rate of convergence for Hilbert space valued processes. *Bernoulli* **24** 202–230. MR3706754
[32] Kolmogorov, A.N. and Rozanov, J.A. (1960). On a strong mixing condition for stationary Gaussian processes. *Theory Probab*. *Appl*. **5** 204–208. MR0133175
[33] Kontorovich, L. (2006). Metric and mixing sufficient conditions for concentration of measure. Available at arxiv.org/abs/math/0610427.
[34] Kontorovich, L. and Ramanan, K. (2008). Concentration inequalities for dependent random variables via the martingale method. *Ann*. *Probab.* **36** 2126–2158. MR2478678

[35] Marton, K. (2004). Measure concentration for Euclidean distance in the case of dependent random variables. *Ann. Probab.* **32** 2526–2544. MR2078549

[36] Maume-Deschamps, V. (2006). Exponential inequalities and functional estimations for weak dependent data; applications to dynamical systems. *Stoch. Dyn.* **6** 535–560. MR2285515

[37] Mc Leish, D. (1975). Invariance principles and mixing random variables. *Econometric Theory* **4** 165–178.

[38] Merlevède, F., Peligrad, M. and Rio, E. (2009). Bernstein inequality and moderate deviations under strong mixing conditions. In *High Dimensional Probability V: The Luminy Volume. Inst. Math. Stat. (IMS) Collect.* **5** 273–292. Beachwood, OH: IMS. MR2797953

[39] Micchelli, C.A. and Pontil, M. (2004). A function representation for learning in Banach spaces. In *Learning Theory. Lecture Notes in Computer Science* **3120** 255–269. Berlin: Springer. MR2177914

[40] Pinelis, I. (1992). An approach to inequalities for the distributions of infinite-dimensional martingales. In *Probability in Banach Spaces*, 8 (*Brunswick, ME*, 1991). *Progress in Probability* **30** 128–134. Boston, MA: Birkhäuser. MR1227615

[41] Pinelis, I. (1994). Optimum bounds for the distributions of martingales in Banach spaces. *Ann. Probab.* **22** 1679–1706. MR1331198

[42] Pinelis, I.F. and Sakhanenko, A.I. (1986). Remarks on inequalities for probabilities of large deviations. *Theory Probab. Appl.* **30** 143–148.

[43] Potapov, D. and Sukochev, F. (2014). Fréchet differentiability of $\mathcal{S}^p$ norms. *Adv. Math.* **262** 436–475. MR3228433

[44] Rio, E. (1996). Sur le théorème de Berry–Esseen pour les suites faiblement dépendantes. *Probab. Theory Related Fields* **104** 255–282. MR1373378

[45] Rio, E. (2013). Extensions of the Hoeffding–Azuma inequalities. *Electron. Commun. Probab.* **18** no. 54, 6. MR3078017

[46] Rosasco, L., Belkin, M. and De Vito, E. (2010). On learning with integral operators. *J. Mach. Learn. Res.* **2** 905–934.

[47] Rosenblatt, M. (1956). A central limit theorem and a strong mixing condition. *Proc. Natl. Acad. Sci. USA* **42** 43–47. MR0074711

[48] Samson, P.-M. (2000). Concentration of measure inequalities for Markov chains and Φ-mixing processes. *Ann. Probab.* **28** 416–461. MR1756011

[49] Song, G. and Zhang, H. (2011). Reproducing kernel Banach spaces with the ℓ^1 norm II: Error analysis for regularized least square regression. *Neural Comput.* **23** 2713–2729. MR2866617

[50] Sriperumbudur, B., Fukumizu, K. and Lanckriet, G. (2011). Learning in Hilbert vs. Banach spaces: A measure embedding viewpoint. In *Advances in Neural Information Processing Systems* 24 (*NIPS* 2011) (J. Shawe-Taylor, R.S. Zemel, P.L. Bartlett, F. Pereira and K.Q. Weinberger, eds.) 1773–1781.

[51] Steinwart, I. (2009). Two oracle inequalities for regularized boosting classifiers. *Stat. Interface* **2** 271–284. MR2540086

[52] van de Geer, S.A. (2002). On Hoeffding's inequality for dependent random variables. In *Empirical Process Techniques for Dependent Data* 161–169. Boston, MA: Birkhäuser. MR1958780

[53] Wintenberger, O. (2010). Deviation inequalities for sums of weakly dependent time series. *Electron. Commun. Probab.* **15** 489–503. MR2733373

[54] Yurinskyi, V. (1970). The infinite-dimensional version of S.N. Bernšteĭn's inequalities. *Theory Probab. Appl.* **15** 108–109.

[55] Yurinsky, V. (1995). *Sums and Gaussian Vectors. Lecture Notes in Math.* **1617**. Berlin: Springer. MR1442713

[56] Zhang, H., Xu, Y. and Zhang, J. (2009). Reproducing kernel Banach spaces for machine learning. *J. Mach. Learn. Res.* **10** 2741–2775. MR2579912

[57] Zhang, H. and Zhang, J. (2013). Vector-valued reproducing kernel Banach spaces with applications to multi-task learning. *J. Complexity* **29** 195–215. MR3018139
[58] Zhang, T. (2002). On the dual formulation of regularized learning schemes with convex risks. *Mach. Learn.* **46** 91–129.

Bernoulli **25**(4B), 2019, 3459–3478
https://doi.org/10.3150/18-BEJ1096

Nonparametric empirical Bayes improvement of shrinkage estimators with applications to time series

EITAN GREENSHTEIN[1], ARIEL MANTZURA[2] and YA'ACOV RITOV[3]

[1]*Israel Central Bureau of Statistics, Kanfei Nesharim 66, 9546456 Jerusalem, Israel. E-mail: eitan.greenshtein@gmail.com*
[2]*Bank of Israel; Eliezer Kaplan 3, P.O. Box 780, 91007 Jerusalem, Israel. E-mail: ariel.mansura@boi.org.il*
[3]*University of Michigan and the Hebrew University of Jerusalem; Department of Statistics, University of Michigan, 1085 South University, Ann Arbor, MI 48109-1107, USA. E-mail: yritov@umich.edu*

We consider the problem of estimating a vector $\boldsymbol{\mu} = (\mu_1, \dots, \mu_n)$ under a squared loss, based on independent observations $Y_i \sim N(\mu_i, 1)$, $i = 1, \dots, n$, and possibly extra structural assumptions. We argue that many estimators are asymptotically equal to $\hat{\mu}_i = \alpha\tilde{\mu}_i + (1-\alpha)Y_i + \xi_i = \tilde{\mu}_i + (1-\alpha)(Y_i - \tilde{\mu}_i) + \xi_i$, where $\alpha \in [0, 1]$ and $\tilde{\mu}_i$ may depend on the data, but is not a function of Y_i, and $\sum \xi_i^2 = o_p(n)$.

We consider the optimal estimator of the form $\tilde{\mu}_i + g(Y_i - \tilde{\mu}_i)$ for a general, possibly random, function g, and approximate it using nonparametric empirical Bayes ideas and techniques. We consider both the retrospective and the sequential estimation problems. We elaborate and demonstrate our results on the case where $\hat{\mu}_i$ are Kalman filter estimators. Simulations and a real data analysis are also provided.

Keywords: empirical Bayes; exchangeable; Kalman filter; shrinkage estimators

References

[1] Banarejee, T., Mukherjee, G. and Sun, W. (2018). Adaptive sparse estimation with side information.
[2] Brockwell, P.J. and Davis, R.A. (1991). *Time Series*: *Theory and Methods*, 2nd ed. *Springer Series in Statistics*. New York: Springer. MR1093459
[3] Brown, L., Cai, T., Zhang, R., Zhao, L. and Zhou, H. (2010). The root-unroot algorithm for density estimation as implemented via wavelet block thresholding. *Probab. Theory Related Fields* **146** 401–433. MR2574733
[4] Brown, L.D. (1971). Admissible estimators, recurrent diffusions, and insoluble boundary value problems. *Ann. Math. Stat.* **42** 855–903. MR0286209
[5] Brown, L.D. and Greenshtein, E. (2009). Nonparametric empirical Bayes and compound decision approaches to estimation of a high-dimensional vector of normal means. *Ann. Statist.* **37** 1685–1704. MR2533468
[6] Brown, L.D., Greenshtein, E. and Ritov, Y. (2013). The Poisson compound decision problem revisited. *J. Amer. Statist. Assoc.* **108** 741–749. MR3174656
[7] Brown, L.D., Mukherjee, G. and Weinstein, A. (2018). Empirical Bayes estimates for a two-way cross-classified model. *Ann. Statist.* **46** 1693–1720. MR3819114
[8] Cohen, N., Greenshtein, E. and Ritov, Y. (2013). Empirical Bayes in the presence of explanatory variables. *Statist. Sinica* **23** 333–357. MR3076170

[9] Copas, J.B. (1969). Compound decisions and empirical Bayes. (With discussion.). *J. Roy. Statist. Soc. Ser. B* **31** 397–425. MR0269013

[10] Efron, B. (2010). *Large-Scale Inference*. *Institute of Mathematical Statistics (IMS) Monographs* **1**. Cambridge: Cambridge Univ. Press. MR2724758

[11] Efron, B. and Morris, C. (1973). Stein's estimation rule and its competitors – an empirical Bayes approach. *J. Amer. Statist. Assoc.* **68** 117–130. MR0388597

[12] Fay, R.E. III and Herriot, R.A. (1979). Estimates of income for small places: An application of James–Stein procedures to census data. *J. Amer. Statist. Assoc.* **74** 269–277. MR0548019

[13] Jiang, W. and Zhang, C.-H. (2009). General maximum likelihood empirical Bayes estimation of normal means. *Ann. Statist.* **37** 1647–1684. MR2533467

[14] Koenker, R. and Mizera, I. (2014). Convex optimization, shape constraints, compound decisions, and empirical Bayes rules. *J. Amer. Statist. Assoc.* **109** 674–685. MR3223742

[15] Lehmann, E.L. and Casella, G. (2003). *Theory of Point Estimation*, *Springer Texts in Statistics*. New York: Springer.

[16] Rao, J.N.K. (2003). *Small Area Estimation*. *Wiley Series in Survey Methodology*. Hoboken, NJ: Wiley Interscience. MR1953089

[17] Robbins, H. (1951). Asymptotically subminimax solutions of compound statistical decision problems. In *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, 1950 131–148. Berkeley and Los Angeles: Univ. California Press. MR0044803

[18] Robbins, H. (1956). An empirical Bayes approach to statistics. In *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability*, 1954–1955, *Vol. I* 157–163. Berkeley and Los Angeles: Univ. California Press. MR0084919

[19] Robbins, H. (1964). The empirical Bayes approach to statistical decision problems. *Ann. Math. Stat.* **35** 1–20. MR0163407

[20] Samuel, E. (1965). Sequential compound estimators. *Ann. Math. Stat.* **36** 879–889. MR0183055

[21] Weinstein, A., Ma, Z., Brown, L.D. and Zhang, C.-H. (2018). Group-linear empirical Bayes estimates for a heteroscedastic normal mean. *J. Amer. Statist. Assoc.* **113** 698–710. MR3832220

[22] Xie, X., Kou, S.C. and Brown, L.D. (2012). SURE estimates for a heteroscedastic hierarchical model. *J. Amer. Statist. Assoc.* **107** 1465–1479. MR3036408

[23] Zhang, C.-H. (2003). Compound decision theory and empirical Bayes methods. *Ann. Statist.* **31** 379–390. MR1983534

Bernoulli **25**(4B), 2019, 3479–3495
https://doi.org/10.3150/18-BEJ1097

Two-sided infinite-bin models and analyticity for Barak–Erdős graphs

BASTIEN MALLEIN[1] and SANJAY RAMASSAMY[2]

[1]*Laboratoire Analyse, Géométrie et Applications, Institut Galilée, 99 avenue Jean-Baptiste Clément, 93430 Villetaneuse, France. E-mail: mallein@math.univ-paris13.fr; url: http://www.math.univ-paris13.fr/~mallein/*
[2]*Département de Mathématiques et Applications, École normale supérieure, 45 rue d'Ulm, 75005 Paris, France. E-mail: sanjay.ramassamy@ens.fr; url: http://www.normalesup.org/~ramassamy/*

In this article, we prove that for any probability distribution μ on $\mathbb{N}$ one can construct a two-sided stationary version of the infinite-bin model – an interacting particle system introduced by Foss and Konstantopoulos – with move distribution μ. Using this result, we obtain a new formula for the speed of the front of infinite-bin models, as a series of positive terms. This implies that the growth rate $C(p)$ of the longest path in a Barak–Erdős graph of parameter p is analytic on $(0, 1]$.

Keywords: Barak–Erdős graphs; infinite-bin model; interacting particle systems; longest path; random graphs; two-sided Markov chains

References

[1] Aldous, D. and Pitman, J. (1983). The asymptotic speed and shape of a particle system. In *Probability, Statistics and Analysis*. *London Mathematical Society Lecture Note Series* **79** 1–23. Cambridge: Cambridge Univ. Press. MR0696018
[2] Barak, A.B. and Erdős, P. (1984). On the maximal number of strongly independent vertices in a random acyclic directed graph. *SIAM J. Algebr. Discrete Methods* **5** 508–514. MR0763980
[3] Bérard, J. and Gouéré, J.-B. (2010). Brunet–Derrida behavior of branching-selection particle systems on the line. *Comm. Math. Phys.* **298** 323–342. MR2669438
[4] Brunet, E. and Derrida, B. (1997). Shift in the velocity of a front due to a cutoff. *Phys. Rev. E* (3) **56** 2597–2604. MR1473413
[5] Chernysh, K. and Ramassamy, S. (2017). Coupling any number of balls in the infinite-bin model. *J. Appl. Probab.* **54** 540–549. MR3668482
[6] Cohen, J.E. and Newman, C.M. (1991). Community area and food-chain length: Theoretical predictions. *Amer. Nat.* **138** 1542–1554.
[7] Comets, F., Fernández, R. and Ferrari, P.A. (2002). Processes with long memory: Regenerative construction and perfect simulation. *Ann. Appl. Probab.* **12** 921–943. MR1925446
[8] Denisov, D., Foss, S. and Konstantopoulos, T. (2012). Limit theorems for a random directed slab graph. *Ann. Appl. Probab.* **22** 702–733. MR2953567
[9] Erdős, P. and Rényi, A. (1959). On random graphs. I. *Publ. Math. Debrecen* **6** 290–297. MR0120167
[10] Foss, S. and Konstantopoulos, T. (2003). Extended renovation theory and limit theorems for stochastic ordered graphs. *Markov Process. Related Fields* **9** 413–468. MR2028221
[11] Foss, S., Martin, J.B. and Schmidt, P. (2014). Long-range last-passage percolation on the line. *Ann. Appl. Probab.* **24** 198–234. MR3161646

[12] Foss, S. and Zachary, S. (2013). Stochastic sequences with a regenerative structure that may depend both on the future and on the past. *Adv. in Appl. Probab.* **45** 1083–1110. MR3161298

[13] Gelenbe, E., Nelson, R., Philips, T. and Tantawi, A. (1986). An approximation of the processing time for a random graph model of parallel computation. In *Proceedings of* 1986 *ACM Fall Joint Computer Conference*, *Los Alamitos*, *CA*, *USA* 691–697.

[14] Isopi, M. and Newman, C.M. (1994). Speed of parallel processing for random task graphs. *Comm. Pure Appl. Math.* **47** 361–376. MR1266246

[15] Konstantopoulos, T. and Trinajstić, K. (2013). Convergence to the Tracy–Widom distribution for longest paths in a directed random graph. *ALEA Lat. Am. J. Probab. Math. Stat.* **10** 711–730. MR3108812

[16] Mallein, B. (2017). Branching random walk with selection at critical rate. *Bernoulli* **23** 1784–1821. MR3624878

[17] Mallein, B. and Ramassamy, S. (2016). Barak–Erdős graphs and the infinite-bin model. ArXiv preprint. Available at arXiv:1610.04043.

[18] Newman, C.M. (1992). Chain lengths in certain random directed graphs. *Random Structures Algorithms* **3** 243–253. MR1164838

[19] Newman, C.M. and Cohen, J.E. (1986). A stochastic theory of community food webs IV: Theory of food chain lengths in large webs. *Proc. R. Soc. Lond., B Biol. Sci.* **228** 355–377.

[20] Propp, J.G. and Wilson, D.B. (1996). Exact sampling with coupled Markov chains and applications to statistical mechanics. Random structures algorithms, 1–2. In *Proceedings of the Seventh International Conference on Random Structures and Algorithms* (*Atlanta*, *GA*, 1995) **9** 223–252. MR1611693

Bernoulli **25**(4B), 2019, 3496–3526
https://doi.org/10.3150/18-BEJ1099

Moving block and tapered block bootstrap for functional time series with an application to the K-sample mean problem

DIMITRIOS PILAVAKIS, EFSTATHIOS PAPARODITIS and THEOFANIS SAPATINAS

Department of Mathematics and Statistics, University of Cyprus, P.O. Box 20537, CY 1678 Nicosia, Cyprus. E-mail: stathisp@ucy.ac.cy

We consider infinite-dimensional Hilbert space-valued random variables that are assumed to be temporal dependent in a broad sense. We prove a central limit theorem for the moving block bootstrap and for the tapered block bootstrap, and show that these block bootstrap procedures also provide consistent estimators of the long run covariance operator. Furthermore, we consider block bootstrap-based procedures for fully functional testing of the equality of mean functions between several independent functional time series. We establish validity of the block bootstrap methods in approximating the distribution of the statistic of interest under the null and show consistency of the block bootstrap-based tests under the alternative. The finite sample behaviour of the procedures is investigated by means of simulations. An application to a real-life dataset is also discussed.

Keywords: functional time series; K-sample mean problem; mean function; moving block bootstrap; spectral density operator; tapered block bootstrap

References

[1] Benko, M., Härdle, W. and Kneip, A. (2009). Common functional principal components. *Ann. Statist.* **37** 1–34. MR2488343
[2] Beutner, E. and Zähle, H. (2016). Functional delta-method for the bootstrap of quasi-Hadamard differentiable functionals. *Electron. J. Stat.* **10** 1181–1222. MR3499525
[3] Billingsley, P. (1999). *Convergence of Probability Measures*, 2nd ed. *Wiley Series in Probability and Statistics: Probability and Statistics*. New York: Wiley. A Wiley-Interscience Publication. MR1700749
[4] Cerovecki, C. and Hörmann, S. (2017). On the CLT for discrete Fourier transforms of functional time series. *J. Multivariate Anal.* **154** 282–295. MR3588570
[5] Dehling, H., Sharipov, O.Sh. and Wendler, M. (2015). Bootstrap for dependent Hilbert space-valued random variables with application to von Mises statistics. *J. Multivariate Anal.* **133** 200–215. MR3282026
[6] Giné, E. and Zinn, J. (1990). Bootstrapping general empirical measures. *Ann. Probab.* **18** 851–869. MR1055437
[7] Hörmann, S. and Kokoszka, P. (2010). Weakly dependent functional data. *Ann. Statist.* **38** 1845–1884. MR2662361
[8] Horváth, L. and Kokoszka, P. (2012). *Inference for Functional Data with Applications*. *Springer Series in Statistics*. New York: Springer. MR2920735

[9] Horváth, L., Kokoszka, P. and Reeder, R. (2013). Estimation of the mean of functional time series and a two-sample problem. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **75** 103–122. MR3008273

[10] Horváth, L. and Rice, G. (2015). Testing equality of means when the observations are from functional time series. *J. Time Series Anal.* **36** 84–108. MR3300207

[11] Horváth, L., Rice, G. and Whipple, S. (2016). Adaptive bandwidth selection in the long run covariance estimator of functional time series. *Comput. Statist. Data Anal.* **100** 676–693. MR3505826

[12] Künsch, H.R. (1989). The jackknife and the bootstrap for general stationary observations. *Ann. Statist.* **17** 1217–1241. MR1015147

[13] Laha, R.G. and Rohatgi, V.K. (1979). *Probability Theory. Wiley Series in Probability and Mathematical Statistics*. New York–Chichester–Brisbane: Wiley. MR0534143

[14] Lahiri, S.N. (2003). *Resampling Methods for Dependent Data. Springer Series in Statistics*. New York: Springer. MR2001447

[15] Liu, R.Y. and Singh, K. (1992). Moving blocks jackknife and bootstrap capture weak dependence. In *Exploring the Limits of Bootstrap* (*East Lansing*, *MI*, 1990). *Wiley Ser. Probab. Math. Statist. Probab. Math. Statist.* 225–248. New York: Wiley. MR1197787

[16] Panaretos, V.M. and Tavakoli, S. (2013). Cramér–Karhunen–Loève representation and harmonic principal component analysis of functional time series. *Stochastic Process. Appl.* **123** 2779–2807. MR3054545

[17] Panaretos, V.M. and Tavakoli, S. (2013). Fourier analysis of stationary time series in function space. *Ann. Statist.* **41** 568–603. MR3099114

[18] Paparoditis, E. (2018). Sieve bootstrap for functional time series. *Ann. Statist.* **46** 3510–3538. MR3852660

[19] Paparoditis, E. and Politis, D.N. (2001). Tapered block bootstrap. *Biometrika* **88** 1105–1119. MR1872222

[20] Paparoditis, E. and Sapatinas, T. (2016). Bootstrap-based testing of equality of mean functions or equality of covariance operators for functional data. *Biometrika* **103** 727–733. MR3551795

[21] Pilavakis, D., Paparoditis, E. and Sapatinas, T. (2019). Supplement to "Moving block and tapered block bootstrap for functional time series with an application to the K-sample mean problem." DOI:10.3150/18-BEJ1099SUPP.

