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Models as Approximations I: Consequences Illustrated with Linear Regression

Andreas Buja, Lawrence Brown, Richard Berk, Edward George, Emil Pitkin, Mikhail Traskin, Kai Zhang and Linda Zhao

Abstract. In the early 1980s, Halbert White inaugurated a "model-robust" form of statistical inference based on the "sandwich estimator" of standard error. This estimator is known to be "heteroskedasticity-consistent," but it is less well known to be "nonlinearity-consistent" as well. Nonlinearity, however, raises fundamental issues because in its presence regressors are not ancillary, hence cannot be treated as fixed. The consequences are deep: (1) population slopes need to be reinterpreted as statistical functionals obtained from OLS fits to largely arbitrary joint x-y distributions; (2) the meaning of slope parameters needs to be rethought; (3) the regressor distribution affects the slope parameters; (4) randomness of the regressors becomes a source of sampling variability in slope estimates of order $1/\sqrt{N}$; (5) inference needs to be based on model-robust standard errors, including sandwich estimators or the x-y bootstrap. In theory, model-robust and model-trusting standard errors can deviate by arbitrary magnitudes either way. In practice, significant deviations between them can be detected with a diagnostic test.

Key words and phrases: Ancillarity of regressors, misspecification, econometrics, sandwich estimator, bootstrap.

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Models as Approximations II: A Model-Free Theory of Parametric Regression

Andreas Buja, Lawrence Brown, Arun Kumar Kuchibhotla, Richard Berk, Edward George and Linda Zhao

Abstract. We develop a model-free theory of general types of parametric regression for i.i.d. observations. The theory replaces the parameters of parametric models with statistical functionals, to be called "regression functionals," defined on large nonparametric classes of joint x-y distributions, without assuming a correct model. Parametric models are reduced to heuristics to suggest plausible objective functions. An example of a regression functional is the vector of slopes of linear equations fitted by OLS to largely arbitrary x-y distributions, without assuming a linear model (see Part I). More generally, regression functionals can be defined by minimizing objective functions, solving estimating equations, or with ad hoc constructions. In this framework, it is possible to achieve the following: (1) define a notion of "well-specification" for regression functionals that replaces the notion of correct specification of models, (2) propose a well-specification diagnostic for regression functionals based on reweighting distributions and data, (3) decompose sampling variability of regression functionals into two sources, one due to the conditional response distribution and another due to the regressor distribution interacting with misspecification, both of order $N^{-1/2}$, (4) exhibit plug-in/sandwich estimators of standard error as limit cases of x-y bootstrap estimators, and (5) provide theoretical heuristics to indicate that x-y bootstrap standard errors may generally be preferred over sandwich estimators.

Key words and phrases: Ancillarity of regressors, misspecification, econometrics, sandwich estimator, bootstrap, bagging.

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Discussion of Models as Approximations I & II

Sara van de Geer

Abstract. We discuss the papers "Models as Approximations" I & II, by A. Buja, R. Berk, L. Brown, E. George, E. Pitkin, M. Traskin, L. Zao and K. Zhang (Part I) and A. Buja, L. Brown, A. K. Kuchibhota, R. Berk, E. George and L. Zhao (Part II). We present a summary with some details for the generalized linear model.

Key words and phrases: Misspecification, sandwich formula.

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Comment on Models as Approximations, Parts I and II, by Buja et al.

Jerald F. Lawless

Abstract. I comment on the papers Models as Approximations I and II, by A. Buja, R. Berk, L. Brown, E. George, E. Pitkin, M. Traskin, L. Zhao and K. Zhang.

Key words and phrases: Covariate distributions, misspecification, regression models, transportability.

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Comment: Models as Approximations

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Discussion of Models as Approximations I & II

Dag Tjøstheim

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Comment: “Models as Approximations I: Consequences Illustrated with Linear Regression” by A. Buja, R. Berk, L. Brown, E. George, E. Pitkin, L. Zhan and K. Zhang

Roderick J. Little

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Comment: Models Are Approximations!

Anthony C. Davison, Erwan Koch and Jonathan Koh

Abstract. This discussion focuses on areas of disagreement with the papers, particularly the target of inference and the case for using the robust 'sandwich' variance estimator in the presence of moderate mis-specification. We also suggest that existing procedures may be appreciably more powerful for detecting mis-specification than the authors' RAV statistic, and comment on the use of the pairs bootstrap in balanced situations.

Key words and phrases: Bootstrap, designed experiment, infinitesimal jackknife, model mis-specification, regression diagnostics, sandwich variance estimator.

