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CONTENTS

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CONTENTS

(continued)

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Aims and Scope
BERNOULLI is the journal of the Bernoulli Society for Mathematical Statistics and Probability, issued four times per year. The journal provides a comprehensive account of important developments in the fields of statistics and probability, offering an international forum for both theoretical and applied work.

Bernoulli Society for Mathematical Statistics and Probability
The Bernoulli Society was founded in 1973. It is an autonomous Association of the International Statistical Institute, ISI. According to its statutes, the object of the Bernoulli Society is the advancement, through international contacts, of the sciences of probability (including the theory of stochastic processes) and mathematical statistics and of their applications to all those aspects of human endeavour which are directed towards the increase of natural knowledge and the welfare of mankind.

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The Society holds a World Congress every four years; more frequent meetings, coordinated by the Society's standing committees and often organised in collaboration with other organisations, are the European Meeting of Statisticians, the Conference on Stochastic Processes and their Applications, the CLAPEM meeting (Latin-American Congress on Probability and Mathematical Statistics), the European Young Statisticians Meeting, and various meetings on special topics – in the physical sciences in particular. The Society, as an association of the ISI, also collaborates with other ISI associations in the organization of the biennial ISI World Statistics Congresses (formerly ISI Sessions).

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 Further, the Society has a twelve member Council and a number of standing committees to carry out the tasks outlined above. Final authority is the general assembly of members of the Society, meeting at least biennially at the ISI World Statistics Congresses.

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Sojourn time dimensions of fractional Brownian motion

IVAN NOURDIN[1,*], GIOVANNI PECCATI[1,**] and STÉPHANE SEURET[2]

[1]*Université du Luxembourg, Unité de Recherche en Mathématiques, Maison du Nombre, 6 avenue de la Fonte, L-4364 Esch-sur-Alzette, Grand Duché du Luxembourg.*
E-mail: [*]*ivan.nourdin@uni.lu;* [**]*giovanni.peccati@uni.lu*
[2]*Université Paris-Est, LAMA (UMR 8050), UPEMLV, UPEC, CNRS, F-94010, Créteil, France.*
E-mail: seuret@u-pec.fr

We describe the size of the sets of sojourn times $E_\gamma = \{t \geq 0 : |B_t| \leq t^\gamma\}$ associated with a fractional Brownian motion B in terms of various large scale dimensions.

Keywords: fractional Brownian motion; logarithmic density; macroscopic Hausdorff dimension; pixel density; sojourn time

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Estimating the number of connected components in a graph via subgraph sampling

JASON M. KLUSOWSKI[1] and YIHONG WU[2]

[1]*Department of Statistics, Rutgers University – New Brunswick, 110 Frelinghuysen Road, Piscataway, NJ, 8019, USA. E-mail: jason.klusowski@rutgers.edu*
[2]*Department of Statistics and Data Science, Yale University, 24 Hillhouse Avenue, New Haven, CT 06511, USA. E-mail: yihong.wu@yale.edu*

Learning properties of large graphs from samples has been an important problem in statistical network analysis since the early work of Goodman (*Ann. Math. Stat.* **20** (1949) 572–579) and Frank (*Scand. J. Stat.* **5** (1978) 177–188). We revisit a problem formulated by Frank (*Scand. J. Stat.* **5** (1978) 177–188) of estimating the number of connected components in a large graph based on the subgraph sampling model, in which we randomly sample a subset of the vertices and observe the induced subgraph. The key question is whether accurate estimation is achievable in the *sublinear* regime where only a vanishing fraction of the vertices are sampled. We show that it is impossible if the parent graph is allowed to contain high-degree vertices or long induced cycles. For the class of chordal graphs, where induced cycles of length four or above are forbidden, we characterize the optimal sample complexity within constant factors and construct linear-time estimators that provably achieve these bounds. This significantly expands the scope of previous results which have focused on unbiased estimators and special classes of graphs such as forests or cliques.

Both the construction and the analysis of the proposed methodology rely on combinatorial properties of chordal graphs and identities of induced subgraph counts. They, in turn, also play a key role in proving minimax lower bounds based on construction of random instances of graphs with matching structures of small subgraphs.

Keywords: chordal graph; minimax lower bound; network sampling; perfect elimination ordering; subgraph counts; subgraph sampling model

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Influence of the seed in *affine* preferential attachment trees

DAVID CORLIN MARCHAND[1,2,*] and IOAN MANOLESCU[2,**]

[1]*Laboratoire de Mathématiques d'Orsay, Univ. Paris-Sud, CNRS, Université Paris-Saclay, 91405 Orsay, France.*
[2]*Université de Fribourg, 23 Chemin du Musée, CH-1700 Fribourg, Switzerland.*
E-mail: [*]*david.marchand@u-psud.fr;* [**]*ioan.manolescu@unifr.ch*

We study randomly growing trees governed by the affine preferential attachment rule. Starting with a seed tree S, vertices are attached one by one, each linked by an edge to a random vertex of the current tree, chosen with a probability proportional to an affine function of its degree. This yields a one-parameter family of preferential attachment trees $(T_n^S)_{n\geq|S|}$, of which the linear model is a particular case. Depending on the choice of the parameter, the power-laws governing the degrees in T_n^S have different exponents.

We study the problem of the asymptotic influence of the seed S on the law of T_n^S. We show that, for any two distinct seeds S and S', the laws of T_n^S and $T_n^{S'}$ remain at uniformly positive total-variation distance as n increases.

This is a continuation of Curien et al. (*J. Éc. Polytech. Math.* **2** (2015) 1–34), which in turn was inspired by a conjecture of Bubeck et al. (*IEEE Trans. Netw. Sci. Eng.* **2** (2015) 30–39). The technique developed here is more robust than previous ones and is likely to help in the study of more general attachment mechanisms.