[22] Politis, D.N. and Romano, J.P. (1994). Limit theorems for weakly dependent Hilbert space valued random variables with application to the stationary bootstrap. *Statist. Sinica* **4** 461–476. MR1309424

[23] Prokhorov, Yu.V. (1956). Convergence of random processes and limit theorems in probability theory. *Teor. Veroyatn. Primen.* **1** 177–238. MR0084896

[24] Ramsay, J.O. and Dalzell, C.J. (1991). Some tools for functional data analysis. *J. Roy. Statist. Soc. Ser. B* **53** 539–572. With discussion and a reply by the authors. MR1125714

[25] Rice, G. and Shang, H.L. (2017). A plug-in bandwidth selection procedure for long-run covariance estimation with stationary functional time series. *J. Time Series Anal.* **38** 591–609. MR3664648

[26] Staicu, A.-M., Lahiri, S.N. and Carroll, R.J. (2015). Significance tests for functional data with complex dependence structure. *J. Statist. Plann. Inference* **156** 1–13. MR3267509

[27] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes. With Applications to Statistics. Springer Series in Statistics*. New York: Springer. MR1385671

[28] Zhang, C., Peng, H. and Zhang, J.-T. (2010). Two samples tests for functional data. *Comm. Statist. Theory Methods* **39** 559–578. MR2745305

[29] Zhang, J.-T. (2013). *Analysis of Variance for Functional Data*. Boca Raton, FL: CRC Press/CRC.

Bernoulli **25**(4B), 2019, 3527–3554
https://doi.org/10.3150/18-BEJ1101

Bernstein-type exponential inequalities in survey sampling: Conditional Poisson sampling schemes

PATRICE BERTAIL[1] and STEPHAN CLÉMENÇON[2]

[1]*Modal'X, UPL, Université Paris-Nanterre, 200 avenue de la République, Nanterre 92100, France. E-mail: patrice.bertail@u-paris10.fr*
[2]*LTCI, Telecom ParisTech, Université Paris-Saclay, 46 rue Barrault, 75013, Paris, France. E-mail: stephan.clemencon@telecom-paristech.fr*

This paper is devoted to establishing exponential bounds for the probabilities of deviation of a sample sum from its expectation, when the variables involved in the summation are obtained by sampling in a finite population according to a rejective scheme, generalizing simple random sampling without replacement, and by using an appropriate normalization. In contrast to Poisson sampling, classical deviation inequalities in the i.i.d. setting do not straightforwardly apply to sample sums related to rejective schemes, due to the inherent dependence structure of the sampled points. We show here how to overcome this difficulty, by combining the formulation of rejective sampling as Poisson sampling conditioned upon the sample size with the Esscher transformation. In particular, the Bennett/Bernstein type bounds thus established highlight the effect of the asymptotic variance of the (properly standardized) sample weighted sum and are shown to be much more accurate than those based on the negative association property shared by the terms involved in the summation. Beyond its interest in itself, such a result for rejective sampling is crucial, insofar as it permit to obtain tail bounds for many other sampling schemes, namely those that can be accurately approximated by rejective plans in the sense of the total variation distance.

Keywords: coupling; Esscher transformation; exponential inequality; Poisson survey scheme; rejective sampling; survey sampling

References

[1] Arratia, R., Goldstein, L. and Langholz, B. (2005). Local central limit theorems, the high-order correlations of rejective sampling and logistic likelihood asymptotics. *Ann. Statist.* **33** 871–914. MR2163162
[2] Barbour, A.D., Holst, L. and Janson, S. (1992). *Poisson Approximation. Oxford Studies in Probability* **2**. New York: Oxford University Press. MR1163825
[3] Bardenet, R. and Maillard, O.-A. (2015). Concentration inequalities for sampling without replacement. *Bernoulli* **21** 1361–1385. MR3352047
[4] Bennett, G. (1962). Probability inequalities for a sum of independent random variables. *J. Amer. Statist. Assoc.* **57** 33–45.
[5] Berger, Y.G. (1998). Rate of convergence to normal distribution for the Horvitz–Thompson estimator. *J. Statist. Plann. Inference* **67** 209–226. MR1624693
[6] Bernstein, S.N. (1964). On a modification of Chebyshev's inequality and on the error in Laplace formula. *Collected Works, Izd-vo 'Nauka', Moscow* (*in Russian*) **4** 71–80.

1350-7265

[7] Bertail, P., Chautru, E. and Clémençon, S. (2017). Empirical processes in survey sampling with (conditional) Poisson designs. *Scand. J. Stat.* **44** 97–111. MR3619696

[8] Bertail, P. and Clémençon, S. (2010). Sharp bounds for the tails of functionals of Markov chains. *Theory Probab. Appl.* **54** 505–515.

[9] Boistard, H., Lopuhaä, H.P. and Ruiz-Gazen, A. (2012). Approximation of rejective sampling inclusion probabilities and application to high order correlations. *Electron. J. Stat.* **6** 1967–1983. MR3020253

[10] Boistard, H., Lopuhaä, H.P. and Ruiz-Gazen, A. (2017). Functional central limit theorems for single-stage sampling designs. *Ann. Statist.* **45** 1728–1758. MR3670194

[11] Bondesson, L., Traat, I. and Lundqvist, A. (2006). Pareto sampling versus Sampford and conditional Poisson sampling. *Scand. J. Stat.* **33** 699–720. MR2300911

[12] Boucheron, S., Lugosi, G. and Massart, P. (2013). *Concentration Inequalities: A Nonasymptotic Theory of Independence*. Oxford: Oxford Univ. Press. MR3185193

[13] Brändén, P. and Jonasson, J. (2012). Negative dependence in sampling. *Scand. J. Stat.* **39** 830–838. MR3000852

[14] Kramer, J.B., Cutler, J. and Radcliffe, A.J. (2011). Negative dependence and Srinivasan's sampling process. *Combin. Probab. Comput.* **20** 347–361. MR2784632

[15] Chen, X.-H., Dempster, A.P. and Liu, J.S. (1994). Weighted finite population sampling to maximize entropy. *Biometrika* **81** 457–469. MR1311090

[16] Choi, K.P. and Xia, A. (2002). Approximating the number of successes in independent trials: Binomial versus Poisson. *Ann. Appl. Probab.* **12** 1139–1148. MR1936586

[17] Deheuvels, P., Puri, M.L. and Ralescu, S.S. (1989). Asymptotic expansions for sums of nonidentically distributed Bernoulli random variables. *J. Multivariate Anal.* **28** 282–303. MR0991952

[18] Deville, J.C. (1987). Réplications d'échantillons, demi-échantillons, Jackknife, bootstrap dans les sondages. In *Les Sondages*. Economica, Ed. Droesbeke, Tassi, Fichet.

[19] Dupačová, J. (1979). A note on rejective sampling. In *Contributions to Statistics* 71–78. Dordrecht-Boston, Mass.-London: Reidel. MR0561260

[20] Fuk, D.H. and Nagaev, S.V. (1971). Probabilistic inequalities for sums of independent random variables. *Theory Probab. Appl.* **16** 643–660.

[21] Fuller, W.A. (2009). Some design properties of a rejective sampling procedure. *Biometrika* **96** 933–944. MR2564501

[22] Goodman, L.A. (1949). On the estimation of the number of classes in a population. *Ann. Math. Stat.* **20** 572–579. MR0032165

[23] Hájek, J. (1964). Asymptotic theory of rejective sampling with varying probabilities from a finite population. *Ann. Math. Stat.* **35** 1491–1523. MR0178555

[24] Hoeffding, W. (1963). Probability inequalities for sums of bounded random variables. *J. Amer. Statist. Assoc.* **58** 13–30. MR0144363

[25] Horvitz, D.G. and Thompson, D.J. (1952). A generalization of sampling without replacement from a finite universe. *J. Amer. Statist. Assoc.* **47** 663–685. MR0053460

[26] Janson, S. (1994). Large deviation inequalities for sums of indicator variables. Unpublished manuscript. Available at www2.math.uu.se/~svante/papers/sj107.ps.

[27] Joag-Dev, K. and Proschan, F. (1983). Negative association of random variables, with applications. *Ann. Statist.* **11** 286–295. MR0684886

[28] Percus, O.E. and Percus, J.K. (1985). Probability bounds on the sum of independent nonidentically distributed binomial random variables. *SIAM J. Appl. Math.* **45** 621–640. MR0796099

[29] Robinson, P.M. (1982). On the convergence of the Horvitz–Thompson estimator. *Aust. J. Stat.* **24** 234–238. MR0678263

[30] Särndal, C.-E., Swensson, B. and Wretman, J. (1992). *Model Assisted Survey Sampling*. *Springer Series in Statistics*. New York: Springer. MR1140409

[31] Serfling, R.J. (1974). Probability inequalities for the sum in sampling without replacement. *Ann*. *Statist*. **2** 39–48. MR0420967

[32] Shao, Q.-M. (2000). A comparison theorem on moment inequalities between negatively associated and independent random variables. *J*. *Theoret*. *Probab*. **13** 343–356. MR1777538

[33] Shiganov, I.S. (1986). Refinement of the upper bound of the constant in the central limit theorem. *J*. *Sov*. *Math*. **35** 2545–2550.

[34] Talagrand, M. (1995). The missing factor in Hoeffding's inequalities. *Ann*. *Inst*. *Henri Poincaré Probab*. *Stat*. **31** 689–702. MR1355613

[35] Tan, Z. (2013). Simple design-efficient calibration estimators for rejective and high-entropy sampling. *Biometrika* **100** 399–415. MR3068442

[36] Tillé, Y. (2006). *Sampling Algorithms*. *Springer Series in Statistics*. New York: Springer. MR2225036

Bernoulli **25**(4B), 2019, 3555–3589
https://doi.org/10.3150/18-BEJ1102

Asymptotic equivalence of fixed-size and varying-size determinantal point processes

SIMON BARTHELMÉ, PIERRE-OLIVIER AMBLARD and NICOLAS TREMBLAY

CNRS, Gipsa-lab, Grenoble INP and Université Grenoble Alpes, 11 rue des Mathématiques, Grenoble Campus, BP46, F-38402 Saint Martin d'Heres Cedex, France. E-mail: simon.barthelme@gipsa-lab.fr; pierre-olivier.amblard@gipsa-lab.fr; nicolas.tremblay@gipsa-lab.fr

Determinantal Point Processes (DPPs) are popular models for point processes with repulsion. They appear in numerous contexts, from physics to graph theory, and display appealing theoretical properties. On the more practical side of things, since DPPs tend to select sets of points that are some distance apart (repulsion), they have been advocated as a way of producing random subsets with high diversity. DPPs come in two variants: fixed-size and varying-size. A sample from a varying-size DPP is a subset of random cardinality, while in fixed-size "k-DPPs" the cardinality is fixed. The latter makes more sense in many applications, but unfortunately their computational properties are less attractive, since, among other things, inclusion probabilities are harder to compute. In this work, we show that as the size of the ground set grows, k-DPPs and DPPs become equivalent, in the sense that fixed-order inclusion probabilities converge. As a by-product, we obtain saddlepoint formulas for inclusion probabilities in k-DPPs. These turn out to be extremely accurate, and suffer less from numerical difficulties than exact methods do. Our results also suggest that k-DPPs and DPPs also have equivalent maximum likelihood estimators. Finally, we obtain results on asymptotic approximations of elementary symmetric polynomials which may be of independent interest.

Keywords: determinantal point processes; point processes; saddlepoint approximation

References

[1] Billingsley, P. (2008). *Probability and Measure*. John Wiley & Sons.
[2] Chen, X.-H., Dempster, A.P. and Liu, J.S. (1994). Weighted finite population sampling to maximize entropy. *Biometrika* **81** 457–469. MR1311090
[3] Couillet, R. and Debbah, M. (2011). *Random Matrix Methods for Wireless Communications*. Cambridge: Cambridge Univ. Press. MR2884783
[4] Daniels, H.E. (1954). Saddlepoint approximations in statistics. *Ann. Math. Stat.* **25** 631–650. MR0066602
[5] DasGupta, A. (2008). *Asymptotic Theory of Statistics and Probability*. *Springer Texts in Statistics*. New York: Springer. MR2664452
[6] Deshpande, A. and Rademacher, L. (2010). Efficient volume sampling for row/column subset selection. In 2010 *IEEE* 51*st Annual Symposium on Foundations of Computer Science—FOCS* 2010 329–338. Los Alamitos, CA: IEEE Computer Soc. MR3025206
[7] Deshpande, A., Rademacher, L., Vempala, S. and Wang, G. (2006). Matrix approximation and projective clustering via volume sampling. In *Proceedings of the Seventeenth Annual ACM-SIAM Symposium on Discrete Algorithms* 1117–1126. New York: ACM. MR2373839

[8] Gautier, G., Bardenet, R. and Valko, M. (2017). Zonotope hit-and-run for efficient sampling from projection DPPs. ArXiv Preprint arXiv:1705.10498.
[9] Jozsa, R. and Mitchison, G. (2015). Symmetric polynomials in information theory: Entropy and subentropy. *J. Math. Phys.* **56** 062201, 17. MR3369891
[10] Kulesza, A. and Taskar, B. (2011). k-DPPs: Fixed-size determinantal point processes. In *Proceedings of the* 28*th International Conference on Machine Learning* (*ICML*-11) 1193–1200.
[11] Kulesza, A. and Taskar, B. (2012). Determinantal point processes for machine learning. *Found. Trends Mach. Learn.* **5** 123–286.
[12] Li, C., Jegelka, S. and Sra, S. (2016). Efficient sampling for k-determinantal point processes.
[13] Macchi, O. (1975). The coincidence approach to stochastic point processes. *Adv. in Appl. Probab.* **7** 83–122. MR0380979
[14] Mariet, Z. and Sra, S. (2017). Elementary symmetric polynomials for optimal experimental design. ArXiv Eprint arXiv:1705.09677v1.
[15] Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields*: *Theory and Applications*. *Monographs on Statistics and Applied Probability* **104**. Boca Raton, FL: CRC Press/CRC. MR2130347
[16] Soshnikov, A. (2000). Determinantal random point fields. *Russian Math. Surveys* **55** 923–975.
[17] Sra, S. (2019). Logarithmic inequalities under a symmetric polynomial dominance order. *Proc. Amer. Math. Soc.* **147** 481–486. MR3894886
[18] Touchette, H. (2015). Equivalence and nonequivalence of ensembles: Thermodynamic, macrostate, and measure levels. *J. Stat. Phys.* **159** 987–1016. MR3345408
[19] Tremblay, N., Barthelmé, S. and Amblard, P.-O. (2018). Determinantal point processes for coresets. ArXiv Preprint arXiv:1803.08700.
[20] Tremblay, N., Barthelme, S. and Amblard, P.O. (2018). Optimized algorithms to sample determinantal point processes. ArXiv E-print arXiv:1802.08471.

Bernoulli **25**(4B), 2019, 3590–3622
https://doi.org/10.3150/18-BEJ1103

The eigenstructure of the sample covariance matrices of high-dimensional stochastic volatility models with heavy tails

JOHANNES HEINY[1] and THOMAS MIKOSCH[2]

[1]*Department of Mathematics, Ruhr-University Bochum, Universitätsstrasse 150, D-44801 Bochum, Germany. E-mail: johannes.heiny@rub.de*
[2]*Department of Mathematical Sciences, University of Copenhagen, Universitetsparken 5, DK-2100 Copenhagen, Denmark. E-mail: mikosch@math.ku.dk*

We consider a p-dimensional time series where the dimension p increases with the sample size n. The resulting data matrix $\mathbf{X}$ follows a stochastic volatility model: each entry consists of a positive random volatility term multiplied by an independent noise term. The volatility multipliers introduce dependence in each row and across the rows. We study the asymptotic behavior of the eigenvalues and eigenvectors of the sample covariance matrix $\mathbf{X}\mathbf{X}'$ under a regular variation assumption on the noise. In particular, we prove Poisson convergence for the point process of the centered and normalized eigenvalues and derive limit theory for functionals acting on them, such as the trace. We prove related results for stochastic volatility models with additional linear dependence structure and for stochastic volatility models where the time-varying volatility terms are extinguished with high probability when n increases. We provide explicit approximations of the eigenvectors which are of a strikingly simple structure. The main tools for proving these results are large deviation theorems for heavy-tailed time series, advocating a unified approach to the study of the eigenstructure of heavy-tailed random matrices.

