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A rank-based Cramér–von-Mises-type test for two samples

Jamye Curry[a], Xin Dang[b] and Hailin Sang[b]

[a]*Georgia Gwinnett College*
[b]*University of Mississippi*

Abstract. We study a rank based univariate two-sample distribution-free test. The test statistic is the difference between the average of between-group rank distances and the average of within-group rank distances. This test statistic is closely related to the two-sample Cramér–von Mises criterion. They are different empirical versions of a same quantity for testing the equality of two population distributions. Although they may be different for finite samples, they share the same expected value, variance and asymptotic properties. The advantage of the new rank based test over the classical one is its ease to generalize to the multivariate case. Rather than using the empirical process approach, we provide a different easier proof, bringing in a different perspective and insight. In particular, we apply the Hájek projection and orthogonal decomposition technique in deriving the asymptotics of the proposed rank based statistic. A numerical study compares power performance of the rank formulation test with other commonly-used nonparametric tests and recommendations on those tests are provided. Lastly, we propose a multivariate extension of the test based on the spatial rank.

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Key words and phrases. Cramér–von Mises criterion, Hájek projection, nonparametric test, rank, two-sample test.

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L-Logistic regression models: Prior sensitivity analysis, robustness to outliers and applications

Rosineide F. da Paz[a], **Narayanaswamy Balakrishnan**[b] **and Jorge Luis Bazán**[c]

[a]*Universidade Federal do Ceará*
[b]*McMaster University*
[c]*Uninversidade de São Paulo*

Abstract. Tadikamalla and Johnson [*Biometrika* **69** (1982) 461–465] developed the L_B distribution to variables with bounded support by considering a transformation of the standard Logistic distribution. In this manuscript, a convenient parametrization of this distribution is proposed in order to develop regression models. This distribution, referred to here as L-Logistic distribution, provides great flexibility and includes the uniform distribution as a particular case. Several properties of this distribution are studied, and a Bayesian approach is adopted for the parameter estimation. Simulation studies, considering prior sensitivity analysis, recovery of parameters and comparison of algorithms, and robustness to outliers are all discussed showing that the results are insensitive to the choice of priors, efficiency of the algorithm MCMC adopted, and robustness of the model when compared with the beta distribution. Applications to estimate the vulnerability to poverty and to explain the anxiety are performed. The results to applications show that the L-Logistic regression models provide a better fit than the corresponding beta regression models.

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Fractional backward stochastic variational inequalities with non-Lipschitz coefficient

Katarzyna Jańczak-Borkowska

University of Technology and Life Sciences

Abstract. We prove the existence and uniqueness of the solution of backward stochastic variational inequalities with respect to fractional Brownian motion and with non-Lipschitz coefficient. We assume that $H > 1/2$.

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Key words and phrases. Backward stochastic differential equation, fractional Brownian motion, backward stochastic variational inequalities, subdifferential operator.

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Spatially adaptive Bayesian image reconstruction through locally-modulated Markov random field models

Salem M. Al-Gezeri[a] **and Robert G. Aykroyd**[b]

[a]*University of Benghazi*
[b]*University of Leeds*

Abstract. The use of Markov random field (MRF) models has proven to be a fruitful approach in a wide range of image processing applications. It allows local texture information to be incorporated in a systematic and unified way and allows statistical inference theory to be applied giving rise to novel output summaries and enhanced image interpretation. A great advantage of such low-level approaches is that they lead to flexible models, which can be applied to a wide range of imaging problems without the need for significant modification.

This paper proposes and explores the use of conditional MRF models for situations where multiple images are to be processed simultaneously, or where only a single image is to be reconstructed and a sequential approach is taken. Although the *coupling* of image intensity values is a special case of our approach, the main extension over previous proposals is to allow the direct coupling of other properties, such as smoothness or texture. This is achieved using a local modulating function which adjusts the influence of global smoothing without the need for a fully inhomogeneous prior model. Several modulating functions are considered and a detailed simulation study, motivated by remote sensing applications in archaeological geophysics, of conditional reconstruction is presented. The results demonstrate that a substantial improvement in the quality of the image reconstruction, in terms of errors and residuals, can be achieved using this approach, especially at locations with rapid changes in the underlying intensity.