Keywords: Barábasi–Albert trees; preferential attachment; seed recognition

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On the eigenproblem for Gaussian bridges

PAVEL CHIGANSKY[1], MARINA KLEPTSYNA[2,*] and
DMYTRO MARUSHKEVYCH[2,**]

[1]*Department of Statistics, The Hebrew University, Mount Scopus, Jerusalem 91905, Israel.*
E-mail: pchiga@mscc.huji.ac.il
[2]*Laboratoire de Statistique et Processus, Le Mans Université, France.*
*E-mail: *marina.kleptsyna@univ-lemans.fr; **dmytro.marushkevych.etu@univ-lemans.fr*

Spectral decomposition of the covariance operator is one of the main building blocks in the theory and applications of Gaussian processes. Unfortunately, it is notoriously hard to derive in a closed form. In this paper, we consider the eigenproblem for Gaussian bridges. Given a *base* process, its bridge is obtained by conditioning the trajectories to start and terminate at the given points. What can be said about the spectrum of a bridge, given the spectrum of its base process? We show how this question can be answered asymptotically for a family of processes, including the fractional Brownian motion.

Keywords: eigenproblem; fractional Brownian motion; Gaussian processes; Karhunen–Loève expansion

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Local differential privacy: Elbow effect in optimal density estimation and adaptation over Besov ellipsoids

CRISTINA BUTUCEA[1,*], AMANDINE DUBOIS[2,‡], MARTIN KROLL[1,†] and ADRIEN SAUMARD[2,§]

[1]*CREST, ENSAE, Institut Polytechnique de Paris, 5 avenue Henry Le Chatelier, F-91120 Palaiseau, France. E-mail:* [*]*cristina.butucea@ensae.fr;* [†]*martin.kroll@ensae.fr*
[2]*CREST-ENSAI, Campus de Ker-Lann, Rue Blaise Pascal, BP 37203 – 35172 Bruz Cedex, France. E-mail:* [‡]*amandine.dubois@ensai.fr;* [§]*adrien.saumard@ensai.fr*

We address the problem of non-parametric density estimation under the additional constraint that only privatised data are allowed to be published and available for inference. For this purpose, we adopt a recent generalisation of classical minimax theory to the framework of local α-differential privacy and provide a lower bound on the rate of convergence over Besov spaces $\mathcal{B}^s_{pq}$ under mean integrated $\mathbb{L}^r$-risk. This lower bound is deteriorated compared to the standard setup without privacy, and reveals a twofold elbow effect. In order to fulfill the privacy requirement, we suggest adding suitably scaled Laplace noise to empirical wavelet coefficients. Upper bounds within (at most) a logarithmic factor are derived under the assumption that α stays bounded as n increases: A linear but non-adaptive wavelet estimator is shown to attain the lower bound whenever $p \geq r$ but provides a slower rate of convergence otherwise. An adaptive non-linear wavelet estimator with appropriately chosen smoothing parameters and thresholding is shown to attain the lower bound within a logarithmic factor for all cases.

Keywords: adaptive estimation; Besov classes of functions; density estimation; local differential privacy; lower bounds; minimax rates; wavelet thresholding

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A fast algorithm with minimax optimal guarantees for topic models with an unknown number of topics

XIN BING[1,*], FLORENTINA BUNEA[1,†] and MARTEN WEGKAMP[1,2]

[1]*Department of Statistics and Data Science, Cornell University, Ithaca, NY, USA.*
E-mail: [*]*xb43@cornell.edu;* [†]*fb238@cornell.edu*
[2]*Department of Mathematics, Cornell University, Ithaca, NY, USA. E-mail: mhw73@cornell.edu*

Topic models have become popular for the analysis of data that consists in a collection of n independent multinomial observations, with parameters $N_i \in \mathbb{N}$ and $\Pi_i \in [0, 1]^p$ for $i = 1, \dots, n$. The model links all cell probabilities, collected in a $p \times n$ matrix Π, via the assumption that Π can be factorized as the product of two nonnegative matrices $A \in [0, 1]^{p\times K}$ and $W \in [0, 1]^{K\times n}$. Topic models have been originally developed in text mining, when one browses through n documents, based on a dictionary of p words, and covering K topics. In this terminology, the matrix A is called the word-topic matrix, and is the main target of estimation. It can be viewed as a matrix of conditional probabilities, and it is uniquely defined, under appropriate separability assumptions, discussed in detail in this work. Notably, the unique A is required to satisfy what is commonly known as the anchor word assumption, under which A has an unknown number of rows respectively proportional to the canonical basis vectors in $\mathbb{R}^K$. The indices of such rows are referred to as anchor words. Recent computationally feasible algorithms, with theoretical guarantees, utilize constructively this assumption by linking the estimation of the set of anchor words with that of estimating the K vertices of a simplex. This crucial step in the estimation of A requires K to be known, and cannot be easily extended to the more realistic set-up when K is unknown.

This work takes a different view on anchor word estimation, and on the estimation of A. We propose a new method of estimation in topic models, that is not a variation on the existing simplex finding algorithms, and that estimates K from the observed data. We derive new finite sample minimax lower bounds for the estimation of A, as well as new upper bounds for our proposed estimator. We describe the scenarios where our estimator is minimax adaptive. Our finite sample analysis is valid for any n, N_i, p and K, and both p and K are allowed to increase with n, a situation not handled well by previous analyses.