Keywords: cluster Poisson limit; convergence; dependent entries; Fréchet distribution; infinite variance stable limit; large deviations; largest eigenvalues; point process; regular variation; sample autocovariance matrix; trace

References

[1] Andersen, T.G., Davis, R.A., Kreiss, J.-P. and Mikosch, T.V. (2009). *Handbook of Financial Time Series*. Berlin: Springer.
[2] Auffinger, A., Ben Arous, G. and Péché, S. (2009). Poisson convergence for the largest eigenvalues of heavy tailed random matrices. *Ann. Inst. Henri Poincaré Probab. Stat.* **45** 589–610. MR2548495
[3] Auffinger, A. and Tang, S. (2016). Extreme eigenvalues of sparse, heavy tailed random matrices. *Stochastic Process. Appl.* **126** 3310–3330. MR3549709
[4] Bai, Z. and Silverstein, J.W. (2010). *Spectral Analysis of Large Dimensional Random Matrices*, 2nd ed. *Springer Series in Statistics*. New York: Springer. MR2567175
[5] Bauwens, L., Hafner, C. and Laurent, S., eds. (2012). *Handbook of Volatility Models and Their Applications*. *Wiley Handbook in Financial Engineering and Econometrics*. Hoboken, NJ: Wiley.
[6] Bhatia, R. (1997). *Matrix Analysis*. *Graduate Texts in Mathematics* **169**. New York: Springer. MR1477662

[7] Billingsley, P. (2012). *Probability and Measure*. *Wiley Series in Probability and Statistics*. Hoboken, NJ: Wiley. Anniversary edition [of MR1324786], With a foreword by Steve Lalley and a brief biography of Billingsley by Steve Koppes. MR2893652
[8] Black, F. and Scholes, M. (1972). The valuation of option contracts and a test of market efficiency. *J. Finance* **27** 399–417.
[9] Bollerslev, T., Todorov, V. and Xu, L. (2015). Tail risk premia and return predictability. *J. Financ. Econ.* **118** 113–134.
[10] Buraczewski, D., Damek, E. and Mikosch, T. (2016). *Stochastic Models with Power-Law Tails*: *The Equation* $X = AX + B$. *Springer Series in Operations Research and Financial Engineering*. Cham: Springer. MR3497380
[11] Cavicchioli, M. (2017). Estimation and asymptotic covariance matrix for stochastic volatility models. *Stat. Methods Appl.* **26** 437–452. MR3687233
[12] Cui, Z., Feng, R. and MacKay, A. (2017). Variable annuities with VIX-linked fee structure under a Heston-type stochastic volatility model. *N. Am. Actuar. J.* **21** 458–483. MR3691487
[13] Davis, R.A., Heiny, J., Mikosch, T. and Xie, X. (2016). Extreme value analysis for the sample autocovariance matrices of heavy-tailed multivariate time series. *Extremes* **19** 517–547. MR3535965
[14] Davis, R.A. and Mikosch, T. (2001). Point process convergence of stochastic volatility processes with application to sample autocorrelation. *J. Appl. Probab.* **38A** 93–104. Probability, statistics and seismology. MR1915537
[15] Davis, R.A., Mikosch, T. and Pfaffel, O. (2016). Asymptotic theory for the sample covariance matrix of a heavy-tailed multivariate time series. *Stochastic Process. Appl.* **126** 767–799. MR3452812
[16] Embrechts, P. and Goldie, C.M. (1980). On closure and factorization properties of subexponential and related distributions. *J. Aust. Math. Soc. A* **29** 243–256. MR0566289
[17] Embrechts, P., Klüppelberg, C. and Mikosch, T. (1997). *Modelling Extremal Events*: *For Insurance and Finance*. *Applications of Mathematics* (*New York*) **33**. Berlin: Springer. MR1458613
[18] Fama, E.F. (1965). The behaviour of stock market prices. *J. Bus.* **38** 34–105.
[19] Fouque, J.-P., Papanicolaou, G., Sircar, R. and Sølna, K. (2011). *Multiscale Stochastic Volatility for Equity*, *Interest Rate*, *and Credit Derivatives*. Cambridge: Cambridge Univ. Press. MR2867464
[20] Fouque, J.-P., Sircar, R. and Zariphopoulou, T. (2017). Portfolio optimization and stochastic volatility asymptotics. *Math. Finance* **27** 704–745. MR3668155
[21] Heiny, J. and Mikosch, T. (2017). Eigenvalues and eigenvectors of heavy-tailed sample covariance matrices with general growth rates: The iid case. *Stochastic Process. Appl.* **127** 2179–2207. MR3652410
[22] Hill, J.B. (2013). Expected shortfall estimation and Gaussian inference for infinite variance time series. *J. Financ. Econom.* **13** 1–44.
[23] Hill, J.B. (2013). Least tail-trimmed squares for infinite variance autoregressions. *J. Time Series Anal.* **34** 168–186. MR3028364
[24] Janssen, A., Mikosch, T., Rezapour, M. and Xie, X. (2018). The eigenvalues of the sample covariance matrix of a multivariate heavy-tailed stochastic volatility model. *Bernoulli* **24** 1351–1393. MR3706796
[25] Kelly, B. and Jiang, H. (2014). Tail risk and asset prices. *Rev. Financ. Stud.* **27** 2841–2871.
[26] Li, D., Rong, X. and Zhao, H. (2017). Equilibrium excess-of-loss reinsurance–investment strategy for a mean–variance insurer under stochastic volatility model. *Comm. Statist. Theory Methods* **46** 9459–9475. MR3691404
[27] Ling, S. (2005). Self-weighted least absolute deviation estimation for infinite variance autoregressive models. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **67** 381–393. MR2155344
[28] Mandelbrot, B. (1963). The variation of certain speculative prices. *J. Bus.* **36** 394–419.
[29] Mikosch, T. and Rezapour, M. (2013). Stochastic volatility models with possible extremal clustering. *Bernoulli* **19** 1688–1713. MR3129030

[30] Mikosch, T. and Wintenberger, O. (2013). Precise large deviations for dependent regularly varying sequences. *Probab. Theory Related Fields* **156** 851–887. MR3078288
[31] Mikosch, T. and Wintenberger, O. (2016). A large deviations approach to limit theory for heavy-tailed time series. *Probab. Theory Related Fields* **166** 233–269. MR3547739
[32] Paul, D. and Aue, A. (2014). Random matrix theory in statistics: A review. *J. Statist. Plann. Inference* **150** 1–29. MR3206718
[33] Petrov, V.V. (1995). *Limit Theorems of Probability Theory: Sequences of Independent Random Variables. Oxford Studies in Probability* **4**. New York: The Clarendon Press, Oxford Univ. Press. MR1353441
[34] Resnick, S.I. (2007). *Heavy-Tail Phenomena: Probabilistic and Statistical Modeling. Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2271424
[35] Resnick, S.I. (2008). *Extreme Values, Regular Variation and Point Processes. Springer Series in Operations Research and Financial Engineering*. New York: Springer. Reprint of the 1987 original. MR2364939
[36] Rio, E. (2013). Inequalities and limit theorems for weakly dependent sequences. *HAL*. Available at https://cel.archives-ouvertes.fr/cel-00867106, 177.
[37] Shephard, N., ed. (2005) *Stochastic Volatility: Selected Readings. Advanced Texts in Econometrics*. Oxford: Oxford Univ. Press. MR2203295
[38] Shephard, N. and Andersen, T.G. (2009). Stochastic volatility: Origins and overview. In *Handbook of Financial Time Series* 233–254. Springer.
[39] Soshnikov, A. (2004). Poisson statistics for the largest eigenvalues of Wigner random matrices with heavy tails. *Electron. Commun. Probab.* **9** 82–91. MR2081462
[40] Soshnikov, A. (2006). Poisson statistics for the largest eigenvalues in random matrix ensembles. In *Mathematical Physics of Quantum Mechanics. Lecture Notes in Physics* **690** 351–364. Berlin: Springer. MR2234922
[41] Wachter, K.W. (1980). The limiting empirical measure of multiple discriminant ratios. *Ann. Statist.* **8** 937–957. MR0585695
[42] Yin, Y.Q., Bai, Z.D. and Krishnaiah, P.R. (1983). Limiting behavior of the eigenvalues of a multivariate F matrix. *J. Multivariate Anal.* **13** 508–516. MR0727036

Bernoulli **25**(4B), 2019, 3623–3651
https://doi.org/10.3150/19-BEJ1104

Gaps and interleaving of point processes in sampling from a residual allocation model

JIM PITMAN[1] and YURI YAKUBOVICH[2]

[1]*Statistics Department, University of California, 367 Evans Hall # 3860, Berkeley, CA 94720-3860, USA. E-mail: pitman@stat.berkeley.edu*
[2]*St. Petersburg State University, Universitetskaya nab. 7-9, St. Petersburg 199034, Russia. E-mail: y.yakubovich@spbu.ru*

This article presents a limit theorem for the gaps $\widehat{G}_{i:n} := X_{n-i+1:n} - X_{n-i:n}$ between order statistics $X_{1:n} \le \cdots \le X_{n:n}$ of a sample of size n from a random discrete distribution on the positive integers $(P_1, P_2, \ldots)$ governed by a residual allocation model (also called a Bernoulli sieve) $P_j := H_j \prod_{i=1}^{j-1}(1 - H_i)$ for a sequence of independent random hazard variables H_i which are identically distributed according to some distribution of $H \in (0, 1)$ such that $-\log(1 - H)$ has a non-lattice distribution with finite mean $\mu_{\log}$. As $n \to \infty$ the finite dimensional distributions of the gaps $\widehat{G}_{i:n}$ converge to those of limiting gaps G_i which are the numbers of points in a stationary renewal process with i.i.d. spacings $-\log(1 - H_j)$ between times T_{i-1} and T_i of births in a Yule process, that is $T_i := \sum_{k=1}^{i} \varepsilon_k / k$ for a sequence of i.i.d. exponential variables ε_k with mean 1. A consequence is that the mean of $\widehat{G}_{i:n}$ converges to the mean of G_i, which is $1/(i\mu_{\log})$. This limit theorem simplifies and extends a result of Gnedin, Iksanov and Roesler for the Bernoulli sieve.

Keywords: GEM distribution; interleaving of simple point processes; residual allocation model; stars and bars duality; stationary renewal process; Yule process

References

[1] Arratia, R., Barbour, A.D. and Tavaré, S. (1992). Poisson process approximations for the Ewens sampling formula. *Ann. Appl. Probab.* **2** 519–535. MR1177897
[2] Athreya, K.B. and Ney, P.E. (1972). *Branching Processes. Die Grundlehren der Mathematischen Wissenschaften* **176**. New York: Springer. MR0373040
[3] Birkner, M., Geiger, J. and Kersting, G. (2005). Branching processes in random environment—A view on critical and subcritical cases. In *Interacting Stochastic Systems* 269–291. Berlin: Springer. MR2118578
[4] Bogachev, L.V., Gnedin, A.V. and Yakubovich, Y.V. (2008). On the variance of the number of occupied boxes. *Adv. in Appl. Math.* **40** 401–432. MR2412154
[5] Borel, É. (1947). Sur les développements unitaires normaux. *C. R. Acad. Sci. Paris* **225** 51. MR0023007
[6] Bruss, F.T. and Rogers, L.C.G. (1991). Pascal processes and their characterization. *Stochastic Process. Appl.* **37** 331–338. MR1102879
[7] Carmona, P., Petit, F. and Yor, M. (1998). Beta–gamma random variables and intertwining relations between certain Markov processes. *Rev. Mat. Iberoam.* **14** 311–367. MR1654531
[8] Daley, D.J. and Vere-Jones, D. (1988). *An Introduction to the Theory of Point Processes. Springer Series in Statistics*. New York: Springer. MR0950166

1350-7265

[9] Donnelly, P. and Tavaré, S. (1986). The ages of alleles and a coalescent. *Adv. in Appl. Probab.* **18** 1–19. MR0827330

[10] Duchamps, J.-J., Pitman, J. and Tang, W. Renewal sequences and record chains related to multiple zeta sums. *Trans. Amer. Math. Soc.* Published online: September 18, 2018.

[11] Engen, S. (1975). A note on the geometric series as a species frequency model. *Biometrika* **62** 697–699. MR0411097

[12] Erdős, P., Rényi, A. and Szüsz, P. (1958). On Engel's and Sylvester's series. *Ann. Univ. Sci. Budapest. Eötvös Sect. Math.* **1** 7–32. MR0102496

[13] Feigin, P.D. (1979). On the characterization of point processes with the order statistic property. *J. Appl. Probab.* **16** 297–304. MR0531764

[14] Feller, W. (1968). *An Introduction to Probability Theory and Its Applications*, *Vol. I*, 3rd ed. New York: Wiley. MR0228020

[15] Gnedin, A., Hansen, B. and Pitman, J. (2007). Notes on the occupancy problem with infinitely many boxes: General asymptotics and power laws. *Probab. Surv.* **4** 146–171. MR2318403

[16] Gnedin, A., Iksanov, A. and Marynych, A. (2010). The Bernoulli sieve: An overview. In 21*st International Meeting on Probabilistic*, *Combinatorial*, *and Asymptotic Methods in the Analysis of Algorithms* (*AofA*'10). *Discrete Math. Theor. Comput. Sci. Proc.*, *AM* 329–341. Nancy: Assoc. Discrete Math. Theor. Comput. Sci. MR2735350

[17] Gnedin, A., Iksanov, A. and Roesler, U. (2008). Small parts in the Bernoulli sieve. In *Fifth Colloquium on Mathematics and Computer Science*. *Discrete Math. Theor. Comput. Sci. Proc.*, *AI* 235–242. Nancy: Assoc. Discrete Math. Theor. Comput. Sci. MR2508790

[18] Gnedin, A. and Pitman, J. (2005). Regenerative composition structures. *Ann. Probab.* **33** 445–479. MR2122798

[19] Gnedin, A.V. (2004). The Bernoulli sieve. *Bernoulli* **10** 79–96. MR2044594

[20] Gnedin, A.V., Iksanov, A.M., Negadajlov, P. and Rösler, U. (2009). The Bernoulli sieve revisited. *Ann. Appl. Probab.* **19** 1634–1655. MR2538083

[21] Grandell, J. (1997). *Mixed Poisson Processes*. *Monographs on Statistics and Applied Probability* **77**. London: CRC Press. MR1463943

[22] Greenwood, P. and Pitman, J. (1980). Construction of local time and Poisson point processes from nested arrays. *J. Lond. Math. Soc.* (2) **22** 182–192. MR0579823

[23] Halmos, P.R. (1944). Random alms. *Ann. Math. Stat.* **15** 182–189. MR0010342

[24] Hambly, B. (1992). On the limiting distribution of a supercritical branching process in a random environment. *J. Appl. Probab.* **29** 499–518. MR1174427

[25] Hunt, G.A. (1960). Markoff chains and Martin boundaries. *Illinois J. Math.* **4** 313–340. MR0123364

[26] Ignatov, T. (1982). A constant arising in the asymptotic theory of symmetric groups, and Poisson–Dirichlet measures. *Teor. Veroyatn. Primen.* **27** 129–140. MR0645134

[27] Iksanov, A. (2012). On the number of empty boxes in the Bernoulli sieve II. *Stochastic Process. Appl.* **122** 2701–2729. MR2926172

[28] Iksanov, A. (2013). On the number of empty boxes in the Bernoulli sieve I. *Stochastics* **85** 946–959. MR3176494

[29] Iksanov, A.M., Marynych, A.V. and Vatutin, V.A. (2015). Weak convergence of finite-dimensional distributions of the number of empty boxes in the Bernoulli sieve. *Theory Probab. Appl.* **59** 87–113. MR3416065

[30] Johnson, N.L., Kemp, A.W. and Kotz, S. (2005). *Univariate Discrete Distributions*, 3rd ed. *Wiley Series in Probability and Statistics*. Hoboken, NJ: Wiley Interscience. MR2163227

[31] Kallenberg, O. (2017). *Random Measures*, *Theory and Applications*. *Probability Theory and Stochastic Modelling* **77**. Cham: Springer. MR3642325

[32] Karlin, S. (1967). Central limit theorems for certain infinite urn schemes. *J. Math. Mech.* **17** 373–401. MR0216548
[33] Kendall, D.G. (1966). Branching processes since 1873. *J. Lond. Math. Soc.* **41** 385–406 (1 plate). MR0198551
[34] Lévy, P. (1947). Remarques sur un théorème de M. Émile Borel. *C. R. Acad. Sci. Paris* **225** 918–919. MR0023008
[35] Lundberg, O. (1940). On random processes and their application to sickness and accident statistics. Ph.D. thesis, Univ. Stockholm, Uppsala. MR0003509
[36] Neuts, M.F. and Resnick, S.I. (1971). On the times of births in a linear birthprocess. *J. Aust. Math. Soc.* **12** 473–475. MR0293732
[37] Nevzorov, V.B. (2001). *Records: Mathematical Theory*. *Translations of Mathematical Monographs* **194**. Providence, RI: Amer. Math. Soc. Translated from the Russian manuscript by D.M. Chibisov. MR1791071
[38] Norris, J.R. (1998). *Markov Chains*. *Cambridge Series in Statistical and Probabilistic Mathematics* **2**. Cambridge: Cambridge Univ. Press. Reprint of 1997 original. MR1600720
[39] Pitman, J. and Yakubovich, Y. (2017). Extremes and gaps in sampling from a GEM random discrete distribution. *Electron. J. Probab.* **22** Paper No. 44, 26. MR3646070
[40] Pitman, J. and Yor, M. (2001). On the distribution of ranked heights of excursions of a Brownian bridge. *Ann. Probab.* **29** 361–384. MR1825154
[41] Pitman, J.W. (1975). One-dimensional Brownian motion and the three-dimensional Bessel process. *Adv. in Appl. Probab.* **7** 511–526. MR0375485
[42] Rényi, A. (1953). On the theory of order statistics. *Acta Math. Acad. Sci. Hung.* **4** 191–231. MR0061792
[43] Sawyer, S. and Hartl, D. (1985). A sampling theory for local selection. *J. Genet.* **64** 21–29.
[44] Smith, W.L. and Wilkinson, W.E. (1969). On branching processes in random environments. *Ann. Math. Stat.* **40** 814–827. MR0246380
[45] Stanley, R.P. (2012). *Enumerative Combinatorics*, *Vol.* 1, 2nd ed. *Cambridge Studies in Advanced Mathematics* **49**. Cambridge: Cambridge Univ. Press. MR2868112
[46] Sukhatme, P.V. (1937). Tests of significance for samples of the χ^2-population with two degrees of freedom. *Ann. Eugen.* **8** 52–56.
[47] Vervaat, W. (1973). Limit theorems for records from discrete distributions. *Stochastic Process. Appl.* **1** 317–334. MR0362457
[48] Waugh, W.A.O'N. (1970). Transformation of a birth process into a Poisson process. *J. Roy. Statist. Soc. Ser. B* **32** 418–431. MR0287620

Bernoulli **25**(4B), 2019, 3652–3672
https://doi.org/10.3150/19-BEJ1106

Harmonic measure for biased random walk in a supercritical Galton–Watson tree

SHEN LIN

Sorbonne Université, Laboratoire de Probabilités, Statistique et Modélisation, UMR 8001, 4 place Jussieu, Boîte courrier 158, 75252 Paris Cedex 05, France. E-mail: shen.lin@sorbonne-universite.fr

We consider random walks λ-biased towards the root on a Galton–Watson tree, whose offspring distribution $(p_k)_{k\geq 1}$ is non-degenerate and has finite mean $m > 1$. In the transient regime $0 < \lambda < m$, the loop-erased trajectory of the biased random walk defines the λ-harmonic ray, whose law is the λ-harmonic measure on the boundary of the Galton–Watson tree. We answer a question of Lyons, Pemantle and Peres (In *Classical and Modern Branching Processes (Minneapolis, MN, 1994)* (1997) 223–237 Springer) by showing that the λ-harmonic measure has a.s. strictly larger Hausdorff dimension than the visibility measure, which is the harmonic measure corresponding to the simple forward random walk. We also prove that the average number of children of the vertices along the λ-harmonic ray is a.s. bounded below by m and bounded above by $m^{-1}\sum k^2 p_k$. Moreover, at least for $0 < \lambda \leq 1$, the average number of children of the vertices along the λ-harmonic ray is a.s. strictly larger than that of the λ-biased random walk trajectory. We observe that the latter is not monotone in the bias parameter λ.

Keywords: Galton–Watson tree; harmonic measure; random walk; stationary measure

References

[1] Aïdékon, E. (2014). Speed of the biased random walk on a Galton–Watson tree. *Probab. Theory Related Fields* **159** 597–617. MR3230003
[2] Ben Arous, G., Hu, Y., Olla, S. and Zeitouni, O. (2013). Einstein relation for biased random walk on Galton–Watson trees. *Ann. Inst. Henri Poincaré Probab. Stat.* **49** 698–721. MR3112431
[3] Ben-Hamou, A. and Salez, J. (2017). Cutoff for nonbacktracking random walks on sparse random graphs. *Ann. Probab.* **45** 1752–1770. MR3650414
[4] Berestycki, N., Lubetzky, E., Peres, Y. and Sly, A. (2018). Random walks on the random graph. *Ann. Probab.* **46** 456–490. MR3758735
[5] Lyons, R. (1990). Random walks and percolation on trees. *Ann. Probab.* **18** 931–958. MR1062053
[6] Lyons, R., Pemantle, R. and Peres, Y. (1995). Ergodic theory on Galton–Watson trees: Speed of random walk and dimension of harmonic measure. *Ergodic Theory Dynam. Systems* **15** 593–619. MR1336708
[7] Lyons, R., Pemantle, R. and Peres, Y. (1996). Biased random walks on Galton–Watson trees. *Probab. Theory Related Fields* **106** 249–264. MR1410689
[8] Lyons, R., Pemantle, R. and Peres, Y. (1997). Unsolved problems concerning random walks on trees. In *Classical and Modern Branching Processes* (*Minneapolis*, *MN*, 1994). *IMA Vol. Math. Appl.* **84** 223–237. New York: Springer. MR1601753
[9] Lyons, R. and Peres, Y. (2016). *Probability on Trees and Networks*. *Cambridge Series in Statistical and Probabilistic Mathematics* **42**. New York: Cambridge Univ. Press. MR3616205

[10] Rousselin, P. (2018). Invariant measures, Hausdorff dimension and dimension drop of some harmonic measures on Galton–Watson trees. *Electron. J. Probab.* **23** Paper No. 46, 31. MR3814240

[11] Virág, B. (2000). On the speed of random walks on graphs. *Ann. Probab.* **28** 379–394. MR1756009

Bernoulli **25**(4B), 2019, 3673–3713
https://doi.org/10.3150/19-BEJ1107

Integral expression for the stationary distribution of reflected Brownian motion in a wedge

SANDRO FRANCESCHI[1] and KILIAN RASCHEL[2]

[1]*Laboratoire de Probabilités, Statistique et Modélisation, Sorbonne Université, 4 Place Jussieu, 75252 Paris Cedex 05, France. E-mail: sandro.franceschi@sorbonne-universite.fr*
[2]*CNRS and Institut Denis Poisson, Université de Tours and Université d'Orléans, Parc de Grandmont, 37200 Tours, France. E-mail: raschel@math.cnrs.fr*

For Brownian motion in a (two-dimensional) wedge with negative drift and oblique reflection on the axes, we derive an explicit formula for the Laplace transform of its stationary distribution (when it exists), in terms of Cauchy integrals and generalized Chebyshev polynomials. To that purpose, we solve a Carleman-type boundary value problem on a hyperbola, satisfied by the Laplace transforms of the boundary stationary distribution.

