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Bayesian robustness to outliers in linear regression and ratio estimation

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Abstract. Whole robustness is a nice property to have for statistical models. It implies that the impact of outliers gradually vanishes as they approach plus or minus infinity. So far, the Bayesian literature provides results that ensure whole robustness for the location-scale model. In this paper, we make two contributions. First, we generalise the results to attain whole robustness in simple linear regression through the origin, which is a necessary step towards results for general linear regression models. We allow the variance of the error term to depend on the explanatory variable. This flexibility leads to the second contribution: we provide a simple Bayesian approach to robustly estimate finite population means and ratios. The strategy to attain whole robustness is simple since it lies in replacing the traditional normal assumption on the error term by a super heavy-tailed distribution assumption. As a result, users can estimate the parameters as usual, using the posterior distribution.

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Key words and phrases. Built-in robustness, simple linear regression, ratio estimator, finite populations, population means, super heavy-tailed distributions.

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A brief review of optimal scaling of the main MCMC approaches and optimal scaling of additive TMCMC under non-regular cases

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Abstract. Transformation based Markov Chain Monte Carlo (TMCMC) was proposed by Dutta and Bhattacharya (*Statistical Methodology* **16** (2014) 100–116) as an efficient alternative to the Metropolis–Hastings algorithm, especially in high dimensions. The main advantage of this algorithm is that it simultaneously updates all components of a high dimensional parameter using appropriate move types defined by deterministic transformation of a single random variable. This results in reduction in time complexity at each step of the chain and enhances the acceptance rate.

In this paper, we first provide a brief review of the optimal scaling theory for various existing MCMC approaches, comparing and contrasting them with the corresponding TMCMC approaches.The optimal scaling of the simplest form of TMCMC, namely *additive TMCMC*, has been studied extensively for the Gaussian proposal density in Dey and Bhattacharya (2017a). Here, we discuss diffusion-based optimal scaling behavior of additive TMCMC for non-Gaussian proposal densities—in particular, uniform, Student's *t* and Cauchy proposals. Although we could not formally prove our diffusion result for the Cauchy proposal, simulation based results lead us to *conjecture* that at least the recipe for obtaining general optimal scaling and optimal acceptance rate holds for the Cauchy case as well. We also consider diffusion based optimal scaling of TMCMC when the target density is discontinuous. Such non-regular situations have been studied in the case of Random Walk Metropolis Hastings (RWMH) algorithm by Neal and Roberts (*Methodology and Computing in Applied Probability* **13** (2011) 583–601) using expected squared jumping distance (ESJD), but the diffusion theory based scaling has not been considered.

We compare our diffusion based optimally scaled TMCMC approach with the ESJD based optimally scaled RWM with simulation studies involving several target distributions and proposal distributions including the challenging Cauchy proposal case, showing that additive TMCMC outperforms RWMH in almost all cases considered.

Key words and phrases. Additive transformation, diffusion, Itô formula, optimal scaling, non-regular, transformation based Markov chain Monte Carlo.

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The coreset variational Bayes (CVB) algorithm for mixture analysis

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Abstract. The pressing need for improved methods for analysing and coping with big data has opened up a new area of research for statisticians. Image analysis is an area where there is typically a very large number of data points to be processed per image, and often multiple images are captured over time. These issues make it challenging to design methodology that is reliable and yet still efficient enough to be of practical use. One promising emerging approach for this problem is to reduce the amount of data that actually has to be processed by extracting what we call coresets from the full dataset; analysis is then based on the coreset rather than the whole dataset. Coresets are representative subsamples of data that are carefully selected via an adaptive sampling approach. We propose a new approach called coreset variational Bayes (CVB) for mixture modelling; this is an algorithm which can perform a variational Bayes analysis of a dataset based on just an extracted coreset of the data. We apply our algorithm to weed image analysis.

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Key words and phrases. Mixture modelling, coresets, variational Bayes, image analysis, Bayesian statistics.

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Modified information criterion for testing changes in skew normal model

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Abstract. In this paper, we study the change point problem for the skew normal distribution model from the view of model selection problem. The detection procedure based on the modified information criterion (MIC) for change problem is proposed. Such a procedure has advantage in detecting the changes in early and late stage of a data comparing to the one based on the traditional Schwarz information criterion which is well known as Bayesian information criterion (BIC) by considering the complexity of the models. Due to the difficulty in deriving the analytic asymptotic distribution of the test statistic based on the MIC procedure, the bootstrap simulation is provided to obtain the critical values at the different significance levels. Simulations are conducted to illustrate the comparisons of performance between MIC, BIC and likelihood ratio test (LRT). Such an approach is applied on two stock market data sets to indicate the detection procedure.