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Comment: Models as (Deliberate) Approximations

David Whitney, Ali Shojaie and Marco Carone

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Comment: Statistical Inference from a Predictive Perspective

Alessandro Rinaldo, Ryan J. Tibshirani and Larry Wasserman

Abstract. What is the meaning of a regression parameter? Why is this the de facto standard object of interest for statistical inference? These are delicate issues, especially when the model is misspecified. We argue that focusing on predictive quantities may be a desirable alternative.

Key words and phrases: Regression, prediction, variable importance, effect sizes.

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Discussion: Models as Approximations

Dalia Ghanem and Todd A. Kuffner

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Models as Approximations—Rejoinder

Andreas Buja, Arun Kumar Kuchibhotla, Richard Berk, Edward George, Eric Tchetgen Tchetgen and Linda Zhao

Abstract. We respond to the discussants of our articles emphasizing the importance of inference under misspecification in the context of the reproducibility/replicability crisis. Along the way, we discuss the roles of diagnostics and model building in regression as well as connections between our well-specification framework and semiparametric theory.

Key words and phrases: Well-specification, reproducibility/replicability, proper scoring rules, causal inference, semiparametrics, diagnostics.

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Larry Brown's Contributions to Parametric Inference, Decision Theory and Foundations: A Survey

James O. Berger and Anirban DasGupta

Abstract. This article gives a panoramic survey of the general area of parametric statistical inference, decision theory and foundations of statistics for the period 1965–2010 through the lens of Larry Brown's contributions to varied aspects of this massive area. The article goes over sufficiency, shrinkage estimation, admissibility, minimaxity, complete class theorems, estimated confidence, conditional confidence procedures, Edgeworth and higher order asymptotic expansions, variational Bayes, Stein's SURE, differential inequalities, geometrization of convergence rates, asymptotic equivalence, aspects of empirical process theory, inference after model selection, unified frequentist and Bayesian testing, and Wald's sequential theory. A reasonably comprehensive bibliography is provided.

Key words and phrases: Admissibility, ancillary, asymptotic equivalence, Bayes, conditional confidence, differential inequality, Edgeworth expansions, estimated confidence, minimax, sequential, shrinkage, sufficiency.

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Gaussianization Machines for Non-Gaussian Function Estimation Models

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Abstract. A wide range of nonparametric function estimation models have been studied individually in the literature. Among them the homoscedastic nonparametric Gaussian regression is arguably the best known and understood. Inspired by the asymptotic equivalence theory, Brown, Cai and Zhou (*Ann. Statist.* **36** (2008) 2055–2084; *Ann. Statist.* **38** (2010) 2005–2046) and Brown et al. (*Probab. Theory Related Fields* **146** (2010) 401–433) developed a unified approach to turn a collection of non-Gaussian function estimation models into a standard Gaussian regression and any good Gaussian nonparametric regression method can then be used.

These Gaussianization Machines have two key components, binning and transformation. When combined with BlockJS, a wavelet thresholding procedure for Gaussian regression, the procedures are computationally efficient with strong theoretical guarantees. Technical analysis given in Brown, Cai and Zhou (*Ann. Statist.* **36** (2008) 2055–2084; *Ann. Statist.* **38** (2010) 2005–2046) and Brown et al. (*Probab. Theory Related Fields* **146** (2010) 401–433) shows that the estimators attain the optimal rate of convergence adaptively over a large set of Besov spaces and across a collection of non-Gaussian function estimation models, including robust nonparametric regression, density estimation, and nonparametric regression in exponential families. The estimators are also spatially adaptive.

The Gaussianization Machines significantly extend the flexibility and scope of the theories and methodologies originally developed for the conventional nonparametric Gaussian regression. This article aims to provide a concise account of the Gaussianization Machines developed in Brown, Cai and Zhou (*Ann. Statist.* **36** (2008) 2055–2084; *Ann. Statist.* **38** (2010) 2005–2046), Brown et al. (*Probab. Theory Related Fields* **146** (2010) 401–433).

Key words and phrases: Adaptivity, asymptotic equivalence, block thresholding, density estimation, exponential family, mean matching, nonparametric function estimation, quadratic variance function, quantile coupling, robust regression, variance stabilizing transformation, wavelets.

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Larry Brown's Work on Admissibility

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Abstract. Many papers in the early part of Brown's career focused on the admissibility or otherwise of estimators of a vector parameter. He established that inadmissibility of invariant estimators in three and higher dimensions is a general phenomenon, and found deep and beautiful connections between admissibility and other areas of mathematics. This review touches on several of his major contributions, with a focus on his celebrated 1971 paper connecting admissibility, recurrence and elliptic partial differential equations.