Keywords: boundary value problem with shift; Carleman-type boundary value problem; conformal mapping; Laplace transform; reflected Brownian motion in a wedge; stationary distribution

References

[1] Aspandiiarov, S., Iasnogorodski, R. and Menshikov, M. (1996). Passage-time moments for nonnegative stochastic processes and an application to reflected random walks in a quadrant. *Ann. Probab.* **24** 932–960. MR1404537

[2] Baccelli, F. and Fayolle, G. (1987). Analysis of models reducible to a class of diffusion processes in the positive quarter plane. *SIAM J. Appl. Math.* **47** 1367–1385. MR0916246

[3] Bernardi, O., Bousquet-Mélou, M. and Raschel, K. (2016). Counting quadrant walks via Tutte's invariant method. In *Proceedings of FPSAC* 2016. *Discrete Math. Theor. Comput. Sci. Proc.* 203–214. Nancy: Assoc. Discrete Math. Theor. Comput. Sci.

[4] Bostan, A., Bousquet-Mélou, M., Kauers, M. and Melczer, S. (2016). On 3-dimensional lattice walks confined to the positive octant. *Ann. Comb.* **20** 661–704. MR3572381

[5] Bousquet-Mélou, M. and Mishna, M. (2010). Walks with small steps in the quarter plane. In *Algorithmic Probability and Combinatorics*. *Contemp. Math.* **520** 1–39. Providence, RI: Amer. Math. Soc. MR2681853

[6] Bramson, M. (2011). Positive recurrence for reflecting Brownian motion in higher dimensions. *Queueing Syst.* **69** 203–215. MR2886468

[7] Bramson, M., Dai, J.G. and Harrison, J.M. (2010). Positive recurrence of reflecting Brownian motion in three dimensions. *Ann. Appl. Probab.* **20** 753–783. MR2650048

[8] Burdzy, K., Chen, Z.-Q., Marshall, D. and Ramanan, K. (2017). Obliquely reflected Brownian motion in nonsmooth planar domains. *Ann. Probab.* **45** 2971–3037. MR3706737

[9] Chen, H. (1996). A sufficient condition for the positive recurrence of a semimartingale reflecting Brownian motion in an orthant. *Ann. Appl. Probab.* **6** 758–765. MR1410114

1350-7265

[10] Chen, Y., Boucherie, R.J. and Goseling, J. (2015). The invariant measure of random walks in the quarter-plane: Representation in geometric terms. *Probab. Engrg. Inform. Sci.* **29** 233–251. MR3326439
[11] Chen, Y., Boucherie, R.J. and Goseling, J. (2016). Invariant measures and error bounds for random walks in the quarter-plane based on sums of geometric terms. *Queueing Syst.* **84** 21–48. MR3548676
[12] Cohen, J. (1984). On a functional relation in three complex variables; three coupled processors Technical Report Mathematical, Institute Utrecht 359, Utrecht University.
[13] Dai, J. (1990). *Steady-State Analysis of Reflected Brownian Motions: Characterization, Numerical Methods and Queueing Applications*. Ann Arbor, MI: ProQuest LLC. Thesis (Ph.D.)–Stanford University. MR2685786
[14] Dai, J. and Kurtz, T. (1994). Characterization of the stationary distribution for a semimartingale reflecting brownian motion in a convex polyhedron. 1–31. Preprint.
[15] Dai, J.G. and Harrison, J.M. (1992). Reflected Brownian motion in an orthant: Numerical methods for steady-state analysis. *Ann. Appl. Probab.* **2** 65–86. MR1143393
[16] Dai, J.G. and Harrison, J.M. (2012). Reflecting Brownian motion in three dimensions: A new proof of sufficient conditions for positive recurrence. *Math. Methods Oper. Res.* **75** 135–147. MR2904627
[17] Dai, J.G. and Miyazawa, M. (2011). Reflecting Brownian motion in two dimensions: Exact asymptotics for the stationary distribution. *Stoch. Syst.* **1** 146–208. MR2948920
[18] Dai, J.G. and Miyazawa, M. (2013). Stationary distribution of a two-dimensional SRBM: Geometric views and boundary measures. *Queueing Syst.* **74** 181–217. MR3054655
[19] Dieker, A.B. and Moriarty, J. (2009). Reflected Brownian motion in a wedge: Sum-of-exponential stationary densities. *Electron. Commun. Probab.* **14** 1–16. MR2472171
[20] Doetsch, G. (1974). *Introduction to the Theory and Application of the Laplace Transformation*. New York: Springer. Translated from the second German edition by Walter Nader. MR0344810
[21] Dreyfus, T., Hardouin, C., Roques, J. and Singer, M.F. (2018). On the nature of the generating series of walks in the quarter plane. *Invent. Math.* **213** 139–203. MR3815564
[22] Dubédat, J. (2004). Reflected planar Brownian motions, intertwining relations and crossing probabilities. *Ann. Inst. Henri Poincaré Probab. Stat.* **40** 539–552. MR2086013
[23] Dupuis, P. and Williams, R.J. (1994). Lyapunov functions for semimartingale reflecting Brownian motions. *Ann. Probab.* **22** 680–702. MR1288127
[24] Fayolle, G. and Iasnogorodski, R. (1979). Two coupled processors: The reduction to a Riemann–Hilbert problem. *Z. Wahrsch. Verw. Gebiete* **47** 325–351. MR0525314
[25] Fayolle, G., Iasnogorodski, R. and Malyshev, V. (1999). *Random Walks in the Quarter-Plane. Algebraic Methods, Boundary Value Problems and Applications. Applications of Mathematics (New York)* **40**. Berlin: Springer. MR1691900
[26] Foddy, M.E. (1984). *Analysis of Brownian Motion with Drift, Confined to a Quadrant by Oblique Reflection (Diffusions, Riemann–Hilbert Problem)*. Ann Arbor, MI: ProQuest LLC. Thesis (Ph.D.)–Stanford University. MR2633392
[27] Foschini, G.J. (1982). Equilibria for diffusion models of pairs of communicating computers—symmetric case. *IEEE Trans. Inform. Theory* **28** 273–284. MR0651627
[28] Franceschi, S. and Kourkova, I. (2017). Asymptotic expansion of stationary distribution for reflected Brownian motion in the quarter plane via analytic approach. *Stoch. Syst.* **7** 32–94. MR3663338
[29] Franceschi, S. and Raschel, K. (2017). Tutte's invariant approach for Brownian motion reflected in the quadrant. *ESAIM Probab. Stat.* **21** 220–234. MR3743912
[30] Harrison, J.M. and Hasenbein, J.J. (2009). Reflected Brownian motion in the quadrant: Tail behavior of the stationary distribution. *Queueing Syst.* **61** 113–138. MR2485885
[31] Harrison, J.M. and Reiman, M.I. (1981). On the distribution of multidimensional reflected Brownian motion. *SIAM J. Appl. Math.* **41** 345–361. MR0628959

[32] Harrison, J.M. and Reiman, M.I. (1981). Reflected Brownian motion on an orthant. *Ann. Probab.* **9** 302–308. MR0606992
[33] Harrison, J.M. and Williams, R.J. (1987). Brownian models of open queueing networks with homogeneous customer populations. *Stochastics* **22** 77–115. MR0912049
[34] Harrison, J.M. and Williams, R.J. (1987). Multidimensional reflected Brownian motions having exponential stationary distributions. *Ann. Probab.* **15** 115–137. MR0877593
[35] Hobson, D.G. and Rogers, L.C.G. (1993). Recurrence and transience of reflecting Brownian motion in the quadrant. *Math. Proc. Cambridge Philos. Soc.* **113** 387–399. MR1198420
[36] Jackson, J.R. (1957). Networks of waiting lines. *Oper. Res.* **5** 518–521. MR0093061
[37] Kella, O. and Ramasubramanian, S. (2012). Asymptotic irrelevance of initial conditions for Skorohod reflection mapping on the nonnegative orthant. *Math. Oper. Res.* **37** 301–312. MR2931282
[38] Kella, O. and Whitt, W. (1996). Stability and structural properties of stochastic storage networks. *J. Appl. Probab.* **33** 1169–1180. MR1416235
[39] Kurkova, I.A. and Suhov, Y.M. (2003). Malyshev's theory and JS-queues. Asymptotics of stationary probabilities. *Ann. Appl. Probab.* **13** 1313–1354. MR2023879
[40] Lakner, P., Reed, J. and Zwart, B. (2016). A Dirichlet process characterization of RBM in a wedge. 1–37. Preprint. Available at arXiv:1605.02020.
[41] Latouche, G. and Miyazawa, M. (2014). Product-form characterization for a two-dimensional reflecting random walk. *Queueing Syst.* **77** 373–391. MR3225816
[42] Le Gall, J.-F. (1987). Mouvement brownien, cônes et processus stables. *Probab. Theory Related Fields* **76** 587–627. MR0917681
[43] Lépingle, D. (2017). A two-dimensional oblique extension of Bessel processes. *Markov Process. Related Fields* **23** 233–266. MR3701543
[44] Litvinchuk, G.S. (2000). *Solvability Theory of Boundary Value Problems and Singular Integral Equations with Shift*. *Mathematics and Its Applications* **523**. Dordrecht: Kluwer Academic. MR1885495
[45] Malyšev, V.A. (1972). An analytic method in the theory of two-dimensional positive random walks. *Sibirsk. Mat. Zh.* **13** 1314–1329, 1421. MR0336823
[46] Muskhelishvili, N.I. (1972). *Singular Integral Equations. Boundary Problems of Functions Theory and Their Applications to Mathematical Physics*. Groningen: Wolters-Noordhoff Publishing. Revised translation from the Russian, edited by J. R. M. Radok, Reprinted. MR0355494
[47] O'Connell, N. and Ortmann, J. (2014). Product-form invariant measures for Brownian motion with drift satisfying a skew-symmetry type condition. *ALEA Lat. Am. J. Probab. Math. Stat.* **11** 307–329. MR3225980
[48] Reiman, M.I. and Williams, R.J. (1988). A boundary property of semimartingale reflecting Brownian motions. *Probab. Theory Related Fields* **77** 87–97. MR0921820
[49] Sarantsev, A. (2013). Comparison techniques for competing brownian particles. 1–31. Preprint. Available at arXiv:1305.1653.
[50] Sarantsev, A. (2015). Triple and simultaneous collisions of competing Brownian particles. *Electron. J. Probab.* **20** no. 29, 28. MR3325099
[51] Sarantsev, A. (2017). Reflected Brownian motion in a convex polyhedral cone: Tail estimates for the stationary distribution. *J. Theoret. Probab.* **30** 1200–1223. MR3687255
[52] Takayama, N. (1992). An approach to the zero recognition problem by Buchberger algorithm. *J. Symbolic Comput.* **14** 265–282. MR1187235
[53] Taylor, L.M. and Williams, R.J. (1993). Existence and uniqueness of semimartingale reflecting Brownian motions in an orthant. *Probab. Theory Related Fields* **96** 283–317. MR1231926
[54] Tutte, W.T. (1995). Chromatic sums revisited. *Aequationes Math.* **50** 95–134. MR1336864
[55] Varadhan, S.R.S. and Williams, R.J. (1985). Brownian motion in a wedge with oblique reflection. *Comm. Pure Appl. Math.* **38** 405–443. MR0792398

[56] Williams, R.J. (1985). Recurrence classification and invariant measure for reflected Brownian motion in a wedge. *Ann*. *Probab*. **13** 758–778. MR0799421
[57] Williams, R.J. (1985). Reflected Brownian motion in a wedge: Semimartingale property. *Z*. *Wahrsch*. *Verw*. *Gebiete* **69** 161–176. MR0779455
[58] Williams, R.J. (1995). Semimartingale reflecting Brownian motions in the orthant. In *Stochastic Networks*. *IMA Vol*. *Math*. *Appl*. **71** 125–137. New York: Springer. MR1381009

Bernoulli **25**(4B), 2019, 3714–3733
https://doi.org/10.3150/19-BEJ1108

Equivalence of some subcritical properties in continuum percolation

JEAN-BAPTISTE GOUÉRÉ[1] and MARIE THÉRET[2]

[1] *Institut Denis-Poisson – UMR CNRS 7013, Université de Tours, Parc de Grandmont, 37200 Tours, France. E-mail: jean-baptiste.gouere@lmpt.univ-tours.fr*

[2] *Modal'X, UPL, Univ Paris Nanterre, F92000 Nanterre France. E-mail: marie.theret@parisnanterre.fr*

We consider the Boolean model on $\mathbb{R}^d$. We prove some equivalences between subcritical percolation properties. Let us introduce some notations to state one of these equivalences. Let C denote the connected component of the origin in the Boolean model. Let $|C|$ denotes its volume. Let ℓ denote the maximal length of a chain of random balls from the origin. Under optimal integrability conditions on the radii, we prove that $\mathbb{E}(|C|)$ is finite if and only if there exists $A, B > 0$ such that $\mathbb{P}(\ell \geq n) \leq Ae^{-Bn}$ for all $n \geq 1$.

Keywords: Boolean model; continuum percolation; critical point

References

[1] Ahlberg, D., Tassion, V. and Teixeira, A. (2018). Existence of an unbounded vacant set for subcritical continuum percolation. *Electron. Commun. Probab.* **23** Paper No. 63, 8. MR3863919

[2] Ahlberg, D., Tassion, V. and Teixeira, A. (2018). Sharpness of the phase transition for continuum percolation in $\mathbb{R}^2$. *Probab. Theory Related Fields* **172** 525–581. MR3851838

[3] Aizenman, M. and Barsky, D.J. (1987). Sharpness of the phase transition in percolation models. *Comm. Math. Phys.* **108** 489–526. MR0874906

[4] Duminil-Copin, H., Raoufi, A. and Tassion, V. (2018). Subcritical phase of d-dimensional Poisson–Boolean percolation and its vacant set. Preprint. Available at arXiv:1805.00695.

[5] Duminil-Copin, H., Raoufi, A. and Tassion, V. (2019). Exponential decay of connection probabilities for subcritical Voronoi percolation in $\mathbb{R}^d$. *Probab. Theory Related Fields* **173** 479–490. MR3916112

[6] Duminil-Copin, H., Raoufi, A. and Tassion, V. (2019). Sharp phase transition for the random-cluster and Potts models via decision trees. *Ann. of Math.* (2) **189** 75–99. MR3898174

[7] Duminil-Copin, H. and Tassion, V. (2016). A new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model. *Comm. Math. Phys.* **343** 725–745. MR3477351

[8] Duminil-Copin, H. and Tassion, V. (2016). A new proof of the sharpness of the phase transition for Bernoulli percolation on $\mathbb{Z}^d$. *Enseign. Math.* **62** 199–206. MR3605816

[9] Gouéré, J.-B. (2008). Subcritical regimes in the Poisson Boolean model of continuum percolation. *Ann. Probab.* **36** 1209–1220. MR2435847

[10] Gouéré, J.-B. and Théret, M. (2017). Positivity of the time constant in a continuous model of first passage percolation. *Electron. J. Probab.* **22** Paper No. 49, 21. MR3661663

[11] Grimmett, G. (1999). *Percolation*, 2nd ed. *Grundlehren der Mathematischen Wissenschaften* [*Fundamental Principles of Mathematical Sciences*] **321**. Berlin: Springer. MR1707339

[12] Kesten, H. (1982). *Percolation Theory for Mathematicians*. *Progress in Probability and Statistics* **2**. Boston, MA: Birkhäuser. MR0692943

1350-7265

[13] Last, G. and Penrose, M. (2018). *Lectures on the Poisson Process*. *Institute of Mathematical Statistics Textbooks* **7**. Cambridge: Cambridge Univ. Press. MR3791470
[14] Meester, R. and Roy, R. (1996). *Continuum Percolation*. *Cambridge Tracts in Mathematics* **119**. Cambridge: Cambridge Univ. Press. MR1409145
[15] Men'shikov, M.V. (1986). Coincidence of critical points in percolation problems. *Dokl*. *Akad*. *Nauk SSSR* **288** 1308–1311. MR0852458
[16] Møller, J. (1994). *Lectures on Random Voronoĭ Tessellations*. *Lecture Notes in Statistics* **87**. New York: Springer. MR1295245
[17] Penrose, M.D. (2018). Non-triviality of the vacancy phase transition for the Boolean model. *Electron*. *Commun*. *Probab*. **23** Paper No. 49, 8. MR3841410
[18] Schneider, R. and Weil, W. (2008). *Stochastic and Integral Geometry*. *Probability and Its Applications* (*New York*). Berlin: Springer. MR2455326
[19] van den Berg, J. (1996). A note on disjoint-occurrence inequalities for marked Poisson point processes. *J*. *Appl*. *Probab*. **33** 420–426. MR1385351
[20] Ziesche, S. (2018). Sharpness of the phase transition and lower bounds for the critical intensity in continuum percolation on $\mathbb{R}^d$. *Ann*. *Inst*. *Henri Poincaré Probab*. *Stat*. **54** 866–878. MR3795069

Bernoulli **25**(4B), 2019, 3734–3761
https://doi.org/10.3150/19-BEJ1109

Estimating the input of a Lévy-driven queue by Poisson sampling of the workload process

LIRON RAVNER[1,2], ONNO BOXMA[2] and MICHEL MANDJES[1]

[1]*University of Amsterdam, Korteweg-de Vries Institute for Mathematics, Science Park, Amsterdam, 1098XG, Netherlands. E-mail: l.ravner@uva.nl*
[2]*Eindhoven University of Technology, Department of Mathematics and Computer Science, 5600MB, Eindhoven, Netherlands.*

This paper aims at semi-parametrically estimating the input process to a Lévy-driven queue by sampling the workload process at Poisson times. We construct a method-of-moments based estimator for the Lévy process' characteristic exponent. This method exploits the known distribution of the workload sampled at an exponential time, thus taking into account the dependence between subsequent samples. Verifiable conditions for consistency and asymptotic normality are provided, along with explicit expressions for the asymptotic variance. The method requires an intermediate estimation step of estimating a constant (related to both the input distribution and the sampling rate); this constant also features in the asymptotic analysis. For subordinator Lévy input, a partial MLE is constructed for the intermediate step and we show that it satisfies the consistency and asymptotic normality conditions. For general spectrally-positive Lévy input a biased estimator is proposed that only uses workload observations above some threshold; the bias can be made arbitrarily small by appropriately choosing the threshold.