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Key words and phrases. Skew normal distribution, change points, model selection, Bayesian information criterion, modified information criterion, likelihood ratio test.

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Failure rate of Birnbaum–Saunders distributions: Shape, change-point, estimation and robustness

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Abstract. The Birnbaum–Saunders (BS) distribution has been largely studied and applied. A random variable with BS distribution is a transformation of another random variable with standard normal distribution. Generalized BS distributions are obtained when the normally distributed random variable is replaced by another symmetrically distributed random variable. This allows us to obtain a wide class of positively skewed models with lighter and heavier tails than the BS model. Its failure rate admits several shapes, including the unimodal case, with its change-point being able to be used for different purposes. For example, to establish the reduction in a dose, and then in the cost of the medical treatment. We analyze the failure rates of generalized BS distributions obtained by the logistic, normal and Student-t distributions, considering their shape and change-point, estimating them, evaluating their robustness, assessing their performance by simulations, and applying the results to real data from different areas.

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A new log-linear bimodal Birnbaum–Saunders regression model with application to survival data

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Abstract. The log-linear Birnbaum–Saunders model has been widely used in empirical applications. We introduce an extension of this model based on a recently proposed version of the Birnbaum–Saunders distribution which is more flexible than the standard Birnbaum–Saunders law since its density may assume both unimodal and bimodal shapes. We show how to perform point estimation, interval estimation and hypothesis testing inferences on the parameters that index the regression model we propose. We also present a number of diagnostic tools, such as residual analysis, local influence, generalized leverage, generalized Cook's distance and model misspecification tests. We investigate the usefulness of model selection criteria and the accuracy of prediction intervals for the proposed model. Results of Monte Carlo simulations are presented. Finally, we also present and discuss an empirical application.

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Necessary and sufficient conditions for the convergence of the consistent maximal displacement of the branching random walk

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Abstract. Consider a supercritical branching random walk on the real line. The consistent maximal displacement is the smallest of the distances between the trajectories followed by individuals at the nth generation and the boundary of the process. Fang and Zeitouni, and Faraud, Hu and Shi proved that under some integrability conditions, the consistent maximal displacement grows almost surely at rate $\lambda^* n^{1/3}$ for some explicit constant λ^*. We obtain here a necessary and sufficient condition for this asymptotic behaviour to hold.

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Hierarchical modelling of power law processes for the analysis of repairable systems with different truncation times: An empirical Bayes approach

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Abstract. In the data analysis from multiple repairable systems, it is usual to observe both different truncation times and heterogeneity among the systems. Among other reasons, the latter is caused by different manufacturing lines and maintenance teams of the systems. In this paper, a hierarchical model is proposed for the statistical analysis of multiple repairable systems under different truncation times. A reparameterization of the power law process is proposed in order to obtain a quasi-conjugate bayesian analysis. An empirical Bayes approach is used to estimate model hyperparameters. The uncertainty in the estimate of these quantities are corrected by using a parametric bootstrap approach. The results are illustrated in a real data set of failure times of power transformers from an electric company in Brazil.

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A temporal perspective on the rate of convergence in first-passage percolation under a moment condition

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Abstract. We study the rate of convergence in the celebrated Shape Theorem in first-passage percolation, obtaining the precise asymptotic rate of decay for the probability of linear order deviations under a moment condition. Our results are presented from a temporal perspective and complement previous work by the same author, in which the rate of convergence was studied from the standard spatial perspective.

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Influence measures for the Waring regression model

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Abstract. In this paper, we present a regression model where the response variable is a count data that follows a Waring distribution. The Waring regression model allows for analysis of phenomena where the Geometric regression model is inadequate, because the probability of success on each trial, p, is different for each individual and p has an associated distribution. Estimation is performed by maximum likelihood, through the maximization of the Q-function using EM algorithm. Diagnostic measures are calculated for this model. To illustrate the results, an application to real data is presented. Some specific details are given in the Appendix of the paper.

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