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Truncated sequential Monte Carlo test with exact power

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Abstract. Monte Carlo hypothesis testing is extensively used for statistical inference. Surprisingly, despite the many theoretical advances in the field, statistical power performance of Monte Carlo tests remains an open question. Because the last assertion may sound questionable for some, the first goal in this paper is to show that the power performance of truncated Monte Carlo tests is still an unsolved question. The second goal here is to present a solution for this issue, that is, we introduce a truncated sequential Monte Carlo procedure with statistical power arbitrarily close to the power of the theoretical exact test. The most significant contribution of this work is the validity of our method for the general case of any test statistic.

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Key words and phrases. Exact hypothesis testing, power loss upper bounds, p -value, sequential Monte Carlo design.

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Bayesian analysis of multiple-inflation Poisson models and its application to infection data

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Abstract. In this article we propose a multiple-inflation Poisson regression to model count response data containing excessive frequencies at more than one non-negative integer values. To handle multiple excessive count responses, we generalize the zero-inflated Poisson regression by replacing its binary regression with the multinomial regression, while Su et al. [*Statist. Sinica* **23** (2013) 1071–1090] proposed a multiple-inflation Poisson model for consecutive count responses with excessive frequencies. We give several properties of our proposed model, and do statistical inference under the fully Bayesian framework. We perform simulation studies and also analyze the data related to the number of infections collected in five major hospitals in Turkey, using our methodology.

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Key words and phrases. Bayesian generalized linear model, EM algorithm, excessive count response, likelihood function, zero-inflated poisson model.

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Poisson–Lindley INAR(1) model with applications

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Abstract. The paper focuses on a new stationary integer-valued autoregressive model of first order with Poisson–Lindley marginal distribution. Several statistical properties of the model are established, like spectral density function, multi-step ahead conditional measures, stationarity, ergodicity and irreducibility. We consider several methods for estimating the unknown parameters of the model and investigate properties of the estimators. The performances of these estimators are compared via simulation. The model is motivated by some applications to two real count time series data.

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Key words and phrases. Compound Poisson distribution, count data, first order autoregressive process, parametric estimation, thinning.

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The exponentiated logarithmic generated family of distributions and the evaluation of the confidence intervals by percentile bootstrap

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Abstract. We study some mathematical properties of a new generator of continuous distributions with three additional parameters, called the exponentiated logarithmic generated family, to extend the normal, Weibull, gamma and Gumbel distributions, among other well-known models. Some special models are discussed. Many properties of this family are studied, some inference procedures developed and a simulation study performed to verify the adequacy of the estimators of the model parameters. We prove empirically the potentiality of the new family by means of two real data sets. The simulation study for different samples sizes assesses the performance of the maximum likelihood estimates obtained by the Swarm Optimization method. We also evaluate the performance of single and dual bootstrap methods in constructing interval estimates for the parameters. Because of the intensive simulations, we use parallel computing on a supercomputer.

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Key words and phrases. Bootstrap, generalized distribution, lifetime, logarithmic distribution, mixture.

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On the number of unobserved and observed categories when sampling from a multivariate hypergeometric population

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Abstract. Consider taking a random sample of size n from a finite population that consists of N categories with M_i copies in the i th category for $i = 1, \dots, N$. Each observed unit in a sample is presumed to have a probability $1 - p$ ($0 < p < 1$) of getting lost from the sample. Let S denote the number of categories not observed in the sample and S_j denote the number of categories where j samples are observed for $j = 1, \dots, n$. In this paper, the probability distribution and factorial moments of S and S_j are studied. A matrix inversion algorithm is used in order to facilitate numerical computations in obtaining the probabilities and factorial moments. A couple of examples of the problem considered in this paper may include a filing or storage process, where objects are randomly assigned to files or storage bins, and from time to time, objects may be missing or have disappeared, species as categories in a capture-recapture problem, or DNA sequence study.

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Key words and phrases. Factorial moments, matrix inversion method, multivariate hypergeometric distribution, multinomial distribution, occupancy problem, Sterling's number of the second kind.