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Density for solutions to stochastic differential equations with unbounded drift

Christian Olivera[a] and Ciprian Tudor[b]

[a]*Universidade Estadual de Campinas*
[b]*Université de Lille 1 and ISMMA*

Abstract. Via a special transform and by using the techniques of the Malliavin calculus, we analyze the density of the solution to a stochastic differential equation with unbounded drift.

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A Jackson network under general regime

Yair Y. Shaki

The Jerusalem College of Technology

Abstract. We consider a Jackson network in a general heavy traffic diffusion regime with the α-*parametrization*. We also assume that each customer may abandon the system while waiting. We show that in this regime the queue-length process converges to a multi-dimensional regulated Ornstein–Uhlenbeck process.

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Fake uniformity in a shape inversion formula

Christian Rau

Shantou University

Abstract. We revisit a shape inversion formula derived by Panaretos in the context of a particle density estimation problem with unknown rotation of the particle. A distribution is presented which imitates, or "fakes", the uniformity or Haar distribution that is part of that formula.

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Stochastic monotonicity from an Eulerian viewpoint

Davide Gabrielli and Ida Germana Minelli

University of L'Aquila

Abstract. Stochastic monotonicity is a well-known partial order relation between probability measures defined on the same partially ordered set. Strassen theorem establishes equivalence between stochastic monotonicity and the existence of a coupling compatible with respect to the partial order. We consider the case of a countable set and introduce the class of *finitely decomposable flows* on a directed acyclic graph associated to the partial order. We show that a probability measure stochastically dominates another probability measure if and only if there exists a finitely decomposable flow having divergence given by the difference of the two measures. We illustrate the result with some examples.

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Unions of random walk and percolation on infinite graphs

Kazuki Okamura
Shinshu University

Abstract. We consider a random object that is associated with both random walks and random media, specifically, the superposition of a configuration of subcritical Bernoulli percolation on an infinite connected graph and the trace of the simple random walk on the same graph. We investigate asymptotics for the number of vertices of the enlargement of the trace of the walk until a fixed time, when the time tends to infinity. This process is more highly self-interacting than the range of random walk, which yields difficulties. We show a law of large numbers on vertex-transitive transient graphs. We compare the process on a vertex-transitive graph with the process on a finitely modified graph of the original vertex-transitive graph and show their behaviors are similar. We show that the process fluctuates almost surely on a certain non-vertex-transitive graph. On the two-dimensional integer lattice, by investigating the size of the boundary of the trace, we give an estimate for variances of the process implying a law of large numbers. We give an example of a graph with unbounded degrees on which the process behaves in a singular manner. As by-products, some results for the range and the boundary, which will be of independent interest, are obtained.

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Estimation of parameters in the DDRCINAR(p) model

Xiufang Liu and Dehui Wang

Jilin University

Abstract. This paper discusses a pth-order dependence-driven random coefficient integer-valued autoregressive time series model (DDRCINAR(p)). Stationarity and ergodicity properties are proved. Conditional least squares, weighted least squares and maximum quasi-likelihood are used to estimate the model parameters. Asymptotic properties of the estimators are presented. The performances of these estimators are investigated and compared via simulations. In certain regions of the parameter space, simulative analysis shows that maximum quasi-likelihood estimators perform better than the estimators of conditional least squares and weighted least squares in terms of the proportion of within-Ω estimates. At last, the model is applied to two real data sets.

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Key words and phrases. Conditional least squares, maximum quasi-likelihood, DDRCINAR(p) model, weighted conditional least squares, asymptotic distribution.

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A note on monotonicity of spatial epidemic models

Achillefs Tzioufas

Universidade de São Paulo

Abstract. The epidemic process on a graph is considered for which infectious contacts occur at rate which depends on whether a susceptible is infected for the first time or not. We show that the Vasershtein coupling extends if and only if secondary infections occur at rate which is greater than that of initial ones. Nonetheless we show that, with respect to the probability of occurrence of an infinite epidemic, the said proviso may be dropped regarding the totally asymmetric process in one dimension, thus settling in the affirmative this special case of the conjecture for arbitrary graphs due to [*Ann. Appl. Probab.* **13** (2003) 669–690].

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