We complement our theoretical results with a detailed simulation study. We illustrate that the new algorithm is faster and more accurate than the current ones, although we start out with a computational and theoretical disadvantage of not knowing the correct number of topics K, while we provide the competing methods with the correct value in our simulations.

Keywords: adaptive estimation; anchor words; high dimensional estimation; identification; latent model; minimax estimation; nonnegative matrix factorization; overlapping clustering; separability; topic model

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Optimal functional supervised classification with separation condition

SÉBASTIEN GADAT[1], SÉBASTIEN GERCHINOVITZ[2] and CLÉMENT MARTEAU[3]

[1]*Toulouse School of Economics, Institut Universitaire de France.*
E-mail: sebastien.gadat@math.univ-toulouse.fr
[2]*Institut Mathématiques de Toulouse & IRT Saint-Exupéry, 3 rue Tarfaya, 31405 Toulouse, France.*
E-mail: sebastien.gerchinovitz@math.univ-toulouse.fr
[3]*Univ Lyon, Université Claude Bernard Lyon 1, CNRS UMR 5208, Institut Camille Jordan F-69622 Villeurbanne, France. E-mail: marteau@math.univ-lyon1.fr*

We consider the binary supervised classification problem with the Gaussian functional model introduced in (*Math. Methods Statist.* **22** (2013) 213–225). Taking advantage of the Gaussian structure, we design a natural plug-in classifier and derive a family of upper bounds on its worst-case excess risk over Sobolev spaces. These bounds are parametrized by a separation distance quantifying the difficulty of the problem, and are proved to be optimal (up to logarithmic factors) through matching minimax lower bounds. Using the recent works of (In *Advances in Neural Information Processing Systems* (2014) 3437–3445 Curran Associates) and (*Ann. Statist.* **44** (2016) 982–1009), we also derive a logarithmic lower bound showing that the popular k-nearest neighbors classifier is far from optimality in this specific functional setting.

Keywords: functional data; supervised classification

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Kernel and wavelet density estimators on manifolds and more general metric spaces

GALATIA CLEANTHOUS[1], ATHANASIOS G. GEORGIADIS[2], GERARD KERKYACHARIAN[3], PENCHO PETRUSHEV[4] and DOMINIQUE PICARD[5]

[1]*Department of Mathematics, Statistics and Physics, Newcastle University, Newcastle Upon Tyne, NE1 7RU, UK. E-mail: galatia.cleanthous@newcastle.ac.uk*
[2]*Department of Mathematics and Statistics, University of Cyprus, 1678 Nicosia, Cyprus. E-mail: gathana@ucy.ac.cy*
[3]*Université de Paris, LPSM, CREST, F-75013 Paris, France. E-mail: kerk@math.univ-paris-diderot.fr*
[4]*Department of Mathematics, University of South Carolina. E-mail: pencho@math.sc.edu*
[5]*Université de Paris, LPSM, F-75013 Paris, France. E-mail: picard@math.univ-paris-diderot.fr*

We consider the problem of estimating the density of observations taking values in classical or nonclassical spaces such as manifolds and more general metric spaces. Our setting is quite general but also sufficiently rich in allowing the development of smooth functional calculus with well localized spectral kernels, Besov regularity spaces, and wavelet type systems. Kernel and both linear and nonlinear wavelet density estimators are introduced and studied. Convergence rates for these estimators are established and discussed.

Keywords: adaptive density estimators; Ahlfors regularity; Besov space; heat kernel; non-parametric estimators; sample kernel density estimators; wavelet density estimators

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Logarithmic Sobolev inequalities for finite spin systems and applications

HOLGER SAMBALE* and ARTHUR SINULIS†

Faculty of Mathematics, Bielefeld University, Postfach 100 131, 33501 Bielefeld, Germany.
E-mail: *hsambale@math.uni-bielefeld.de; †asinulis@math.uni-bielefeld.de

We derive sufficient conditions for a probability measure on a finite product space (a *spin system*) to satisfy a (modified) logarithmic Sobolev inequality. We establish these conditions for various examples, such as the (vertex-weighted) exponential random graph model, the random coloring and the hard-core model with fugacity.

This leads to two separate branches of applications. The first branch is given by mixing time estimates of the Glauber dynamics. The proofs do not rely on coupling arguments, but instead use functional inequalities. As a byproduct, this also yields exponential decay of the relative entropy along the Glauber semigroup. Secondly, we investigate the concentration of measure phenomenon (particularly of higher order) for these spin systems. We show the effect of better concentration properties by centering not around the mean, but around a stochastic term in the exponential random graph model. From there, one can deduce a central limit theorem for the number of triangles from the CLT of the edge count. In the Erdős–Rényi model the first-order approximation leads to a quantification and a proof of a central limit theorem for subgraph counts.

Keywords: central limit theorem; concentration of measure; exponential random graph model; finite product spaces; logarithmic Sobolev inequality; mixing time; spin systems

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Functional weak limit theorem for a local empirical process of non-stationary time series and its application

ULRIKE MAYER[1,*], HENRYK ZÄHLE[1,†] and ZHOU ZHOU[2]

[1]*Department of Mathematics, Saarland University, 66123 Saarbrücken, Germany.*
E-mail: [*]*mayer@math.uni-sb.de;* [†]*zaehle@math.uni-sb.de*
[2]*Department of Statistics, 100 St. George St., Toronto, ON, M5S 3G3 Canada.*
E-mail: zhou@utstat.toronto.edu

We derive a functional weak limit theorem for a local empirical process of a wide class of piece-wise locally stationary (PLS) time series. The latter result is applied to derive the asymptotics of weighted empirical quantiles and weighted V-statistics of non-stationary time series. The class of admissible underlying time series is illustrated by means of PLS linear processes and PLS ARCH processes.