Mixture models applied to heterogeneous populations

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Abstract. Mixture models provide a flexible representation of heterogeneity in a finite number of latent classes. From the Bayesian point of view, Markov Chain Monte Carlo methods provide a way to draw inferences from these models. In particular, when the number of subpopulations is considered unknown, more sophisticated methods are required to perform Bayesian analysis. The Reversible Jump Markov Chain Monte Carlo is an alternative method for computing the posterior distribution by simulation in this case. Some problems associated with the Bayesian analysis of these class of models are frequent, such as the so-called “label-switching” problem. However, as the level of heterogeneity in the population increases, these problems are expected to become less frequent and the model’s performance to improve. Thus, the aim of this work is to evaluate the normal mixture model fit using simulated data under different settings of heterogeneity and prior information about the mixture proportions. A simulation study is also presented to evaluate the model’s performance considering the number of components known and estimating it. Finally, the model is applied to a censored real dataset containing antibody levels of Cytomegalovirus in individuals.

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Key words and phrases. Identifiability, sensitivity analysis, subpopulations, frequentist properties, NHANES.

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Identifiability of structural characteristics: How relevant is it for the Bayesian approach?

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Abstract. The role of identification in the Bayesian approach is still debatable. Since Lindley [Bayesian Statistics. A Review (1971) Philadelphia], most Bayesian statisticians pretend that unidentifiability causes no real difficulty in their approach. Recently, Wechsler, Izbicki and Esteves [*Amer. Statist.* **67** (2013) 90–93] provide a simple example illustrating this perspective. By critically reading Wechsler, Izbicki and Esteves [*Amer. Statist.* **67** (2013) 90–93], we intend to show that the Bayesian approach is far from being free of the identification problems, provided that the interest is focused on the interpretation of the parameters. It is written using a rather ancient style, the so-called Platonic dialogues. In modern times, there are beautiful examples of that, particularly in Foundations of Mathematics, where debatable subjects are discussed: let us refer Heyting [Intuitionism. An Introduction (1971) North-Holland Publishing Company], where the debate between a formalist and an intuitionist is presented as a dialogue; or Lakatos [Proofs and Refutations. The Logic of Mathematical Discovery (1976) Cambridge University Press], where the relationship between proofs and conjectures is magnificently illustrated. We hope that this style will help to understand why identifiability really matters in the Bayesian approach.

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Nonlinear filtering with correlated Lévy noise characterized by copulas

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Abstract. The objective in stochastic filtering is to reconstruct the information about an unobserved (random) process, called the signal process, given the current available observations of a certain noisy transformation of that process.

Usually X and Y are modeled by stochastic differential equations driven by a Brownian motion or a jump (or Lévy) process. We are interested in the situation where both the state process X and the observation process Y are perturbed by coupled Lévy processes. More precisely, $L = (L_1, L_2)$ is a 2-dimensional Lévy process in which the structure of dependence is described by a Lévy copula. We derive the associated Zakai equation for the density process and establish sufficient conditions depending on the copula and L for the solvability of the corresponding solution to the Zakai equation. In particular, we give conditions of existence and uniqueness of the density process, if one is interested to estimate quantities like $\mathbb{P}(X(t) > a)$, where a is a threshold.

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Some unified results on stochastic properties of residual lifetimes at random times

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Abstract. The residual life of a random variable X at random time Θ is defined to be a random variable X_{Θ} having the same distribution as the conditional distribution of $X - \Theta$ given $X > \Theta$ (denoted by $X_{\Theta} = (X - \Theta | X > \Theta)$). Let (X, Θ_1) and (Y, Θ_2) be two pairs of jointly distributed random variables, where X and Θ_1 (and, Y and Θ_2) are not necessarily independent. In this paper, we compare random variables X_{Θ_1} and Y_{Θ_2} by providing sufficient conditions under which X_{Θ_1} and Y_{Θ_2} are stochastically ordered with respect to various stochastic orderings. These comparisons have been made with respect to hazard rate, likelihood ratio and mean residual life orders. We also study various ageing properties of random variable X_{Θ_1} . By considering this generalized model, we generalize and unify several results in the literature on stochastic properties of residual lifetimes at random times. Some examples to illustrate the application of the results derived in the paper are also presented.

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Products of normal, beta and gamma random variables: Stein operators and distributional theory

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Abstract. In this paper, we extend Stein’s method to products of independent beta, gamma, generalised gamma and mean zero normal random variables. In particular, we obtain Stein operators for mixed products of these distributions, which include the classical beta, gamma and normal Stein operators as special cases. These operators lead us to closed-form expressions involving the Meijer G -function for the probability density function and characteristic function of the mixed product of independent beta, gamma and central normal random variables.

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Key words and phrases. Stein’s method, normal distribution, beta distribution, gamma distribution, generalised gamma distribution, products of random variables distribution, Meijer G -function.

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