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Two classes of dynamic binomial integer-valued ARCH models

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Abstract. This paper introduces two classes of binomial integer-valued ARCH models with dynamic survival probabilities, each of which is controlled by a stochastic recurrence equation. Stationarity and ergodicity of the process are established, and stochastic properties are given. Conditional least squares and conditional maximum likelihood estimators for the parameters of interest are considered, and their large-sample properties are established. The performances of these estimators are compared via simulation studies. Finally, we demonstrate the usefulness of the proposed models by analyzing real datasets.

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Bi-objective mathematical model for optimal sequencing of two-level factorial designs

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Abstract. Conducting sequencing experiments with good statistical properties and low cost is a crucial challenge for both researchers and practitioners. The main reason for this challenge is the combinatorial nature of the problem and the possible conflicts among objectives. The problem was addressed by proposing a mathematical programming formulation aimed at generating minimum-cost run orders with the best statistical properties for 2^k full-factorial and fractional-factorial designs. The approach performance is evaluated using designs of up to 64 experiments with different levels of resolution. The results indicate that the approach can yield optimal or sub-optimal solutions, depending on the objectives established for a given design matrix.

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Key words and phrases. Design of experiments, systematic sequencing, linear time trend, mathematical programming, combinatorial optimization.

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Proper Bayes minimax estimation of parameters of Poisson distributions in the presence of unbalanced sample sizes

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Abstract. In this paper, we consider the problem of simultaneously estimating parameters of independent Poisson distributions in the presence of possibly unbalanced sample sizes under weighted standardized squared error loss. A class of heterogeneous Bayesian shrinkage estimators that utilize the unbalanced nature of sample sizes is proposed. To provide a theoretical justification, we first derive a necessary and sufficient condition for an estimator in the class to be proper Bayes and hence admissible and then obtain sufficient conditions for minimaxity that are compatible with the admissibility condition. Heterogeneous and homogeneous shrinkage estimators are compared by simulation. Several estimation methods are applied to data relating to the standardized mortality ratio.

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On multiple imputation for unbalanced ranked set samples with applications in quantile estimation

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Abstract. We consider multiple imputation (MI) for unbalanced ranked set samples (URSS) by considering them as data sets with missing values. We replace each missing value with a set of plausible values drawn from a predictive distribution that represents the uncertainty about the appropriate value to impute. Using the structure of the MI dataset, we develop algorithms that imitate the structure of URSS to carry out the desired statistical inference. We provide results for the convergence of the empirical distribution functions of imputed samples to the population distribution function, under both URSS and simple random sampling (SRS). We obtain the variances of the imputed URSS, and the expected values of the variance estimators. We also study the problem of quantile estimation using an imputed URSS and propose a hybrid method based on the bootstrap and imputation of URSS data. We apply our results to estimate the mean and quantiles of the mercury in contaminated fish under perfect and imperfect URSS.

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Generalized partially linear single index model with measurement error, instruments and binary response

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Abstract. Partially linear generalized single index models are widely used and have attracted much attention in the literature. However, when the covariates are subject to measurement error, the problem is much less studied. On the other hand, instrumental variables are important elements in studying many errors-in-variables problems. We use the relation between the unobservable variables and the instruments to devise consistent estimators for partially linear generalized single index models with binary response. We establish the consistency, asymptotic normality of the estimator and illustrate the numerical performance of the method through simulation studies and a data example. Despite the connection to (*Scand. J. Statist.* **42** (2015) 104–117) in its general layout, the mathematical derivations are much more challenging in the context studied here.

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Some new Stein operators for product distributions

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Abstract. We provide a general result for finding Stein operators for the product of two independent random variables whose Stein operators satisfy a certain assumption, extending a recent result of (*Journal of Mathematical Analysis and Applications* **469** (2019) 260–279). This framework applies to non-centered normal and non-centered gamma random variables, as well as a general sub-family of the variance-gamma distributions. Curiously, there is an increase in complexity in the Stein operators for products of independent normals as one moves, for example, from centered to non-centered normals. As applications, we give a simple derivation of the characteristic function of the product of independent normals, and provide insight into why the probability density function of this distribution is much more complicated in the non-centered case than the centered case.