Keywords: local empirical process; piece-wise locally stationary time series; weak convergence; weighted empirical quantile; weighted V-statistic

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On the best constant in the martingale version of Fefferman's inequality

ADAM OSĘKOWSKI

Faculty of Mathematics, Informatics and Mechanics, University of Warsaw, Banacha 2, 02-097 Warsaw, Poland. E-mail: ados@mimuw.edu.pl

Let $X = (X_t)_{t\geq 0} \in H^1$ and $Y = (Y_t)_{t\geq 0} \in \mathrm{BMO}$ be arbitrary continuous-path martingales. The paper contains the proof of the inequality

$$\mathbb{E}\int_0^\infty \left|d\langle X, Y\rangle_t\right| \leq \sqrt{2}\|X\|_{H^1}\|Y\|_{\mathrm{BMO}_2},$$

and the constant $\sqrt{2}$ is shown to be the best possible. The proof rests on the construction of a certain special function, enjoying appropriate size and concavity conditions.

Keywords: BMO; best constants; duality; martingale; maximal

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Busemann functions and semi-infinite O'Connell–Yor polymers

TOM ALBERTS[1,*], FIRAS RASSOUL-AGHA[1,†] and MACKENZIE SIMPER[2]

[1]*Department of Mathematics, University of Utah, 155 S 1400 E Rm 233, SLC, UT 84112, USA.*
E-mail: [*]*alberts@math.utah.edu;* [†]*firas@math.utah.edu*
[2]*Department of Mathematics, Stanford University, Building 380, Stanford, CA 94305, USA.*
E-mail: msimper@stanford.edu

We prove that given any fixed asymptotic velocity, the finite length O'Connell–Yor polymer has an infinite length limit satisfying the law of large numbers with this velocity. By a Markovian property of the quenched polymer this reduces to showing the existence of *Busemann functions*: almost sure limits of ratios of random point-to-point partition functions. The key ingredients are the Burke property of the O'Connell–Yor polymer and a comparison lemma for the ratios of partition functions. We also show the existence of infinite length limits in the Brownian last passage percolation model.

Keywords: Busemann functions; O'Connell–Yor polymer; semi-infinite quenched path measures

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On sampling from a log-concave density using kinetic Langevin diffusions

ARNAK S. DALALYAN[1] and LIONEL RIOU-DURAND[2]

[1]*CREST, ENSAE, Institut Polytechnique de Paris, 5 avenue Le Chatelier, 91129 Palaiseau, France. E-mail: arnak.dalalyan@ensae.fr*
[2]*Department of Statistics, University of Warwick, Coventry, CV4 7AL, United Kingdom. E-mail: lionel.riou-durand@warwick.ac.uk*

Langevin diffusion processes and their discretizations are often used for sampling from a target density. The most convenient framework for assessing the quality of such a sampling scheme corresponds to smooth and strongly log-concave densities defined on $\mathbb{R}^p$. The present work focuses on this framework and studies the behavior of the Monte Carlo algorithm based on discretizations of the kinetic Langevin diffusion. We first prove the geometric mixing property of the kinetic Langevin diffusion with a mixing rate that is optimal in terms of its dependence on the condition number. We then use this result for obtaining improved guarantees of sampling using the kinetic Langevin Monte Carlo method, when the quality of sampling is measured by the Wasserstein distance. We also consider the situation where the Hessian of the log-density of the target distribution is Lipschitz-continuous. In this case, we introduce a new discretization of the kinetic Langevin diffusion and prove that this leads to a substantial improvement of the upper bound on the sampling error measured in Wasserstein distance.

Keywords: Hamiltonian Monte Carlo; kinetic Langevin; Langevin algorithm; Markov Chain Monte Carlo; mixing rate

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On estimation of nonsmooth functionals of sparse normal means

O. COLLIER[1,3,*], L. COMMINGES[2,3,†] and A.B. TSYBAKOV[3,‡]

[1]*MODAL'X, Université Paris-Nanterre. E-mail:* [*]*olivier.collier@parisnanterre.fr*
[2]*CEREMADE, Université Paris-Dauphine PSL. E-mail:* [†]*comminges@ceremade.dauphine.fr*
[3]*CREST, ENSAE. E-mail:* [‡]*alexandre.tsybakov@ensae.fr*

We study the problem of estimation of $N_\gamma(\theta) = \sum_{i=1}^d |\theta_i|^\gamma$ for $\gamma > 0$ and of the ℓ_γ-norm of θ for $\gamma \geq 1$ based on the observations $y_i = \theta_i + \varepsilon\xi_i$, $i = 1, \dots, d$, where $\theta = (\theta_1, \dots, \theta_d)$ are unknown parameters, $\varepsilon > 0$ is known, and ξ_i are i.i.d. standard normal random variables. We find the non-asymptotic minimax rate for estimation of these functionals on the class of s-sparse vectors θ and we propose estimators achieving this rate.

