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Improved estimation of location parameters of two exponential distributions with ordered scale

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Abstract. In the usual statistical inference problem, we estimate an unknown parameter of a statistical model using the information in the random sample. A priori information about the parameter is also known in several real-life situations. One such information is the order restriction between the parameters. This prior information improves the estimation quality. In this paper, we deal with the componentwise estimation of location parameters of two exponential distributions with ordered scale parameters under a bowl-shaped affine invariant loss function and generalized Pitman closeness criterion. We have shown that several benchmark estimators, such as maximum likelihood estimators (MLE), uniformly minimum variance unbiased estimators (UMVUE) and the best affine equivariant estimators (BAEE) are inadmissible. We have given sufficient conditions under which the dominating estimators are derived. Under the generalized Pitman closeness criterion, a Stein-type improved estimator is proposed. As an application, we have considered special sampling schemes such as type-II censoring, progressive type-II censoring and record values. We conducted a simulation study to compare the risk performance of the improved estimators. Finally, we performed a real-life data analysis to demonstrate the practical applications of our findings.

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Optimal designs for comparing multivariate linear models with known covariance matrix

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Abstract. This article is concerned with the optimal design problem of efficient statistical inference for comparing multivariate linear models estimated from samples of independent measurements. The objective is to find the μ_D-optimal designs that minimize the volume of the confidence tube for the multivariate linear models. The definitions of the volume for the confidence tube are obtained. General equivalence theorems are established to verify the μ_D-optimality in the set of all approximate designs. Two examples are presented to illustrate the applications of the obtained results.

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A new bivariate integer-valued autoregressive model with interaction effect

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Abstract. In this paper, we propose a new bivariate integer-valued autoregressive process with interaction effects, referred to as the IEBINAR(1) model. We consider three estimation methods for the unknown parameters of interest: conditional least squares, conditional maximum likelihood, and a two-step estimation approach. The asymptotic properties of the estimators are established. The performance of these estimation methods is compared through simulation experiments. Furthermore, we conduct hypothesis testing to examine the existence of interaction effects in the IEBINAR(1) model. Finally, a real data application is presented to evaluate the performance of the proposed model.

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Key words and phrases. Thinning models, Bivariate integer-valued autoregressive models, interaction effect, hypothesis test.

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Local linear correlation for measuring local association

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Abstract. In simple linear regression with response variable Y and covariate X, the classical Pearson correlation measures the strength of the linear association between Y and X, and corresponds to the standardized slope of the regression line. This paper explores the concept of local linear correlation to capture the locally linear association between Y and X as a function of X, while preserving key properties of the Pearson correlation. Without assuming a parametric form for the joint distribution of (X, Y), we show that the kernel-weighted local linear correlation measures the strength of locally linear association, and is connected to local linear regression through its interpretation as a locally standardized slope. We derive the finite-sample and asymptotic properties of the population and sample versions of local linear correlations and the optimal order of the bandwidth is provided. Numerical results confirm the asymptotic theory and a baseball data example is given for illustration.

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Integral fractional Ornstein–Uhlenbeck process model for animal movement

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Abstract. The study of animal telemetry is crucial in ecology, providing valuable information on movement patterns, behavior, and habitat use of various species, which is essential for conservation and management efforts. Numerous models in the literature address animal telemetry by modeling velocity, telemetry data itself, or both processes jointly through a Markovian approach. In this work, we propose a novel approach by modeling the velocity of each coordinate axis for animal telemetry data using a fractional Ornstein–Uhlenbeck (fOU) process. The integral of the fOU process models the position data in animal telemetry. This proposed model is particularly flexible in capturing long-range memory effects. The Hurst parameter $H \in (0, 1)$ plays a crucial role in the integral fOU process, determining the long-range memory. The integral fOU process is nonstationary; a higher Hurst parameter ($H > 0.5$) indicates stronger memory, leading to trajectories with transient tendencies, while a lower Hurst parameter ($H < 0.5$) implies weaker memory, resulting in trajectories with recurring trends. When $H = 0.5$, the process reduces to a standard integral Ornstein–Uhlenbeck process. We develop a simulation algorithm for telemetry trajectories using finite-dimensional distributions and employ the maximum likelihood method for parameter estimation, with its performance evaluated through simulation studies. Finally, we present a telemetry application involving Fin Whales dispersing throughout the Gulf of California.

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Berry–Esseen bound for the mean estimator of a supercritical branching process in a random environment

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Abstract. Let (Z_n) be a supercritical branching process in an independent and identically distributed (i.i.d.) random environment ξ. For the mean estimator $M_n = n^{-1}\sum_{k=0}^{n-1}(Z_{k+1}/Z_k)$ introduced by (*The Annals of Statistics* **7** (1979) 680–685), we establish a Berry–Esseen bound and an algebraic nonuniform Berry–Esseen bound, and some applications of the nonuniform Berry–Esseen bound to confidence interval estimation are discussed.

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Correction to: The Bayes estimator of a conditional density: Asymptotic behavior

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Abstract. In this note, we provide corrections to some calculation errors in the Examples 5.2 and 5.3 of the paper: *A.G. Nogales, The Bayes Estimator of a Conditional Density: Asymptotic Behavior, Brazilian Journal of Probability and Statistics, 38 (4), 531–548, (2024). https://doi.org/10.1214/24-BJPS617*

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