Keywords: Lévy-driven queue; nonparametric estimation; Poisson probing; transient queueing; queue input estimation

References

[1] Afèche, P. and Ata, B. (2013). Bayesian dynamic pricing in queueing systems with unknown delay cost characteristics. *Manufacturing & Service Operations Management* **15** 292–304.
[2] Antunes, N., Jacinto, G. and Pacheco, A. (2014). Probing a M/G/1 queue with general input and service times. *ACM SIGMETRICS Perform. Eval. Rev.* **41** 34–36.
[3] Antunes, N., Jacinto, G., Pacheco, A. and Wichelhaus, C. (2016). Estimation of the traffic intensity in a piecewise-stationary $M_t/G_t/1$ queue with probing. *ACM SIGMETRICS Perform. Eval. Rev.* **44** 3–5.
[4] Armony, M. and Plambeck, E.L. (2005). The impact of duplicate orders on demand estimation and capacity investment. *Manage. Sci.* **51** 1505–1518.
[5] Asanjarani, A., Nazarathy, Y. and Pollett, P.K. (2017). Parameter and state estimation in queues and related stochastic models: A bibliography. Available at arXiv:1701.08338.
[6] Bekker, R., Boxma, O.J. and Resing, J.A.C. (2009). Lévy processes with adaptable exponent. *Adv. in Appl. Probab.* **41** 177–205. MR2514950
[7] Belomestny, D. (2011). Statistical inference for time-changed Lévy processes via composite characteristic function estimation. *Ann. Statist.* **39** 2205–2242. MR2893866
[8] Bingham, N.H. and Pitts, S.M. (1999). Nonparametric inference from $M/G/1$ busy periods. *Comm. Statist. Stochastic Models* **15** 247–272. MR1683605

1350-7265

[9] Blanghaps, N., Nov, Y. and Weiss, G. (2013). Sojourn time estimation in an M/G/∞ queue with partial information. *J. Appl. Probab.* **50** 1044–1056. MR3161372
[10] Buchmann, B. and Grübel, R. (2003). Decompounding: An estimation problem for Poisson random sums. *Ann. Statist.* **31** 1054–1074. MR2001642
[11] Dębicki, K. and Mandjes, M. (2015). *Queues and Lévy Fluctuation Theory*. *Universitext*. Springer. MR3379923
[12] Duffie, D. and Glynn, P. (2004). Estimation of continuous-time Markov processes sampled at random time intervals. *Econometrica* **72** 1773–1808. MR2095532
[13] Ferguson, T.S. (1996). *A Course in Large Sample Theory*. *Texts in Statistical Science Series*. London: CRC Press. MR1699953
[14] Glynn, P.W., Melamed, B. and Whitt, W. (1993). Estimating customer and time averages. *Oper. Res.* **41** 400–408.
[15] Goldenshluger, A. (2016). Nonparametric estimation of the service time distribution in the M/G/∞ queue. *Adv. in Appl. Probab.* **48** 1117–1138. MR3595768
[16] Goldenshluger, A. (2018). The $M/G/\infty$ estimation problem revisited. *Bernoulli* **24** 2531–2568. MR3779694
[17] Gugushvili, S. (2009). Nonparametric estimation of the characteristic triplet of a discretely observed Lévy process. *J. Nonparametr. Stat.* **21** 321–343. MR2530929
[18] Hall, P. and Heyde, C.C. (1980). *Martingale Limit Theory and Its Application*. New York-London: Academic Press [Harcourt Brace Jovanovich, Publishers]. Probability and Mathematical Statistics. MR0624435
[19] Hansen, M.B. (2005). Nonparametric estimation of the stationary M/G/1 workload distribution function. In *Proceedings of the* 37*th Conference on Winter Simulation* 869–877. Winter Simulation Conference.
[20] Hansen, M.B. and Pitts, S.M. (2006). Nonparametric inference from the $M/G/1$ workload. *Bernoulli* **12** 737–759. MR2248235
[21] Hassin, R. (2016). *Rational Queueing*. Boca Raton, FL: CRC Press. MR3467608
[22] Hassin, R. and Haviv, M. (2003). *To Queue or Not to Queue*: *Equilibrium Behavior in Queueing Systems*. *International Series in Operations Research & Management Science* **59**. Boston, MA: Kluwer Academic. MR2006433
[23] Heijmans, R.D.H. and Magnus, J.R. (1986). Consistent maximum-likelihood estimation with dependent observations. The general (nonnormal) case and the normal case. *J. Econometrics* **32** 253–285. MR0864928
[24] Heijmans, R.D.H. and Magnus, J.R. (1986). On the first-order efficiency and asymptotic normality of maximum likelihood estimators obtained from dependent observations. *Stat. Neerl.* **40** 169–188. MR0868075
[25] Heyde, C.C. (1997). *Quasi-Likelihood and Its Application*: *A General Approach to Optimal Parameter Estimation*. *Springer Series in Statistics*. New York: Springer. MR1461808
[26] Kella, O., Boxma, O. and Mandjes, M. (2006). A Lévy process reflected at a Poisson age process. *J. Appl. Probab.* **43** 221–230. MR2225062
[27] Kyprianou, A.E. (2006). *Introductory Lectures on Fluctuations of Lévy Processes with Applications*. *Universitext*. Berlin: Springer. MR2250061
[28] Neumann, M.H. and Reiß, M. (2009). Nonparametric estimation for Lévy processes from low-frequency observations. *Bernoulli* **15** 223–248. MR2546805
[29] Novak, A., Taylor, P. and Veitch, D. (2006). The distribution of the number of arrivals in a subinterval of a busy period of a single server queue. *Queueing Syst.* **53** 105–114. MR2243923
[30] Pitts, S.M. (1994). Nonparametric estimation of the stationary waiting time distribution function for the $GI/G/1$ queue. *Ann. Statist.* **22** 1428–1446. MR1311982

[31] Ravner, L., Boxma, O. and Mandjes, M. (2019). Supplement to "Estimating the input of a Lévy-driven queue by Poisson sampling of the workload process." DOI:10.3150/19-BEJ1109SUPP.
[32] Ross, J.V., Taimre, T. and Pollett, P.K. (2007). Estimation for queues from queue length data. *Queueing Syst*. **55** 131–138. MR2311210
[33] Schweer, S. and Wichelhaus, C. (2015). Nonparametric estimation of the service time distribution in the discrete-time $GI/G/\infty$ queue with partial information. *Stochastic Process*. *Appl*. **125** 233–253. MR3274698
[34] Smith, D.E. and Li, M. (2002). Estimating data delays from Poisson probe delays, Aug. 6 2002. US Patent 6,430,160.
[35] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247
[36] Yao, D.D. (1992). On Wolff's PASTA martingale. *Oper. Res*. **40** S352–S355.

Bernoulli **25**(4B), 2019, 3762–3795
https://doi.org/10.3150/19-BEJ1110

Estimation of fully nonparametric transformation models

BENJAMIN COLLING* and INGRID VAN KEILEGOM**

*Research Centre for Operations Research and Business Statistics (ORSTAT), KU Leuven, Naamsestraat 69, Leuven, 3000, Belgium. E-mail: *benjamin.colling@uclouvain.be; **ingrid.vankeilegom@kuleuven.be*

Consider the following nonparametric transformation model $\Lambda(Y) = m(X) + \varepsilon$, where X is a d-dimensional covariate, Y is a continuous univariate dependent variable and ε is an error term with zero mean and which is independent of X. We assume that the unknown transformation Λ is strictly increasing and that m is an unknown regression function. Our goal is to develop two new nonparametric estimators of the transformation Λ, the first one based on the least squares loss and the second one based on the least absolute deviation loss, and to compare their performance with that of the estimators developed by Chiappori, Komunjer and Kristensen (*J. Econometrics* **188** (2015) 22–39). Our proposed estimators are based on an estimator of the conditional distribution of U given X, where U is an appropriate transformation of Y that is uniformly distributed. The main motivation for working with U instead of Y is that, in transformation models, the response Y is often skewed with very long tails, and so kernel smoothing based on Y does not work well. Hence, we expect to obtain better estimators if we pre-transform Y before applying kernel smoothing. We establish the asymptotic normality of the two proposed estimators. We also carry out a simulation study to illustrate the performance of our estimators, to compare these new estimators with the ones of Chiappori, Komunjer and Kristensen (*J. Econometrics* **188** (2015) 22–39) and to see under which model conditions which estimators behave the best.

Keywords: asymptotic properties; identification; kernel smoothing; nonparametric regression; nonparametric transformation

References

[1] Bickel, P.J. and Doksum, K.A. (1981). An analysis of transformations revisited. *J. Amer. Statist. Assoc.* **76** 296–311. MR0624332

[2] Box, G.E.P. and Cox, D.R. (1964). An analysis of transformations. (With discussion). *J. Roy. Statist. Soc. Ser. B* **26** 211–252. MR0192611

[3] Breiman, L. and Friedman, J.H. (1985). Estimating optimal transformations for multiple regression and correlation. *J. Amer. Statist. Assoc.* **80** 580–619. With discussion and with a reply by the authors. MR0803258

[4] Carroll, R.J. and Ruppert, D. (1988). *Transformation and Weighting in Regression*. *Monographs on Statistics and Applied Probability*. New York: CRC Press. MR1014890

[5] Chen, S. (2002). Rank estimation of transformation models. *Econometrica* **70** 1683–1697. MR1929984

[6] Chernozhukov, V., Fernández-Val, I. and Galichon, A. (2010). Quantile and probability curves without crossing. *Econometrica* **78** 1093–1125. MR2667913

[7] Chiappori, P.-A., Komunjer, I. and Kristensen, D. (2015). Nonparametric identification and estimation of transformation models. *J. Econometrics* **188** 22–39. MR3371659

[8] Colling, B., Heuchenne, C., Samb, R. and Van Keilegom, I. (2015). Estimation of the error density in a semiparametric transformation model. *Ann. Inst. Statist. Math.* **67** 1–18. MR3297856
[9] Colling, B. and Van Keilegom, I. (2016). Goodness-of-fit tests in semiparametric transformation models. *TEST* **25** 291–308. MR3493520
[10] Colling, B. and Van Keilegom, I. (2017). Goodness-of-fit tests in semiparametric transformation models using the integrated regression function. *J. Multivariate Anal.* **160** 10–30. MR3688687
[11] Colling, B. and Van Keilegom, I. (2019). Supplement to "Estimation of fully nonparametric transformation models." DOI:10.3150/19-BEJ1110SUPP.
[12] Dette, H., Neumeyer, N. and Pilz, K.F. (2006). A simple nonparametric estimator of a strictly monotone regression function. *Bernoulli* **12** 469–490. MR2232727
[13] Heuchenne, C., Samb, R. and Van Keilegom, I. (2015). Estimating the error distribution in semiparametric transformation models. *Electron. J. Stat.* **9** 2391–2419. MR3417187
[14] Horowitz, J.L. (1996). Semiparametric estimation of a regression model with an unknown transformation of the dependent variable. *Econometrica* **64** 103–137. MR1366143
[15] Horowitz, J.L. (2001). Nonparametric estimation of a generalized additive model with an unknown link function. *Econometrica* **69** 499–513. MR1819761
[16] Jacho-Chávez, D., Lewbel, A. and Linton, O. (2010). Identification and nonparametric estimation of a transformed additively separable model. *J. Econometrics* **156** 392–407. MR2609941
[17] John, J.A. and Draper, N.R. (1980). An alternative family of transformations. *J. R. Stat. Soc. Ser. C. Appl. Stat.* **29** 190–197.
[18] Linton, O., Sperlich, S. and Van Keilegom, I. (2008). Estimation of a semiparametric transformation model. *Ann. Statist.* **36** 686–718. MR2396812
[19] MacKinnon, J.G. and Magee, L. (1990). Transforming the dependent variable in regression models. *Internat. Econom. Rev.* **31** 315–339.
[20] Neumeyer, N., Noh, H. and Van Keilegom, I. (2016). Heteroscedastic semiparametric transformation models: Estimation and testing for validity. *Statist. Sinica* **26** 925–954. MR3559937
[21] Sakia, R.M. (1992). The Box–Cox transformation technique: A review. *Statistician* **41** 169–178.
[22] Vanhems, A. and Van Keilegom, I. (2019). Estimation of a semiparametric transformation model in the presence of endogeneity. *Econometric Theory* **35** 73–110. MR3904172
[23] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes: With Applications to Statistics. Springer Series in Statistics*. New York: Springer. MR1385671
[24] Yeo, I.-K. and Johnson, R.A. (2000). A new family of power transformations to improve normality or symmetry. *Biometrika* **87** 954–959. MR1813988
[25] Zellner, A. and Revankar, N.S. (1969). Generalized production functions. *Rev. Econ. Stud.* **36** 241–250.

Bernoulli **25**(4B), 2019, 3796–3831
https://doi.org/10.3150/19-BEJ1111

Long-time heat kernel estimates and upper rate functions of Brownian motion type for symmetric jump processes

YUICHI SHIOZAWA[1] and JIAN WANG[2]

[1]*Department of Mathematics, Graduate School of Science, Osaka University, Toyonaka, Osaka, 560-0043, Japan. E-mail: shiozawa@math.sci.osaka-u.ac.jp*
[2]*College of Mathematics and Informatics & Fujian Key Laboratory of Mathematical Analysis and Applications (FJKLMAA), Fujian Normal University, 350007 Fuzhou, P.R. China. E-mail: jianwang@fjnu.edu.cn*

Let X be a symmetric jump process on $\mathbb{R}^d$ such that the corresponding jumping kernel $J(x,y)$ satisfies

$$J(x,y) \le \frac{c}{|x-y|^{d+2}\log^{1+\varepsilon}(e+|x-y|)}$$

for all $x, y \in \mathbb{R}^d$ with $|x-y| \ge 1$ and some constants $c, \varepsilon > 0$. Under additional mild assumptions on $J(x,y)$ for $|x-y| < 1$, we show that $C\sqrt{r \log\log r}$ with some constant $C > 0$ is an upper rate function of the process X, which enjoys the same form as that for Brownian motions. The approach is based on heat kernel estimates of large time for the process X. As a by-product, we also obtain two-sided heat kernel estimates of large time for symmetric jump processes whose jumping kernels are comparable to

$$\frac{1}{|x-y|^{d+2+\varepsilon}}$$

for all $x, y \in \mathbb{R}^d$ with $|x-y| \ge 1$ and some constant $\varepsilon > 0$.

Keywords: Dirichlet form; heat kernel; symmetric jump process; upper rate function

References

[1] Bae, J., Kang, J., Kim, P. and Lee, J. (2018). Heat kernel estimates for symmetric jump processes with mixed polynomial growths. Preprint. Available at arXiv:1804.06918.
[2] Barlow, M.T., Bass, R.F., Chen, Z.-Q. and Kassmann, M. (2009). Non-local Dirichlet forms and symmetric jump processes. *Trans. Amer. Math. Soc.* **361** 1963–1999. MR2465826
[3] Barlow, M.T., Grigor'yan, A. and Kumagai, T. (2009). Heat kernel upper bounds for jump processes and the first exit time. *J. Reine Angew. Math.* **626** 135–157. MR2492992
[4] Bass, R.F. and Kumagai, T. (2008). Symmetric Markov chains on $\mathbb{Z}^d$ with unbounded range. *Trans. Amer. Math. Soc.* **360** 2041–2075. MR2366974
[5] Carlen, E.A., Kusuoka, S. and Stroock, D.W. (1987). Upper bounds for symmetric Markov transition functions. *Ann. Inst. Henri Poincaré Probab. Stat.* **23** 245–287. MR0898496
[6] Chen, Z.-Q., Kim, P. and Kumagai, T. (2008). Weighted Poincaré inequality and heat kernel estimates for finite range jump processes. *Math. Ann.* **342** 833–883. MR2443765

1350-7265

[7] Chen, Z.-Q., Kim, P. and Kumagai, T. (2011). Global heat kernel estimates for symmetric jump processes. *Trans. Amer. Math. Soc.* **363** 5021–5055. MR2806700
[8] Chen, Z.-Q. and Kumagai, T. (2008). Heat kernel estimates for jump processes of mixed types on metric measure spaces. *Probab. Theory Related Fields* **140** 277–317. MR2357678
[9] Chen, Z.-Q. and Kumagai, T. (2010). A priori Hölder estimate, parabolic Harnack principle and heat kernel estimates for diffusions with jumps. *Rev. Mat. Iberoam.* **26** 551–589. MR2677007
[10] Chen, Z.-Q. and Zhang, X. (2016). Heat kernels and analyticity of non-symmetric jump diffusion semigroups. *Probab. Theory Related Fields* **165** 267–312. MR3500272
[11] Davies, E.B. (1989). *Heat Kernels and Spectral Theory*. *Cambridge Tracts in Mathematics* **92**. Cambridge: Cambridge Univ. Press. MR0990239
[12] Dyda, B. and Kassmann, M. (2013). On weighted Poincaré inequalities. *Ann. Acad. Sci. Fenn. Math.* **38** 721–726. MR3113104
[13] Foondun, M. (2009). Heat kernel estimates and Harnack inequalities for some Dirichlet forms with non-local part. *Electron. J. Probab.* **14** 314–340. MR2480543
[14] Fukushima, M., Oshima, Y. and Takeda, M. (2011). *Dirichlet Forms and Symmetric Markov Processes*, extended ed. *De Gruyter Studies in Mathematics* **19**. Berlin: de Gruyter. MR2778606
[15] Gnedenko, B. (1943). Sur la croissance des processus stochastiques homogènes à accroissements indépendants. *Bull. Acad. Sci. URSS. Sér. Math.* [*Izv. Akad. Nauk SSSR*] **7** 89–110. MR0009265
[16] Itô, K. and McKean, H.P. Jr. (1974). *Diffusion Processes and Their Sample Paths*. Berlin: Springer. MR0345224
[17] Khintchine, A. (1938). Zwei Sätze über stochastische Prozesse mit stabilen Verteilungen. *Rec. Math.* [*Mat. Sb.*] *N.S.* **3** 577–584.
[18] Masamune, J. and Uemura, T. (2011). Conservation property of symmetric jump processes. *Ann. Inst. Henri Poincaré Probab. Stat.* **47** 650–662. MR2841069
[19] Sato, K. (2001). Basic results on Lévy processes. In *Lévy Processes: Theory and Applications* (O. Barndorff-Nielsen, T. Mikosch and S. Resnick, eds.) 3–37. Boston: Birkhäuser.
[20] Sato, K. (2013). *Lévy Processes and Infinitely Divisible Distributions*. *Cambridge Studies in Advanced Mathematics* **68**. Cambridge: Cambridge Univ. Press. MR3185174
[21] Shiozawa, Y. and Wang, J. (2017). Rate functions for symmetric Markov processes via heat kernel. *Potential Anal.* **46** 23–53. MR3595960
[22] Sirao, T. (1953). On some asymptotic properties concerning homogeneous differential processes. *Nagoya Math. J.* **6** 95–107. MR0058907

Bernoulli **25**(4B), 2019, 3832–3863
https://doi.org/10.3150/19-BEJ1112

Consistent estimation of the spectrum of trace class Data Augmentation algorithms

SAPTARSHI CHAKRABORTY* and KSHITIJ KHARE**

Department of Statistics, 102 Griffin Floyd Hall, University of Florida, Gainesville, FL 32611, USA. E-mail: *c7rishi@ufl.edu; **kdkhare@stat.ufl.edu

Markov chain Monte Carlo is widely used in a variety of scientific applications to generate approximate samples from intractable distributions. A thorough understanding of the convergence and mixing properties of these Markov chains can be obtained by studying the spectrum of the associated Markov operator. While several methods to bound/estimate the second largest eigenvalue are available in the literature, very few general techniques for consistent estimation of the entire spectrum have been proposed. Existing methods for this purpose require the Markov transition density to be available in closed form, which is often not true in practice, especially in modern statistical applications. In this paper, we propose a novel method to consistently estimate the entire spectrum of a general class of Markov chains arising from a popular and widely used statistical approach known as Data Augmentation. The transition densities of these Markov chains can often only be expressed as intractable integrals. We illustrate the applicability of our method using real and simulated data.