Keywords: functional estimation; nonsmooth functional; norm estimation; polynomial approximation; sparsity

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Matching strings in encoded sequences

ADRIANA COUTINHO[1], RODRIGO LAMBERT[2] and JÉRÔME ROUSSEAU[3,4]

[1]*Instituto de Matemática e Estatística, Universidade de São Paulo,*
Rua do Matão, 1010, CEP 05508-090, São Paulo-SP, Brazil. E-mail: adcoutinho.s@gmail.com
[2]*Faculdade de Matemática, Universidade Federal de Uberlândia,*
Av. João Naves de Avila, 2121, CEP 38408-100, Uberlândia-MG, Brazil. E-mail: rodrigolambert@ufu.br;
url: sites.google.com/view/rodrigolambert
[3]*Departamento de Matemática, Faculdade de Ciências da Universidade do Porto,*
Rua do Campo Alegre, 687, 4169-007 Porto, Portugal. E-mail: jerome.rousseau@ufba.br;
url: www.sd.mat.ufba.br/~jerome.rousseau
[4]*Departamento de Matemática, Universidade Federal da Bahia, Av. Ademar de Barros s/n, 40170-110 Salvador, Brazil*

We investigate the length of the longest common substring for encoded sequences and its asymptotic behaviour. The main result is a strong law of large numbers for a re-scaled version of this quantity, which presents an explicit relation with the Rényi entropy of the source. We apply this result to the zero-inflated contamination model and the stochastic scrabble. In the case of dynamical systems, this problem is equivalent to the shortest distance between two observed orbits and its limiting relationship with the correlation dimension of the pushforward measure. An extension to the shortest distance between orbits for random dynamical systems is also provided.

Keywords: coding; correlation dimension; random dynamical systems; Rényi entropy; shortest distance; string matching

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First-order covariance inequalities via Stein's method

MARIE ERNST[1], GESINE REINERT[2] and YVIK SWAN[3]

[1]*Université de Liège, Liège, Belgium. E-mail: m.ernst@uliege.be*
[2]*University of Oxford, Oxford, UK. E-mail: reinert@stats.ox.ac.uk*
[3]*Université libre de Bruxelles, Bruxelles, Belgium. E-mail: yvswan@ulb.ac.be*

We propose probabilistic representations for inverse Stein operators (i.e., solutions to Stein equations) under general conditions; in particular, we deduce new simple expressions for the Stein kernel. These representations allow to deduce uniform and nonuniform Stein factors (i.e., bounds on solutions to Stein equations) and lead to new covariance identities expressing the covariance between arbitrary functionals of an arbitrary univariate target in terms of a weighted covariance of the derivatives of the functionals. Our weights are explicit, easily computable in most cases and expressed in terms of objects familiar within the context of Stein's method. Applications of the Cauchy–Schwarz inequality to these weighted covariance identities lead to sharp upper and lower covariance bounds and, in particular, weighted Poincaré inequalities. Many examples are given and, in particular, classical variance bounds due to Klaassen, Brascamp and Lieb or Otto and Menz are corollaries. Connections with more recent literature are also detailed.

Keywords: covariance identities; Cramér–Rao inequality; Stein equation; Stein kernel; variance bounds

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Perfect sampling for Gibbs point processes using partial rejection sampling

SARAT B. MOKA* and DIRK P. KROESE†

School of Mathematics and Physics, The University of Queensland, Brisbane.
E-mail: *s.babumoka@uq.edu.au; †kroese@maths.uq.edu.au

We present a perfect sampling algorithm for Gibbs point processes, based on the partial rejection sampling of Guo, Jerrum and Liu (In *STOC'17 – Proceedings of the 49th Annual ACM SIGACT Symposium on Theory of Computing* (2017) 342–355 ACM). Our particular focus is on pairwise interaction processes, penetrable spheres mixture models and area-interaction processes, with a finite interaction range. For an interaction range $2r$ of the target process, the proposed algorithm can generate a perfect sample with $O(\log(1/r))$ expected running time complexity, provided that the intensity of the points is not too high and $\Theta(1/r^d)$ parallel processor units are available.

Keywords: area-interaction process; hard-core process; pairwise interaction process; partial-rejection sampling; penetrable spheres mixture model; perfect sampling; Strauss process

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Noncommutative Lebesgue decomposition and contiguity with applications in quantum statistics

AKIO FUJIWARA[1] and KOICHI YAMAGATA[2]

[1]*Department of Mathematics, Osaka University, Toyonaka, Osaka 560-0043, Japan.*
E-mail: fujiwara@math.sci.osaka-u.ac.jp
[2]*Graduate School of Informatics and Engineering, The University of Electro-Communications, Chofu, Tokyo 182-8585, Japan. E-mail: koichi.yamagata@uec.ac.jp*

We herein develop a theory of contiguity in the quantum domain based upon a novel quantum analogue of the Lebesgue decomposition. The theory thus formulated is pertinent to the weak quantum local asymptotic normality introduced in the previous paper [Yamagata, Fujiwara, and Gill, *Ann. Statist.* **41** (2013) 2197–2217], yielding substantial enlargement of the scope of quantum statistics.

Keywords: contiguity; Lebesgue decomposition; likelihood ratio; local asymptotic normality; quantum statistics

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Directional differentiability for supremum-type functionals: Statistical applications

JAVIER CÁRCAMO*, ANTONIO CUEVAS† and LUIS-ALBERTO RODRÍGUEZ‡

Departamento de Matemáticas, Universidad Autónoma de Madrid, 28049 Madrid, Spain.
E-mail: *javier.carcamo@uam.es; †antonio.cuevas@uam.es; ‡luisalberto.rodriguez@uam.es

We show that various functionals related to the supremum of a real function defined on an arbitrary set or a measure space are Hadamard directionally differentiable. We specifically consider the supremum norm, the supremum, the infimum, and the amplitude of a function. The (usually non-linear) derivatives of these maps adopt simple expressions under suitable assumptions on the underlying space. As an application, we improve and extend to the multidimensional case the results in Raghavachari (*Ann. Statist.* **1** (1973) 67–73) regarding the limiting distributions of Kolmogorov–Smirnov type statistics under the alternative hypothesis. Similar results are obtained for analogous statistics associated with copulas. We additionally solve an open problem about the Berk–Jones statistic proposed by Jager and Wellner (In *A Festschrift for Herman Rubin* (2004) 319–331 IMS). Finally, the asymptotic distribution of maximum mean discrepancies over Donsker classes of functions is derived.

