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Copula estimation through wavelets

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Abstract. Recently some nonparametric estimation procedures have been proposed using kernels and wavelets to estimate the copula function. In this context, knowing that a copula function can be expanded in a wavelet basis, we propose a new nonparametric copula estimation procedure through wavelets for independent data and times series under an α-mixing condition. The main feature of this estimator is that we make no assumptions on the data distribution and there is no need to use ARMA–GARCH modelling before estimating the copula. Convergence rates for the estimator were computed, showing the estimator consistency. Some simulation studies are presented, as well as analysis of real data sets.

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Improved U-tests for variance components in one-way random effects models

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Abstract. Based on a decomposition of a U-statistic, Nobre, Singer and Silvapulle (In *Beyond Parametrics in Interdisciplinary Research, Festschrift to P.K. Sen* (2008) 197–210 Institute of Mathematical Statistics) proposed a test for the hypothesis that the within-treatment variance component in a one-way random effects model is null, specially useful when very mild assumptions are imposed on the underlying distributions. We consider a bootstrap version of that U-test and evaluate its performance via simulation studies in different scenarios. The bootstrap U-test has better statistical properties than the original test even in small samples. Furthermore, it is easy to implement and has a low computational cost. We consider two examples with unbalanced small sample datasets, for illustrative purposes.

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Moments of truncated scale mixtures of skew-normal distributions

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Abstract. In this work, we consider the problem of finding the moments of a doubly truncated member of the class of scale mixtures of skew-normal (TSMSN) distributions. We obtain a general result and then use it to derive the moments in the case of doubly truncated versions of skew-normal, skew-t, skew-slash and skew-contaminated normal distributions. Many properties of the TSMSN family are studied, inference procedures are developed and a simulation study is performed to assess the procedures. Two applications are also provided, one of them in the context of censored regression models and another in the field of actuarial sciences.

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Key words and phrases. Kurtosis, moments, truncated distributions, scale mixtures of skew-normal distributions, skewness.

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On a bimodal Birnbaum–Saunders distribution with applications to lifetime data

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Abstract. The Birnbaum–Saunders distribution is a flexible and useful model which has been used in several fields. In this paper, a new bimodal version of this distribution based on the alpha-skew-normal distribution is established. We discuss some of its mathematical and inferential properties. We consider likelihood-based methods to estimate the model parameters. We carry out a Monte Carlo simulation study to evaluate the performance of the maximum likelihood estimators. For illustrative purposes, three real data sets are analyzed. The results indicated that the proposed model outperformed some existing models in the literature, in special, a recent bimodal extension of the Birnbaum–Saunders distribution.

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Key words and phrases. Birnbaum–Saunders distribution, alpha-skew-normal distribution, bimodality, maximum likelihood estimation, Monte Carlo simulation.

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Calibration procedures for linear regression models with multiplicative distortion measurement errors

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Abstract. This paper considers linear regression models when neither the response variable nor the covariates can be directly observed, but are measured with multiplicative distortion measurement errors. To eliminate the effect caused by the distortion, we propose two calibration procedures: the conditional absolute mean calibration and the conditional variance calibration. Both calibration procedures avoid using the nonzero expectation conditions imposed on the variables in the literature. Utilizing these calibrated variables, the least squares estimators are obtained, associated with their asymptotic results. The asymptotic normal confidence intervals and empirical likelihood confidence intervals are also proposed. Simulation studies are conducted to compare the proposed calibration procedures and a real example is analyzed to illustrate our proposed method.