Keywords: Data Augmentation algorithms; eigenvalues of Markov operators; MCMC convergence; trace class Markov operators

References

[1] Adamczak, R. and Bednorz, W. (2015). Some remarks on MCMC estimation of spectra of integral operators. *Bernoulli* **21** 2073–2092. MR3378459
[2] Albert, J.H. and Chib, S. (1993). Bayesian analysis of binary and polychotomous response data. *J. Amer. Statist. Assoc.* **88** 669–679. MR1224394
[3] Asmussen, S. and Glynn, P.W. (2011). A new proof of convergence of MCMC via the ergodic theorem. *Statist. Probab. Lett.* **81** 1482–1485. MR2818658
[4] Athreya, K.B. and Atuncar, G.S. (1998). Kernel estimation for real-valued Markov chains. *Sankhya, Ser. A* **60** 1–17. MR1714774
[5] Bennett, C.H. (1976). Efficient estimation of free energy differences from Monte Carlo data. *J. Comput. Phys.* **22** 245–268. MR0471852
[6] Canty, A. and Ripley, B.D. (2017). boot: Bootstrap R (S-Plus) Functions. R package version 1.3-19.
[7] Chakraborty, S. and Khare, K. (2017). Convergence properties of Gibbs samplers for Bayesian probit regression with proper priors. *Electron. J. Stat.* **11** 177–210. MR3604022
[8] Chakraborty, S. and Khare, K. (2019). Supplement to "Consistent estimation of the spectrum of trace class data augmentation algorithms." DOI:10.3150/19-BEJ1112SUPP.
[9] Choi, H.M. and Hobert, J.P. (2013). The Polya-gamma Gibbs sampler for Bayesian logistic regression is uniformly ergodic. *Electron. J. Stat.* **7** 2054–2064. MR3091616
[10] Choi, H.M. and Román, J.C. (2017). Analysis of Polya-Gamma Gibbs sampler for Bayesian logistic analysis of variance. *Electron. J. Stat.* **11** 326–337. MR3606773

[11] Conway, J.B. (1990). *A Course in Functional Analysis*, 2nd ed. *Graduate Texts in Mathematics* **96**. New York: Springer. MR1070713
[12] Davison, A.C. and Hinkley, D.V. (1997). *Bootstrap Methods and Their Application*. *Cambridge Series in Statistical and Probabilistic Mathematics* **1**. Cambridge: Cambridge Univ. Press. MR1478673
[13] Diaconis, P., Khare, K. and Saloff-Coste, L. (2008). Gibbs sampling, exponential families and orthogonal polynomials. *Statist*. *Sci*. **23** 151–178. MR2446500
[14] Diaconis, P. and Saloff-Coste, L. (1993). Comparison techniques for random walk on finite groups. *Ann*. *Probab*. **21** 2131–2156. MR1245303
[15] Diaconis, P. and Saloff-Coste, L. (1996). Nash inequalities for finite Markov chains. *J*. *Theoret*. *Probab*. **9** 459–510. MR1385408
[16] Diaconis, P. and Stroock, D. (1991). Geometric bounds for eigenvalues of Markov chains. *Ann*. *Appl*. *Probab*. **1** 36–61. MR1097463
[17] Eddelbuettel, D. (2013). *Seamless R and C++ Integration with Rcpp*. New York: Springer.
[18] François, O. (2000). Geometric inequalities for the eigenvalues of concentrated Markov chains. *J*. *Appl*. *Probab*. **37** 15–28. MR1761658
[19] Frühwirth-Schnatter, S. (2001). Markov chain Monte Carlo estimation of classical and dynamic switching and mixture models. *J*. *Amer*. *Statist*. *Assoc*. **96** 194–209. MR1952732
[20] Garren, S.T. and Smith, R.L. (2000). Estimating the second largest eigenvalue of a Markov transition matrix. *Bernoulli* **6** 215–242. MR1748720
[21] Hobert, J. P., Jung, Y. J., Khare, K. and Qin, Q. (2015). Convergence analysis of the Data Augmentation algorithm for Bayesian linear regression with non-Gaussian errors. ArXiv e-prints.
[22] Hobert, J.P., Roy, V. and Robert, C.P. (2011). Improving the convergence properties of the data augmentation algorithm with an application to Bayesian mixture modeling. *Statist*. *Sci*. **26** 332–351. MR2918006
[23] Hoffman, A.J. and Wielandt, H.W. (1953). The variation of the spectrum of a normal matrix. *Duke Math*. *J*. **20** 37–39. MR0052379
[24] Jones, G.L. and Hobert, J.P. (2001). Honest exploration of intractable probability distributions via Markov chain Monte Carlo. *Statist*. *Sci*. **16** 312–334. MR1888447
[25] Jörgens, K. (1982). *Linear Integral Operators*. *Surveys and Reference Works in Mathematics* **7**. Boston, MA–London: Pitman. MR0647629
[26] Khare, K. and Hobert, J.P. (2011). A spectral analytic comparison of trace-class data augmentation algorithms and their sandwich variants. *Ann*. *Statist*. **39** 2585–2606. MR2906879
[27] Khare, K. and Zhou, H. (2009). Rates of convergence of some multivariate Markov chains with polynomial eigenfunctions. *Ann*. *Appl*. *Probab*. **19** 737–777. MR2521887
[28] Koltchinskii, V. and Giné, E. (2000). Random matrix approximation of spectra of integral operators. *Bernoulli* **6** 113–167. MR1781185
[29] Lawler, G.F. and Sokal, A.D. (1988). Bounds on the L^2 spectrum for Markov chains and Markov processes: A generalization of Cheeger's inequality. *Trans*. *Amer*. *Math*. *Soc*. **309** 557–580. MR0930082
[30] Liu, J.S., Wong, W.H. and Kong, A. (1994). Covariance structure of the Gibbs sampler with applications to the comparisons of estimators and augmentation schemes. *Biometrika* **81** 27–40. MR1279653
[31] Meng, X.-L. and Wong, W.H. (1996). Simulating ratios of normalizing constants via a simple identity: A theoretical exploration. *Statist*. *Sinica* **6** 831–860. MR1422406
[32] Pal, S., Khare, K. and Hobert, J.P. (2017). Trace class Markov chains for Bayesian inference with generalized double Pareto shrinkage priors. *Scand*. *J*. *Stat*. **44** 307–323. MR3658516
[33] Polson, N.G., Scott, J.G. and Windle, J. (2013). Bayesian inference for logistic models using Pólya-Gamma latent variables. *J*. *Amer*. *Statist*. *Assoc*. **108** 1339–1349. MR3174712
[34] Qin, Q. and Hobert, J.P. (2018). Trace-class Monte Carlo Markov chains for Bayesian multivariate linear regression with non-Gaussian errors. *J*. *Multivariate Anal*. **166** 335–345. MR3799651

[35] Qin, Q., Hobert, J.P. and Khare, K. (2017). Estimating the spectral gap of a trace-class Markov operator. Preprint. Available at arXiv:1704.00850.
[36] R Core Team (2015). R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing.
[37] Raftery, A.E. and Lewis, S. (1992). How many iterations in the Gibbs sampler. *Bayesian Stat*. **4** 763–773.
[38] Rajaratnam, B., Sparks, D., Khare, K. and Zhang, L. (2017). Scalable Bayesian shrinkage and uncertainty quantification in high-dimensional regression. ArXiv e-prints.
[39] Retherford, J.R. (1993). *Hilbert Space*: *Compact Operators and the Trace Theorem*. *London Mathematical Society Student Texts* **27**. Cambridge: Cambridge Univ. Press. MR1237405
[40] Rosenthal, J.S. (1995). Minorization conditions and convergence rates for Markov chain Monte Carlo. *J*. *Amer*. *Statist*. *Assoc*. **90** 558–566. MR1340509
[41] Roy, V. (2012). Convergence rates for MCMC algorithms for a robust Bayesian binary regression model. *Electron*. *J*. *Stat*. **6** 2463–2485. MR3020272
[42] Saloff-Coste, L. (2004). Total variation lower bounds for finite Markov chains: Wilson's lemma. In *Random Walks and Geometry* 515–532. Berlin: de Gruyter. MR2087800
[43] Sinclair, A. and Jerrum, M. (1989). Approximate counting, uniform generation and rapidly mixing Markov chains. *Inform*. *and Comput*. **82** 93–133. MR1003059
[44] Wickham, H. (2007). Reshaping data with the reshape package. *J*. *Stat*. *Softw*. **21** 1–20.
[45] Wickham, H. (2016). *ggplot2*: *Elegant Graphics for Data Analysis*. New York: Springer.
[46] Yuen, W.K. (2000). Applications of geometric bounds to the convergence rate of Markov chains on $\mathbf{R}^n$. *Stochastic Process*. *Appl*. **87** 1–23. MR1751162

Bernoulli **25**(4B), 2019, 3864–3882
https://doi.org/10.3150/19-BEJ1113

Principal components analysis of regularly varying functions

PIOTR KOKOSZKA[1], STILIAN STOEV[2] and QIAN XIONG[1]

[1]*Colorado State University. E-mail: Piotr.Kokoszka@colostate.edu*
[2]*University of Michigan.*

The paper is concerned with asymptotic properties of the principal components analysis of functional data. The currently available results assume the existence of the fourth moment. We develop analogous results in a setting which does not require this assumption. Instead, we assume that the observed functions are regularly varying. We derive the asymptotic distribution of the sample covariance operator and of the sample functional principal components. We obtain a number of results on the convergence of moments and almost sure convergence. We apply the new theory to establish the consistency of the regression operator in a functional linear model.

Keywords: functional data; principal components; regular variation

References

[1] Anderson, P.L. and Meerschaert, M.M. (1997). Periodic moving averages of random variables with regularly varying tails. *Ann. Statist.* **25** 771–785. MR1439323
[2] Bosq, D. (2000). *Linear Processes in Function Spaces: Theory and Applications. Lecture Notes in Statistics* **149**. New York: Springer. MR1783138
[3] Dauxois, J., Pousse, A. and Romain, Y. (1982). Asymptotic theory for the principal component analysis of a vector random function: Some applications to statistical inference. *J. Multivariate Anal.* **12** 136–154. MR0650934
[4] Davis, R. and Resnick, S. (1985). Limit theory for moving averages of random variables with regularly varying tail probabilities. *Ann. Probab.* **13** 179–195. MR0770636
[5] Davis, R. and Resnick, S. (1986). Limit theory for the sample covariance and correlation functions of moving averages. *Ann. Statist.* **14** 533–558. MR0840513
[6] de Acosta, A. (1981). Inequalities for B-valued random vectors with applications to the strong law of large numbers. *Ann. Probab.* **9** 157–161. MR0606806
[7] de Acosta, A. and Giné, E. (1979). Convergence of moments and related functionals in the general central limit theorem in Banach spaces. *Z. Wahrsch. Verw. Gebiete* **48** 213–231. MR0534846
[8] Embrechts, P., Klüppelberg, C. and Mikosch, T. (1997). *Modelling Extremal Events. For Insurance and Finance. Applications of Mathematics (New York)* **33**. Berlin: Springer. MR1458613
[9] Hall, P. and Hosseini-Nasab, M. (2006). On properties of functional principal components analysis. *J. R. Stat. Soc. Ser. B. Stat. Methodol.* **68** 109–126. MR2212577
[10] Hörmann, S. and Kokoszka, P. (2010). Weakly dependent functional data. *Ann. Statist.* **38** 1845–1884. MR2662361
[11] Horváth, L. and Kokoszka, P. (2012). *Inference for Functional Data with Applications. Springer Series in Statistics*. New York: Springer. MR2920735

[12] Horváth, L., Kokoszka, P. and Rice, G. (2014). Testing stationarity of functional time series. *J. Econometrics* **179** 66–82. MR3153649
[13] Hsing, T. and Eubank, R. (2015). *Theoretical Foundations of Functional Data Analysis*, *with an Introduction to Linear Operators*. *Wiley Series in Probability and Statistics*. Chichester: Wiley. MR3379106
[14] Hult, H. and Lindskog, F. (2006). Regular variation for measures on metric spaces. *Publ*. *Inst*. *Math*. (*Beograd*) (*N.S.*) **80(94)** 121–140. MR2281910
[15] Klüppelberg, C. and Mikosch, T. (1994). Some limit theory for the self-normalised periodogram of stable processes. *Scand*. *J*. *Stat*. **21** 485–491. MR1310091
[16] Kokoszka, P. and Reimherr, M. (2013). Asymptotic normality of the principal components of functional time series. *Stochastic Process*. *Appl*. **123** 1546–1562. MR3027890
[17] Kokoszka, P. and Reimherr, M. (2017). *Introduction to Functional Data Analysis*. *Texts in Statistical Science Series*. Boca Raton, FL: CRC Press. MR3793167
[18] Kokoszka, P.S. and Taqqu, M.S. (1996). Parameter estimation for infinite variance fractional ARIMA. *Ann*. *Statist*. **24** 1880–1913. MR1421153
[19] Kokoszka, P., Stoev, S. and Xiong, Q. (2019). Supplement to "Principal components analysis of regularly varying functions." DOI:10.3150/19-BEJ1113SUPP.
[20] Kuelbs, J. and Mandrekar, V. (1974). Domains of attraction of stable measures on a Hilbert space. *Studia Math*. **50** 149–162. MR0345155
[21] Linde, W. (1986). *Probability in Banach Spaces – Stable and Infinitely Divisible Distributions*, 2nd ed. *A Wiley-Interscience Publication*. Chichester: Wiley. MR0874529
[22] Lucca, D.O. and Moench, E. (2015). The pre-FOMC announcement drift. *J. Finance* **70** 329–371.
[23] Meerschaert, M.M. (1984). *Multivariable Domains of Attraction and Regular Variation*. Ann Arbor, MI: ProQuest LLC. Thesis (Ph.D.) – University of Michigan. MR2633908
[24] Meerschaert, M.M. and Scheffler, H.-P. (2001). *Limit Distributions for Sums of Independent Random Vectors*: *Heavy Tails in Theory and Practice*. *Wiley Series in Probability and Statistics*: *Probability and Statistics*. New York: Wiley. MR1840531
[25] Meiguet, T. (2010). Heavy Tailed Functional Time series. Ph.D. Thesis, Universite catholigue de Louvain.
[26] Mikosch, T., Gadrich, T., Klüppelberg, C. and Adler, R.J. (1995). Parameter estimation for ARMA models with infinite variance innovations. *Ann*. *Statist*. **23** 305–326. MR1331670
[27] Ramsay, J.O. and Silverman, B.W. (2005). *Functional Data Analysis*, 2nd ed. *Springer Series in Statistics*. New York: Springer. MR2168993
[28] Resnick, S.I. (1987). *Extreme Values*, *Regular Variation*, *and Point Processes*. *Applied Probability. A Series of the Applied Probability Trust* **4**. New York: Springer. MR0900810
[29] Resnick, S.I. (2007). *Heavy-Tail Phenomena*: *Probabilistic and Statistical Modeling*. *Springer Series in Operations Research and Financial Engineering*. New York: Springer. MR2271424
[30] Rvačeva, E.L. (1962). On domains of attraction of multi-dimensional distributions. In *Select*. *Transl*. *Math*. *Statist*. *and Probability*, *Vol*. 2 183–205. Providence, R.I.: Amer. Math. Soc. MR0150795
[31] Samorodnitsky, G. and Taqqu, M.S. (1994). *Stable Non-Gaussian Random Processes*: *Stochastic Models with Infinite Variance*. *Stochastic Modeling*. New York: CRC Press. MR1280932
[32] Yao, F., Müller, H.-G. and Wang, J.-L. (2005). Functional linear regression analysis for longitudinal data. *Ann*. *Statist*. **33** 2873–2903. MR2253106

Bernoulli **25**(4B), 2019, 3883–3911
https://doi.org/10.3150/19-BEJ1114

Structured matrix estimation and completion

OLGA KLOPP[1,3,*], YU LU[2,**], ALEXANDRE B. TSYBAKOV[3,†] and HARRISON H. ZHOU[2,‡]

[1] *ESSEC Business School, Cergy-Pontoise, France. E-mail:* [*] *kloppolga@math.cnrs.fr*
[2] *Yale University, New Haven, USA. E-mail:* [**] *yu.lu@yale.edu;* [‡] *huibin.zhou@yale.edu*
[3] *ENSAE, UMR CNRS 9194, Palaiseau, France. E-mail:* [†] *alexandre.tsybakov@ensae.fr*

We study the problem of matrix estimation and matrix completion under a general framework. This framework includes several important models as special cases such as the Gaussian mixture model, mixed membership model, bi-clustering model and dictionary learning. We establish the optimal convergence rates in a minimax sense for estimation of the signal matrix under the Frobenius norm and under the spectral norm. As a consequence of our general result we obtain minimax optimal rates of convergence for various special models.

Keywords: matrix completion; matrix estimation; minimax optimality; mixture model; stochastic block model

References

[1] Achlioptas, D. (2003). Database-friendly random projections: Johnson–Lindenstrauss with binary coins. *J. Comput. System Sci.* **66** 671–687. Special issue on PODS 2001 (Santa Barbara, CA). MR2005771
[2] Airoldi, E.M., Blei, D.M., Fienberg, S.E. and Xing, E.P. (2008). Mixed membership stochastic blockmodels. *J. Mach. Learn. Res.* **9** 1981–2014.
[3] Azizyan, M., Singh, A. and Wasserman, L. (2013). Minimax theory for high-dimensional Gaussian mixtures with sparse mean separation. In *Advances in Neural Information Processing Systems* (C.J.C. Burges, L. Bottou, M. Welling, Z. Ghahramani and K.Q. Weinberger, eds.) **26** 2139–2147. Curran Associates.
[4] Bandeira, A.S. and van Handel, R. (2016). Sharp nonasymptotic bounds on the norm of random matrices with independent entries. *Ann. Probab.* **44** 2479–2506. MR3531673
[5] Belkin, M. and Sinha, K. (2010). Polynomial learning of distribution families. In 2010 *IEEE* 51*st Annual Symposium on Foundations of Computer Science—FOCS* 2010 103–112. Los Alamitos, CA: IEEE Computer Soc. MR3024780
[6] Borgs, C., Chayes, J. and Smith, A. (2015). Private graphon estimation for sparse graphs. In *Advances in Neural Information Processing Systems* 1369–1377.
[7] Bunea, F., She, Y. and Wegkamp, M.H. (2011). Optimal selection of reduced rank estimators of high-dimensional matrices. *Ann. Statist.* **39** 1282–1309. MR2816355
[8] Cai, T.T. and Zhou, W.-X. (2016). Matrix completion via max-norm constrained optimization. *Electron. J. Stat.* **10** 1493–1525. MR3507371
[9] Candes, E.J. and Plan, Y. (2010). Matrix completion with noise. *Proc. IEEE* **98** 925–936.
[10] Candès, E.J. and Tao, T. (2010). The power of convex relaxation: Near-optimal matrix completion. *IEEE Trans. Inform. Theory* **56** 2053–2080. MR2723472
[11] Chan, S.H. and Airoldi, E.M. (2014). A consistent histogram estimator for exchangeable graph models. In *Proceedings of the* 31*st International Conference on Machine Learning* 208–216.