Keywords: Berk–Jones statistic; copulas; Delta method; empirical processes; Hadamard directional derivative; Kolmogorov distance; Kolmogorov–Smirnov statistic; Kuiper statistic; maximum mean discrepancy

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Scaling limits for super-replication with transient price impact

PETER BANK[1] and YAN DOLINSKY[2]

[1]*Institut für Mathematik, Technische Universität Berlin, Straße des 17. Juni 136, 10623 Berlin, Germany. E-mail:* bank@math.tu-berlin.de
[2]*Department of Statistics, Hebrew University, Mount Scopus, Jerusalem 91905, Israel. E-mail:* yan.dolinsky@mail.huji.ac.il

We prove a scaling limit theorem for the super-replication cost of options in a Cox–Ross–Rubinstein binomial model with transient price impact. The correct scaling turns out to keep the market depth parameter constant while resilience over fixed periods of time grows in inverse proportion with the duration between trading times. For vanilla options, the scaling limit is found to coincide with the one obtained by PDE-methods in (*Math. Finance* **22** (2012) 250–276) for models with purely temporary price impact. These models are a special case of our framework and so our probabilistic scaling limit argument allows one to expand the scope of the scaling limit result to path-dependent options.

Keywords: binomial model; liquidity; scaling limit; super-replication; transient price impact

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Exponential integrability and exit times of diffusions on sub-Riemannian and metric measure spaces

ANTON THALMAIER* and JAMES THOMPSON†

Department of Mathematics, Campus Belval – Maison du Nombre, Université du Luxembourg, 6, avenue de la Fonte, L-4364 Esch-sur-Alzette, Grand Duchy of Luxembourg.
E-mail: *anton.thalmaier@uni.lu; †james.thompson@uni.lu

In this article, we derive moment estimates, exponential integrability, concentration inequalities and exit times estimates for canonical diffusions firstly on sub-Riemannian limits of Riemannian foliations and secondly in the nonsmooth setting of $\mathrm{RCD}^*(K,N)$ spaces. In each case, the necessary ingredients are Itô's formula and a comparison theorem for the Laplacian, for which we refer to the recent literature. As an application, we derive pointwise Carmona-type estimates on eigenfunctions of Schrödinger operators.

Keywords: concentration inequality; eigenfunction; exit time; exponential integrability; Kato; RCD space; Schrödinger; sub-Riemannian

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On Sobolev tests of uniformity on the circle with an extension to the sphere

SREENIVASA RAO JAMMALAMADAKA[1], SIMOS MEINTANIS[2] and THOMAS VERDEBOUT[3]

[1]*Department of Statistics and Applied Probability, University of California Santa Barbara, Santa Barbara, CA 93106-3110, USA. E-mail: sreenivas@ucsb.edu*
[2]*Department of Economics, National and Kapodistrian University of Athens,1, Sofokleous and Aristidou street, 10559 Athens, Greece, and Unit for Business Mathematics and Informatics, North-West University, Vaal Triangle Campus, PO Box 1174, Vanderbijlpark 1900, South Africa. E-mail: simosmei@econ.uoa.gr*
[3]*ECARES and Département de Mathématique, Université libre de Bruxelles, Avenue F.D. Roosevelt, 50, ECARES, CP114/04, B-1050, Brussels, Belgium. E-mail: tverdebo@ulb.ac.be*

Circular and spherical data arise in many applications, especially in biology, Earth sciences and astronomy. In dealing with such data, one of the preliminary steps before any further inference, is to test if such data is isotropic, that is, uniformly distributed around the circle or the sphere. In view of its importance, there is a considerable literature on the topic. In the present work, we provide new tests of uniformity on the circle based on original asymptotic results. Our tests are motivated by the shape of locally and asymptotically maximin tests of uniformity against generalized von Mises distributions. We show that they are uniformly consistent. Empirical power comparisons with several competing procedures are presented via simulations. The new tests detect particularly well multimodal alternatives such as mixtures of von Mises distributions. A practically-oriented combination of the new tests with already existing Sobolev tests is proposed. An extension to testing uniformity on the sphere, along with some simulations, is included. The procedures are illustrated on a real dataset.

Keywords: directional data; goodness-of-fit tests; Sobolev tests; testing uniformity on spheres

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Concentration of the spectral norm of Erdős–Rényi random graphs

GÁBOR LUGOSI[1,2,3], SHAHAR MENDELSON[4] and NIKITA ZHIVOTOVSKIY[5]

[1]*Department of Economics and Business, Pompeu Fabra University, Barcelona, Spain. E-mail: gabor.lugosi@upf.edu*
[2]*ICREA, Pg. Lluís Companys 23, 08010 Barcelona, Spain*
[3]*Barcelona Graduate School of Economics, Ramon Trias Fargas, 25-27, 08005 Barcelona, Spain*
[4]*The Australian National University, ACT 2601, Canberra, Australia. E-mail: shahar.mendelson@anu.edu.au*
[5]*Google Research, Brain Team, Brandschenkestrasse 110, 8002 Zürich, Switzerland E-mail: nikita.zhivotovskiy@phystech.edu*

We present results on the concentration properties of the spectral norm $\|A_p\|$ of the adjacency matrix A_p of an Erdős–Rényi random graph $G(n, p)$. First, we consider the Erdős–Rényi random graph *process* and prove that $\|A_p\|$ is *uniformly* concentrated over the range $p \in [C \log n/n, 1]$. The analysis is based on delocalization arguments, uniform laws of large numbers, together with the entropy method to prove concentration inequalities. As an application of our techniques, we prove sharp sub-Gaussian moment inequalities for $\|A_p\|$ for all $p \in [c \log^3 n/n, 1]$ that improve the general bounds of Alon, Krivelevich, and Vu (*Israel J. Math.* **131** (2002) 259–267) and some of the more recent results of Erdős et al. (*Ann. Probab.* **41** (2013) 2279–2375). Both results are consistent with the asymptotic result of Füredi and Komlós (*Combinatorica* **1** (1981) 233–241) that holds for fixed p as $n \to \infty$.