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Key words and phrases. Stein's method, Stein operators, product distributions, product of independent normal random variables.

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On testing exponentiality based on a new estimator for the scale parameter

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Abstract. A test of fit for the exponential distribution is presented, which is based on transformed observations and a new estimator for the scale parameter. The asymptotic null distribution of the test statistic is obtained and the consistency of the test is discussed. Monte Carlo simulation results on a power comparison study show that the proposed test is competitive under the considered families of alternatives and sample sizes.

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Discrete line integral on uniform grids: Probabilistic interpretation and applications

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Abstract. Following the methodology developed by (*Comput. Math. Appl.* **33** (1997) 81–104), we define a discrete version of gradient vector and associated line integral along arbitrary path connecting two nodes of uniform grid. An exponential representation of joint survival function of bivariate discrete non-negative integer-valued random variables in terms of discrete line integral is established. We apply it to generate a discrete analogue of the Sibuya-type aging property, incorporating many classical and new bivariate discrete models. Several characterizations and closure properties of this class of bivariate discrete distributions are presented.

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Key words and phrases. Bivariate geometric distributions, characterization, discrete bivariate lack of memory and aging properties, discrete gradient vector, failure rate, line integral, Marshall–Olkin model, reliability, Sibuya's dependence function.

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A note on jump Atlas models

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Abstract. The market weight of a stock is its capitalization (cap) divided by the total market cap. Rank these weights from top to bottom. The capital distribution curve is a plot of weights versus ranks. For the US stock market, it is linear on a double logarithmic scale, and stable with respect to time (*Stochastic Portfolio Theory* (2002) Springer). This property has been captured by models with rank-dependent dynamics: Each stock's cap logarithm is a Brownian motion with drift and diffusion coefficients depending on its current rank (*Probability Theory and Related Fields* **147** (2010) 123–159). However, short-term stock movements have heavy tails. One can add jumps to Brownian motions to capture this. Observed time stability follows from a long-term stability result, stated and proved here. Via simulations, we find which properties of continuous models are preserved after adding jumps.

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Asymptotics for heavy-tailed renewal–reward processes and applications to risk processes and heavy traffic networks

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Abstract. Consider a renewal–reward process $S_{N(t)} = \sum_{k=1}^{N(t)} X_k$ and let $\{\tau_n\}$ be the interarrival times. It is well known that, under regularity conditions, $S_{N(t)}$ is asymptotically Gaussian provided X_n and τ_n have finite second moment. However, in modelling risk processes or heavy traffic networks, the assumption of the finiteness of the second moment may not be compatible. Also, the independency of the processes $\{S_n\}$ and $\{N(t)\}$ might be not realistic. In this situation, heavy-tailed distributions arise as a proper alternative and dependency between τ_n and the reward X_n should be allowed. By making use of the Mallows–Wasserstein distance we derive CLT type results for heavy-tailed renewal–reward dependent processes. Applications to risk processes and heavy traffic networks are exhibited.

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Asymptotic behavior of the maximum of multivariate order statistics in a norm sense

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Abstract. In this work, we investigate the asymptotic behavior of the extremes of a multivariate data by using the Reduced Ordering Principle (R-ordering). When, the sup-norm is used, we reveal the interrelation between the R-ordering principle and Marginal Ordering Principle (M-ordering). The asymptotic behavior of the maximum sup-norms corresponding to the bivariate data is completely determined. Finally, an application to real data illustrates and corroborates the theoretical results.

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Reduction principle for functionals of strong-weak dependent vector random fields

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Abstract. We prove the reduction principle for asymptotics of functionals of vector random fields with weakly and strongly dependent components. These functionals can be used to construct new classes of random fields with skewed and heavy-tailed distributions. Contrary to the case of scalar long-range dependent random fields, it is shown that the asymptotic behaviour of such functionals is not necessarily determined by the terms at their Hermite rank. The results are illustrated by an application to the first Minkowski functional of the Student random fields. Some simulation studies based on the theoretical findings are also presented.

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