[12] Chatterjee, S. (2015). Matrix estimation by universal singular value thresholding. *Ann. Statist.* **43** 177–214. MR3285604
[13] Chaudhuri, K., Dasgupta, S. and Vattani, A. (2009). Learning mixtures of gaussians using the k-means algorithm. Preprint. Available at arXiv:0912.0086.
[14] Cheng, Y. and Church, G.M. (2000). Biclustering of expression data. In *ISMB* **8** 93–103.
[15] Dasgupta, S. (1999). Learning mixtures of Gaussians. In 40*th Annual Symposium on Foundations of Computer Science* (*New York*, 1999) 634–644. Los Alamitos, CA: IEEE Computer Soc. MR1917603
[16] Dasgupta, S. and Schulman, L.J. (2000). A two-round variant of em for Gaussian mixtures. In *Proceedings of the Sixteenth Conference on Uncertainty in Artificial Intelligence*, *UAI* '00 152–159. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc.
[17] Du, S.S., Wang, Y. and Singh, A. (2017). On the power of truncated SVD for general high-rank matrix estimation problems. In *Advances in Neural Information Processing Systems* 30 (*NIPS*) 2017 445–455.
[18] Foygel, R. and Srebro, N. (2011). Concentration-based guarantees for low-rank matrix reconstruction. In 24*nd Annual Conference on Learning Theory* (*COLT*).
[19] Gao, C., Lu, Y., Ma, Z. and Zhou, H.H. (2016). Optimal estimation and completion of matrices with biclustering structures. *J. Mach. Learn. Res.* **17** Paper No. 161, 29. MR3569248
[20] Gao, C., Lu, Y. and Zhou, H.H. (2015). Rate-optimal graphon estimation. *Ann. Statist.* **43** 2624–2652. MR3405606
[21] Giraud, C. (2015). *Introduction to High-Dimensional Statistics*. *Monographs on Statistics and Applied Probability* **139**. Boca Raton, FL: CRC Press. MR3307991
[22] Gross, D. (2011). Recovering low-rank matrices from few coefficients in any basis. *IEEE Trans. Inform. Theory* **57** 1548–1566. MR2815834
[23] Hardt, M. (2014). Understanding alternating minimization for matrix completion. In 55*th Annual IEEE Symposium on Foundations of Computer Science—FOCS* 2014 651–660. Los Alamitos, CA: IEEE Computer Soc. MR3344916
[24] Hartigan, J.A. (1972). Direct clustering of a data matrix. *J. Amer. Statist. Assoc.* **67** 123–129.
[25] Holland, P.W., Laskey, K.B. and Leinhardt, S. (1983). Stochastic blockmodels: First steps. *Soc. Netw.* **5** 109–137. MR0718088
[26] Hsu, D. and Kakade, S.M. (2013). Learning mixtures of spherical Gaussians: Moment methods and spectral decompositions. In *ITCS*'13—*Proceedings of the* 2013 *ACM Conference on Innovations in Theoretical Computer Science* 11–19. New York: ACM. MR3385380
[27] Hsu, D., Kakade, S.M. and Zhang, T. (2012). A tail inequality for quadratic forms of subgaussian random vectors. *Electron. Commun. Probab.* **17** no. 52, 6. MR2994877
[28] Karrer, B. and Newman, M.E.J. (2011). Stochastic blockmodels and community structure in networks. *Phys. Rev. E* (3) **83** 016107, 10. MR2788206
[29] Keshavan, R.H., Montanari, A. and Oh, S. (2010). Matrix completion from noisy entries. *J. Mach. Learn. Res.* **11** 2057–2078. MR2678022
[30] Klopp, O. (2011). Rank penalized estimators for high-dimensional matrices. *Electron. J. Stat.* **5** 1161–1183. MR2842903
[31] Klopp, O. (2014). Noisy low-rank matrix completion with general sampling distribution. *Bernoulli* **20** 282–303. MR3160583
[32] Klopp, O., Lu, Y., Tsybakov, A. and Zhou, H. (2019). Supplement to "Structured matrix estimation and completion." DOI:10.3150/19-BEJ1114SUPP.
[33] Klopp, O., Tsybakov, A.B. and Verzelen, N. (2017). Oracle inequalities for network models and sparse graphon estimation. *Ann. Statist.* **45** 316–354. MR3611494
[34] Koltchinskii, V., Lounici, K. and Tsybakov, A.B. (2011). Nuclear-norm penalization and optimal rates for noisy low-rank matrix completion. *Ann. Statist.* **39** 2302–2329. MR2906869

[35] Negahban, S. and Wainwright, M.J. (2012). Restricted strong convexity and weighted matrix completion: Optimal bounds with noise. *J. Mach. Learn. Res.* **13** 1665–1697. MR2930649
[36] Olshausen, B.A. and Field, D.J. (1997). Sparse coding with an overcomplete basis set: A strategy employed by v1? *Vis. Res.* **37** 3311–3325.
[37] Rigollet, P. and Tsybakov, A. (2011). Exponential screening and optimal rates of sparse estimation. *Ann. Statist.* **39** 731–771. MR2816337
[38] Soni, A., Jain, S., Haupt, J. and Gonella, S. (2016). Noisy matrix completion under sparse factor models. *IEEE Trans. Inform. Theory* **62** 3636–3661. MR3506754
[39] Tsybakov, A.B. (2009). *Introduction to Nonparametric Estimation*. *Springer Series in Statistics*. New York: Springer. MR2724359
[40] Vempala, S. and Wang, G. (2004). A spectral algorithm for learning mixture models. *J. Comput. System Sci.* **68** 841–860. MR2059647
[41] Wolfe, P.J. and Olhede, S.C. (2013). Nonparametric graphon estimation. Preprint. Available at arXiv:1309.5936.
[42] Xu, J., Massoulié, L. and Lelarge, M. (2014). Edge label inference in generalized stochastic block models: From spectral theory to impossibility results. In *Conference on Learning Theory* 903–920. Barcelona, Spain.
[43] Yang, Y. and Barron, A. (1999). Information-theoretic determination of minimax rates of convergence. *Ann. Statist.* **27** 1564–1599. MR1742500

Bernoulli **25**(4B), 2019, 3912–3938
https://doi.org/10.3150/19-BEJ1115

Rademacher complexity for Markov chains: Applications to kernel smoothing and Metropolis–Hastings

PATRICE BERTAIL[1] and FRANÇOIS PORTIER[2]

[1] *Modal'X, UPL, University of Paris-Nanterre, France. E-mail: patrice.bertail@gmail.com*
[2] *Télécom Paris, Institut polytechnique de Paris, France. E-mail: francois.portier@gmail.com*

The concept of Rademacher complexity for independent sequences of random variables is extended to Markov chains. The proposed notion of "regenerative block Rademacher complexity" (of a class of functions) follows from renewal theory and allows to control the expected values of suprema (over the class of functions) of empirical processes based on Harris Markov chains as well as the excess probability. For classes of Vapnik–Chervonenkis type, bounds on the "regenerative block Rademacher complexity" are established. These bounds depend essentially on the sample size and the probability tails of the regeneration times. The proposed approach is employed to obtain convergence rates for the kernel density estimator of the stationary measure and to derive concentration inequalities for the Metropolis–Hastings algorithm.

Keywords: concentration inequalities; Kernel smoothing; Markov chains; Metropolis Hastings; Rademacher complexity

References

[1] Adamczak, R. (2008). A tail inequality for suprema of unbounded empirical processes with applications to Markov chains. *Electron. J. Probab.* **13** 1000–1034. MR2424985
[2] Akritas, M.G. and Van Keilegom, I. (2001). Non-parametric estimation of the residual distribution. *Scand. J. Stat.* **28** 549–567. MR1858417
[3] Athreya, K.B. and Ney, P. (1978). A new approach to the limit theory of recurrent Markov chains. *Trans. Amer. Math. Soc.* **245** 493–501. MR0511425
[4] Azaïs, R., Delyon, B. and Portier, F. (2018). Integral estimation based on Markovian design. *Adv. in Appl. Probab.* **50** 833–857. MR3877256
[5] Bartlett, P.L., Bousquet, O. and Mendelson, S. (2005). Local Rademacher complexities. *Ann. Statist.* **33** 1497–1537. MR2166554
[6] Bartlett, P.L. and Mendelson, S. (2002). Rademacher and Gaussian complexities: Risk bounds and structural results. *J. Mach. Learn. Res.* **3** 463–482. MR1984026
[7] Bednorz, W., Łatuszyński, K. and Latała, R. (2008). A regeneration proof of the central limit theorem for uniformly ergodic Markov chains. *Electron. Commun. Probab.* **13** 85–98. MR2386065
[8] Berbee, H.C.P. (1979). *Random Walks with Stationary Increments and Renewal Theory*. *Mathematical Centre Tracts* **112**. Amsterdam: Mathematisch Centrum. MR0547109
[9] Bertail, P. and Clémençon, S. (2004). Edgeworth expansions of suitably normalized sample mean statistics for atomic Markov chains. *Probab. Theory Related Fields* **130** 388–414. MR2095936
[10] Bertail, P. and Clémençon, S. (2006). Regenerative block bootstrap for Markov chains. *Bernoulli* **12** 689–712. MR2248233

1350-7265

[11] Bertail, P. and Clémençon, S. (2009). Sharp bounds for the tails of functionals of Markov chains. *Theory Probab. Appl.* **54** 609–619. MR2766354

[12] Bolthausen, E. (1980). The Berry–Esseen theorem for functionals of discrete Markov chains. *Z. Wahrsch. Verw. Gebiete* **54** 59–73. MR0595481

[13] Bolthausen, E. (1982). On the central limit theorem for stationary mixing random fields. *Ann. Probab.* **10** 1047–1050. MR0672305

[14] Boucheron, S., Lugosi, G. and Massart, P. (2013). *Concentration Inequalities: A Nonasymptotic Theory of Independence*. Oxford: Oxford Univ. Press. MR3185193

[15] Bousquet, O., Boucheron, S. and Lugosi, G. (2004). Introduction to statistical learning theory. In *Advanced Lectures on Machine Learning* 169–207. Springer.

[16] Bradley, R.C. (2005). Basic properties of strong mixing conditions. A survey and some open questions. *Probab. Surv.* **2** 107–144. MR2178042

[17] Chen, X. (1999). Limit theorems for functionals of ergodic Markov chains with general state space. *Mem. Amer. Math. Soc.* **139** xiv+203. MR1491814

[18] de la Peña, V.H. and Giné, E. (1999). *Decoupling*. *Probability and Its Applications* (*New York*). New York: Springer. From dependence to independence, Randomly stopped processes. U-statistics and processes. Martingales and beyond. MR1666908

[19] Dedecker, J. and Gouëzel, S. (2015). Subgaussian concentration inequalities for geometrically ergodic Markov chains. *Electron. Commun. Probab.* **20** 64. MR3407208

[20] Douc, R., Guillin, A. and Moulines, E. (2008). Bounds on regeneration times and limit theorems for subgeometric Markov chains. *Ann. Inst. Henri Poincaré Probab. Stat.* **44** 239–257. MR2446322

[21] Douc, R., Moulines, E. and Rosenthal, J.S. (2004). Quantitative bounds on convergence of time-inhomogeneous Markov chains. *Ann. Appl. Probab.* **14** 1643–1665. MR2099647

[22] Einmahl, U. and Mason, D.M. (2000). An empirical process approach to the uniform consistency of kernel-type function estimators. *J. Theoret. Probab.* **13** 1–37. MR1744994

[23] Einmahl, U. and Mason, D.M. (2005). Uniform in bandwidth consistency of kernel-type function estimators. *Ann. Statist.* **33** 1380–1403. MR2195639

[24] Giné, E. and Guillou, A. (2001). On consistency of kernel density estimators for randomly censored data: Rates holding uniformly over adaptive intervals. *Ann. Inst. Henri Poincaré Probab. Stat.* **37** 503–522. MR1876841

[25] Giné, E. and Guillou, A. (2002). Rates of strong uniform consistency for multivariate kernel density estimators. *Ann. Inst. Henri Poincaré Probab. Stat.* **38** 907–921. En l'honneur de J. Bretagnolle, D. Dacunha-Castelle, I. Ibragimov. MR1955344

[26] Giné, E. and Nickl, R. (2008). Uniform central limit theorems for kernel density estimators. *Probab. Theory Related Fields* **141** 333–387. MR2391158

[27] Giné, E. and Nickl, R. (2016). *Mathematical Foundations of Infinite-Dimensional Statistical Models*. *Cambridge Series in Statistical and Probabilistic Mathematics*. New York: Cambridge Univ. Press. MR3588285

[28] Haario, H., Saksman, E. and Tamminen, J. (2001). An adaptive Metropolis algorithm. *Bernoulli* **7** 223–242. MR1828504

[29] Hansen, B.E. (2008). Uniform convergence rates for kernel estimation with dependent data. *Econometric Theory* **24** 726–748. MR2409261

[30] Jain, N. and Jamison, B. (1967). Contributions to Doeblin's theory of Markov processes. *Z. Wahrsch. Verw. Gebiete* **8** 19–40. MR0221591

[31] Jarner, S.F. and Hansen, E. (2000). Geometric ergodicity of Metropolis algorithms. *Stochastic Process. Appl.* **85** 341–361. MR1731030

[32] Joulin, A. and Ollivier, Y. (2010). Curvature, concentration and error estimates for Markov chain Monte Carlo. *Ann. Probab.* **38** 2418–2442. MR2683634

[33] Koltchinskii, V. (2006). Local Rademacher complexities and oracle inequalities in risk minimization. *Ann. Statist.* **34** 2593–2656. MR2329442
[34] Koltchinskii, V. (2011). *Oracle Inequalities in Empirical Risk Minimization and Sparse Recovery Problems. Lecture Notes in Math.* **2033**. Heidelberg: Springer. MR2829871
[35] Łatuszyński, K., Miasojedow, B. and Niemiro, W. (2013). Nonasymptotic bounds on the estimation error of MCMC algorithms. *Bernoulli* **19** 2033–2066. MR3129043
[36] Levental, S. (1988). Uniform limit theorems for Harris recurrent Markov chains. *Probab. Theory Related Fields* **80** 101–118. MR0970473
[37] Malinovskiĭ, V.K. (1986). Limit theorems for Harris Markov chains. I. *Theory Probab. Appl.* **31** 315–332. MR0850991
[38] Malinovskiĭ, V.K. (1989). Limit theorems for Harris Markov chains. II. *Theory Probab. Appl.* **34** 289–303. MR1005734
[39] McDiarmid, C. (1989). On the method of bounded differences. In *Surveys in Combinatorics*, 1989 (*Norwich*, 1989). *London Mathematical Society Lecture Note Series* **141** 148–188. Cambridge: Cambridge Univ. Press. MR1036755
[40] Mengersen, K.L. and Tweedie, R.L. (1996). Rates of convergence of the Hastings and Metropolis algorithms. *Ann. Statist.* **24** 101–121. MR1389882
[41] Meyn, S. and Tweedie, R.L. (2009). *Markov Chains and Stochastic Stability*, 2nd ed. Cambridge: Cambridge Univ. Press. With a prologue by Peter W. Glynn. MR2509253
[42] Mohri, M. and Rostamizadeh, A. (2010). Rademacher complexity for non i.i.d. processes. *Adv. Neural Inf. Process. Syst.* 1097–1104.
[43] Mohri, M. and Rostamizadeh, A. (2010). Stability bounds for stationary ϕ-mixing and β-mixing processes. *J. Mach. Learn. Res.* **11** 789–814. MR2600630
[44] Nickl, R. and Söhl, J. (2017). Nonparametric Bayesian posterior contraction rates for discretely observed scalar diffusions. *Ann. Statist.* **45** 1664–1693. MR3670192
[45] Nolan, D. and Pollard, D. (1987). U-processes: Rates of convergence. *Ann. Statist.* **15** 780–799. MR0888439
[46] Nummelin, E. (1978). A splitting technique for Harris recurrent Markov chains. *Z. Wahrsch. Verw. Gebiete* **43** 309–318. MR0501353
[47] Nummelin, E. (1984). *General Irreducible Markov Chains and Nonnegative Operators. Cambridge Tracts in Mathematics* **83**. Cambridge: Cambridge Univ. Press. MR0776608
[48] Paulin, D. (2015). Concentration inequalities for Markov chains by Marton couplings and spectral methods. *Electron. J. Probab.* **20** 79. MR3383563
[49] Peligrad, M. (1992). Properties of uniform consistency of the kernel estimators of density and of regression functions under dependence assumptions. *Stoch. Stoch. Rep.* **40** 147–168. MR1275130
[50] Portier, F. (2016). On the asymptotics of Z-estimators indexed by the objective functions. *Electron. J. Stat.* **10** 464–494. MR3466190
[51] Portier, F. and Segers, J. (2018). On the weak convergence of the empirical conditional copula under a simplifying assumption. *J. Multivariate Anal.* **166** 160–181. MR3799641
[52] Rakhlin, A. and Sridharan, K. (2015). Festschrift for Alexey Chervonenkis. In *Measures of Complexity* 171–185. Cham: Springer.
[53] Robert, C.P. and Casella, G. (2004). *Monte Carlo Statistical Methods*, 2nd ed. *Springer Texts in Statistics*. New York: Springer. MR2080278
[54] Roberts, G.O. and Rosenthal, J.S. (2004). General state space Markov chains and MCMC algorithms. *Probab. Surv.* **1** 20–71. MR2095565
[55] Roberts, G.O. and Tweedie, R.L. (1996). Geometric convergence and central limit theorems for multidimensional Hastings and Metropolis algorithms. *Biometrika* **83** 95–110. MR1399158

[56] Shorack, G.R. and Wellner, J.A. (2009). *Empirical Processes with Applications to Statistics*. *Classics in Applied Mathematics* **59**. Philadelphia, PA: SIAM. MR3396731

[57] Smith, W.L. (1955). Regenerative stochastic processes. *Proc*. *R*. *Soc*. *Lond*. *Ser. A* **232** 6–31. MR0073877

[58] Stute, W. (1982). A law of the logarithm for kernel density estimators. *Ann*. *Probab.* **10** 414–422. MR0647513

[59] Talagrand, M. (1994). Sharper bounds for Gaussian and empirical processes. *Ann*. *Probab.* **22** 28–76. MR1258865

[60] van der Vaart, A.W. (1998). *Asymptotic Statistics*. *Cambridge Series in Statistical and Probabilistic Mathematics* **3**. Cambridge: Cambridge Univ. Press. MR1652247

[61] van der Vaart, A.W. and Wellner, J.A. (1996). *Weak Convergence and Empirical Processes*. *With Applications to Statistics*. *Springer Series in Statistics*. New York: Springer. MR1385671

[62] van der Vaart, A.W. and Wellner, J.A. (2007). Empirical processes indexed by estimated functions. In *Asymptotics*: *Particles*, *Processes and Inverse Problems*. *Institute of Mathematical Statistics Lecture Notes—Monograph Series* **55** 234–252. Beachwood, OH: IMS. MR2459942

[63] Vapnik, V.N. (1998). *Statistical Learning Theory*. *Adaptive and Learning Systems for Signal Processing*, *Communications*, *and Control*. New York: Wiley. MR1641250

[64] Vapnik, V.N. and Červonenkis, A.J. (1971). The uniform convergence of frequencies of the appearance of events to their probabilities. *Theory Probab. Appl*. **16** 264–279. MR0288823

[65] Walter, G. and Blum, J. (1979). Probability density estimation using delta sequences. *Ann*. *Statist*. **7** 328–340. MR0520243

[66] Wintenberger, O. (2017). Exponential inequalities for unbounded functions of geometrically ergodic Markov chains: Applications to quantitative error bounds for regenerative Metropolis algorithms. *Statistics* **51** 222–234. MR3600471

Bernoulli **25**(4B), 2019, 3939–3977
https://doi.org/10.3150/19-BEJ1116

Inverse exponential decay: Stochastic fixed point equation and ARMA models

KRZYSZTOF BURDZY[1], BARTOSZ KOŁODZIEJEK[2] and TVRTKO TADIĆ[3]

[1]*Department of Mathematics, University of Washington, Box 354350, Seattle, WA 98195, USA. E-mail: burdzy@uw.edu*

[2]*Faculty of Mathematics and Information Science, Warsaw University of Technology, Koszykowa 75, 00-662 Warsaw, Poland. E-mail: b.kolodziejek@mini.pw.edu.pl*

[3]*Microsoft Corporation, One Microsoft Way, Redmond, WA 98052, USA. E-mail: tvrtko@math.hr*

We study solutions to the stochastic fixed point equation $X \stackrel{d}{=} AX + B$ when the coefficients are nonnegative and B is an "inverse exponential decay" (IED) random variable. We provide theorems on the left tail of X which complement well-known tail results of Kesten and Goldie. We generalize our results to ARMA processes with nonnegative coefficients whose noise terms are from the IED class. We describe the lower envelope for these ARMA processes.