Keywords: concentration; empirical processes; random graphs

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Convergence of persistence diagrams for topological crackle

TAKASHI OWADA[1] and OMER BOBROWSKI[2]

[1]*Department of Statistics, Purdue University, West Lafayette, IN 47907, USA. E-mail: owada@purdue.edu*
[2]*Department of Electrical Engineering, Technion – Israel Institute of Technology, Haifa, 32000, Israel. E-mail: omer@ee.technion.ac.il*

In this paper, we study the persistent homology associated with topological crackle generated by distributions with an unbounded support. Persistent homology is a topological and algebraic structure that tracks the creation and destruction of topological cycles (generalizations of loops or holes) in different dimensions. Topological crackle is a term that refers to topological cycles generated by random points far away from the bulk of other points, when the support is unbounded. We establish weak convergence results for persistence diagrams – a point process representation for persistent homology, where each topological cycle is represented by its (*birth*, *death*) coordinates. In this work, we treat persistence diagrams as random closed sets, so that the resulting weak convergence is defined in terms of the Fell topology. Using this framework, we show that the limiting persistence diagrams can be divided into two parts. The first part is a deterministic limit containing a densely-growing number of persistence pairs with a shorter lifespan. The second part is a two-dimensional Poisson process, representing persistence pairs with a longer lifespan.

Keywords: extreme value theory; fell topology; persistent homology; point process; topological crackle

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Weighted Lépingle inequality

PAVEL ZORIN-KRANICH

Mathematical Institute, University of Bonn, Bonn, Germany. E-mail: pzorin@uni-bonn.de

We prove an estimate for weighted pth moments of the pathwise r-variation of a martingale in terms of the A_p characteristic of the weight. The novelty of the proof is that we avoid real interpolation techniques.

Keywords: p-variation; Burkholder–Davis–Gundy inequality; Muckenhoupt A_p weight

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A refined Cramér-type moderate deviation for sums of local statistics

XIAO FANG[1,*], LI LUO[1,†] and QI-MAN SHAO[1,2]

[1]*Department of Statistics, The Chinese University of Hong Kong, Hong Kong.*
E-mail: [*]*xfang@sta.cuhk.edu.hk;* [†]*luoli@link.cuhk.edu.hk*
[2]*Department of Statistics and Data Science, Southern University of Science and Technology, China.*
E-mail: qmshao@sta.cuhk.edu.hk

We prove a refined Cramér-type moderate deviation result by taking into account of the skewness in normal approximation for sums of local statistics of independent random variables. We apply the main result to k-runs, U-statistics and subgraph counts in the Erdős–Rényi random graph. To prove our main result, we develop exponential concentration inequalities and higher-order tail probability expansions via Stein's method.

Keywords: Cramér-type moderate deviation; Erdős–Rényi random graph; k-runs; local dependence; skewness correction; Stein's method; U-statistic

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Bayesian linear regression for multivariate responses under group sparsity

BO NING[1], SEONGHYUN JEONG[2] and SUBHASHIS GHOSAL[3]

[1]*Department of Statistics and Data Science, Yale University, 24 Hillhouse Avenue, New Haven, CT 06511, USA. E-mail: bo.ning@yale.edu*
[2]*Booth School of Business, University of Chicago, 5807 S Woodlawn Ave, Chicago, IL 60637, USA. E-mail: seonghyun.jeong@chicagobooth.edu*
[3]*Department of Statistics, North Carolina State University, 4276 SAS Hall, 2311 Stinson Drive, Raleigh, NC 27695, USA. E-mail: sghosal@ncsu.edu*

We study frequentist properties of a Bayesian high-dimensional multivariate linear regression model with correlated responses. The predictors are separated into many groups and the group structure is pre-determined. Two features of the model are unique: (i) group sparsity is imposed on the predictors; (ii) the covariance matrix is unknown and its dimensions can also be high. We choose a product of independent spike-and-slab priors on the regression coefficients and a new prior on the covariance matrix based on its eigendecomposition. Each spike-and-slab prior is a mixture of a point mass at zero and a multivariate density involving the $\ell_{2,1}$-norm. We first obtain the posterior contraction rate, the bounds on the effective dimension of the model with high posterior probabilities. We then show that the multivariate regression coefficients can be recovered under certain compatibility conditions. Finally, we quantify the uncertainty for the regression coefficients with frequentist validity through a Bernstein–von Mises type theorem. The result leads to selection consistency for the Bayesian method. We derive the posterior contraction rate using the general theory by constructing a suitable test from the first principle using moment bounds for certain likelihood ratios. This leads to posterior concentration around the truth with respect to the average Rényi divergence of order $1/2$. This technique of obtaining the required tests for posterior contraction rate could be useful in many other problems.