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Nonparametric Bayesian estimation of a Hölder continuous diffusion coefficient

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Abstract. We consider a nonparametric Bayesian approach to estimate the diffusion coefficient of a stochastic differential equation given discrete time observations over a fixed time interval. As a prior on the diffusion coefficient, we employ a histogram-type prior with piecewise constant realisations on bins forming a partition of the time interval. Specifically, these constants are realizations of independent inverse Gamma distributed randoma variables. We justify our approach by deriving the rate at which the corresponding posterior distribution asymptotically concentrates around the data-generating diffusion coefficient. This posterior contraction rate turns out to be optimal for estimation of a Hölder-continuous diffusion coefficient with smoothness parameter $0 < \lambda \leq 1$. Our approach is straightforward to implement, as the posterior distributions turn out to be inverse Gamma again, and leads to good practical results in a wide range of simulation examples. Finally, we apply our method on exchange rate data sets.

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Improved estimators of the entropy in scale mixture of exponential distributions

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Abstract. In the present communication, the problem of estimating entropy of a scale mixture of exponential distributions is considered under the squared error loss. Inadmissibility of the best affine equivariant estimator(BAEE) is established by deriving an improved estimator which is not smooth. Using the integral expression of risk difference (IERD) approach of Kubokawa (*The Annals of Statistics* **22** (1994) 290–299), classes of estimators are obtained which improve upon the BAEE. The boundary estimator of this class is the Brewster and Zidek-type estimator and this estimator is smooth. We have shown that the Brewster and Zidek-type estimator is a generalized Bayes estimator. As an application of these results, we have obtained improved estimators for the entropy of a multivariate Lomax distribution. Finally, percentage risk reduction of the improved estimators for the entropy of a multivariate Lomax distribution is plotted to compare the risk performance of the improved estimators.

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The cone percolation model on Galton–Watson and on spherically symmetric trees

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Abstract. We study a rumor model from a percolation theory and branching process point of view. It is defined according to the following rules: (1) at time zero, only the root (a fixed vertex of the tree) is declared informed, (2) at time $n+1$, an ignorant vertex gets the information if it is, at a graph distance, at most R_v of some its ancestral vertex v, previously informed. We present relevant lower and upper bounds for the probability of that event, according to the distribution of the random variables that defines the radius of influence of each individual. We work with (homogeneous and non-homogeneous) Galton–Watson branching trees and spherically symmetric trees which includes homogeneous and k-periodic trees. We also present bounds for the expected size of the connected component in the subcritical case for homogeneous trees and homogeneous Galton–Watson branching trees.

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Galton–Watson processes in varying environment and accessibility percolation

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Abstract. This paper deals with branching processes in varying environment with selection, where the offspring distribution depends on the generation and every particle has a random fitness which can only increase along genealogical lineages (descendants with small fitness do not survive). We view the branching process in varying environment (BPVE) as a particular example of branching random walk. We obtain conditions for the survival or extinction of a BPVE (with or without selection), using fixed point techniques for branching random walks. These conditions rely only on the first and second moments of the offspring distributions. Our results can be interpreted in terms of accessibility percolation on Galton-Watson trees. In particular, we obtain that there is no accessibility percolation on almost every Galton-Watson tree where the expected number of offspring grows sublinearly in time, while superlinear growths allows percolation. This result is in agreement with what was found for deterministic trees in Nowak and Krug (*Europhysics Letters* **101** (2013) 66004).

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On classical and Bayesian asymptotics in state space stochastic differential equations

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Abstract. In this article, we investigate consistency and asymptotic normality of the maximum likelihood and the posterior distribution of the parameters in the context of state space stochastic differential equations (*SDE*s). We then extend our asymptotic theory to random effects models based on systems of state space *SDE*s, covering both independent and identical and independent but non-identical collections of state space *SDE*s. We also address asymptotic inference in the case of multidimensional linear random effects, and in situations where the data are available in discretized forms. It is important to note that asymptotic inference, either in the classical or in the Bayesian paradigm, has not been hitherto investigated in state space *SDE*s.

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Exponential ergodicity for a class of non-Markovian stochastic processes

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Abstract. The existence of an invariant probability measure is proven for a class of solutions of stochastic differential equations with finite delay. This is done, in this non-Markovian setting, using the cluster expansion method, from Gibbs field theory. It holds for small perturbations of ergodic diffusions.

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