Keywords: ARMA models; inverse-gamma distribution; iterated random sequences; stochastic fixed point equation; tail estimates; time series

References

[1] Bieniek, M., Burdzy, K. and Pal, S. (2012). Extinction of Fleming–Viot-type particle systems with strong drift. *Electron. J. Probab.* **17** no. 11, 15 pp. MR2878790

[2] Bingham, N.H., Goldie, C.M. and Teugels, J.L. (1987). *Regular Variation. Encyclopedia of Mathematics and Its Applications* **27**. Cambridge: Cambridge Univ. Press. MR0898871

[3] Brockwell, P.J. and Davis, R.A. (1991). *Time Series: Theory and Methods*, 2nd ed. *Springer Series in Statistics*. New York: Springer. MR1093459

[4] Buraczewski, D., Damek, E. and Mikosch, T. (2016). *Stochastic Models with Power-Law Tails: The Equation $X = AX + B$. Springer Series in Operations Research and Financial Engineering*. Cham: Springer. MR3497380

[5] Burdzy, K., Hołyst, R. and March, P. (2000). A Fleming–Viot particle representation of the Dirichlet Laplacian. *Comm. Math. Phys.* **214** 679–703. MR1800866

[6] Diaconis, P. and Freedman, D. (1999). Iterated random functions. *SIAM Rev.* **41** 45–76. MR1669737

[7] Goldie, C.M. (1991). Implicit renewal theory and tails of solutions of random equations. *Ann. Appl. Probab.* **1** 126–166. MR1097468

[8] Hoff, P.D. (2009). *A First Course in Bayesian Statistical Methods. Springer Texts in Statistics*. New York: Springer. MR2648134

[9] Kesten, H. (1973). Random difference equations and renewal theory for products of random matrices. *Acta Math.* **131** 207–248. MR0440724

[10] Kołodziejek, B. (2018). On perpetuities with light tails. *Adv. in Appl. Probab.* **50** 1119–1154. MR3881112

[11] Tchen, A.H. (1980). Inequalities for distributions with given marginals. *Ann. Probab.* **8** 814–827. MR0577318

1350-7265

Bernoulli **25**(4B), 2019, 3978–4006
https://doi.org/10.3150/19-BEJ1117

Weighted Poincaré inequalities, concentration inequalities and tail bounds related to Stein kernels in dimension one

ADRIEN SAUMARD

CREST, ENSAI, Université Bretagne Loire, Campus de ker-lann, Rue Blaise Pascal – BP 37203 35712 Bruz Cedex, France. E-mail: asaumard@gmail.com

We investigate links between the so-called Stein's density approach in dimension one and some functional and concentration inequalities. We show that measures having a finite first moment and a density with connected support satisfy a weighted Poincaré inequality with the weight being the Stein kernel, that indeed exists and is unique in this case. Furthermore, we prove weighted log-Sobolev and asymmetric Brascamp–Lieb type inequalities related to Stein kernels. We also show that existence of a uniformly bounded Stein kernel is sufficient to ensure a positive Cheeger isoperimetric constant. Then we derive new concentration inequalities. In particular, we prove generalized Mills' type inequalities when a Stein kernel is uniformly bounded and sub-gamma concentration for Lipschitz functions of a variable with a sub-linear Stein kernel. More generally, when some exponential moments are finite, the Laplace transform of the random variable of interest is shown to bounded from above by the Laplace transform of the Stein kernel. Along the way, we prove a general lemma for bounding the Laplace transform of a random variable, that may be of independent interest. We also provide density and tail formulas as well as tail bounds, generalizing previous results that where obtained in the context of Malliavin calculus.

Keywords: concentration inequality; covariance identity; isoperimetric constant; Stein kernel; tail bound; weighted log-Sobolev inequality; weighted Poincaré inequality

References

[1] Ané, C., Blachère, S., Chafaï, D., Fougères, P., Gentil, I., Malrieu, F., Roberto, C. and Scheffer, G. (2000). *Sur les Inégalités de Sobolev Logarithmiques*. *Panoramas et Synthèses* [*Panoramas and Syntheses*] **10**. Paris: Société Mathématique de France. With a preface by Dominique Bakry and Michel Ledoux. MR1845806

[2] Bakry, D., Cattiaux, P. and Guillin, A. (2008). Rate of convergence for ergodic continuous Markov processes: Lyapunov versus Poincaré. *J. Funct. Anal.* **254** 727–759. MR2381160

[3] Bakry, D., Gentil, I. and Ledoux, M. (2014). *Analysis and Geometry of Markov Diffusion Operators*. *Grundlehren der Mathematischen Wissenschaften* [*Fundamental Principles of Mathematical Sciences*] **348**. Cham: Springer. MR3155209

[4] Barbour, A.D., Ross, N. and Wen, Y. (2018). Central moment inequalities using Stein's method. Preprint. Available at arXiv:1802.10225.

[5] Barthe, F. and Roberto, C. (2003). Sobolev inequalities for probability measures on the real line. *Studia Math.* **159** 481–497. Dedicated to Professor Aleksander Pełczyński on the occasion of his 70th birthday (Polish). MR2052235

1350-7265

[6] Bobkov, S.G. and Götze, F. (1999). Exponential integrability and transportation cost related to logarithmic Sobolev inequalities. *J. Funct. Anal.* **163** 1–28. MR1682772

[7] Bobkov, S.G., Götze, F. and Houdré, C. (2001). On Gaussian and Bernoulli covariance representations. *Bernoulli* **7** 439–451. MR1836739

[8] Bobkov, S.G. and Houdré, C. (1997). Isoperimetric constants for product probability measures. *Ann. Probab.* **25** 184–205. MR1428505

[9] Bobkov, S. and Ledoux, M. (1997). Poincaré's inequalities and Talagrand's concentration phenomenon for the exponential distribution. *Probab. Theory Related Fields* **107** 383–400. MR1440138

[10] Bobkov, S.G. and Ledoux, M. (2009). On weighted isoperimetric and Poincaré-type inequalities. In *High Dimensional Probability V*: *The Luminy Volume*. *Inst. Math. Stat.* (*IMS*) *Collect.* **5** 1–29. Beachwood, OH: IMS. MR2797936

[11] Bobkov, S.G. and Ledoux, M. (2009). Weighted Poincaré-type inequalities for Cauchy and other convex measures. *Ann. Probab.* **37** 403–427. MR2510011

[12] Bobkov, S. and Ledoux, M. (2014). One-dimensional empirical measures, order statistics, and Kantorovich transport distances. *Mem. Amer. Math. Soc.* To appear.

[13] Bonnefont, M. and Joulin, A. (2014). Intertwining relations for one-dimensional diffusions and application to functional inequalities. *Potential Anal.* **41** 1005–1031. MR3269712

[14] Bonnefont, M., Joulin, A. and Ma, Y. (2016). A note on spectral gap and weighted Poincaré inequalities for some one-dimensional diffusions. *ESAIM Probab. Stat.* **20** 18–29. MR3519678

[15] Bonnefont, M., Joulin, A. and Ma, Y. (2016). Spectral gap for spherically symmetric log-concave probability measures, and beyond. *J. Funct. Anal.* **270** 2456–2482. MR3464047

[16] Boucheron, S., Lugosi, G. and Massart, P. (2013). *Concentration Inequalities*. Oxford: Oxford Univ. Press. A nonasymptotic theory of independence, With a foreword by Michel Ledoux. MR3185193

[17] Cattiaux, P., Guillin, A. and Wu, L.-M. (2011). Some remarks on weighted logarithmic Sobolev inequality. *Indiana Univ. Math. J.* **60** 1885–1904. MR3008255

[18] Cattiaux, P., Gozlan, N., Guillin, A. and Roberto, C. (2010). Functional inequalities for heavy tailed distributions and application to isoperimetry. *Electron. J. Probab.* **15** 346–385. MR2609591

[19] Chatterjee, S. (2007). Stein's method for concentration inequalities. *Probab. Theory Related Fields* **138** 305–321. MR2288072

[20] Chatterjee, S. (2012). A new approach to strong embeddings. *Probab. Theory Related Fields* **152** 231–264. MR2875758

[21] Chatterjee, S. (2014). A short survey of Stein's method. In *Proceedings of the International Congress of Mathematicians – Seoul* 2014. *Vol. IV* 1–24. Seoul: Kyung Moon Sa. MR3727600

[22] Chatterjee, S. and Dey, P.S. (2010). Applications of Stein's method for concentration inequalities. *Ann. Probab.* **38** 2443–2485. MR2683635

[23] Chen, L.H.Y., Goldstein, L. and Shao, Q.-M. (2011). *Normal Approximation by Stein's Method*. *Probability and Its Applications* (*New York*). Heidelberg: Springer. MR2732624

[24] Cook, N., Goldstein, L. and Johnson, T. (2018). Size biased couplings and the spectral gap for random regular graphs. *Ann. Probab.* **46** 72–125. MR3758727

[25] Cordero-Erausquin, D. and Gozlan, N. (2017). Transport proofs of weighted Poincaré inequalities for log-concave distributions. *Bernoulli* **23** 134–158. MR3556769

[26] Courtade, T.A., Fathi, M. and Pananjady, A. (2017). Existence of Stein Kernels under a Spectral Gap, and Discrepancy Bound. Preprint. Available at arXiv:1703.07707.

[27] Döbler, C. (2015). Stein's method of exchangeable pairs for the beta distribution and generalizations. *Electron. J. Probab.* **20** 109. MR3418541

[28] Dümbgen, L. (2010). Bounding Standard Gaussian Tail Probabilities. Technical Report.

[29] Eden, R. and Viens, F. (2013). General upper and lower tail estimates using Malliavin calculus and Stein's equations. In *Seminar on Stochastic Analysis*, *Random Fields and Applications VII*. *Progress in Probability* **67** 55–84. Basel: Birkhäuser/Springer. MR3380095

[30] Fathi, M. (2018). Stein kernels and moment maps. *Ann. Probab.* To appear. Available at arXiv:1804.04699.

[31] Fathi, M. and Nelson, B. (2017). Free Stein kernels and an improvement of the free logarithmic Sobolev inequality. *Adv. Math.* **317** 193–223. MR3682667

[32] Ghosh, S. and Goldstein, L. (2011). Applications of size biased couplings for concentration of measures. *Electron. Commun. Probab.* **16** 70–83. MR2763529

[33] Ghosh, S. and Goldstein, L. (2011). Concentration of measures via size-biased couplings. *Probab. Theory Related Fields* **149** 271–278. MR2773032

[34] Ghosh, S., Goldstein, L. and Raič, M. (2011). Concentration of measure for the number of isolated vertices in the Erdős–Rényi random graph by size bias couplings. *Statist. Probab. Lett.* **81** 1565–1570. MR2832913

[35] Goldstein, L. and Işlak, Ü. (2014). Concentration inequalities via zero bias couplings. *Statist. Probab. Lett.* **86** 17–23. MR3162712

[36] Gozlan, N. (2010). Poincaré inequalities and dimension free concentration of measure. *Ann. Inst. Henri Poincaré Probab. Stat.* **46** 708–739. MR2682264

[37] Latała, R. and Oleszkiewicz, K. (2000). Between Sobolev and Poincaré. In *Geometric Aspects of Functional Analysis. Lecture Notes in Math.* **1745** 147–168. Berlin: Springer. MR1796718

[38] Ledoux, M., Nourdin, I. and Peccati, G. (2015). Stein's method, logarithmic Sobolev and transport inequalities. *Geom. Funct. Anal.* **25** 256–306. MR3320893

[39] Ledoux, M., Nourdin, I. and Peccati, G. (2017). A Stein deficit for the logarithmic Sobolev inequality. *Sci. China Math.* **60** 1163–1180. MR3665794

[40] Ley, C., Reinert, G. and Swan, Y. (2017). Stein's method for comparison of univariate distributions. *Probab. Surv.* **14** 1–52. MR3595350

[41] Liu, Q., Lee, J. and Jordan, M. (2016). A kernelized Stein discrepancy for goodness-of-fit tests. In *International Conference on Machine Learning* 276–284.

[42] Mackey, L. and Gorham, J. (2016). Multivariate Stein factors for a class of strongly log-concave distributions. *Electron. Commun. Probab.* **21** 56. MR3548768

[43] Mackey, L., Jordan, M.I., Chen, R.Y., Farrell, B. and Tropp, J.A. (2014). Matrix concentration inequalities via the method of exchangeable pairs. *Ann. Probab.* **42** 906–945. MR3189061

[44] Menz, G. and Otto, F. (2013). Uniform logarithmic Sobolev inequalities for conservative spin systems with super-quadratic single-site potential. *Ann. Probab.* **41** 2182–2224. MR3098070

[45] Miclo, L. (2008). Quand est-ce que des bornes de Hardy permettent de calculer une constante de Poincaré exacte sur la droite? *Ann. Fac. Sci. Toulouse Math.* (6) **17** 121–192. MR2464097

[46] Muckenhoupt, B. (1972). Hardy's inequality with weights. *Studia Math.* **44** 31–38. Collection of articles honoring the completion by Antoni Zygmund of 50 years of scientific activity, I. MR0311856

[47] Nourdin, I. and Viens, F.G. (2009). Density formula and concentration inequalities with Malliavin calculus. *Electron. J. Probab.* **14** 2287–2309. MR2556018

[48] Paulin, D., Mackey, L. and Tropp, J.A. (2016). Efron–Stein inequalities for random matrices. *Ann. Probab.* **44** 3431–3473. MR3551202

[49] Saumard, A. Supplement to "Weighted Poincaré inequalities, concentration inequalities and tail bounds related to Stein kernels in dimension one." DOI:10.3150/19-BEJ1117SUPP.

[50] Saumard, A. and Wellner, J.A. (2018). Efron's monotonicity property for measures on $\mathbb{R}^2$. *J. Multivariate Anal.* **166** 212–224. MR3799644

[51] Saumard, A. and Wellner, J.A. (2018). On the isoperimetric constant, covariance inequalities and L_p-Poincaré inequalities in dimension one. *Bernoulli*, To appear. Available at arXiv:1711.00668.

[52] Stein, C. (1972). A bound for the error in the normal approximation to the distribution of a sum of dependent random variables. In *Proceedings of the Sixth Berkeley Symposium on Mathematical Statistics and Probability* (*Univ. California, Berkeley, Calif.*, 1970/1971), *Vol. II: Probability theory* 583–602. Berkeley, CA: Univ. California Press. MR0402873

[53] Stein, C. (1986). *Approximate Computation of Expectations*. *Institute of Mathematical Statistics Lecture Notes – Monograph Series* **7**. Hayward, CA: IMS. MR0882007

[54] Treilhard, J. and Mansouri, A.-R. (2015). Concentration inequalities via Malliavin calculus with applications. *Electron*. *Commun*. *Probab*. **20** 36. MR3352331

[55] Viens, F.G. (2009). Stein's lemma, Malliavin calculus, and tail bounds, with application to polymer fluctuation exponent. *Stochastic Process*. *Appl*. **119** 3671–3698. MR2568291

Bernoulli Forthcoming Papers

THAI, M.-N.
Birth and death process in mean field type interaction

HAUSER, R.A. and MATZINGER, H.
Microscopic path structure of optimally aligned random sequences

CHENG, D., CAMMAROTA, V., FANTAYE, Y., MARINUCCI, D. and SCHWARTZMAN, A.
Multiple testing of local maxima for detection of peaks on the (celestial) sphere

BERNHARD, H. and DAS, B.
Heavy-tailed random walks, buffered queues and hidden large deviations

HO, N., NGUYEN, X. and RITOV, Y.
Robust estimation of mixing measures in finite mixture models

CÉNAC, P., DE LOYNES, B., OFFRET, Y. and ROUSSELLE, A.
Recurrence of multidimensional persistent random walks. Fourier and series criteria

FAN, J.Y., HAMZA, K., JAGERS, P. and KLEBANER, F.
Convergence of the age structure of general schemes of population processes

GÖTZE, F. and SAMBALE, H.
Second order concentration via logarithmic Sobolev inequalities

CASTILLO, I. and SZABÓ, B.
Spike and slab empirical Bayes sparse credible sets

GEISS, C., LABART, C. and LUOTO, A.
Random walk approximation of BSDEs with Hölder continuous terminal condition

HIRSCH, C. and MÖNCH, C.
Distances and large deviations in the spatial preferential attachment model

BELITSER, E. and NURUSHEV, N.
Needles and straw in a haystack: Robust confidence for possibly sparse sequences

BONNET, G. and CHENAVIER, N.
The maximal degree in a Poisson–Delaunay graph

MAZUR, S., OTRYAKHIN, D. and PODOLSKIJ, M.
Estimation of the linear fractional stable motion

BOGERD, K., CASTRO, R.M. and VAN DER HOFSTAD, R.
Cliques in rank-1 random graphs: The role of inhomogeneity

GAYNANOVA, I.
Prediction and estimation consistency of sparse multi-class penalized optimal scoring

ZWINGMANN, T. and HOLZMANN, H.
Weak convergence of quantile and expectile processes under general assumptions

Continues

Bernoulli Forthcoming Papers—*Continued*

GIORDANO, L.M., JOLIS, M. and QUER-SARDANYONS, L.
SPDEs with fractional noise in space: Continuity in law with respect to the Hurst index

DING, X.
High dimensional deformed rectangular matrices with applications in matrix denoising

SHI, C. and TANG, B.
Construction results for strong orthogonal arrays of strength three

MA, Z. and LI, X.
Subspace perspective on canonical correlation analysis: Dimension reduction and minimax rates

FOKIANOS, K., STØVE, B., TJØSTHEIM, D. and DOUKHAN, P.
Multivariate count autoregression

SHEN, Y. and WANG, Y.
Operator-scaling Gaussian random fields via aggregation

DÖRING, L. and WEISSMANN, P.
Stable processes conditioned to hit an interval continuously from the outside

MUKHERJEE, S.
Degeneracy in sparse ERGMs with functions of degrees as sufficient statistics

DAOUIA, A., GIRARD, S. and STUPFLER, G.
Tail expectile process and risk assessment

CONTI, P.L., MARELLA, D., MECATTI, F. and ANDREIS, F.
A unified principled framework for resampling based on pseudo-populations: Asymptotic theory

MARIUCCI, E., RAY, K. and SZABÓ, B.
A Bayesian nonparametric approach to log-concave density estimation

BEUTNER, E., BORDES, L. and DOYEN, L.
Consistent semiparametric estimators for recurrent event times models with application to virtual age models

PRIVAULT, N. and SERAFIN, G.
Normal approximation for sums of weighted U-statistics – application to Kolmogorov bounds in random subgraph counting

YANO, K. and KATO, K.
On frequentist coverage errors of Bayesian credible sets in moderately high dimensions

ALETTI, G., CRIMALDI, I. and GHIGLIETTI, A.
Interacting reinforced stochastic processes: Statistical inference based on the weighted empirical means

Continues

Bernoulli Forthcoming Papers—*Continued*

SCHNURR, A.
The fourth characteristic of a semimartingale

LIU, Y. and PAGÈS, G.
Characterization of probability distribution convergence in Wasserstein distance by L^p- quantization error function

LIANG, M. SCHILLING, R.L. and WANG, J.
A unified approach to coupling SDEs driven by Levy noise and some applications

MINSKER, S. and WEI, X.
Robust Modifications of U-statistics and applications to covariance estimation problems

ISSOGLIO, E. and RUSSO, F.
A Feynman-Kac result via Markov BSDEs with generalised drivers

LEI, J.
Convergence and concentration of empirical measures under Wasserstein distance in unbounded functional spaces

MARCHAND, D.C. and MANOLESCU, I.
Influence of the seed in affine preferential attachment trees

DAVIS, R.A., NIELSEN, M.S. and ROHDE, V.
Stochastic differential equations with a fractionally filtered delay: A semimartingale model for long-range dependent processes

NOURDIN, I., PECCATI, G. and SEURET, S.
Sojourn time dimensions of fractional Brownian motion

GAO, C.
Robust regression via mutivariate regression depth

KLUSOWSKI, J. M. and WU, Y.
Estimating the number of connected components in a graph via subgraph sampling

SCHWEINBERGER, M.
Consistent structure estimation of exponential-family random graph models with block structure

JIANG, B., LENG, C. and CHEN, Z.
Dynamic linear discriminant analysis in high dimensional space

GANTERT, N., HEYDENREICH, M. and HIRSCHER, T.
Strictly weak consensus in the uniform compass model on Z