Keywords: Bayesian variable selection; covariance matrix; group sparsity; multivariate linear regression; posterior contraction rate; Rényi divergence; spike-and-slab prior

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Frequency domain theory for functional time series: Variance decomposition and an invariance principle

PIOTR KOKOSZKA[1] and NEDA MOHAMMADI JOUZDANI[2]

[1]*Department of Statistics, Colorado State University, Fort Collins, CO 80523, USA.*
E-mail: *Piotr.Kokoszka@colostate.edu*
[2]*Department of Mathematics, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland. E-mail:* *neda.mohammadijouzdani@epfl.ch*

This paper is concerned with frequency domain theory for functional time series, which are temporally dependent sequences of functions in a Hilbert space. We consider a variance decomposition, which is more suitable for such a data structure than the variance decomposition based on the Karhunen–Loéve expansion. The decomposition we study uses eigenvalues of spectral density operators, which are functional analogs of the spectral density of a stationary scalar time series. We propose estimators of the variance components and derive convergence rates for their mean square error as well as their asymptotic normality. The latter is derived from a frequency domain invariance principle for the estimators of the spectral density operators. This principle is established for a broad class of linear time series models. It is a main contribution of the paper.

Keywords: functional data; invariance principle; spectral analysis; time series; variance decomposition

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Local law and Tracy–Widom limit for sparse stochastic block models

JONG YUN HWANG*, JI OON LEE† and WOOSEOK YANG‡

Department of Mathematical Sciences, KAIST, Daejeon, 34141, Korea.
E-mail: *jyh9006@kaist.ac.kr; †jioon.lee@kaist.edu; ‡ws.yang@kaist.ac.kr

We consider the spectral properties of sparse stochastic block models, where N vertices are partitioned into K balanced communities. Under an assumption that the intra-community probability and inter-community probability are of similar order, we prove a local semicircle law up to the spectral edges, with an explicit formula on the deterministic shift of the spectral edge. We also prove that the fluctuation of the extremal eigenvalues is given by the GOE Tracy–Widom law after rescaling and centering the entries of sparse stochastic block models. Applying the result to sparse stochastic block models, we rigorously prove that there is a large gap between the outliers and the spectral edge without centering.

Keywords: local law; random matrices; stochastic block model; Tracy–Widom distribution

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Stratonovich type integration with respect to fractional Brownian motion with Hurst parameter less than 1/2

JORGE A. LEÓN

Depto. de Control Automático, Cinvestav-IPN, Apartado Postal 14-740, 07000 Ciudad de México, Mexico. E-mail: jleon@ctrl.cinvestav.mx

Let B^H be a fractional Brownian motion with Hurst parameter $H \in (0, 1/2)$ and $p : \mathbb{R} \to \mathbb{R}$ a polynomial function. The main purpose of this paper is to introduce a Stratonovich type stochastic integral with respect to B^H, whose domain includes the process $p(B^H)$. That is, an integral that allows us to integrate $p(B^H)$ with respect to B^H, which does not happen with the symmetric integral given by Russo and Vallois (*Probab. Theory Related Fields* **97** (1993) 403–421) in general. Towards this end, we combine the approaches utilized by León and Nualart (*Stochastic Process. Appl.* **115** (2005) 481–492), and Russo and Vallois (*Probab. Theory Related Fields* **97** (1993) 403–421), whose aims are to extend the domain of the divergence operator for Gaussian processes and to define some stochastic integrals, respectively. Then, we study the relation between this Stratonovich integral and the extension of the divergence operator (see León and Nualart (*Stochastic Process. Appl.* **115** (2005) 481–492)), an Itô formula and the existence of a unique solution of some Stratonovich stochastic differential equations. These last results have been analyzed by Alòs, León and Nualart (*Taiwanese J. Math.* **5** (2001) 609–632), where the Hurst paramert H belongs to the interval $(1/4, 1/2)$.

Keywords: derivative and divergence operators in the Malliavin calculus sense; Doss transformation; fractional integrals and derivatives; Itô formula; Malliavin calculus for fBm; Stratonovich stochastic differential equation; symmetric stochastic integration

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HÜTTER, J.C., MAO, C., RIGOLLET, P. and ROBEVA, E.
Estimation of Monge matrices

TODINO, A.P.
Nodal lengths in shrinking domains for random eigenfunctions on S^2

LOPUSANSCHI, O. and SIMON, D.
Area anomaly in the rough path Brownian scaling limit of hidden Markov walks

VERSHYNIN, R.
Concentration inequalities for random tensors

BARKHAGEN, M., CHAU, N.H., MOULINES, E., RASONYI, M., SABANIS, S. and ZHANG, Y.
On stochastic gradient Langevin dynamics with dependent data streams in the log-concave case

TRUQUET, L.
A perturbation analysis of Markov chains models with time-varying parameters

PAINDAVEINE, D., REMY, J. and VERDEBOUT, T.
Sign tests for weak principal directions

CAN, S.U., EINMAHL, J.H.L. and LAEVEN, R.J.A.
Goodness-of-fit testing for copulas: A distribution-free approach

GRZYWNY, T. and SZCZYPKOWSKI, K.
Lévy processes: Concentration function and heat kernel bounds

MARYNYCH, A.V. and MOLCHANOV, I.
Sieving random iterative function systems

KATO, S. and MCCULLAGH, P.
Some properties of a Cauchy family on the sphere derived from the Möbius transformations

BELLEC, P.C.
Adaptive confidence sets in shape restricted regression

CARPENTIER, A. and VERZELEN, N.
Optimal sparsity testing in linear regression model

TRUQUET, L.
Coupling and perturbation techniques for categorical time series

ENIKEEVA, F., MUNK, A., POHLMANN, M. and WERNER, F.
Bump detection in the presence of dependency: Does it ease or does it load?

CHEN, Q. and GOLDSCHMIDT, C.
Parking on a random rooted plane tree