Key words and phrases: Complete class theorems, inadmissibility, Blyth's method, elliptic partial differential equation, recurrence, Brownian diffusion, loss function, best invariant estimator, James–Stein estimator, variational problem, differential inequality.

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Statistical Theory Powering Data Science

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Dedicated to Lawrence D. Brown

Abstract. Statisticians are finding their place in the emerging field of data science. However, many issues considered "new" in data science have long histories in statistics. Examples of using statistical thinking are illustrated, which range from exploratory data analysis to measuring uncertainty to accommodating nonrandom samples. These examples are then applied to service networks, baseball predictions and official statistics.

Key words and phrases: Service networks, queueing theory, empirical Bayes, nonparametric estimation, sports statistics, decennial census, house price index.

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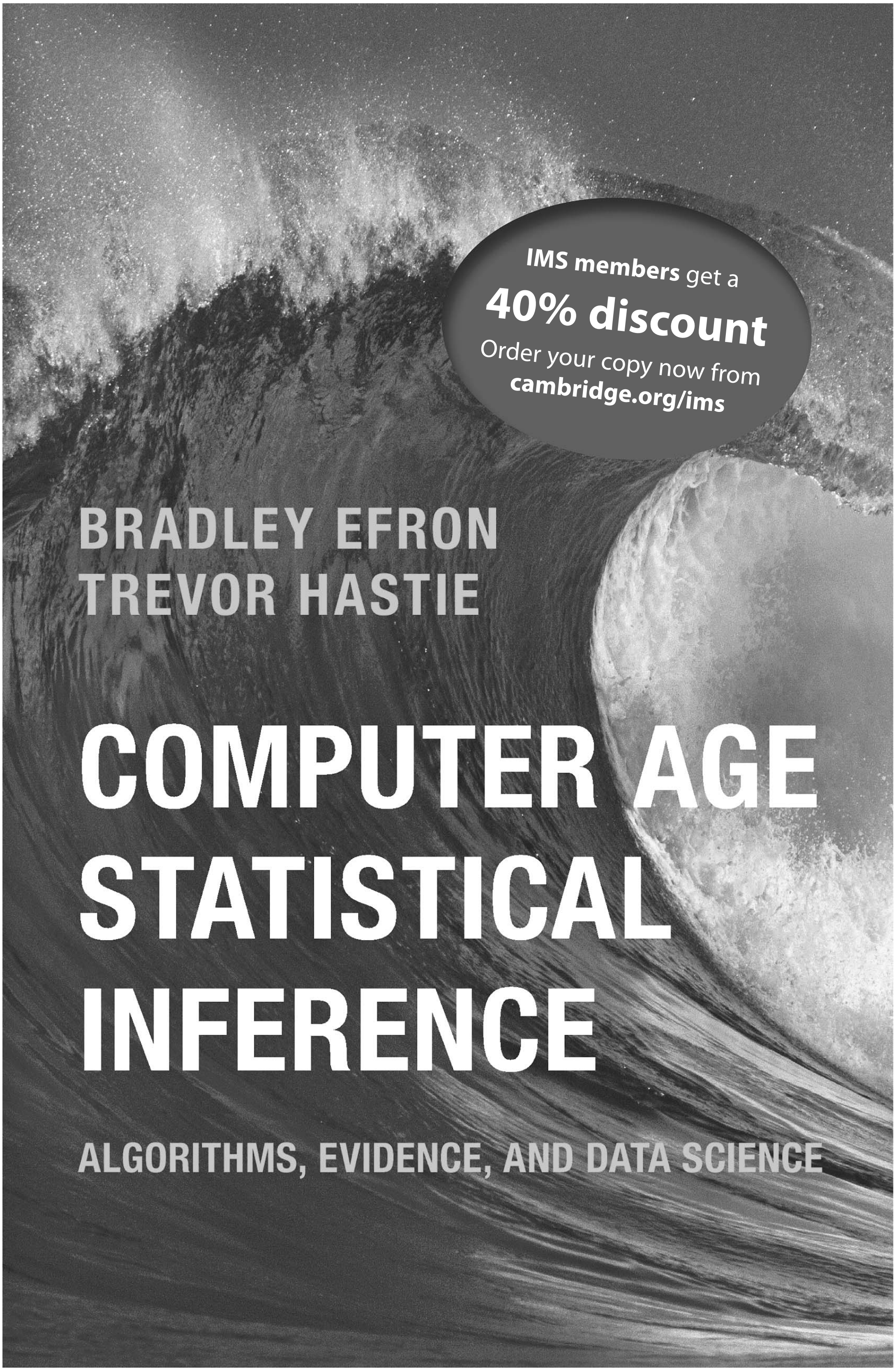
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