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The integral of the product of a power and Bessel's K_ν function

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Abstract. Withers and Nadarajah [*Braz. J. Probab. Stat.* **28** (2014) 140–149] gave new expressions for hypergeometric functions when two arguments differ by an integer. Here, we give new expressions for $\int_{x_0}^{x} x^\mu K_\nu(x)\,dx$ when $\mu \pm \nu$ is an integer, where $K_\nu(\cdot)$ denotes the modified Bessel function of order ν. Each new expression is a finite sum of terms involving only the gamma function and the modified Bessel function.

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Withers, C. S. and Nadarajah, S. (2014). Hypergeometric functions where two arguments differ by an integer. *Brazilian Journal of Probability and Statistics* **28**, 140–149. MR3165433

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Bayesian skew-probit regression for binary response data

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Abstract. Since many authors have emphasized the need of asymmetric link functions to fit binary regression models, we propose in this work two new skew-probit link functions for the binary response variables. These link functions will be named power probit and reciprocal power probit due to the relation between them including the probit link as a special case. Also, the probit regressions are special cases of the models considered in this work. A Bayesian inference approach using MCMC is developed for real data suggesting that the link functions proposed here are more appropriate than other link functions used in the literature. In addition, simulation study show that the use of probit model will lead to biased estimate of the regression coefficient.

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On Bayesian D-optimal design criteria and the General Equivalence Theorem in joint generalized linear models for the mean and dispersion

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Abstract. The joint modeling of mean and dispersion has been used to model many problems in statistics, especially in industry, where not only the mean of response, but also the dispersion depends on the covariates. In scientific research, one of the crucial points is the experimental design, which when properly implemented, will create a reliable structure, essential to improve the statistical inference and for the development of the next phases of the experimental process. The theory of optimal design of experiments is a powerful and flexible approach to generate efficient experimental designs. In the context of optimal designs, the General Equivalence Theorem plays a fundamental role, because it permits to check if a design found is optimal. In this article, we investigated the validity of the General Equivalence Theorem for obtaining Bayesian D and D_S optimal designs in joint generalized linear models for the mean and dispersion.

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Key words and phrases. General Equivalence Theorem, D and D_S-optimal designs, Bayesian designs, joint modeling of mean and dispersion.

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Order statistics and exceedances for some models of INID random variables

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Abstract. Order statistics and exceedances for some general models of independent but not necessarily identically distributed (INID) random variables are considered. The distributions of order statistics from INID sample are described in terms of symmetric functions. Some exceedance models based on order statistics from INID random variables are considered, the limit distributions of exceedance statistics are obtained. For the model of INID random variables referred as F^{α}-scheme introduced by Nevzorov (*Zapiski Nauchnykh Seminarov LOMI* **142** (1985) 109–118) the limiting distribution of exceedance statistic has been derived. This distribution is expressed in terms of permutations with inversions, Gaussian Hypergeometric function and incomplete beta functions.

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A Mermin–Wagner theorem on Lorentzian triangulations with quantum spins

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Abstract. We consider infinite random causal Lorentzian triangulations emerging in quantum gravity for critical values of parameters. With each vertex of the triangulation we associate a Hilbert space representing a bosonic particle moving in accordance with the standard laws of Quantum Mechanics. The particles interact via two-body potentials decaying with the graph distance. A Mermin–Wagner type theorem is proven for infinite-volume reduced density matrices related to solutions to DLR equations in the Feynman–Kac (FK) representation.

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Key words and phrases. Causal Lorentzian triangulations, size-biased critical Galton–Watson branching process, quantum bosonic system with continuous spins, compact Lie group action, the Feynman–Kac representation, FK-DLR equations, reduced density matrix, invariance.

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Change in the mean in the domain of attraction of the normal law via Darling–Erdős theorems

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Abstract. This paper studies the problem of testing the null assumption of no-change in the mean of chronologically ordered independent observations on a random variable X versus the at most one change in the mean alternative hypothesis. The approach taken is via a Darling–Erdős type self-normalized maximal deviation between sample means before and sample means after possible times of a change in the expected values of the observations of a random sample. Asymptotically, the thus formulated maximal deviations are shown to have a standard Gumbel distribution under the null assumption of no change in the mean. A first such result is proved under the condition that $EX^2 \log\log(|X|+1) < \infty$, while in the case of a second one, X is assumed to be in a specific class of the domain of attraction of the normal law, possibly with infinite variance.

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Key words and phrases. Change in the mean, domain of attraction of the normal law, Darling–Erdős theorems, Gumbel distribution, weighted metrics, Brownian bridge.

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Estimation with improved efficiency in semi-parametric linear longitudinal models

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Abstract. In this article, we revisit the semi-parametric linear models with auto-correlated errors, where the means of the repeated responses of an individual consist of a specified regression function in time dependent covariates as well as a time dependent nonparametric function. The estimation of the regression parameters involved in the specified regression function is of main interest, and most of the existing studies estimate these parameters by using the so-called semi-parametric generalized estimating equations (SGEEs) approach. We offer two main contributions. First, we demonstrate that the existing SGEEs are partly standardized. Second, as opposed to this partly standardized SGEE (PSSGEE) approach, we suggest a fully standardized semi-parametric generalized quasi-likelihood (FSSGQL) approach that provides more efficient regression estimates. This efficiency gain by the FSSGQL approach over the PSSGEE approach is also demonstrated through an empirical study.

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Domains of operator semi-attraction of probability measures on Banach spaces

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Abstract. The paper deals with operator (semi-) stability and domains of operator (semi-) attraction of probability measures on infinite dimensional Banach spaces: characterizations of operator (semi-) stability and of domains of (normal) operator (semi-) attraction are given; it is shown that the set of operator stable probability measures is a closed subset under weak topology; the domain of operator semi-attraction of a given stable probability measure coincides with its domain of operator attraction; and a probability measure is (semi-) stable iff its finite-dimensional projections are (semi-) stable.

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